Discrete Mathematics: Assignment #VIII

11/05/2008

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Problem 1

In how many ways, an, can a sum of 1s and 2s sum up to n? For n = 3 there are 3 ways. $3 = 1 + 1 + 1 = 2 + 1 = 1 + 2$. Find a RR for a and solve it on the computer. What is a_{20}? Plot with command Plot the rst 10 values of a_n

Solution

The recurrence relation was solved using the Fibonacci numbers. Paying attention to the problem, we get that our problem follows the Fibonacci numbers, excluding the first one. The formula to get the following numbers in the relation is:

$$a_n = a_{n-1} + a_{n-2}$$

When our initial values are $a_1 = 1$ and $a_2 = 2$.

When applying the formula in Mathematica, we get that for $a_{20} = 10946$. The plot of the first 10 numbers of this sequence are shown in the plot in 1.

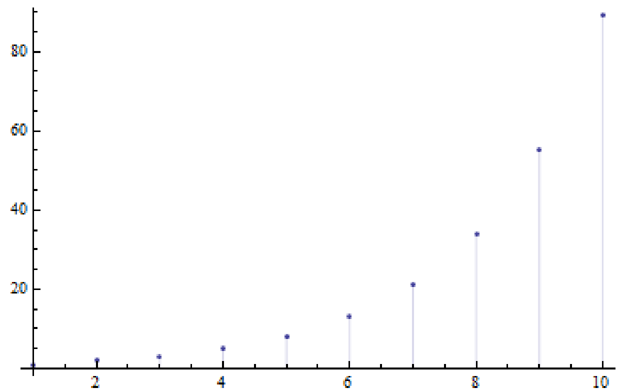

Figure 1: Exercise 1 - First 10 values of n

Problem 2

Find and plot an unstable 2-cycle in the logistic map for a = 4. Iterate a couple of times to illustrate the instability. You can locate the 2-cycle by the fix points of g(g(x)).

Justification

When a = 4 in the logistic map the limiting behavior is chaotic. There is infinite number of initial conditions that lead to cycles but unfortunately we were no able to understand the plot and for that reason, we could not find a 2-cycle in it clearly. As seen in Figure 2, Mathematica solves the possible limit cycles of period two as 0.345492 and 0.969846 but we have no further explanation for this solution more than that.

period	iterates	linear stability
1	0	unstable
1	0.75	unstable
2	0.345492, 0.904508	unstable
3	0.116978, 0.413176, 0.969846	unstable
3	0.188255, 0.61126, 0.950484	unstable
4	0.0337639, 0.130496, 0.453866, 0.991487	unstable
4	0.0432273, 0.165435, 0.552264, 0.989074	unstable
4	0.277131, 0.801317, 0.636831, 0.925109	unstable

Figure 2: Exercise 2 - Possible limit cycles for this choice of parameter

We are sorry we are not able to succeed finding a 2-cycle for a = 4 as expected. During our investigation we found a formula for finding cycles of any length when a = 4, but we may have not understand the exercise. We asked different assistants from the math department and looked the documentation provided by Mathematica, we run off of time to complete this exercise as we think it should be done. We will add all the information we got from Mathematica.

n	0	1	2	3	4
x_n	0.050000	0.19000	0.61560	0.94655	0.20238

Figure 3: Exercise 2 - Iteration of the Logistic map for a = 4

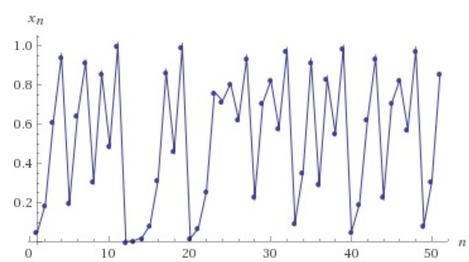

Figure 4: Exercise 2 - Iteration of the logistic map for 50 iterations

$$x_n = \sin^2\left(2^{n-1}\cos^{-1}(1-2x_0)\right) \approx \sin^2\left(0.225513 \times 2^n\right)$$

