LendBook
a Lending Limit Order Book

prevert [*]

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Abstract

A lending limit order book is a non-custodial, peer to peer, permissionless, high LTV lending protocol which takes the form of an order book and allows the borrowing of assets backing limit orders. This new primitive brings users multiple benefits: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, and high leverage for leveraged traders and interest-bearing limit orders for makers. In addition, the protocol is immune to the risk of bad debt and can be run with minimized governance.

Introduction

Lending protocols offer users the opportunity to lend and borrow cryptoassets in a decentralized, permissionless, and trustless manner. However, despite the numerous benefits they present, their expansion has been hindered by a common birth defect - the risk of accumulating bad debts. A bad debt appears when the

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price of the deposited collateral rapidly falls and its value no longer covers the outstanding loan, putting the protocol at risk of insolvency and bank run. The loss can be exacerbated by the lack of sufficient liquidity in the pools in which the collateral assets are exchanged to cover borrowed assets during liquidation events.[1]

To mitigate this existential risk, lending protocols impose various constraints on the borrowers side, such as high collateral-to-debt ratios and high liquidation costs. They also limit lending markets to high-quality assets. These constraints impair users' experience by making borrowing more expensive and riskier and by restricting the range of borrowable assets.

Despite many innovative features introduced in the sector since its inception in 2018, significant improvement of users experience is still awaiting a foolproof solution for the risk of bad debt. LendBook, a lending limit order book, is a new primitive which eliminates the insolvency risk and brings along the way many benefits for lenders and borrowers.

A lending limit order book (LLOB) is a non-custodial, peer to peer, decentralized exchange merged with a fully-fledged lending protocol in which (i) the assets backing the limit orders can be borrowed and (ii) the borrowed assets of the bid side are collateralized by the assets in the ask side, and reciprocally. Figure 1 presents the double queue organization of a typical central limit order book. On the left-hand side, makers place limit buy orders which traders find profitable to take if the price decreases. On the right-hand side, they place limit sell orders which will be filled if the price increases. Rectangles' height indicate how much makers have deposited in their limit orders.

Fig. 2 shows how a lending primitive is attached to a limit order book. Makers allow other traders to borrow assets deposited in the book in exchange of an interest rate. The rectangles with a blue and orange background represent the assets borrowed from the orders at the same limit price.

The markets are organized around a golden rule: any position which bor-

[1] See also RiskDAO.

