LendBook
a Lending Limit Order Book

prevert *

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Abstract

A lending limit order book is a non-custodial, peer to peer, permissionless, high loan-to-value lending protocol that enables users to borrow limit orders' assets collateralized by their own limit orders. This new financial primitive offers users multiple benefits: stop loss orders with guaranteed stop price, zero liquidation penalty, high leverage strategies and interest-bearing limit orders. In addition, the protocol is immune to the risk of bad debt and can operate without the need for off-chain governance.

Current issues with lending protocols

Lending protocols provide users with the ability to lend and borrow cryptoassets in a decentralized, permissionless, and trustless manner. However, their growth has been impeded by a common birth flaw: the risk of incurring bad debt. A bad debt appears when the price of the deposited collateral rapidly falls and its value no longer covers the outstanding loan, putting the protocol at risk of insolvency

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and bank run. The loss can be exacerbated by the lack of sufficient liquidity in the pools in which the collateral assets are exchanged to cover borrowed assets during liquidation events.[1]

To mitigate this existential risk, lending protocols impose various constraints on the borrowers side, such as high collateral-to-debt ratios and high liquidation costs. They also limit lending markets to high-quality assets. These constraints impair users' experience by making borrowing more expensive and riskier and by restricting the range of borrowable assets. Despite many innovative features introduced in the sector since its emergence in 2018, significant improvement of users experience is still awaiting a foolproof solution for the risk of bad debt.

Furthermore, as regulatory pressures are mounting, the decentralized finance sector is expected to fracture into two main areas: compliant and accredited protocols, implementing Know Your Customer (KYC) processes and reporting to relevant authorities for Anti-Money Laundering (AML) and taxation; and fully decentralized protocols that operate beyond the reach of regulatory agencies.

LendBook, a lending limit order book, is a new financial primitive which eliminates the insolvency risk, achieve full decentralization and brings along the way a host of new benefits for lenders and borrowers, like high Loan-To-Value (LTV) and leverage, borrowing programmability and interest-bearing limit orders.

1 What is a lending limit order book

A lending limit order book (LLOB) is a non-custodial, peer to peer and permissionless lending protocol that enables users to borrow limit orders' assets. Borrowing positions are collateralized by limit orders posted the other side of the order book. Borrowed assets from buy orders are collateralized by the assets deposited in sell orders, and reciprocally.

Figure 1 presents the double queue organization of a typical central limit

[1]See RiskDAO for examples of bad debt events.

order book. On the left-hand side, makers place buy orders which traders find profitable to take if the price decreases below their limit prices. On the right-hand side, they place sell orders which are filled if the price increases above their limit prices. Rectangles' height indicate how much makers have deposited in their limit orders.

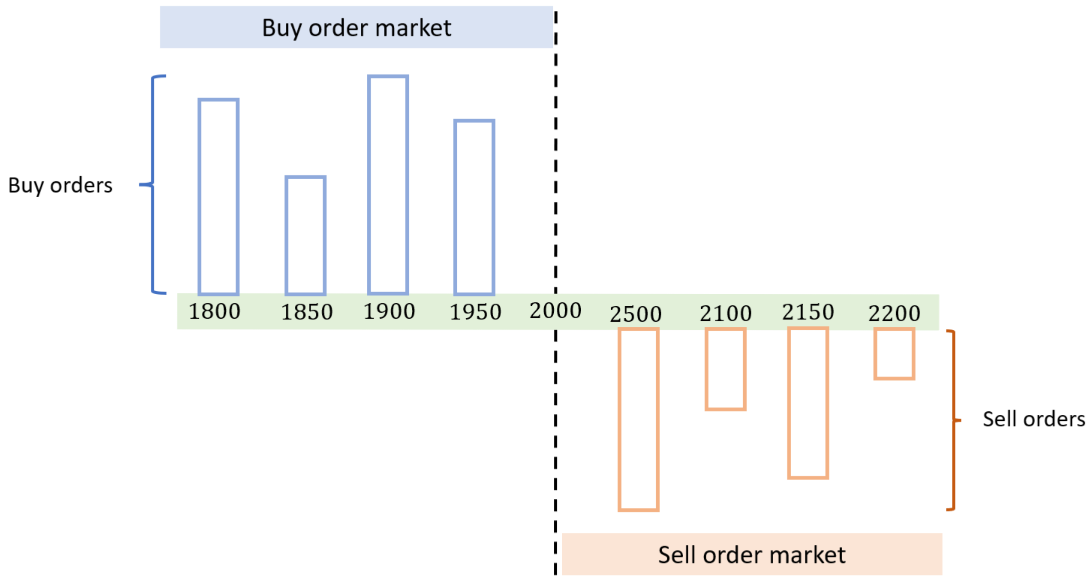

Figure 1: A graphical representation of a central limit order book.

Fig. 2 shows how a lending primitive is attached to a limit order book. Makers allow traders to borrow their assets deposited in the book in exchange of an interest rate. The rectangles with a blue and orange background represent the assets borrowed from the orders at the same limit price.

The main rule governing the protocol stipulates that any position borrowing against a limit order must be closed when that limit order is executed. The alignment of the two events significantly streamlines the settlement process for both parties.

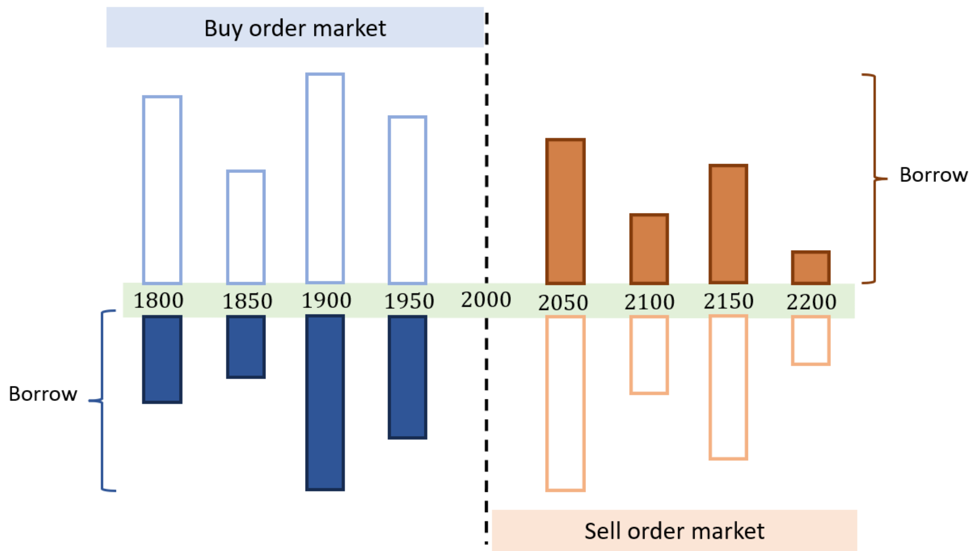

Figure 2: A central limit order book with lending functionalities.

