χ Solutions to Exams with theory

1. About graphs and trees:

What is the chromatic number for $K_{4,3}$?

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707 A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color. The chromatic number of a graph is the least number of colors needed for a coloring of this graph, or chi χ. The chromatic number of a planar graph (no edges cross each other) is no greater than 4.

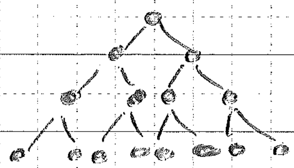
A simple graph is bipartite if its vertex set V can be positioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2.

A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set
630 partitioned into two subsets of m and n vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.

$\chi(K_{4,3}) = 2$ because is bipartite.

Draw a full, rooted, binary tree with 8 leaves 345

A rooted tree is a tree in which one vertex has been designated as 718
the root and every edge is directed away from the root 719
An ordered rooted tree (aka binary tree) is a rooted tree where the children of each internal vertex are ordered. In a binary tree, if an internal vertex has two children, the first child is called the left child and the second child is called the right child.



Find a Hamilton cycle in K_5

A simple path (doesn't contain the same edge more than once) in a graph G that passes through every vertex exactly once is called a Hamilton path, and a simple circuit in a graph G that passes through every vertex exactly once is called a Hamilton circuit. 631

path $= x_0, x_1, \ldots, x_{n-1}, x_n$ in the graph $G=(V,E)$ if $V=\{x_0, x_1, x_{n-1}, x_n\}$

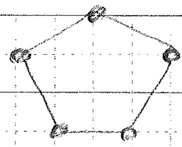
and $x_i \neq x_j$ for $0 \leq i < j \leq n$

circuit if $x_0, x_1, \ldots$ is a Hamilton path.

Dirac's theorem: if G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then G has a HC.

Ore's theorem: if G is a simple graph with n vertices with $n \geq 3$ such as that $\deg(u) + \deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has a HC

Solution for example C_5



for which n have Q_n an Euler circuit?

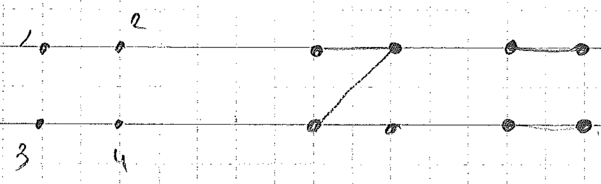
An Euler circuit in a graph is a simple circuit containing every edge of G. An Eulerpath in G is a simple path containing every edge of G.

A connected multigraph (has multiple/parallel edges) with at least two vertices has an Euler circuit if and only iff each of its vertices has even degree.

$n \geq 2$ iff $\deg(v) = 2k$ $\quad k \in \mathbb{Z}$

Draw the non isomorphic simple graph with 4 vertices and 2 edges

643 the simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there
647 exists a one-to-one and onto function (surjective function, $f(x) = y \ \forall y$)
706 f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if f(a) and f(b) are adjacent in G_2, for all a and b in V_1. Such a function f is called an isomorphism. Two simple graphs that are not isomorphic are called nonisomorphic



OBS!
there are two noniso graphs
(1,2) (2,3) – (1,2) (3,4)

COUNTING

How many elements are in the union of four sets if the sets have 45, 60, 70 and 100 elements respectively, each pair of sets share 25 elements, each three of the sets share 10 elements and no element is in all four sets?

An r-combination of elements of a set is an unordered selection of r elements from the set. The number of r-combinations of a set with n distinc elements is denoted by $C(n, r)$, also denoted by $\binom{n}{r}$ and is called a binomial coefficient.

The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \le r \le n$, equals

$$C\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Each pair of sets $\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{4\cdot 3\cdot \cancel{2}\cdot \cancel{1}}{2\cdot 1\cdot \cancel{2}\cdot \cancel{1}} = \frac{12}{2} = 6\cdot 25 = 150$

Each three of sets $\binom{4}{3} = \frac{4!}{3!\,1} = \frac{4}{1} = 4\cdot 10 = 40$