Figure 5: Exercise 2 - Closed form solution

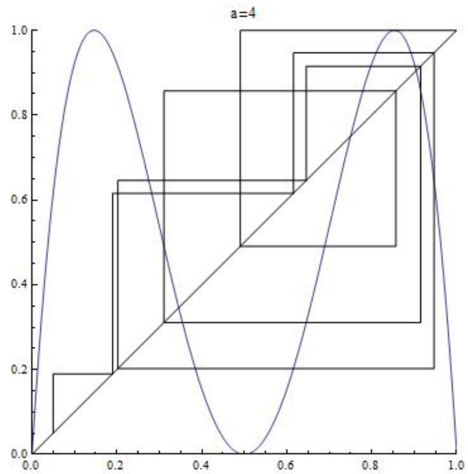

Figure 6: Exercise 2 - Cobweb diagram og g[g[x]] for 10 iterations

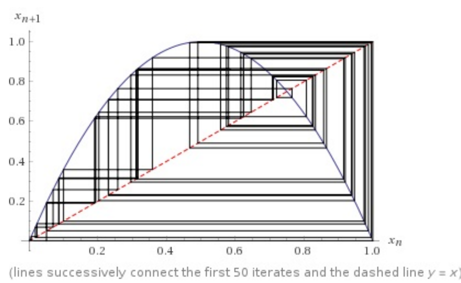

Figure 7: Exercise 2 - Cobweb diagram

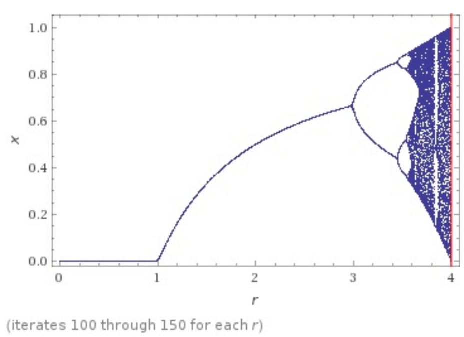

Figure 8: Exercise 2 - Bifurcation Diagram

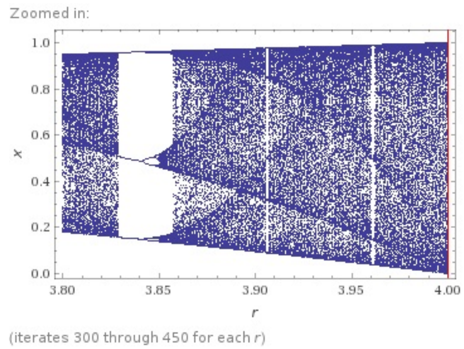

Figure 9: Exercise 2 - Bifurcation Diagram between 3.8 and 4

All the figures above were made using Wolfram Alpha Mathematica Widgets.

Problem 3

Consider the CA called Majority Action. The majority of the 3 cells decides the state of the middle cell in next generation. So 2-3 black cells gives black and 0-1 black cells gives white. What is the rule number? Run it for a random seed a couple of times. What happens? Extend to 2 neighbors on each side. What is the rule number for majority action this time? Any difference compared to 2 neighbors?

Solution

We start defining the rule number of the CA defined in the exercise. The binary number of the second generation is 11101000 or in decimal number, 232. That is the number of the CA we are working with. 11. As we run rule 232 in Mathematica, we can see how this rule works. The first generation of cells will

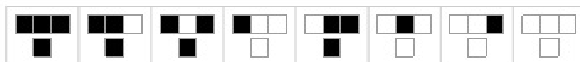

Figure 10: Exercise 2 - Majority Action rule 232

change depending of the majority of their neighbors. Cells that have close neighbors, will have children, each generation closer and closer to most of their alive friends. The same way, if the majority of the cells around an alive cell are dead, the living cell will join the dead ones. Otherwise, the cells will keep going generation to generation, packed in groups. At a large amount of generations, the plot gets a pattern of white and black bars as seen inf Figure 11.

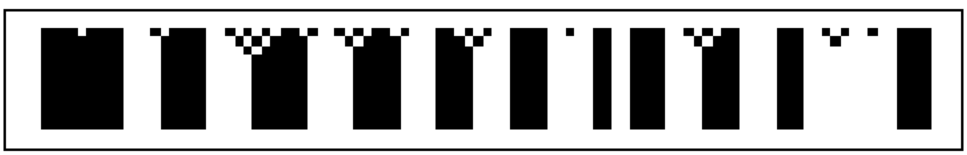

Figure 11: Exercise 2 - Majority Action rule 232, Random seed

If the first generation in rule 232 is a pattern created by dead and alive cells, for instance a large amount of 1,0,1,0,1,0 we get a curious figure. While trying to follow the majority of their neighborhood, the cells make a checkered inverted triangle until all of them die as seen in Figure 12. As extra information, this will happen when the triangle stars and ends in zero. If the pattern starts and ends in one, the triangle tends to alive and the plot would be black instead of white.