Example

Let's illustrate how a lending operation works. Alice posts a buy order of 3 ETH at price 1900 USDC while market price is 2000. To do so, she deposits $3 \times 1900 = 5700$ USDC in the protocol's USDC vault. Bob is willing to borrow 3800 USDC from Alice's buy order. He places a sell order of 2 ETH at 2200 (or any price greater than market price) and deposits 2 ETH in the ETH vault. With 2 ETH of collateral, he can then borrow $2 \times 1900 = 3800$ USDC from Alice. The financial flows are summarized in Fig. 3.

If the price decreases to 1900, a taker swaps Alice's remaining 1900 USDC for 1 ETH, which triggers the closing of Bob's position. Bob keeps the borrowed USDC and Alice is given 2 ETH taken from Bob's collateral (see Fig. 4).

Nothing changes for Alice compared to a non-borrowed buy order. Importantly, Bob's assets are not market sold but directly transferred to Alice who is paid back with the collateral.

Before reviewing further the benefits of implementing a lending order book, let's dive into the details of its functioning.

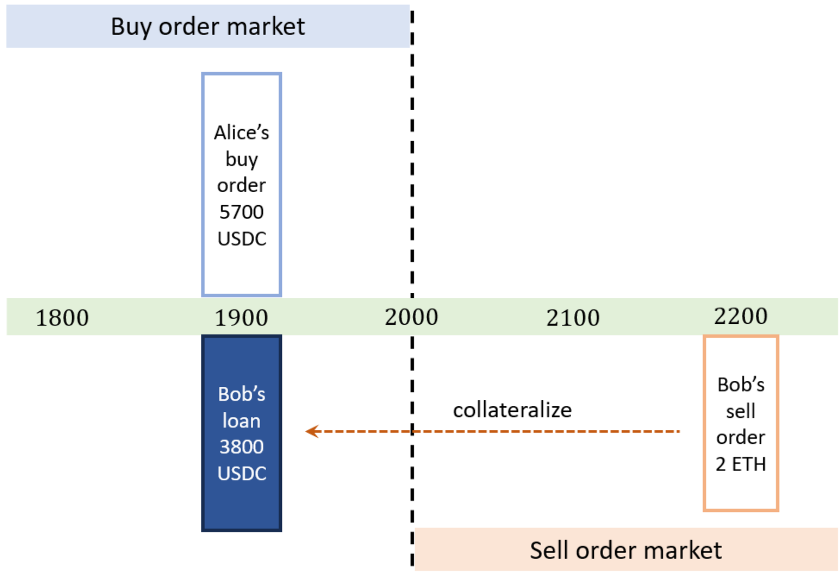

Figure 3: Bob borrows 3800 USDC from Alice, collateralized by his sell order at 2200.

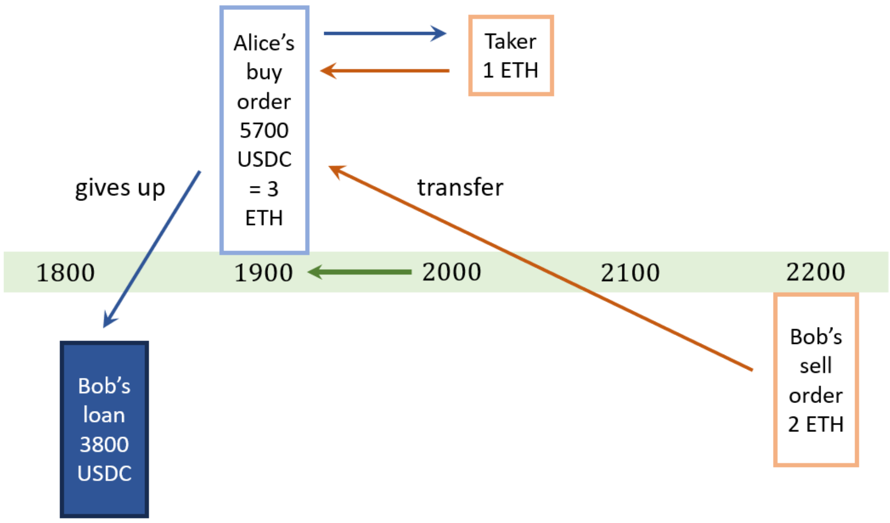

Figure 4: If the price decreases to 1900, Alice's remaining assets are taken. Bob's collateral is transferred to Alice to complete the filling.

2 Functioning

2.1 Lending Limit Order book's Rules

The operations are governed by six rules. Appendix A presents a formal specification of the rules.

R1. When a limit order is taken, all positions borrowing from the order are closed out.

In the example, Bob's position is closed out when Alice's buy order is taken.

R2. Limit orders' assets which serve as collateral for borrowing positions cannot be borrowed.

Example: Bob places a sell order at 2200 USDC and deposits 2 ETH. He then borrows 3800 USDC from Alice's buy order. His 2 ETH serve as collateral and cannot be borrowed.

The prohibition against borrowing collateral prevents situations in which collateral assets could be unavailable in case of liquidation. In addition, not all assets deposited in an order can be borrowed, as sufficient incentives must be preserved for traders to take the non-borrowed assets when the price hits the limit price.

R3. If the borrower's own limit order, which assets serve as collateral, is filled, his borrowing position is closed out.

The closing of the borrowing position guarantees that the type of assets serving as collateral always matches the type needed in case of liquidation.

Example (variant): Bob's sell order is filled first for $2 \times 2200 = 4400$ USDC, of which 3800 are kept by the protocol to close his borrowing position from Alice's buy order.

R4. Makers are unable to withdraw their borrowed assets until one of the following events occurs: 1) the borrower repays the loan, 2) his order is taken, or 3) the borrower's order, which collateralize his position, is taken.

Makers whose deposited assets are not borrowed can withdraw their assets when they wish. However, the withdrawal of borrowed assets is conditional on one of the three events listed in Rule 4. In the interim, lenders earn an interest as compensation for their assets being temporarily inaccessible. They have also the possibility to mark their orders as non-borrowable, which prevents their orders to be borrowed again once the positions on them are paid back or liquidated.

R5. Orders which assets are taken are automatically replaced on the opposite side of the order book.

The converted assets are replaced at a limit price specified by the maker, or, in the absence of such specification, at a default limit price. This default is set at 10% above or below the previous limit price, depending on the nature of the order.

Example: Alice's buy order is filled for 3 ETH. The protocol relocates the ETH in a sell order which limit price is by default 1900 + 10% = 2090.

R6. Orders which assets are borrowed cannot be executed at a loss.

This restriction safeguards against premature liquidation by external parties who might otherwise execute orders at an unfair price. While potentially costly, the cost could be mitigated by filling a small quantity, or even negated if the taker is the maker/lender. To guarantee that orders are always executed at a profit, a price feed is pulled prior to any transaction.

Example: Bob borrows 3,800 USDC against Alice's buy order with a limit price of 1,900. Alice's limit order cannot be executed if the price feed indicates a market price above 1,900.

