

HW 5

Monday, November 06, 2023 3:29 PM

4.6: Solution of Equations by iterative Methods

1b) Program the Gauss-Seidel method and test it on the following

$$\begin{cases} 3x + y + z = 5 \\ 3x + y - 5z = -1 \\ x + 3y - z = 3 \end{cases}$$

Analyze what happens when these systems are solved by simple Gaussian elimination without pivoting.

```

1
2
3 def gauss_seidel(A, b, x0, tol=1e-10, max_iterations=1000):
4     n = len(A)
5     x = x0.copy()
6
7     for k in range(max_iterations):
8         x_old = x.copy()
9
10        for i in range(n):
11            sum = 0
12            for j in range(n):
13                if j != i:
14                    sum += A[i][j] * x[j]
15            x[i] = (b[i] - sum) / A[i][i]
16
17        if max(abs(x[i] - x_old[i]) for i in range(n)) < tol:
18            return x
19        print(x)
20    return x
21
22
23
24
25 A=[[3,1,1],[3,1,-5],[1,3,-1]]
26 b=[5,-1,3]
27
28
29 print(gauss_seidel(A,b,[2,2,1]))

```

We get [nan, nan, nan]

```

1 def gauss(A, b):
2     n = len(b)
3
4     for i in range(n):
5         max_row = i
6         for k in range(i+1, n):
7             if abs(A[k][i]) > abs(A[max_row][i]):
8                 max_row = k
9         A[i], A[max_row] = A[max_row], A[i]
10        b[i], b[max_row] = b[max_row], b[i]
11
12        pivot = A[i][i]
13        for j in range(i, n):
14            A[i][j] /= pivot
15            b[i] /= pivot
16
17        for k in range(i+1, n):
18            factor = A[k][i]
19            for j in range(i, n):
20                A[k][j] -= factor * A[i][j]
21            b[k] -= factor * b[i]
22
23    x = [0] * n
24    for i in range(n-1, -1, -1):
25        x[i] = b[i]
26        for j in range(i+1, n):
27            x[i] -= A[i][j] * x[j]
28
29    return x

```

We get [1.0000000000000002, 1.0, 1.0]

So with the Gauss elimination without pivoting we get a solution and for the Gauss Seidel Method we only get a solution if we start with [1,1,1]. Otherwise it diverges.