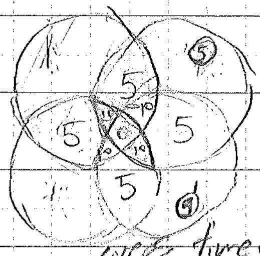
the four sets $\binom{4}{4} = 1\cdot 0 = 0$

$$45+60+70+100-150+40=165$$

Menos restado de mas

Ocorre las 4

Zonas de contacto entre 2



wedo times

4 - 2

4 - 3

4 - 1

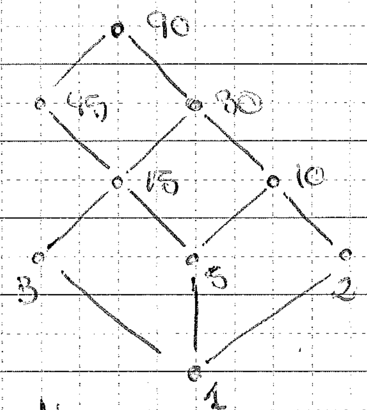
1 - 4

POSET

Draw the Hasse diagram for divisibility on the set
601 $\{1, 2, 3, 5, 10, 15, 30, 45, 90\}$. Is it lattice?
512 A relation R on a set S is called a partial ordering if it is:
- Reflexive: xRx for $\forall x$
- Antisymmetric: $\forall a \forall b \; ((a,b) \in R \wedge (b,a) \in R) \rightarrow (a=b))$
  $aRb \wedge bRa$ iff $a=b$
- transitive: $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

A set S together with a partial ordering R is called a partially ordered set or poset, and is denoted by (S,R)
Hasse diagrams of a poset don't show loops because they are assumed, because they are innherent to a poset. The Hass diagrams do not show the transitive edges because the same reason, they are innherent to a poset.
Finally arrange each edge so that its initial vertex is below its terminal vertex. Remove arrows, because all edges point "forward" to the terminal edges.

A partially ordered set in which every pair of elements has both a least upper bound (join) and a greater lower bound (meet) is called a lattice.



Hasse diagram

The set is lattice because, for example
join $\{15, 10\} = 30$
meet $\{15, 10\} = 5$
$\forall (a,b)$ have join and meet vertices.

EQ, R

Determine whether the relation represented by this zero-one matrix is an equivalence relation

$$M_R = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

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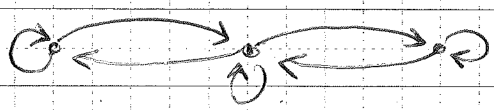
A relation on a set A is called an equivalence relation if it is reflexive ($xRx, \forall x$), symmetric ($\forall a \forall b \ (a,b) \in R \rightarrow (b,a) \in R$) and transitive ($(a,b) \wedge (b,c) \rightarrow (a,c)$)

two elements a and b that are related by an equivalence relation are called equivalent the notation $a \sim b$ denote that a and b are equivalent elements with respect to a particular equivalence relation. Let R be an equivalence relation on a set A. the set of all elements that are related to an element a of A is called the equivalence class of a. the equivalence class of a with respect to R is denoted by $[a]_R$. When only one relation is under consideration, we can write $[a]$

the Matrix is an equivalent relation with two classes
$[a] = \{11001\}$ and $[b] = \{00110\}$

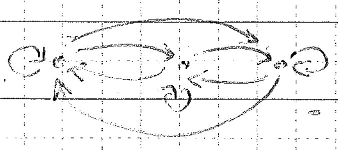
Determine whether the relation represented by this direct graph is an equivalent relation



A relation is called an equivalent relation if it is reflexive ($xRx, \forall x$) symmetric ($\forall a \forall b \ (a,b) \in R \rightarrow (b,a) \in R$) and transitive $(a,b) \wedge (b,c) \vdash (a,c)$

the graph is reflexive, symmetric but not transitibe.

$(a,b) \in R \ \wedge \ (b,c) \in R \ \vdash \ (a,c) \notin R$

 } transitive

GENERATING FUNCTIONS

Find the generating function, $G(x)$ for the following problem:
In how many ways can 20 identical ballons be distributed to 4 children so that each children gets at least 3 ballons but no one gets more than 7 ballons. Express $G(x)$ on closed form. Explain how to proceed to solve the problem.

Limits $3 \le x \le 7$ $\quad G(x) = (x^3 + x^4 + x^5 + x^6 + x^7)^4$ 4 childs
min max

$= (x^3)^4 (1 + x + x^2 + x^3 + x^4)^4 = x^{12}\left(\dfrac{1-x^5}{1-x}\right)^4$

TABLE PP 526
$1 + x + x^2 + \dots + x^n = \dfrac{1-x^{n+1}}{1-x}$

we want the coefficient in front of (x^{20}) 5 ghe

NOMINATOR
$(1-x^5)^4 = x^{20} - 4x^{15} + 6x^{10} - 4x^5 + 1$

Binomial expansion
$(1-x)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} (-x)^k$