3 Interest rate model

Two interest rates coexist in a LLOB: one for the buy order market and one for the sell order market. They are algorithmically adjusted to the real-time supply and demand of assets. Only borrowed assets earn an interest rate.

The market's utilization rate (UR) is the ratio of the total borrowed assets to the total supplied assets. The higher the UR, the higher the interest rate. As assets cannot be simultaneously lent out and used as collateral, the sum of the buy order market's UR and the sell order market's UR cannot exceed 100%: $\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}} \leq 1$ (see Appendix C for details). As a result, the two interest rates $\mathrm{IR}_{\mathrm{BO}}$ and $\mathrm{IR}_{\mathrm{SO}}$ depend on the two measures of utilization rate:

$$\mathrm{IR}_{\mathrm{BO}} = \alpha + \beta \mathrm{UR}_{\mathrm{BO}} + \gamma(\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}})$$

$$\mathrm{IR}_{\mathrm{SO}} = \alpha + \beta \mathrm{UR}_{\mathrm{SO}} + \gamma(\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}})$$

with α, β, γ three positive parameters.

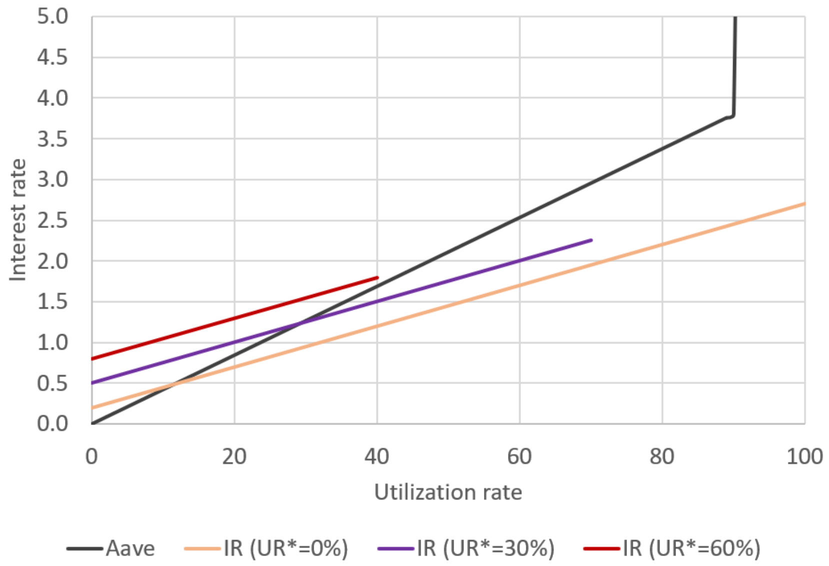

Figure 5: Interest rate schedule with respect to pool's utilization rate in Aave and LendBook. Parameter values (see Section 3): $\alpha = 0.002$, $\beta = 0.015$ and $\gamma = 0.01$ (UR*: utilization rate in the paired asset market).

Fig. 5 compares the interest rate schedule in the ETH market with respect to the utilization rate in Aave and the one in LendBook.[2] The higher the utilization rate in the two markets, the higher the interest rate. The curve is flatter in Lendbook than in Aave and does not spike after 90% UR.Variations in pool utilization cannot cause large swings in interest rates, which is a source of uncertainty for borrowers and lenders in other protocols, especially at times of high market volatility.

4 Benefits of using LendBook

The benefits of a lending order book are multiple: stop loss orders with guaranteed stop price, zero liquidation penalty, high leverage, programmability of leverage strategies, automated market making and minimized governance. Let's review them one by one.

4.1 Zero cost liquidation

The prevalent liquidation mechanism in lending protocols allows a liquidator to repay a fraction of the borrower's debt and acquire its collateral at a discount. In LendBook, the closing of a borrowing position does not rely on the active monitoring of liquidators but on that of takers. A minimal part of limit orders are kept non-borrowable and reserved to traders who, by taking the non-borrowed part of the assets, initiate the internal transfer from the borrowers to the lenders.

Example (continued): suppose the price crosses 1900. Alice's remaining buy order of 1 ETH is filled by a taker. Bob's collateral is simultaneously transferred to Alice's account as if Bob were Alice's counterparty to the trade.

Since lenders agree to receive the collateral as a payment, the protocol does not need to incentivize bots to liquidate unhealthy positions on time. The bor-

[2] Interest rate model is subject to future adjustments.

rower does not incur liquidation costs as a result.

4.2 High leverage

LendBook enables leverage factors a magnitude higher than what other lending protocols offer. To understand how, let's examine how traders leverage their position. As in other protocols, they can borrow Y and swap them to amplify their position in X, or they can borrow X and swap them to short X. Borrowers can easily do loops of borrowing and swapping to amplify their leverage.[3]

Example (continued): After Bob borrowed 3800 USDC from Alice, he converts the USDC for 3800/2000 = 1.9 ETH. His leverage factor is 3.9/2 = 1.95. He can increase further his leverage by doing a second borrowing loop. By depositing 1.9 ETH in his sell order, he can borrow 1.9 × 1900 = 3610 additional USDC from the same or another buy order (assuming the same limit price of 1900 USDC). After exchanging at market price 3610 USDC for 3610/2000 = 1.8 additional ETH, his total leverage is now (2 + 1.9 + 1.8)/2 = 2.85.

4.2.1 Loan to value

The Loan to Value (LTV) ratio defines the maximum amount of assets that can be borrowed with a specific collateral. For example, a maximum LTV of 80% in the ETH/USDC market means that users can borrow at most 0.80 USDC worth of ETH for every USDC deposited as collateral.

In LendBook, traders can obtain LTV close to 100% by borrowing from orders which limit prices are close enough to market price. The LTV is (limit price)/(current price) if the order from which assets are borrowed is a buy order and (current price)/(limit price) if the borrowed order is a sell order. Both LTV approaches 100% when the market price gets closer to the limit price.

Example (continued): For every USDC Bob borrows from Alice's buy order, he

[3]see Appendix B for a formal analysis of this section.

has to deposit at least 1/1900 ETH which, at current price, is worth 2000/1900 = 1,053 USDC. His LTV is 1900/2000 = 95%. Had Bob borrowed from a buy order which limit price is 1990 USDC, his LTV would be 1990/2000 = 99.5%.

4.2.2 Maximum leverage

High LTV translates into high leverage. Abstracting from gas and swap costs, the n-loop leverage factor is:

$$1 + \mathrm{LTV} + \mathrm{LTV}^2 + ... + \mathrm{LTV}^n$$

Assuming borrowers could infinitely loop at the same limit price, their maximum leverage would be:

$$\frac{1}{1 - \mathrm{LTV}}$$

As the limit price gets closer to the market price, the LTV tends to 1 and the theoretical maximum leverage to infinity.

Example: When Bob borrows from a buy order which limit price is 1900, his maximum leverage is 21. Had he borrowed from a limit price of 1990 USDC, his maximum leverage would be 210.