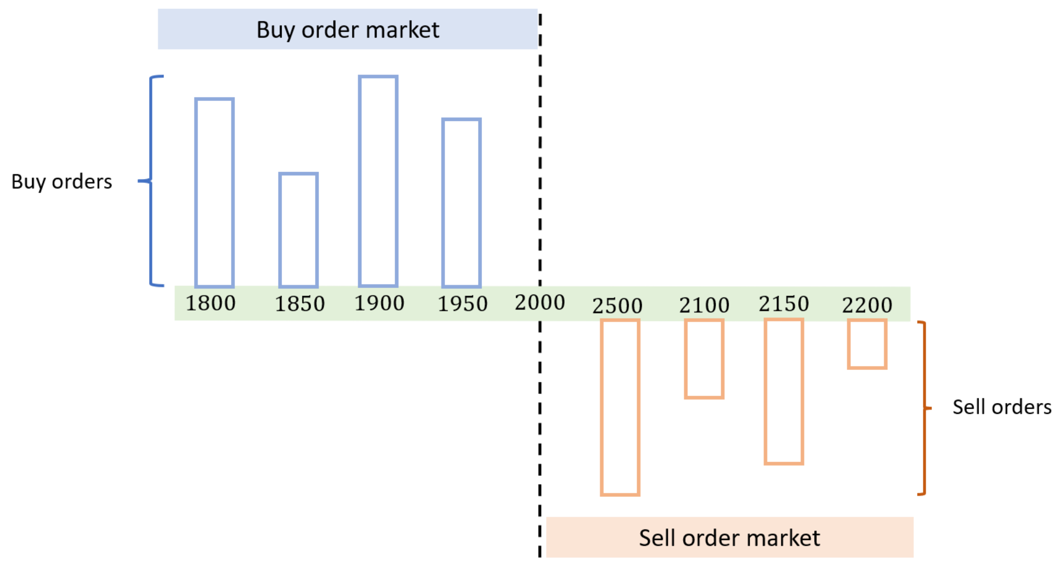

Figure 1: A graphical representation of a central limit order book

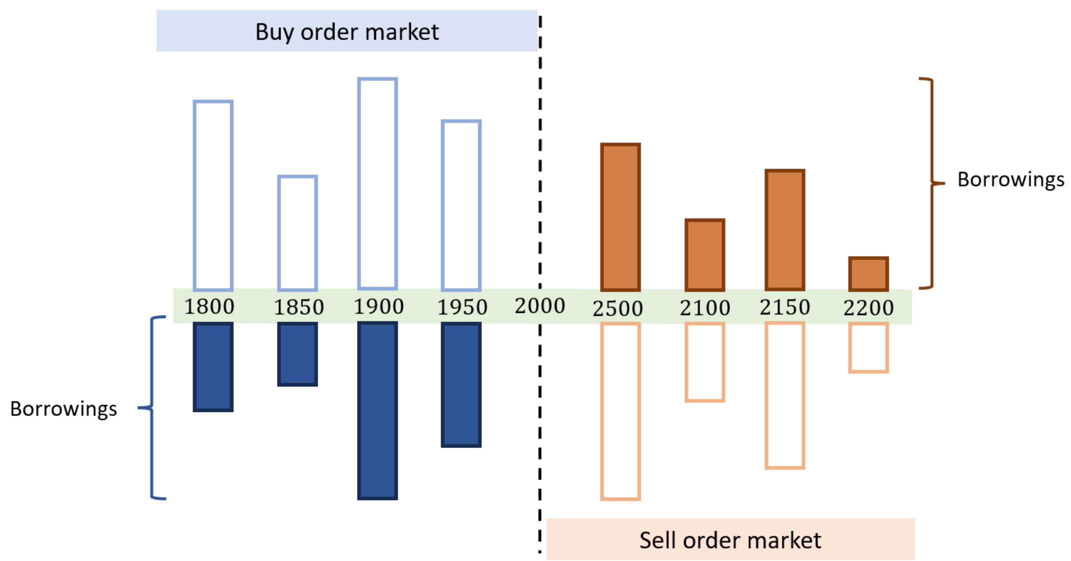

Figure 2: A central limit order book to which lending functionalities are added.

rows from a limit order is liquidated when the limit order is filled. The strict coincidence of the two events greatly simplifies the settlement process on both sides. The benefits are multiple: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, high leverage, minimized loss ratio and programmable strategies for leveraged traders and interest-bearing limit orders for makers. On top of those benefits, the protocol is immune from the risk of bad debt and can be run with minimized governance.

Example

Let us begin by illustrating how a lending operation works. Suppose Alice posts a buy order of 3 ETH at price 1900 USDC while market price is 2000. To do so, she deposits $3 \times 1900 = 5700$ USDC in the protocol's USDC vault. Bob is willing to borrow 3800 USDC from Alice's buy order. He places a sell order of 2 ETH at 2200 and deposits 2 ETH in the ETH vault. With 2 ETH of collateral, he can then borrow $2 \times 1900 = 3800$ USDC from Alice. The financial flows are summarized in Fig. 3.

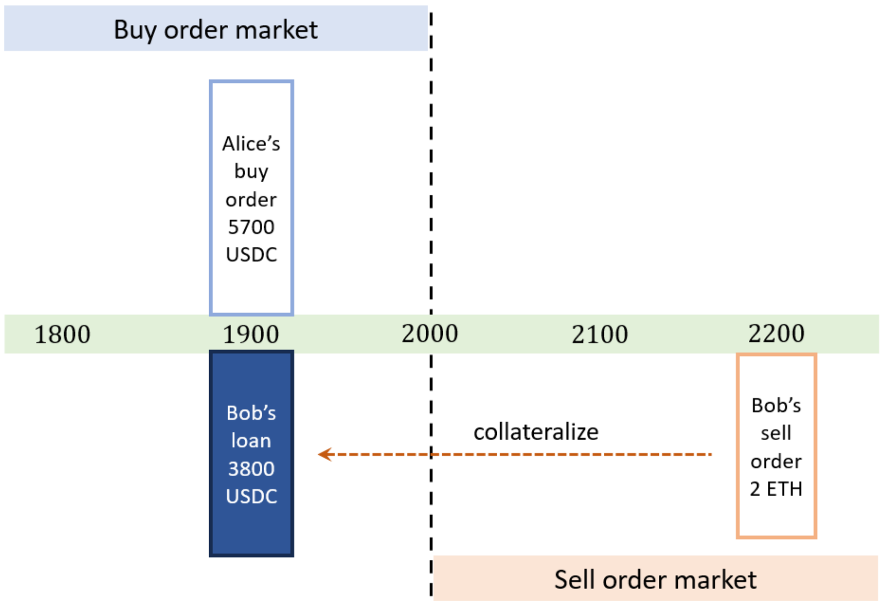

Figure 3: Bob borrows 3800 USDC from Alice, collateralized by his sell order at 2200.

If the price decreases to 1900, a taker swaps Alice's remaining USDC for ETH. The coincidence constraint imposes the liquidation of Bob's position. Bob keeps the borrowed USDC and Alice is given 2 ETH taken from Bob's collateral (see Fig. 4).