DENOMINATOR
$\dfrac{1}{(1-x)^4} = 1 + \binom{4}{1}x + \binom{5}{2}x^2 + \binom{6}{3}x^3 + \binom{7}{4}x^4 + \binom{8}{5}x^5 + \binom{9}{6}x^6 + \binom{10}{7}x^7 + \binom{11}{8}x^8 + \dots$

GF TABLE 526
$\dfrac{1}{(1+x)^n}$

GF $\quad G(x) = x^{12} \cdot (x^{20} - 4x^{15} + 6x^{10} - 4x^5 + 1)\left(1 + \binom{4}{1}x + \dots + \binom{6}{3}x^3 + \dots + \binom{11}{8}x^8 + \dots\right)$

$12 + 5 + 3 = 20$

$$\left.\begin{aligned}[x^{20}] &= x^{12} \cdot (-4x^5) \cdot \binom{6}{3} x^3 = -4\binom{6}{3} \\ &= x^{12} \cdot 1 \cdot \binom{11}{8} x^8 = \binom{11}{8}\end{aligned}\right\} \binom{11}{8} - 4\binom{6}{3}$$

PLANAR

Investigate simple connected graphs G with 15 vertices such that five of them have degree 4 and the remaining 10 vertices have degree 1. Use the following questions.

629 How many edges?

Let $G=(V,E)$ be an undirected graph with m edges. then

$$2m = \sum_{v \in V} \deg(v)$$

this applies even if multiple edges and loops are present.

$$2m = 5\text{vertex} \deg(4) + 10\text{vertex} \deg(1)$$

$$= (5\times4)+(10\times1) = 20+10 = 30$$

$$m = 30/2 = 15 \text{ edges}$$

689
207 Is G planar? How many regions?

A graph is called planar if it can be drawn in the plane without any edges crossing (where a crossing of edges is the intersection of the lines or arcs representing them at a point other than their common endpoint)

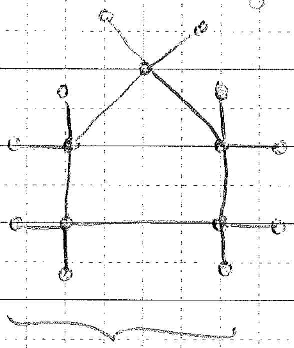
A planar representation of a graph splits the plane into regions, including an unbounded region. Euler showed that all planar representations of a graph split the plane in the same number of regions. Euler's formula $r = e - v + 2$ v = vertices e = edges, r = number of regions. Graphs that contain the same number of vertices and edges contain a cycle.

If it's planar, then regions $= 15 - 15 + 2 = 2$ regions

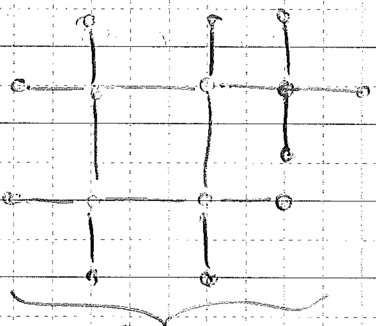
Are there nonisomorphic graphs?

the graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ are isomorphic if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 iff $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1

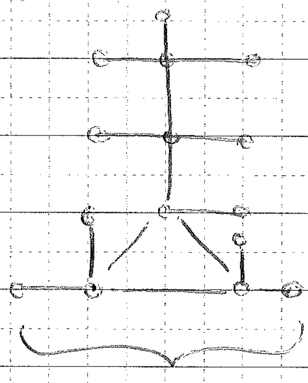
there is a cycle, and we have 5 vertex with degree (4) so our cycle must be length $3 \leq N \leq 5$



5cycle



C_4



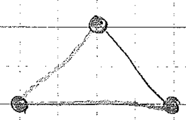
C_3

Show that O_k is k-regular for all $k \geq 2$, that is, show that all vertices in O_k has degree k

The odd graph O_k, k is an integer ≥ 2, is defined in the following way: the vertices represent the subset with $k-1$ elements that can be obtained from a set with $2k-1$ elements. Two vertices are joined with an edge iff the corresponding subsets are disjoint.

Two sets are called disjoint if their intersection is the empty set.