Fig. 6 shows the maximum leverage factor as a function of the distance of the limit price to the market price. Buy (or sell) orders, which limit prices are lower (higher) than market price, give traders a way to leverage-long (-short) the base token. The maximum leverage on the graph is 1000 for a buy order limit price of 1998 and 900 for a sell order limit price of 2002, which are two dollars apart from the market price. Borrowers can still achieve a significant leverage for limit orders far away from current price. The maximum leverage gradually decreases to 2 for limit prices 1000 and 3000.

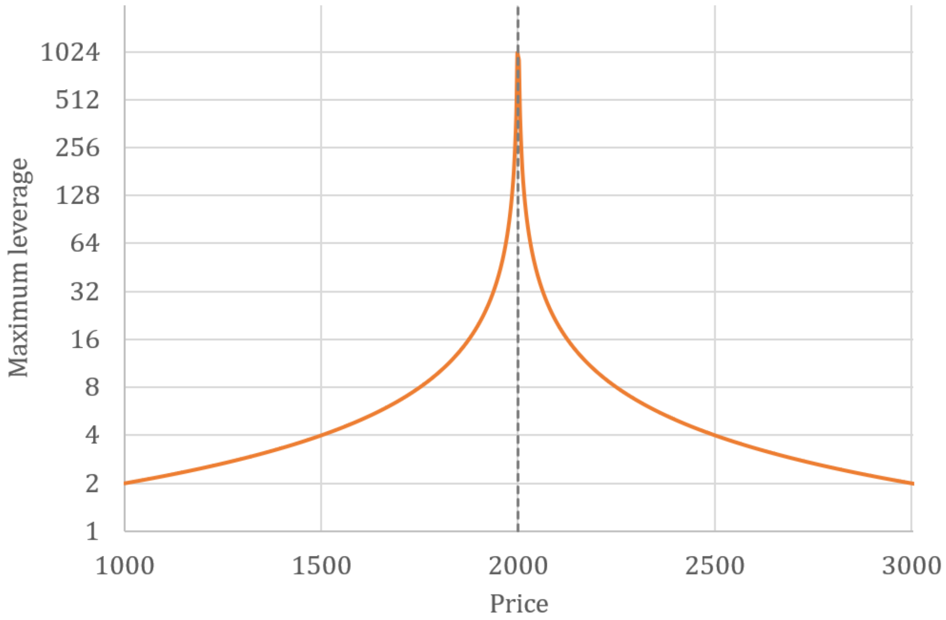

Figure 6: Distance of limit price to market price (2000) and maximum leverage (base-2 log scale).

4.2.3 Application: Liquid staking derivative markets

LendBook is exceptionally well-suited for liquid staking derivative markets in which high LTV is a key feature.

Let us illustrate the efficiency of the protocol with the wstETH/ETH market. A look at the Uniswap V3 market, endowed with 27m liquidity, indicates a capital spread between 1.1458 and 1.1461.[4] Assuming a similar liquidity distribution on LendBook, the order book representation is represented in Fig. 7.

A strategy amplifying the yield consists in depositing wstETH on the sell order side and borrowing ETH on the buy order side. Given a current price of 1.1460, borrowing ETH at the limit price of 1.1458 would give users a LTV of $1.1458/1.1460 = 99.98\%$. Exchanging on Uniswap the ETH for wstETH at a 0.01% fee rate would provide traders with an adjusted LTV of 99.97% (assuming no price impact). If the process could be infinitely repeated, the maximum

[4]See Uniswap's pool address, oberved on Nov. 13th 2023.

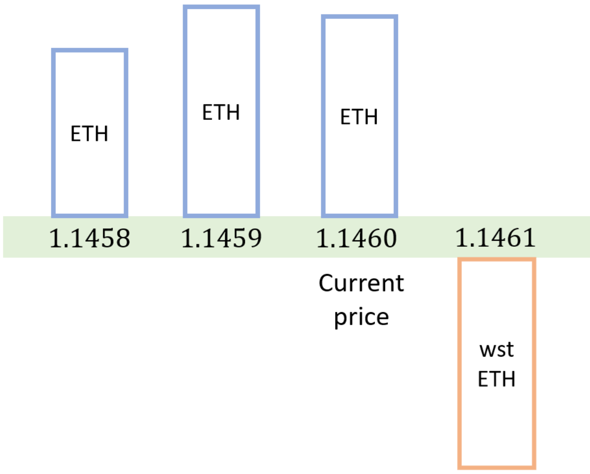

Figure 7: Capital distribution in the wstETH/ETH market similar to the Uniswap liquidity pool.

leverage factor would be

$$\frac{1}{1 - \mathrm{LTV}} = 3642$$

In practice, the leverage factor will be far less due to a finite number of rounds of borrowing, price impact and gas costs. But even a leverage factor of 50 is still 5 times greater than what other lending protocols currently propose. Given a stETH APR of 3.9% and a borrow APY of 3%, leveraged APR would be $50 \times 0.9\% = 45\%$.

4.3 Stop loss and take profit orders

4.3.1 Stop loss

A stop-loss order allows traders to close long positions by selling the assets or a short position by buying the assets. In LendBook, users open stop-loss orders by borrowing assets from limit orders. The stop price in case of price decrease (or increase) is the limit price of the buy (sell) order from which they borrow.

Example (continued): if the price decreases to 1900, Alice's buy order is taken.

Bob keeps the 3800 USDC and gives up his 2 ETH. This is as if he benefits from a stop loss (sell ETH when its price decreases) at the guaranteed price of 1900. His stop price is Alice's limit price.[5]

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when markets move fast. Here the stop price is guaranteed by the filling of the sell order at the limit price.

A guaranteed stop price is particularly useful to hedge against the risk of assets losing their peg or protocol's hacks.

4.3.2 Take profit

In addition, by posting their collateral in the order book, borrowers can program in advance their exit strategy.

Example: Bob benefits from a take profit at the price of 2200. If the price increases to 2200, Bob's sell order is taken first. His 2 ETH are exchanged against 4400 USDC from which 3800 are used to pay back his borrowing position (cf. Rule 3). He keeps the 3800 USDC he borrowed from Alice and earns a profit of 600 USDC.

A take-profit option is an integral part of risk management in case of leveraged position.

Example: Bob borrows 3800 USDC from Alice and exchanges the amount for 1.95 ETH. If the price hits 2200, his sell order is filled and his borrowing position is closed out. The protocol pays back Bob's debt of 3800 USDC with the 2200×2 *ETH = 4400 USDC of his sell order. Bob's leveraged profit is* $3.95 \times 200 = 790$ *USDC.*

Fig. 8 shows the price interval over which Bob makes a profit or a loss and

[5] See Appendix A for a formal presentation of stop loss orders.