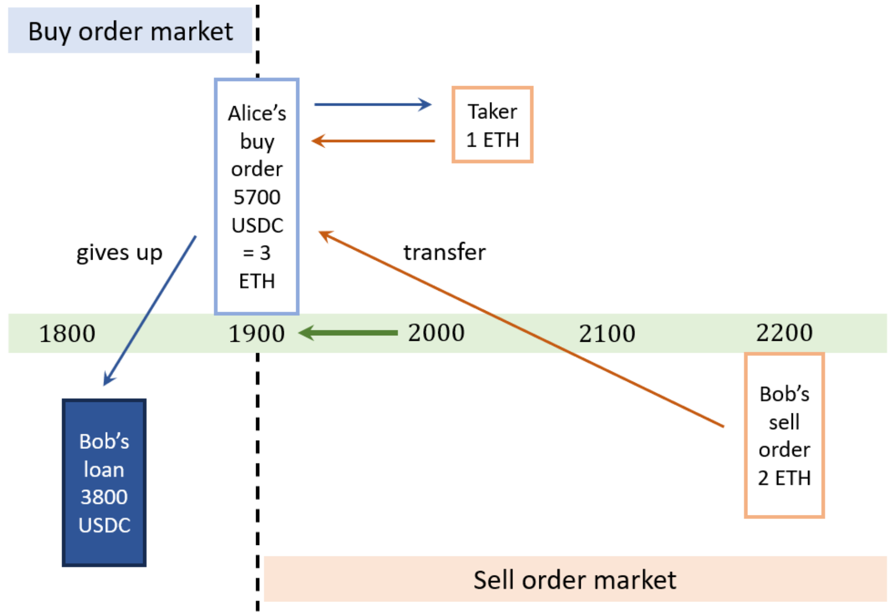

Figure 4: If the price decreases to 1900, Alice's remaining assets in her buy order are taken. Bob's collateral is transferred to Alice.

Nothing changes for Alice compared to a vanilla buy order. Importantly, Bob's assets are not swapped in the market in case of liquidation but are just transferred to Alice who is paid back in the collateral's currency. Before reviewing further the benefits of implementing a lending order book, let's dive into the details of the LLOB functioning.

1 Functioning

1.1 Lending Limit Order book's Rules

The LLOB is organized around four rules. Appendix A presents a formal specification of the rules.

R1. When a limit order is taken, all positions borrowing from the order are liquidated.

This is coincidence of events constraint illustrated in previous example. To avoid premature liquidations by external actors who would take orders at unfair price, a price feed is pulled before any filling to check that an order is not taken at a loss.[2]

R2. Limit orders' assets which serve as collateral for borrowing positions cannot be borrowed.

Example: Bob places a sell order at 2200 USDC and deposits 2 ETH. He then borrows 3800 USDC from Alice's buy order. His 2 ETH serve as collateral and cannot be borrowed.

The prohibition against borrowing collateral strengthens the protocol's safety for lenders. It prevents situations in which collateral assets could be unavailable in case of liquidation. In addition, not all assets deposited in an order can be borrowed, as sufficient incentives must be preserved for traders to take the non-borrowed assets when the price hits the limit price.

R3. In the case the borrower's own limit order, which assets serve as collateral, is filled, his borrowing position is automatically closed out at the time of the filling.

The rule guarantees the absence of mismatch between the type of assets serving as collateral and the type needed in case of liquidation.

Example (continued): Bob's sell order is filled for 2 × 2200 = 4400 USDC, of which 3800 are used to close out his borrowing position from Alice.

R4. Makers cannot remove their borrowed assets until the borrower repays his loan, or his order is taken or the borrower's order, which collateralize his position, is taken.

Borrowers can close their position when they wish. Makers, whose deposited

[2] Taking a buy order at a price higher than its limit price or a sell order at a price lower than its limit price is not costly if the taker exchanges a small amount or if she is the maker/lender herself. In both cases, the borrower is forced to exchange his collateral at an unfair price for the benefit of the lender.

assets are not borrowed can remove their assets and close their orders when they wish. This is different for borrowed assets which removal is conditional on one of the three events mentioned in R4. Meanwhile, lenders are compensated by a slowly increasing interest rate (see next Section) on the temporarily locked assets.

2 Interest rate model

In absence of platform fee, the P2P interest rate is the same for lenders and borrowers. Interest income accrues every second using the block timestamp. It is algorithmically adjusted to the real-time supply and demand of assets in the protocol.

2.1 Utilization rate

The interest rate depends on market's utilization rate (UR), which is the ratio of the total borrowed assets to the total supplied assets. The higher the UR, the higher the interest rate. Two interest rates coexist in a LLOB: one for the buy order market and one for the sell order market. Both markets are intertwined. UR can vary between 0 and 100% but the actual maximum rate depends on the other side's UR.

To understand why, Fig. 5 presents two opposite market conditions. In both conditions, Alice deposits 1999 USDC and Bob 1 ETH. In the left-hand side market, Bob borrows 1999 USDC from Alice backed by 1 ETH. The buy order market's UR is 100% and the sell order market's UR is 0%. In the right-hand side market, Alice borrows 1999/2001 = 0.999 ETH from Bob backed by 1999 USDC. The sell order market's UR is 99.9% and the buy order market's is 0%.