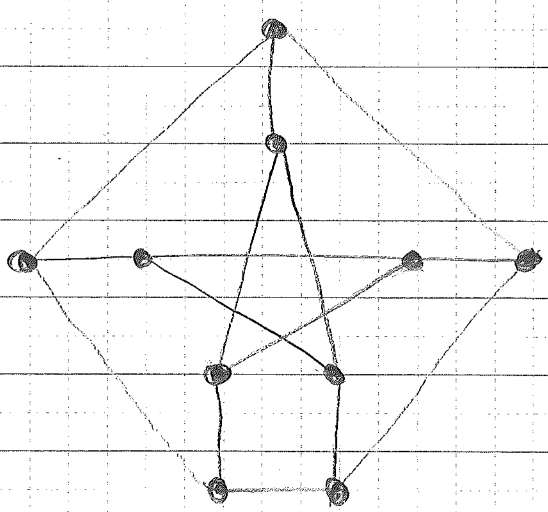
O_2 elements $= k-1 = 1$ from the set $S = 2k-1 = \{1, 2, 3\}$



O_3 elements $= 2$ from $S = \{1, 2, 3, 4, 5\}$

Combination of vertices $= \binom{5}{2} = 10$ vertices

edges $= 10 \cdot 3/2 = 15$



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What is the sum of the nth row in Pascal's triangle?

Binomial theorem

$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

Corollary 1

$\sum_{k=0}^{n} \binom{n}{k} = 2^n$

$(x+y)^3 = \sum_{j=0}^{3} \binom{3}{j} x^{n-j} y^j = \binom{3}{0} x^3 + \binom{3}{1} x^2 y + \binom{3}{2} x y^2 + \binom{3}{3} y^3 =$

$= 1 \cdot x^3 + 3 \cdot x^2 y + 3 x y^2 + 1 y^3$

Only coefficients $= 1+3+3+1 = 8 = 2^3$ {corollary 1

1

1 1

1 2 1

1 3 3 1

$\boxed{\sum [(x+y)^n] = 2^n}$

How many six letter strings can be formed by 2U, 2R and 2O?

Repetition \ Order	YES	NO
YES	n^r	$\binom{n+r-1}{r}$
NO	$P(n,r)$	$\binom{n}{r}$

$$\frac{6!}{2!\ 2!\ 2!} = 90$$

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = C(n,r)$$

$$P(n,r) = \frac{n!}{(n-r)!}$$

Arranging items with repetition, order matters

$$\frac{n!}{x_1!\,x_2!}$$

n = number of items you want to pick
$x_1!$ = Items repeated

How many non negative integer solutions are there to the equation

$x_1 + x_2 + x_3 = 13$?

$\downarrow \quad \downarrow \quad \downarrow$

$+1 \quad +1 \quad +13 = 15$

teacher's trick

$$\binom{15}{13} = \frac{15!}{13!\,(15-13)!} = 15 \cdot 14 / 2 = 105$$

You have to buy 25 bagels, they are three types: plain, tomato and blueberry. The restrictions are: At least five of each sort but no more than 11 of any sort. In how many ways can you do this? Solve the problem with inclusion and exclusion or by using generating function

$5 \leq x \leq 11$ 25 bagels 3 types

$x_p + x_t + x_b = 25$

$$G(x) = (x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11})^{\xrightarrow{\text{types}}3} = x^{15} \cdot \overbrace{(1 + x + x^2 + \dots + x^6)^3}^{\text{table 526}}$$

$$x^{15} \left(\frac{(1-x^7)}{(1-x)}\right)^3$$

Binomial expansion $\sum_{k=0}^{n} \binom{n}{k} 1^{n-k} (-x)^k$

$$(1-x^7)^3 = \binom{3}{0}(-x^7)^0 + \binom{3}{1}(-x^7)^1 + \binom{3}{2}(-x^7)^2 + \binom{3}{3}(-x^7)^3$$

$$= 1 - 3x^7 + 3x^{14} - x^{21}$$

$$\left(\frac{1}{1-x}\right)^3 = \binom{2}{0}x^0 + \binom{3}{1}x^1 + \dots + \binom{5}{3}x^3 + \dots + \binom{12}{10}x^{10} + \dots$$

$$\sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k$$

TABLE 526

$$G(x) = x^{15}(1 - 3x^7 + 3x^{14} - x^{21})\left(\frac{1}{1-x}\right)^3$$

$15 + 7 + 3 = 25$

Solutions: $x^{15}(-3x^7)\left(\binom{5}{3}x^3\right) = -3\binom{5}{3}$

$x^{15} \cdot 1 \cdot \binom{12}{10} x^{10} = \binom{12}{10}$

$\binom{12}{10} - 3\binom{5}{3}$ SOLUTION

$15 + 10 = 25$

Show that the divides relation is a partial order on the positive integers. The divides relation R is defined by xRy iff $x|y$. Here x and y are positive integers