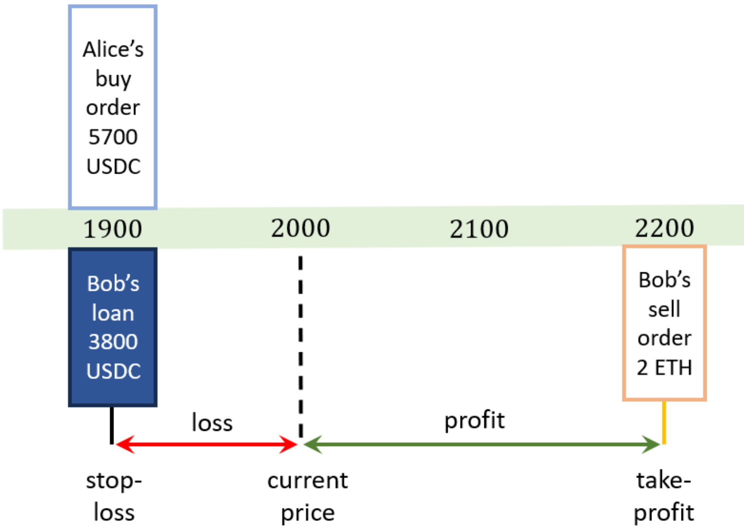

Figure 8: When Bob borrows from Alice at 1900 and places a collateral limit order at 2200, his leverage is closed out for a profit if his limit order is taken first and for a loss if Alice's buy order is taken first.

at which prices his position is closed out in both directions.

4.4 Programmability of borrowing strategies

The combination of an order book with a lending protocol unlocks an infinite set of strategies that borrowers can fine-tune and program in advance. We have already seen that traders are able to program their stop loss by choosing at which limit price they borrow assets, and their take-profit by selecting the limit price of their collateral order on the other side of the book.

In addition, Borrowers can also split their position between several orders with different limit prices. This way, their stop orders can be gradually executed and their borrowing progressively reduced as the price reaches well-specified thresholds.

Example (see Fig. 9): Bob borrows 1 ETH from Alice's buy order which limit price is 1900 and 1 ETH from Clair's buy order which limit price is 1800. Bob's

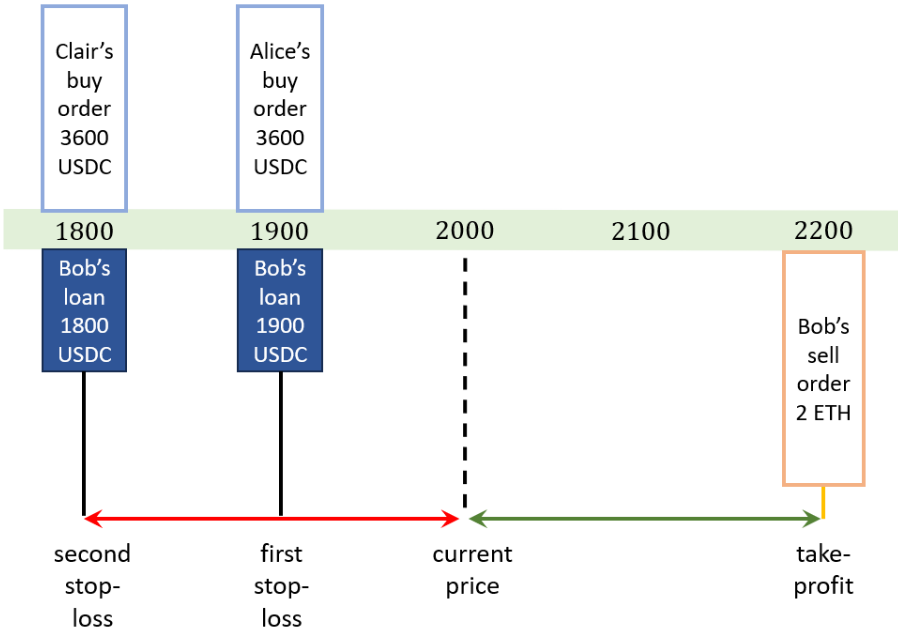

Figure 9: Bob borrows from Alice and Clair at different closing prices.

borrowing is halved at 1900 then closed out at 1800.

Borrowers can also divide their collateral between several limit orders to gradually take their profit.

Example (see Fig. 10.): Bob places a sell order of 1 ETH at 2100 and another sell order of 1 ETH at 2200. He borrows 3800 USDC from Alice and exchanges the amount for 1.95 ETH. If the price hits 2100, his first sell order is filled and his borrowing position is simultaneously closed out for 1900 USDC. The protocol pays back Bob's debt of 1900 USDC with the 2100 USDC of the sell order. Then, if the price crosses 2200, the second sell order is filled and his borrowing position is closed out for the remaining 1900 USDC. After Bob's debt is paid back, Bob makes a profit of 3.95 $\times$ 150 = 592 USDC. Bob takes around half of his profit at 2100 and the other half at 2200.

Another strategy for borrowers consists in reposting their borrowed amounts on the order book to gradually raise their leverage as the price moves.

Example (see Fig. 11): Bob places a sell order of 2 ETH at 2200, borrows 3600

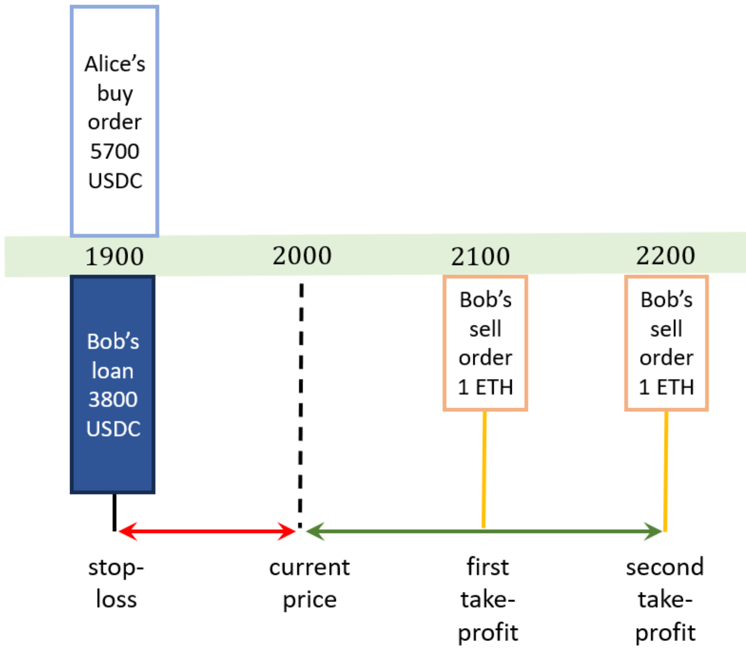

Figure 10: Half of Bob's borrowing position is closed at 2100, the other half at 2200. The two limit prices allow Bob to gradually take his profit.