Because assets cannot be simultaneously lent and used as collateral, Bob and Alice cannot simultaneously borrow all assets from each other. The buy

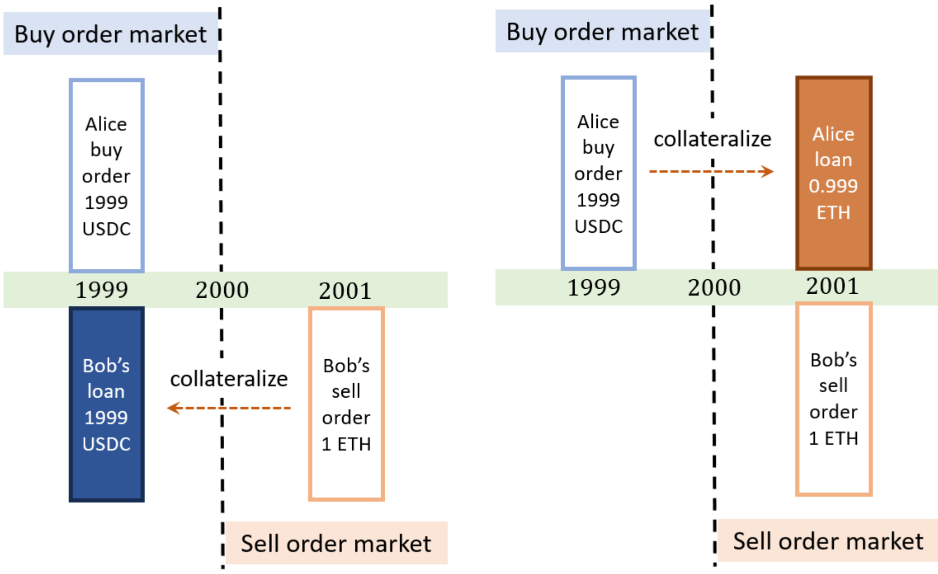

Figure 5: Left-hand side diagram: Utilization rate is 100% in the buy order market and 0% in the sell order market. Right-hand side diagram: Utilization rate is (approximately) 100% in the sell order market and 0% in the buy order market.

order market's UR $\mathrm{UR_{BO}}$ and the sell order market's UR $\mathrm{UR_{SO}}$ are linked by the inequality:

$$\mathrm{UR_{BO}} + \mathrm{UR_{SO}} < 1$$

Fig. 6 presents different market conditions in the UR space. A long market is characterized by optimistic expectations of the base asset price and dominated by leveraged long strategies. The buy order market's UR is consequently higher than the sell order market UR. A short market is characterized by opposite expectations and strategies.

The interest rate depends on the two measures of utilization rate:

$$\mathrm{R_{BO}} = f(\mathrm{UR_{BO}} + h(\mathrm{UR_{BO}} + \mathrm{UR_{SO}})$$

$$\mathrm{R_{SO}} = f(\mathrm{UR_{SO}}) + h(\mathrm{UR_{BO}} + \mathrm{UR_{SO}})$$

with f and h two increasing functions. The higher the markets' UR and the higher the global UR $\mathrm{UR_{BO}} + \mathrm{UR_{SO}}$, the higher the interest rate. Table 1 illus-

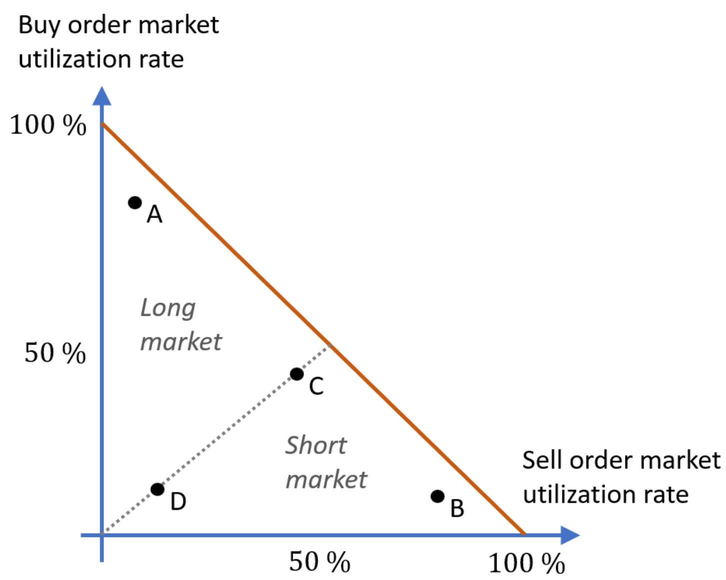

Figure 6: Global UR is high in market conditions A, B and C. Buy order market's UR is high in A, middle in C and low in B and D. Buy order market's UR is high in B, middle in C and low in A and D. The upper part of the triangle is dominated by long positions (traders borrow and swap quote tokens to long the base token) whereas the lower part is dominated by short positions (traders borrow base tokens to short it).

trates possible values for the interest rates based on the linear model:

$$\mathrm{R_{BO}} = 0.005 + 0.05\mathrm{UR_{BO}} + 0.05(\mathrm{UR_{BO}} + \mathrm{UR_{SO}})$$

$$\mathrm{R_{SO}} = 0.005 + 0.05\mathrm{UR_{SO}} + 0.05(\mathrm{UR_{BO}} + \mathrm{UR_{SO}})$$