A relation R on a set A is called reflexive if $(a,a) \in R$ for every element $a \in R$

$x|x$ x divide x

xRx so xRx is included

A relation R on a set A such that for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ is called antisymmetric

if $x|y$ then $y=Kx$

if $y|x$ then $x=Cy$

$y = Kx = KCy$

that means $KC = 1 \in \mathbb{Z}^+$

so $\underline{y = Kx = x}$

A relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$ for all $a,b,c \in A$

$x|y \longleftrightarrow y = Kx$

$y|z \longleftrightarrow z = Cy$

$z = Cy = CKx \Longleftrightarrow xRz$

Define the relation R on the integers $\mathbb{Z}$ by xRx for $x, y \in \mathbb{Z}$ iff $x \equiv y \pmod 7$. Show that this is an equivalence relation with $\mathbb{Z}$

$xRy,\ x, y \in \mathbb{Z} \iff x \equiv y \ (M\ 7)$

Reflexive

$(a, a) \in R,\ \forall a \in R$

is x mod 7 of x? yes

$x \equiv x \pmod 7 \iff xRx$

A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all $a, b \in A$

$(x - y) = K \cdot 7 \quad x \equiv y$

$(y - x) = -K \cdot 7 \quad y \equiv x$

yRx ✓

transitive

$(x - y) = K \cdot 7$

$y - z = C \cdot 7$

$x - y + y - z = (K + C) \cdot 7$

$x - z = (K + C) \cdot 7$

xRz ✓

Draw all non-isomorphic trees with five vertices. Note trees are simple connected graphs

The simple graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2 for all a and b in V_1. Such a function f is called an isomorphism. Two simple graphs that are not isomorphic are called nonisomorphic.

Properties of the trees: A tree with n vertices has $n-1$ edges.

The handshaking theorem: let $G = (V, E)$ be an undirected graph (without arrows) with m edges. then 629

$$2m = \sum_{v \in V} \deg(v)$$

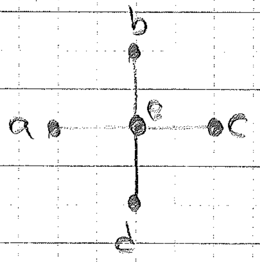
Vertices $V = 5 \longrightarrow$ Edges $= V - 1 = 4$

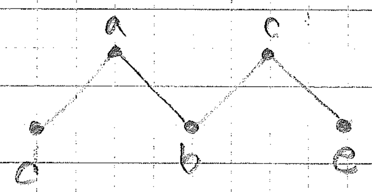
Handshaking $\quad 2 \cdot 4 = \sum_{v \in V} \deg(v)$ $\qquad V = 5$

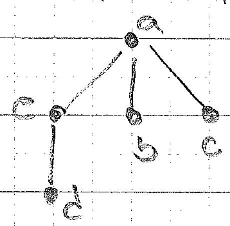
$$8 = \deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) + \deg(v_5)$$

$$8 = a + b + c + d + e$$

$\begin{array}{cccccc} & a & b & c & d & e \\ 8 = & 1+ & 1+ & 1+ & 1+ & 4 \end{array}$ $\qquad \begin{array}{cccccc} & a & b & c & d & e \\ 8 = & 1+ & 1+ & 2+ & 2+ & 2 \end{array}$ $\qquad \begin{array}{cccccc} & a & b & c & d & e \\ 8 = & 3 & 1 & 1 & 1 & 2 \end{array}$







Let G be a simple graph $|V| = n \geq 3$ If $deg(x) + deg(y) \geq n$ for all nonadjacent $x, y \in V$, then G contains a Hamilton cycle

This is sufficient condition for the existence of a Hamilton cycle proved by Ore 1960

Draw a graph for which the condition holds and find the Hamilton cycle. Show also that...

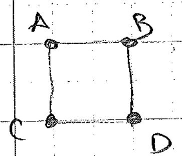
A simple path in graph G that passes through every vertex exactly once is called a Hamilton path, and a simple circuit in a graph G that passes through every vertex exactly once is called a Hamilton circuit

If G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then G has a Hamilton cycle

ORE'S THEOREM

If G is a simple graph with n vertices with $n \geq 3$ such that $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has a Hamilton circuit

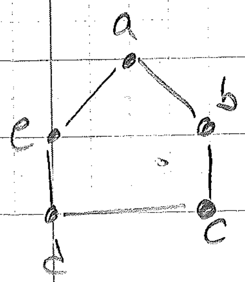
Ore's theorem gives sufficient conditions to be Hamiltonian, some graphs may be still hamiltonian.