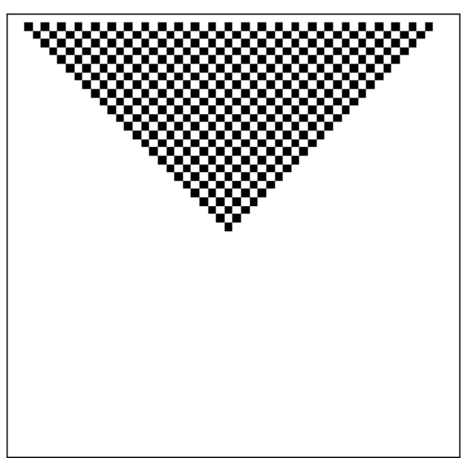

Figure 12: Exercise 2 - Majority Action rule 232, checkered pattern

When we broaden the neighborhood to two extra cells we get rule number 4276676736, or in binary 11111110111010001110100010000000. This rule, that we named MR-5 as a short name of Majority Rule

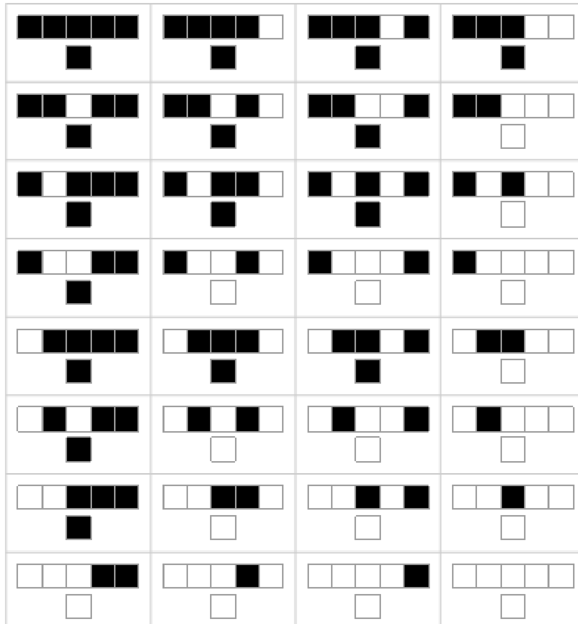

Figure 13: Exercise 2 - Majority Action with 5 cells

5 cells, works in a similar way that rule 232 but with more combinations. We can see that the pattern is similar to 232 but with broader bars as seen in Figure 14.

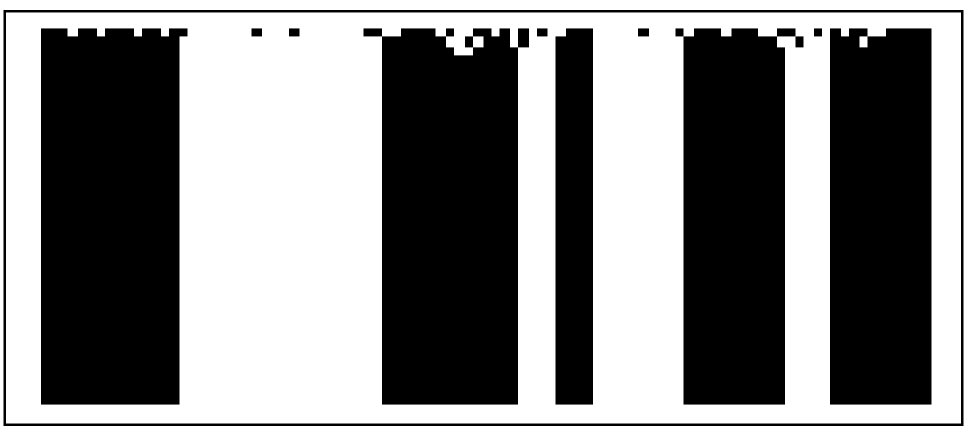

Figure 14: Exercise 2 - Majority Action MR-5, random seed

We can already appreciate a difference with rule 232. The triangles created when a pattern 1,0,1,0,1,0 is found is different than the 232 rule. This triangle is not checkered but formed with lines as seen in Figure 15.

This pattern is formed because of the state 1,0,1,0,1 creates a new generation that colludes with state 0,1,0,1,0 that kills cells. Following these two states, the inverted triangle is created and follows until they die.