2.2 Term spread

Interest rates are typically higher for long maturity than for short ones. This is referred to as the yield curve spread or term spread. The interest rate spread compensates lenders for the longer period over which their capital is tied up. A term spread is programatically replicated by the interest rate formula:

$$r = \max(r_0 + (1+\gamma)^t, R)$$

UR_{BO}/UR_{SO}	0.95 %	0.75	0.5	0.25	0.05
0.95	-	-	-	-	(10.3, 5.8)
0.75	-	-	-	(9.3, 6.8)	(8.3, 4.8)
0.5	-	-	(8.0, 8.0)	(6.8, 5.5)	(5.8, 3.5)
0.25	-	(6.8, 9.3)	(5.5, 6.8)	(4.3, 4.3)	(3.3, 2.3)
0.05	(5.8, 10.3)	(4.8, 8.3)	(3.5, 5.8)	(2.3, 3.3)	(1.3, 1.3)

Table 1: Interest rates (R_{BO}, R_{SO}) in the buy and sell order markets respectively in function of buy order market's UR (rows) and sell order market's UR (columns).

in which r_0 is the interest rate at inception of the borrowing, R is a ceiling for the interest rate and γ is the instantaneous increase rate.

As interest rate gradually increases with time, borrowers are incentivized to roll over their debt to cheaper offers, which benefits to lenders who gain more liquid lending positions.

As an illustration, Table 2 presents a numerical simulation of the interest rate dynamics.

Duration	# blocks	premium rate %	total rate %
1 week	50400	0.10	10.10
3 month	201600	0.40	10.40
6 months	2628000	5.40	15.40
1 year	5256000	11.08	21.08
2 years	10512000	23.40	30.00

Table 2: Interest rate dynamics after the soft exit method is called by the lender. Parameter's values: $r_0 = 10\%$, $\alpha = 0.00000002$, max net rate: $R = 30\%$.

3 Benefits of using LendBook

The benefits of appending a lending protocol to an order book are multiple: stop loss orders with guaranteed stop price, zero liquidation costs, high leverage, programmability of leverage strategies and minimized governance. Let's review them one by one.

3.1 Stop loss orders

A stop-loss order allows a trader to close a long position by selling the asset or a short position by buying the asset.

In the introductory example, Bob keeps the 4000 USDC and gives up his 2 ETH if the price decreases to 2000. This is as if he benefits from a stop loss at the guaranteed price of 2000. His stop price is both Alice's limit price and the price at which his borrowing position is liquidated.[3]

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when markets move fast. Here the stop price is guaranteed by the filling of the sell order at the limit price.

3.2 Zero liquidation costs

In the introduction's example, suppose the price hits 1800 USDC and Alice's buy order of 1 ETH is filled. Bob's collateral is transferred to Alice's wallet as if Bob where Alice's counterparty to the trade. Compared to what happens in other lending protocols, the liquidation of a borrowing position does not rely on the swap of the collateral on a decentralized exchange. Alice is happy to receive the collateral as a payment. The fact that the lender accepts a repayment in

[3]See Appendix A for a formal presentation.

kind (here in ETH) rather than in the currency lent (in USDC) has far-reaching implications.

A first major implication is the dramatic simplification and high safety of the liquidation process. Since no trade is executed, the liquidation doesn't rely on an AMM pool with the risk of a sub-optimal execution. The liquidation cannot create a bad debt for the protocol if the trade size is too large relative to the pool's liquidity. Borrowing positions cannot end under water even in case of strong and rapid price action, gas fee spike, or blockchain congestion/downtime.

The risk of bad debt for lending protocols translates into an execution risk borne by the maker of the limit order. In the example, if the price is rapidly falling, Alice gets her buy order executed for 1800 when the market price may actually be 1780. Or the buy order could be not executed at all if the price rapidly reverses. This creates an opportunity cost for the maker, common to all limit order books, but for which the maker is now compensated by an interest rate.

Another major consequence is that the protocol doesn't need to heavily incentivize bots to liquidate unhealthy positions in a timely manner. If the order's assets are partially loaned out, the takers will initiate the internal transfer from the borrower to the lender by filing the part of the orders not borrowed. If the order's assets are fully borrowed, the protocol will offer external actors a moderate fee to execute the internal transfer. In both cases, the liquidation costs incurred by borrowers will be zero or close to zero.

3.3 High leverage

The main benefits of lending protocols for traders is the possibility of leveraging positions. In LendBook as in other protocols, they can borrow Y and swap them to amplify their position in X, or they can borrow X and swap them to short X. The absence of insolvency risk in LendBook allows leverage factors a magnitude higher.[4]

Borrowers can easily do loops of borrowing and swapping to increase their leverage.

Example continued: After Bob borrowed 3800 USDC from Alice, he converts the USDC for 3800/2000 = 1.9 ETH. His leverage factor is 3.9/2 = 1.95. He can increase further his leverage by doing a second borrowing loop. By depositing the 1.9 ETH in his sell order, he can borrow 1.9 × 1900 = 3610 additional USDC from the same or another buy order (assuming the same limit price of 1900 USDC). After exchanging at market price the 3610 USDC for 3610/2000 = 1.805 additional ETH, his total leverage is now (2 + 1.9 + 1.805)/2 = 2.853.

A key risk parameter is the Maximum Loan to Value (MLTV) ratio, which defines the maximum amount of assets that can be borrowed with a specific collateral. For example, a MLTV of 80% in the ETH/USDC market means that users can borrow at most 0.80 USDC worth of ETH for every USDC deposited as collateral. Once a borrow occurs, the actual LTV evolves with the ETH/USDC price.