$N = 4$ Nonadjacent = AD, CB

$deg(A) + deg(D) = 2 + 2 = 4 \geq N$

$deg(C) + deg(B) = 2 + 2 = 4 \geq N$



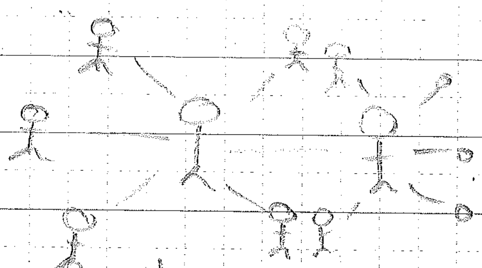
$N = 5$ $deg(a\,c) = 4 < 5$ DOESN'T HOLD

A group of 12 people meet for dinner at a big circular table. In this group everyone knows at least 6 other. Can they be seated around the table in such way that each person knows the person to the left and to the right?

n = vertices $\quad n \geq 3 \quad \deg(x) + \deg(y) \geq n$

n = people = 12

each person = 6 relationships = edges



$\deg(6) + \deg(6) = 12 \geq 12$ persons

there is a Hamilton cycle! that means that yes, they can be seated in a circular table that follows a HC

Consider connected simple graphs G with 11 vertices. Prove that either G or its complement $\bar{G}$ must be nonplanar

A graph is called planar if it can be drawn in the plane without any edges crossing.

Euler's formula

Let G be a connected planar graph with e edges and v vertices. Let r = number of regions in a planar representation of G $\quad r = e - v + 2$

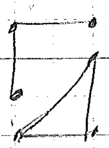
If G is a connected simple graph where $v \geq 3$, then

$e \leq 3v - 6$

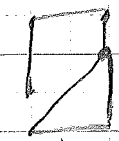
If G is a connected simple graph, then G has a vertex of degree not exceeding five

Path: sequence of edges that begins at a vertex and travels from vertex to vertex

An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph



Not connected



connected

$K_{11} = \binom{11}{2} = \frac{10 \cdot 11}{2} = 55$ edges

to be planar

$e \leq 3 \cdot 11 - 6 \leq 27$

$55 - 27 = 28$

if G has 27 e

$\bar{G}$ has 28 $\bar{e}$

that means that one has to be non planar

Connected graphs with v vertices have $\binom{v}{2}$ (possible?) edges

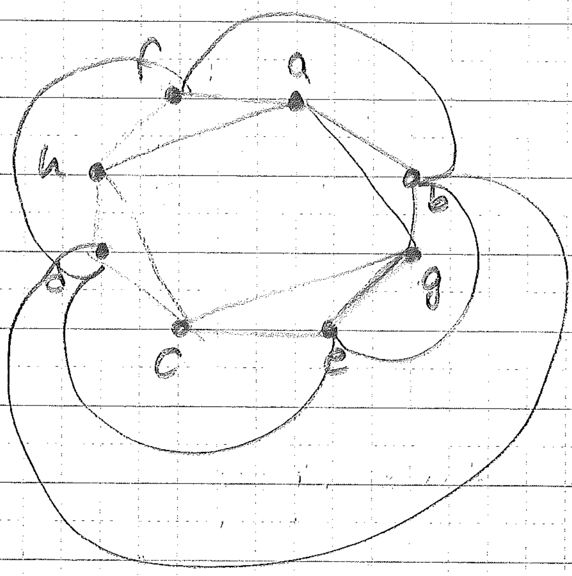
Complementary graph $\bar{G}$ of a simple G has the same vertices as G. Two vertices are adjacent iff they are not in G

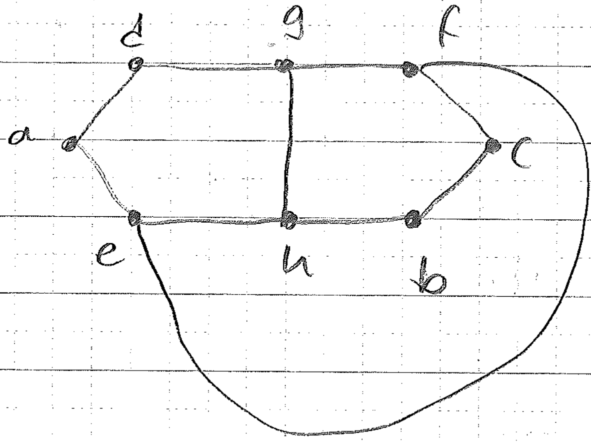
This theorem does not hold for eight vertices. Find a counter example to part a above, that is, find a planar G for which also the complement is planar