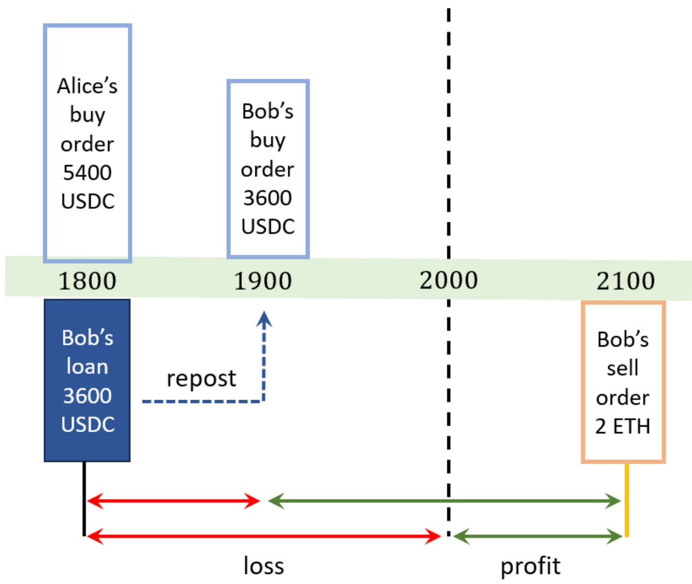

Figure 11: Bob replaces his borrowed USDC on the order book to leverage long his position if the price declines to 1900.

USDC from Alice at 1800 and reposts the USDC in a buy order at 1900. If the price decreases to 1900, Bob's USDC are taken in exchange of 3600/1900 = 1.9 ETH. Bob is now leveraged long ETH. If the price reverts and increases above 1900, Bob profits.

4.5 Automated market making

Orders which assets are filled are automatically replaced in the order book (Rule 5). The new limit price is specified by the maker or, by default, set at the previous one +10% if the taken order is a buy order or -10% if it is a sell order.

Example: Market price is 2000. Alice deposits 5700 in a buy order which limit price is 1900. Once filled, the protocol relocates the 3 ETH in a sell order which limit price is 1900 + 10% = 2090.

This feature allows makers (or protocols built on LendBook) to implement simple or advanced market making strategies. The strategy space is similar to what liquidity providers can implement in Uniswap-type Automated Market Makers (AMMs), with two main exceptions: 1) how liquidity is distributed on the price space is more flexible and granular and 2) instead of earning a fee rate, makers earn the spread on top of the interest rate paid by borrowers.

Example (continued): Alice's sell order is taken for 3 × 2090 = 6270 USDC. Her net profit is 6270 - 5700 = 570 USDC. Her profit rate is 570/5700 = 10%.

The primary risk encountered by market makers is inventory risk, also known as impermanent loss in Automated Market Makers (AMMs). This risk arises in case of a swift decline or increase in price, leading to an imbalance in the market maker's inventory. In LendBook, makers have the option to hedge against this risk by using stop-loss orders at guaranteed price.

Example: Alice wants to market make the DAI/USDT market but is afraid of DAI losing its peg. She deposits 1000 DAI in a sell order at price 1.01 and sets a paired limit price of 0.99 if her sell order is taken and the assets are replaced *in a buy order. To hedge against a price depeg, she borrows 980 USDT from a buy order at 0.98. If the DAI price falls below 0.98, her borrowing position is liquidated and Alice keeps the USDT.*

Two types of lending strategies can be envisioned. In the first one, lenders act as market makers. They are active liquidity providers in the book and post limit orders closed to current price. They constantly replace their filled order on the other side of the market to earn the spread in addition to the lending return rate. In the second type, lenders follow single-sided Aave-style strategies. They earn a return for their deposited assets but minimize the risk of conversion by posting limit orders relatively far from current price.

4.6 Absence of bad debt

A major implication of borrowing assets from limit orders is the dramatic simplification and high safety of the liquidation process. Borrowing positions cannot go under-collateralized even in case of strong and rapid price action, gas fee spike, or blockchain congestion/downtime. The closing of a borrowing position does not rely on trading against an AMM pool with the risk of a sub-optimal execution. The risk of rapid price variation is still present but borne by the maker of the limit order.

Example: If the price is rapidly falling, Alice gets her buy order executed for 1800 when the market price may actually be 1780. Or the buy order could remain unfilled or partially unfilled if the price rapidly reverses.

This creates an opportunity cost for makers. Although the cost is common to all limit order books, makers are compensated in LendBook by an interest rate.

The fact that lending pools stay well collateralized under any market conditions has many benefits at the UX and governance levels. In particular, there is no need for supervision by third party, funding a safety module, liquidation costs, or borrowing restrictions (supply caps, borrowing caps, low loan-to-value) anymore.

4.7 Governance minimization

The governance activity of lending protocols has considerably grown and gained complexity over time. Managing pools' risks has been progressively delegated to experts which mission is to keep in check the pools' risk and update their risk parameters. This involves assessing multiple risk factors like assets' onchain liquidity, price volatility and market capitalization.

Despite committing considerable resources and expertise to risk management, protocols' solvency is still at risk of a lack of due diligence or governance failure. Lending protocols have also implemented and funded sizeable financial buffers to absorb shortfall events and protect lenders from bad debt. Those safety measures mitigate solvency risk at the expense of token holders.

In contrast, the functioning of a LLOB is fully algorithmic and automated. As pools' solvency is encoded at the smart contract level and does not rely on team's interventions or governance by a DAO, full decentralization becomes a realistic objective which LendBook will actively pursue. The protocol will be governance free with non-upgradeable smart contracts and parameters set at the time of contract deployment.

Furthermore, no governance process is needed to whitelist approved tokens. Markets are created permissionlessly by calling a factory contract. The number of assets that could be listed is only limited by the existence of a reliable price feed.[6]

5 Conclusion

Lending protocols are an essential building block for blockchain's applications and have grown to represent tens of billions of dollars in value. However, the

[6]Aave V3 currently lists 20 or so assets on Ethereum. The additional list of admissible tokens with a Chainlink feed includes SHIB, GRT, SAND, APE, CVX, ANKR, SUSHI, RDNT, BADGER and PERP.

vast majority of this value is held in smart contracts which management is still partially centralized. A complete decentralization process has failed so far, due to a persistent risk of insolvency, which management creates points of centralization.

LendBook's immunity to insolvency risk marks a significant advancement in the space. There is no concept of bad debt that might need to be absorbed by a DAO treasury / insurance fund or socialized across lenders. There is no trade-off in case of liquidation between the costs incurred by borrowers, liquidators' incentives and lenders' safety. The radically innovative design unlocks many new features like high LTV and leverage, borrowing programmability and interest-bearing limit orders. This also makes possible the protocol to operate in a fully decentralized way.

Appendix A: Formal analysis of liquidation and collateral constraints

The limit order book trades the asset pair X/Y with X the base token (e.g. ETH) and Y the quote token (e.g. USDC). It is populated with buy orders (y_i, p_i) and sell orders (x_i, p_i) where y_i and x_i are the assets backing the limit orders on both sides of the book and p_i are the limit prices. $\hat{x}_j^k$ represents the assets borrowed by borrower k from sell order (x_j, q_j) and $\hat{y}_j^k$ are assets borrowed by k from buy order (y_j, p_j).