In LendBook, the LTV increases with the distance between the ETH price and the limit price of the order from which the user borrows. Traders can benefit from high LTV by borrowing from orders which limit price is close to market price.

Example continued: For every USDC Bob borrows from Alice's buy order, he has to deposit at least 1/1900 ETH which, at current price, is worth 2000/1900 = 1,053 USDC. His MLTV is as high as 1900/2000 = 95%. Had Bob borrowed from a buy order which limit price is 1990 USDC, his MLTV would be 1990/2000 = 99.5%.

High LTV translates in turn into high leverage.

Example continued: After exchanging 3800 USDC for 1.9 ETH, Bob's leverage factor is 1 + LTV, abstracting away from gas and swap costs. After a second

[4]see Appendix B for a formal analysis of this section.

loop of borrowing, his leverage would increase to 1 + LTV + LTV^2*. Assuming he could infinitely loop, the maximum leverage would be:*

$$\frac{1}{1 - \text{LTV}}$$

which is 21 in Bob's situation.

Had Bob borrowed from a buy order which limit price is 1990 USDC, his max leverage factor would be 210. As the limit price gets closer to the market price, the LTV tends to 1 and the theoretical max leverage to infinity.

3.4 Programmability of borrowing strategies

The combination of an order book with a lending protocol unlocks an infinite set of strategies that borrowers can fine-tune and program in advance.

By choosing the limit price of their collateral orders, borrowers can program at which price their leverage will be closed with a profit.

Example (see Fig. 3.): Bob places a sell order of 2 ETH at 2200, borrows 3800 USDC from Alice and exchanges the amount for 1.95 ETH. If the price hits 2200, his sell order is filled and his borrowing position is simultaneously closed out. The protocol pays back Bob's debt of 3800 USDC with the 2200 $\times$ *2 ETH = 4400 USDC of his sell order. Bob's leveraged profit is 3.95* $\times$ *200 = 790 USDC.*

Borrowers can also easily split their position between several orders with different limit prices. This way, their stop orders can be gradually executed as the price reaches well-specified thresholds.

Or, if they leverage their position, they can manage their liquidation risk by selecting various limit prices at which their borrowing is progressively liquidated.

Example (see Fig. 7): Bob borrows 1 ETH from Alice's buy order which limit price is 1900 and 1 ETH from Clair's buy order which limit price is 1800. Bob is liquidated at 1900 for half his position then possibly fully liquidated at 1800.

Borrowers can also spread their collateral between several limit orders to

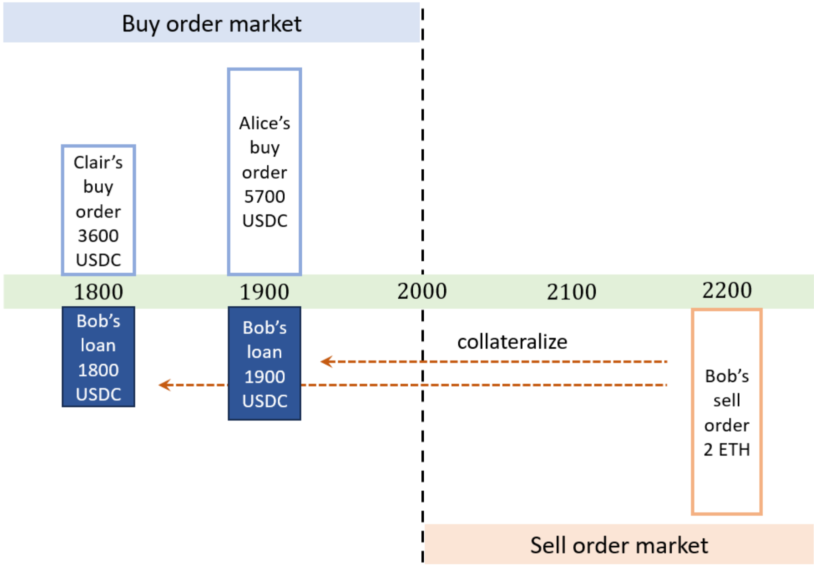

Figure 7: Bob borrows from Alice and Clair at different liquidation prices.

gradually take their profit. In Fig. 8, Bob takes around half of his profit at price 2100 and the other half at price 2200.

Example (see Fig. 8.): Bob places a sell order of 1 ETH at 2100 and another sell order of 1 ETH at 2200. He borrows 3800 USDC from Alice and exchanges the amount for 1.95 ETH. If the price hits 2100, his first sell order is filled and his borrowing position is simultaneously closed out for 1900 USDC. The protocol pays back Bob's debt of 1900 USDC with the 2100 USDC of his sell order. Then, if the price crosses 2200, his second sell order is filled and his borrowing position is closed out for the remaining 1900 USDC. After Bob's debt is paid back, Bob makes a profit of 3.95 $\times$ 150 = 592 USDC.

Another strategy for borrowers consists in reposting their borrowed amounts on the order book to gradually raise their leverage as the price changes.