$G = 8V \qquad K_8 = \frac{(V-1)(V-2)}{2} = \frac{8 \cdot 7}{2} = 28$ edges

planar if $e \leq 3 \cdot 8 - 6 \leq 18$ maximal edges

$28 - 18 = 10 \quad \bar{G} \leq e$ regions $= e - V + 2 = 12$



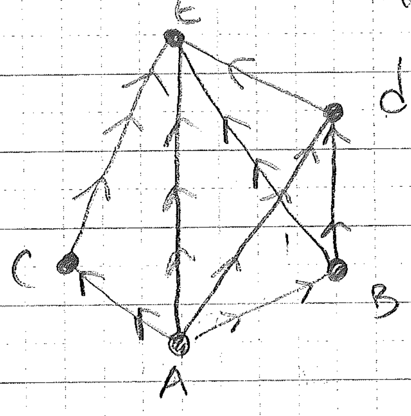


Draw the Hasse Diagram for the poset represented by the following zero-one matrix. label the elements in the set 1, 2, 3, 4 and 5 as in the matrix.

Is this poset lattice?

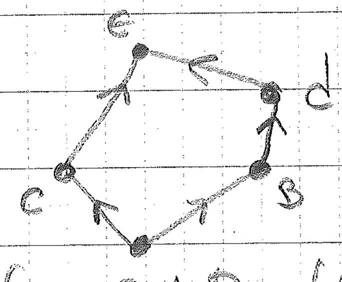
$$M_R = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left.\begin{array}{c|ccccc} & a & b & c & d & e \\ 1=a & 1 & 1 & 1 & 1 & 1 \\ 2=b & 0 & 1 & 0 & 1 & 1 \\ 3=c & 0 & 0 & 1 & 0 & 1 \\ 4=d & 0 & 0 & 0 & 1 & 1 \\ 5=e & 0 & 0 & 0 & 0 & 1 \end{array}\right\}$$

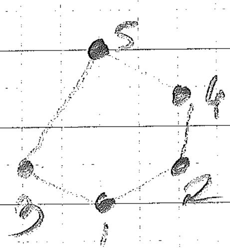


Hasse law 1
↳ don't show loops

Hasse law 2: Don't show transitivity



Hasse law 3: Don't show arrows, point allway forward

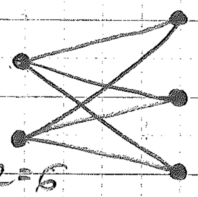


There is allways a join and a meet, so it is lattice

632
706

This question concerns complete bipartite graphs which are denoted $K_{m,n}$ where n and m are positive integers
A simple graph is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2) When this condition holds, we call the pair (V_1, V_2) a bipartition of the vertex set V of G

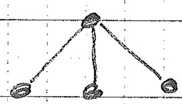
Complete matching from V_1 to V_2 iff
$|N(A)| \geq |A|$ for all A of V_1



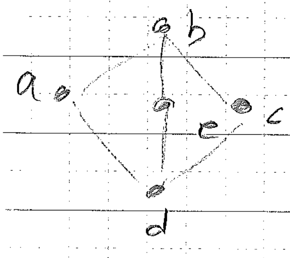
$3 \times 2 = 6$

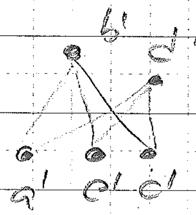
$K_{2,3}$ Complete

How many edges and vertices are there in $K_{m,n}$?
vertices = $m+n$ edges = $m \cdot n$
What is the chromatic number for $K_{m,n}$?
$\chi(K_{m,n}) = 2$ because potato
For which m and n is $K_{m,n}$ a tree?
$n=1$ or $m=1$ or both, for instance