Liquidation

The coincidence of the filling of a limit order and the liquidation of positions borrowing from the order is programatically enforced. If a position j has a loan $\hat{x}_0^j < x_0$ from the sell order (x_0, p_0), the remaining assets $x_0 - \hat{x}_0^j$ can be filled by a taker if the price crosses p_0.

The position borrowing $\hat{x}_0^j$ is sufficiently collateralized if the borrower's available assets deposited in his buy orders are at least worth $p_0\hat{x}_0^j$. In case of filling, the maker receives Y worth $p_0(x_0 - \hat{x}_0^j)$ from the taker and worth $p_0\hat{x}_0^j$ from the liquidation of the borrowing position. The borrower keeps the borrowed amount $\hat{x}_0^j$ and the taker receives the remaining assets $x_0 - \hat{x}_0^j$.

Symmetrically, if a position j has a loan $\hat{y}_1^j$ from the buy order (y_1, p_1), the position is sufficiently collateralized if the borrower's available assets deposited in his sell orders are at least worth $\hat{x}_1^j/p_1$. If the order is taken, his collateral $\tilde{x}_j = \hat{y}_1^j/p_1$ is transferred to the owner of O_1 and his debt $\hat{y}_1^j$ is canceled off.

The coincidence of events transforms the borrower into the owner of a stop order. If j borrows from a buy order (y_1, p_1) and is liquidated at p_1, he keeps the loan $\hat{y}_1^j$ and gives up his collateral worth $\hat{y}_1^j/p_1$. j has a stop-loss at price p_1: this is as if he's buying $\hat{y}_1^j$ when the price decreases to p_1. Symmetrically, if j borrows from a sell order (x_0, p_0) and is liquidated at p_0, this is as if he's buying

$\hat{x}_0^j$ when the price hits p_0.

The smart contract acts as a central clearinghouse. In case of liquidation, the borrower's debt is written off with respect to the contract's pool and the owner of the taken order receives the collateral directly from the contract.

Excess collateral

Excess collateral of a user for a given asset is the sum of all assets deposited in her limit orders minus:

- the assets which collateralize all her borrowing positions
- the assets that other users borrow from her orders.

Excess collateral must always be positive for all users. Failing to be true, user's non-borrowed deposited assets could be insufficient to cover required transfers in case of liquidation of user's own positions.

More formally, suppose user i placed S buy orders $\{(y_s, p_s); s = 1, ..., S\}$ and borrowed from T sell orders $\{(x_t, p_t); t = 1, ..., T\}$. For each position t from which the user has borrowed $\hat{x}_t^i$, she has to deposit in buy orders a total collateral worth at least $p_t \hat{x}_t^i$. Besides, suppose that K positions $\{\hat{y}_s^k; k = 1, ..., K\}$ borrowed from the user's buy orders.

User i's excess collateral in asset Y is:

$$\mathrm{EC}_{i,Y} = \sum_{s=1}^{S} y_s - \sum_{t=1}^{T} p_t \hat{x}_t^i - \sum_{k=1}^{K} \hat{y}_i^k$$

where:

- $\sum_{s=1}^{S} y_s$ is the sum of all user's deposits, available either for borrowing or collateralizing positions
- $\sum_{t=1}^{T} p_t \hat{x}_t^i$ is the sum of collaterals needed in case all user's borrowing positions are liquidated
- $\sum_{k=1}^{K} \hat{y}_s^k$ is the total borrowed amount from user's buy orders

Excess collateral defines is a key concept which positivity constraint determines:

- the maximum amount a user can borrow (also conditional on the availability of assets, see next section)
- the maximum amount which can be borrowed from a user's limit order (this formalizes rule Rule 2)
- the minimum amount a borrower must pay back when his order which serves as collateral is taken (this formalizes Rrule 3)
- the maximum amount a user can remove (this formalizes Rule 4)

Borrowable assets

Not all assets deposited in an order can be borrowed, as sufficient incentives must be preserved for traders to take the non-borrowed part of the assets when the price hits the limit price. The non-borrowable assets are noted $\bar{x}$ in the sell order market and $\bar{y}$ in the buy order market. For the same reason, users must deposit a minimum quantity $\bar{y}$ or $\bar{x}$ of assets when they make a limit order.

Borrowing asset X reduces lender's excess collateral in X as less assets are available for others to borrow. It also reduces borrower's excess collateral in Y as more assets are required to secure the loan. It follows that borrowable assets in sell order (x_i, p_i) are limited by:

- assets X deposited in the order
- user's excess collateral in X
- borrower's excess collateral in Y

Formally, the maximum borrowable asset is:

$$\min(0, x_i - \bar{x} - \sum_{n=1}^{N} \hat{x}_i^k, \mathrm{EC}_{i,Y})$$

where:

- $x_i - \bar{x} - \sum_{n=1}^{N} \hat{x}_i^k$ are order's available assets, net of borrowed amounts and minimum deposit
- $\mathrm{EC}_{i,X}$ is owner of order i's excess collateral

In addition, borrower's excess collateral must also be positive: $\mathrm{EC}_{j,Y} > 0$.

Appendix B: Formal analysis of leverage

In the general case, if a trader borrows $\hat{y}_i^j$ from a buy order (y_i, p_i) and sells the assets at price $p^s > p_i$. His P&L varies with current price p:

$$\hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{1}$$

His leverage is $1 + p_i/p^s$. He can also level up his long by loop-borrowing and swapping more assets. The total amount of leverage, denoted λ_n, is function of the LTV $= p_i/p^s$ and the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p_i}{p^s}\right)^{t-1}$$

His P&L with n loops scales linearly with leverage:

$$\lambda_n \hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{2}$$

Symmetrically, if a trader borrows $\hat{x}_i^j$ from a sell order (x_i, p_i) and sells the assets at price $p^s < p_i$, the amount of leverage λ_n he can obtain is function of the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p^s}{p_i}\right)^{t-1}$$

His P&L after n rounds of borrowing is:

$$\lambda_n \hat{x}_i^j \max(p^s - p, p^s - p_i)$$

Below p_i, the borrower's debt is in X (denominated in Y). Above p_i, his debt is in Y with a maximum loss of $p^s - p_i$. By infinitely iterating the borrowing process, the maximum theoretical leverage is:

$$\lambda_\infty = \frac{p^s}{p^s - p_i}$$

in the buy order market and

$$\lambda_\infty = \frac{p_i}{p_i - p^s}$$

in the sell order market.

The two leverage factors tend to infinity when p^s gets closer to p_i. Traders can therefore attain arbitrarily high leverage by borrowing assets from limit orders which price is as close to current price as possible.

Appendix C: Interest rate setting

6 Interest rate model

The interest rate is symmetric for lenders and borrowers (abstracting from platform fee). It is algorithmically adjusted to the real-time supply and demand of assets in the protocol.

6.1 Utilization rate

Utilization rate (UR) can vary between 0 and 100% but the actual maximum rate depends on the other side's UR. To understand why, Fig. 12 presents two opposite market conditions. In both conditions, Alice deposits 1999 USDC and Bob 1 ETH. In the left-hand side market, Bob borrows 1999 USDC from Alice backed by 1 ETH. The buy order market's UR is 100% and the sell order market's UR is 0%. In the right-hand side market, Alice borrows 1999/2001 = 0.999 ETH from Bob backed by 1999 USDC. The sell order market's UR is 99.9% and the buy order market's is 0%.