Example (see Fig. 9): Bob places a sell order of 2 ETH at 2200, borrows 4000 USDC from Alice and reposts the USDC in a buy order at 1950. If the price decreases to 1950, Bob's USDC are taken in exchange of 4000/1950 = 2.05 ETH. Bob is now leveraged long on ETH. If the price reverts and increases, Bob profits.

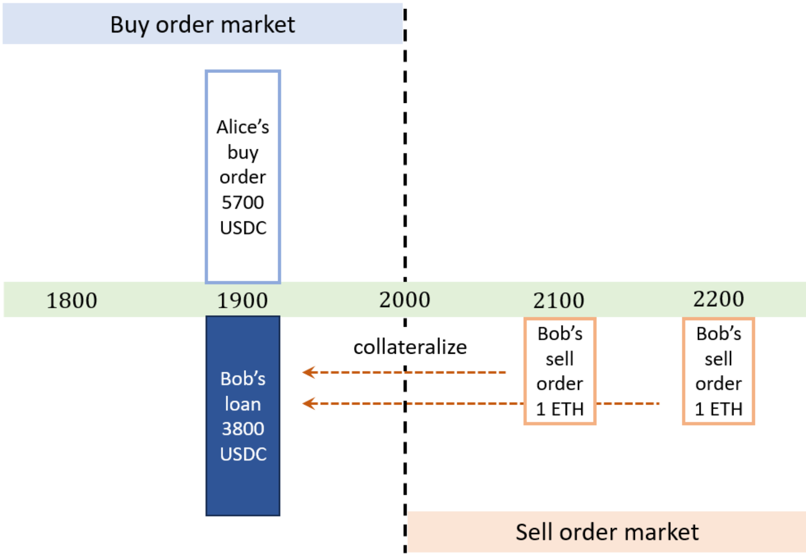

Figure 8: Half of Bob's borrowing position is closed at 2100, the other half at 2200. The two limit prices allow Bob to gradually take his profit from his leveraged position.

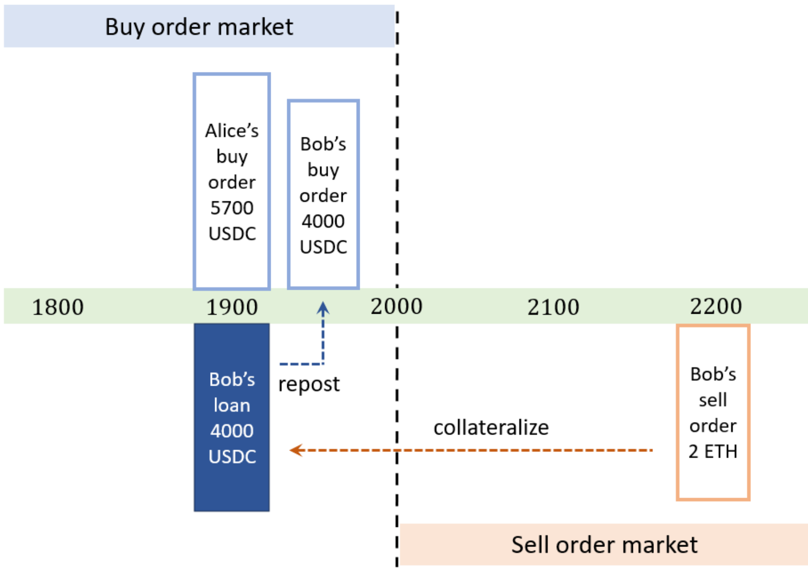

Figure 9: Bob reposts his borrowed USDC on the order book to leverage long his position if the price declines to 1950.

3.5 Minimized governance

The governance activity of lending protocols has considerably grown and gained complexity over time. Managing pools' risks has been progressively delegated to experts which job is to keep in check the pools' risk and update their risk parameters. This involves assessing multiple risk factors like assets' onchain liquidity, price volatility and market capitalization.

Despite committing considerable resources and expertise to risk management, protocols' solvency is still at risk of a lack of due diligence or governance failure. Lending protocols have also implemented and funded sizeable financial buffers to absorb shortfall events and protect lenders from bad debt. Those safety measures mitigate solvency risk at the expense of token holders.

In contrast, lending markets in LendBook stay afloat without the need of supervision by third party, a safety fund or borrowing restrictions. This allows the protocol to avoid supply caps, borrowing caps or liquidation thresholds at the pool level. The governance scope can be limited to strategic decisions, which minimize the risk of organization failures and ultimately unlocks a higher level of decentralization.

4 Conclusion

A lending limit order book is a decentralized exchange to which is appended a minimalist yet fully-fledged lending protocol which efficiently lends assets backed by limit orders. The immunity of the protocol to insolvency risk is a remarkable improvement over existing lending markets which are forced to set many guardrails and constantly update the pools' safety parameters. There is no concept of bad debt that might need to be absorbed by a DAO treasury / insurance fund or socialized across lenders. There is no trade-off in case of liquidation between borrowers' costs, liquidators' incentives and lenders' safety.

The protocol is optimized for traders who will find highly appealing its features: stop-loss orders with guaranteed stop price, high LTV and leverage without liquidation costs.