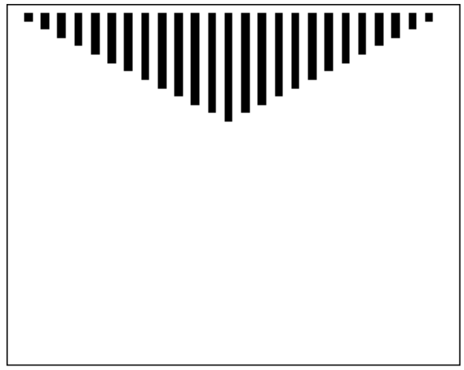

Figure 15: Exercise 2 - Majority Action rule 232, checkered pattern

Problem 4

Run a seed with 3 black cells in a row in Game of Life. What happens? Investigate what happens if you make the initial state longer? That is, 4, 5 and 6 black cells in a row and even longer. Include an illustrative plot. Use ;; when arrange the seed.

Solution

When applying the Game of Life rule to three black cells in a row, the neighborhood iterates in the form of vertical and horizontal 3 cells-line indefinitely. This form is also called a blinker and is shown in Figure 16.

Figure 16: Exercise 4 - Game of Life, Blinker

When applying the Game of Life to four black cells in a row and move one generation, the line changes to a 6 black cells forming a black rectangle. Next step, the rectangle will change to a form called beehive and it stays in form showed in Figure 17 permanently.

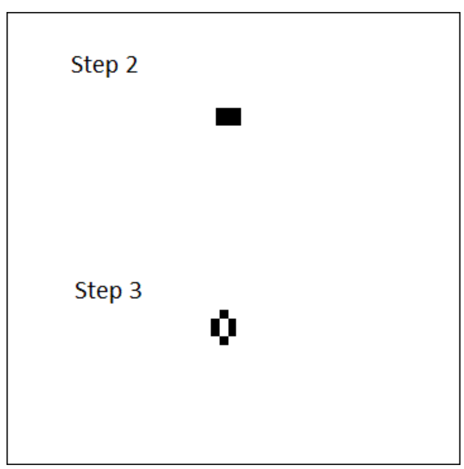

Figure 17: Exercise 4 - Game of Life, Beehive

Problem 4 continued on next page. . .

When applying the Game of Life to five black cells in a row it has seven steps in total until reaching stability. When reaching step 6 it begins to iterate from step 6 to step 7 as shown in Figure 18.

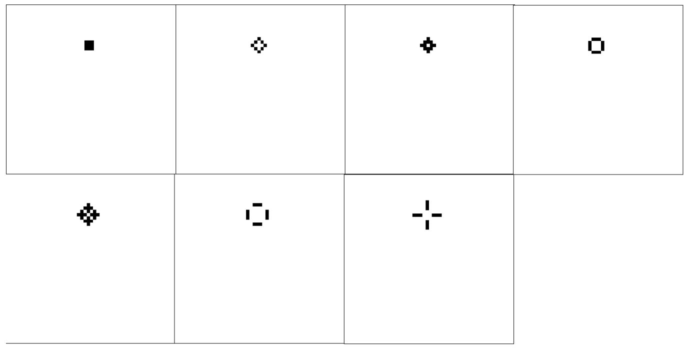

Figure 18: Exercise 4 - Game of Life, Traffic Light with 4 Blinkers

When applying the Game of Life to six black cells in a row, it has ten steps in total until disappearing. The black horizontal line transforms into a vertical figure and ends up fading away as shown in Figure 19.

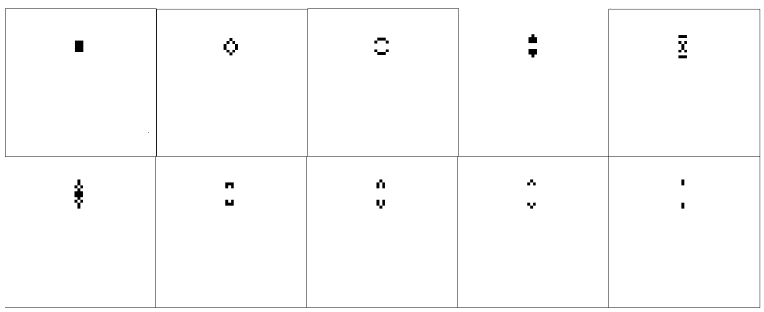

Figure 19: Exercise 4 - Game of Life, Six cells in a row