Because assets cannot be lent and used as collateral at the same time, Bob and Alice cannot simultaneously borrow all assets from each other. The buy order market's UR UR_{BO} and the sell order market's UR UR_{SO} are linked by the inequality:

$$\text{UR}_{\text{BO}} + \text{UR}_{\text{SO}} < 1$$

Fig. 13 presents different market conditions in the UR space. A long market is characterized by optimistic expectations of the base asset price and dominated by leveraged long strategies. The buy order market's UR is consequently higher

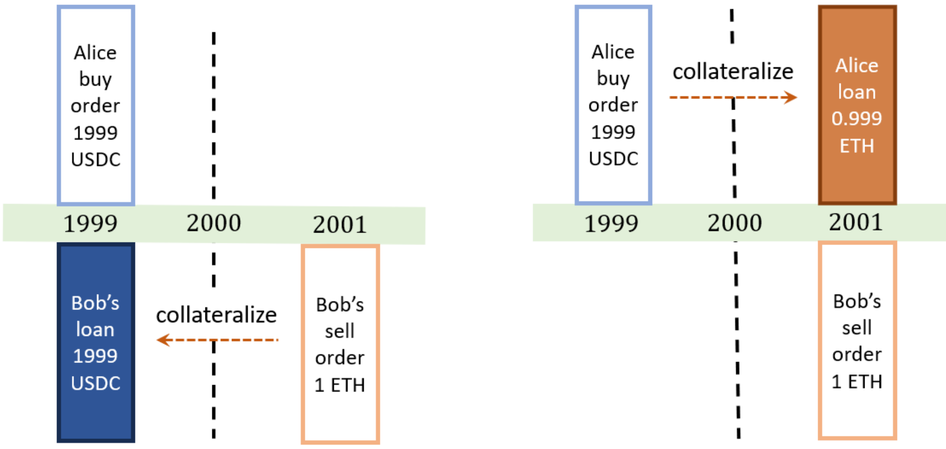

Figure 12: Left-hand side diagram: Utilization rate is 100% in the buy order market and 0% in the sell order market. Right-hand side diagram: Utilization rate is (approximately) 100% in the sell order market and 0% in the buy order market.

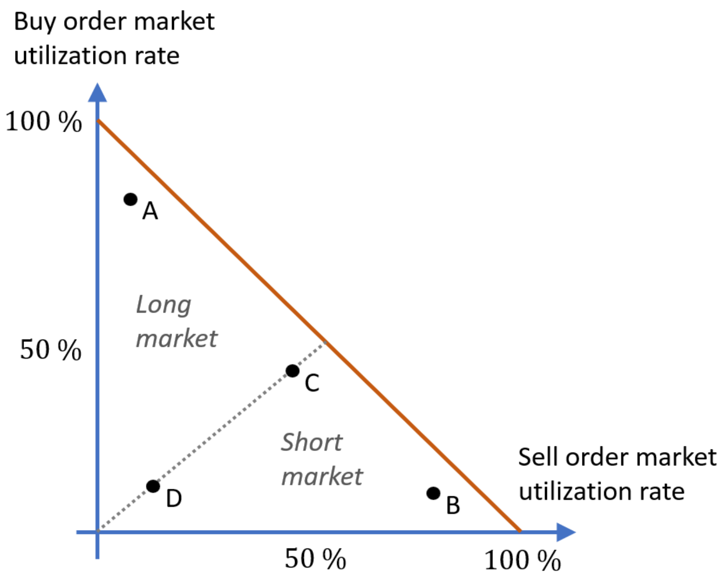

Figure 13: Global UR is high in market conditions A, B and C. Buy order market's UR is high in A, middle in C and low in B and D. Buy order market's UR is high in B, middle in C and low in A and D. The upper part of the triangle is dominated by long positions (traders borrow and swap quote tokens to long the base token) whereas the lower part is dominated by short positions (traders borrow base tokens to short it).

than the sell order market UR. A short market is characterized by opposite expectations and strategies.

The interest rates depend on the two measures of utilization rate:

$$\mathrm{IR}_{\mathrm{BO}} = \alpha + \beta \mathrm{UR}_{\mathrm{BO}} + \gamma(\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}})$$

$$\mathrm{IR}_{\mathrm{SO}} = \alpha + \beta \mathrm{UR}_{\mathrm{SO}} + \gamma(\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}})$$

with $\alpha, \beta, \gamma \geq 0$. The higher the markets' UR and the higher the global UR $\mathrm{UR}_{\mathrm{BO}} + \mathrm{UR}_{\mathrm{SO}}$, the higher the interest rate. Table 1 (end of document) illustrates possible values for the interest rates.

Continuous compounding

Interest income accrues every second using the block timestamp. IR_0 is the initial interest rate at date 0. n_1 seconds later, the interest rate changes to IR_1, n_2 seconds later to IR_2 and so forth.

The continuously compound interest rate over the period $[0, T]$ is $e^{\mathrm{TWIR}_T} - 1$ where TWIR_T is the date T time-weighted sum of interest rates since origin:

$$\mathrm{TWIR}_T = \frac{n_1}{N}\mathrm{IR}_0 + \frac{n_2 - n_1}{N}\mathrm{IR}_1 + ... + \frac{n_T - n_{T-1}}{N}\mathrm{IR}_T - 1$$

with N the number of seconds in a year.

Suppose a user borrows assets between dates $t \geq 0$ and T (present). Borrower's interest rate is $e^{\mathrm{dTWIR}_t} - 1$ with $\mathrm{dTWIR}_t = \mathrm{TWIR}_T - \mathrm{TWIR}_t$. The third-order Taylor expansion is:

$$e^{\mathrm{dTWIR}_t} - 1 \approx \mathrm{dTWIR}_t + \frac{\mathrm{dTWIR}_t^2}{2} + \frac{\mathrm{dTWIR}_t^3}{6}$$

UR_{BO}/UR_{SO}	0.95 %	0.75	0.5	0.25	0.05
0.95	-	-	-	-	(10.3, 5.8)
0.75	-	-	-	(9.3, 6.8)	(8.3, 4.8)
0.5	-	-	(8.0, 8.0)	(6.8, 5.5)	(5.8, 3.5)
0.25	-	(6.8, 9.3)	(5.5, 6.8)	(4.3, 4.3)	(3.3, 2.3)
0.05	(5.8, 10.3)	(4.8, 8.3)	(3.5, 5.8)	(2.3, 3.3)	(1.3, 1.3)

Table 1: Interest rates (IR_{BO}, IR_{SO}) in the buy and sell order markets respectively in function of buy order market's UR (rows) and sell order market's UR (columns). Parameters' values: $\alpha = 0.005$, $\beta = \gamma = 0.05$.