For which m and n has $K_{m,n}$ a Hamilton cycle
$m=n$ and $m>1$
Show that $K_{2,3}$ is isomorphic to the graph G on next page





	a	b	c	d	e
a		1	0	1	0
b	1		1	0	1
c	0	1		1	0
d	1	0	1		1
e	0	1	0	1	

If is symmetric
Then is bipartite

Possible solutions to

$$x_1 + x_2 + x_3 = 14$$

$$2 < x_1 < 6$$
$$6 < x_2 < 10$$
$$0 < x_3 \le 5$$

GENERATING FUNCTION

TABLE 526

$$1 + x + x^2 + \dots + x^n = \frac{1 - x^{n+1}}{1-x}$$

$$\underbrace{(x^3 + x^4 + x^5)}_{x_1} \underbrace{(x^7 + x^8 + x^9)}_{x_2} \underbrace{(x + x^2 + x^3 + x^4)}_{x_3}$$

$$x^3(1 + x + x^2)\; x^7(1 + x + x^2)\; x(1 + x + x^2 + x^3)$$

$$= x^{11}(1 + x + x^2)^2(1 + x + x^2 + x^3) \Rightarrow \text{fold with GF Table 526}$$

$$= x^{11}\left(\frac{1-x^3}{1-x}\right)^2\left(\frac{1-x^4}{1-x}\right)$$

$$= x^{11}\frac{(1-x^3)^2(1-x^4)}{(1-x)^2(1-x)} = x^{11}\frac{(1-x^3)^2(1-x^4)}{(1-x)^3} =$$

TABLE

$$(1+x^r)^n = \sum_{k=0}^{n}\binom{n}{k}x^{rk}$$

1 if k
0 otherwise

$$= x^{11}(1-x^3)^2(1-x^4)\frac{1}{(1-x)^3}$$

→ table 526 expansion

$$(1-x^3)^2 = \sum_{k=0}^{2}\binom{2}{0}x^{3\cdot 0} + \binom{2}{1}(-x^3)^1 + \binom{2}{2}(-x^3)^2 = 1 - 2x^3 + x^6$$

$$x^{11}(1-x^4) = (x^{11} - x^{15})(1 - 2x^3 + x^6) = x^{11} - 2x^{14} + x^{17} - x^{15} + 2x^{18} - x^{21}$$

We want the coefficient 14, we can use x^{11} and $-2x^{14}$

$$\frac{1}{(1-x)^3} = \sum_{k=0}^{\infty}\binom{n+k-1}{k}(-1)^k x^k$$

for x^{11} we need the coefficien of x^3 from this expansion

$$\binom{3+3-1}{3}(+1)^3 x^3 = \binom{5}{3} - x^3 \quad \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 3\cdot 2\cdot 1} = \frac{20}{2} = 10$$

OBS!

AKTA

total possible solutions $= 10 - 2 = 8$

130
382
580

$2 \le x_1 \le 6$ $6 \le x_2 \le 10$ $0 \le x_3 \le 5$ $H = 14$

total = 14

x_1

3 - 7 - 4

3 - 8 - 3

3 - 9 - 2

$X_a + X_b + X_c = 25 \qquad 5 \leq x \leq 11$

$X_a = 5 + y_a$

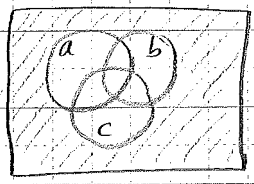
$X_b = 5 + y_b$

$X_c = 5 + y_c$

At least 5 of each are sure the combinations are reduced to:

$y_a + y_b + y_c = 25 - 15 = 10 \qquad 0 \leq x \leq 11 - 5 = \underline{\underline{6}}$

total combination without any restriction of 10 items and 3 slots.
$\binom{10+3-1}{10} = \binom{12}{10}$



$U = \forall$ comb a, b, c

$a, b, c = \forall$ comb. $x \geq 7$

$\bar{a}\,\bar{b}\,\bar{c} = \forall$ comb. $x < 7$

Instead of all combinations of $x \leq 6$ we will look for comb $x < 7$ by using

$a, b, c \geq 7$

$\bar{a}\,\bar{b}\,\bar{c} < 7$

$$N(\bar{a}\bar{b}\bar{c}) = U - (N(a) + N(b) + N(c)) + N(ab) + N(ac) + N(bc) - N(abc)$$

$N(a) \qquad x_a' + x_b + x_c = 10 \qquad x' \geq 7$ — 3 slots, 3 items left

$x_b + x_c = 10 - 7 = 3 \rightarrow \binom{3+3-1}{3} = \binom{5}{3}$

$N(a) = N(b) = N(c) \rightarrow 3\binom{5}{3}$

$N(ab) \qquad x_a' + x_b' + x_c = 10 \qquad x' \geq 7$

$x_c = 10 - 7 - 7 = -4 \rightarrow$ it is not possible because we only have 10 muffins

$N(ab) = N(bc) = N(ac) = 0$

$N(\bar{a}\bar{b}\bar{c}) = \binom{12}{10} - 3\binom{5}{3}$