The protocol also caters to lenders who are paid for posting limit orders. Two distinct lending strategies are possible. In the first type, lenders act as market makers. They are active traders and liquidity providers in the book and post limit orders closed to current price. They constantly replace their filled order on the other side of the market to earn the spread in addition to the lending return rate. In the second type, lenders follow single-sided Aave-style strategies. They earn a return for their deposited assets but minimize the risk of conversion by posting limit orders relatively far from current price.

Appendix A: Formal analysis of liquidation and collateral constraints

The limit order book trades the asset pair X/Y with X the base token (e.g. ETH) and Y the quote token (e.g. USDC). It is populated with buy orders (y_i, p_i) and sell orders (x_i, p_i) where y_i and x_i are the assets backing the limit orders on both sides of the book and p_i are the limit prices. $\hat{x}_j^k$ represents the assets borrowed by borrower k from sell order (x_j, q_j) and $\hat{y}_j^k$ are assets borrowed by k from buy order (y_j, p_j).

Liquidation

The coincidence of the filling of a limit order and the liquidation of positions borrowing from the order is programatically enforced. If a position j has a loan $\hat{x}_0^j < x_0$ from the sell order (x_0, p_0), the remaining assets $x_0 - \hat{x}_0^j$ can be filled by a taker if the price crosses p_0.

The position borrowing $\hat{x}_0^j$ is sufficiently collateralized if the borrower's available assets deposited in his buy orders are at least worth $p_0\hat{x}_0^j$. In case of filling, the maker receives Y worth $p_0(x_0 - \hat{x}_0^j)$ from the taker and worth $p_0\hat{x}_0^j$ from the liquidation of the borrowing position. The borrower keeps the borrowed amount $\hat{x}_0^j$ and the taker receives the remaining assets $x_0 - \hat{x}_0^j$.

Symmetrically, if a position j has a loan $\hat{y}_1^j$ from the buy order (y_1, p_1), the position is sufficiently collateralized if the borrower's available assets deposited in his sell orders are at least worth $\hat{x}_1^j/p_1$. If the order is taken, his collateral $\tilde{x}_j = \hat{y}_1^j/p_1$ is transferred to the owner of O_1 and his debt $\hat{y}_1^j$ is canceled off.

The coincidence of events transforms the borrower into the owner of a stop order. If j borrows from a buy order (y_1, p_1) and is liquidated at p_1, he keeps the loan $\hat{y}_1^j$ and gives up his collateral worth $\hat{y}_1^j/p_1$. j has a stop-loss at price p_1: this is as if he's buying $\hat{y}_1^j$ when the price decreases to p_1. Symmetrically, if j borrows from a sell order (x_0, p_0) and is liquidated at p_0, this is as if he's buying

$\hat{x}_0^j$ when the price hits p_0.

The smart contract acts as a central clearinghouse. The borrower's debt is written off with respect to the contract's pool and the owner of the taken order receives the collateral directly from the contract.

Excess collateral

Excess collateral of a user for a given asset is the sum of all assets deposited in her limit orders minus:

- the assets which collateralize all her borrowing positions
- the assets that other users borrow from her orders.

Excess collateral must always be positive for all users. Failing to be positive, user's non-borrowed deposited assets could be insufficient to cover required transfers in case of liquidation of user's own positions.

More formally, suppose user i placed S buy orders $\{(y_s, p_s); s = 1, ..., S\}$ and borrowed from T sell orders $\{(x_t, p_t); t = 1, ..., T\}$. For each position t from which the user has borrowed $\hat{x}_t^i$, she has to deposit in buy orders a total collateral worth at least $p_t \hat{x}_t^i$. Besides, suppose that K positions $\{\hat{y}_s^k; k = 1, ..., K\}$ borrowed from the user's buy orders.

User i's excess collateral in asset Y is:

$$\mathrm{EC}_{i,Y} = \sum_{s=1}^{S} y_s - \sum_{t=1}^{T} p_t \hat{x}_t^i - \sum_{k=1}^{K} \hat{y}_i^k$$

where:

- $\sum_{s=1}^{S} y_s$ is the sum of all user's deposits, available either for borrowing or collateralizing positions
- $\sum_{t=1}^{T} p_t \hat{x}_t^i$ is the sum of collaterals needed in case all user's borrowing positions are liquidated
- $\sum_{k=1}^{K} \hat{y}_s^k$ is the total borrowed amount from user's buy orders

The excess collateral constraint determines:

- the maximum amount a user can borrow (also conditional on the availability of assets, see next section)
- the maximum amount which can be borrowed from a user's limit order (this formalizes rule R2)
- the maximum amount a user can remove (this formalizes rule R4)
- the minimum amount a borrower must pay back when his order which serves as collateral is taken (this formalizes rule R3)

Borrowable assets

Not all assets deposited in an order can be borrowed, as sufficient incentives must be preserved for traders to take the non-borrowed part of the assets when the price hits the limit price. The non-borrowable assets are noted $\bar{x}$ in the sell order market and $\bar{y}$ in the buy order market. For the same reason, users must deposit a minimum quantity $\bar{y}$ or $\bar{x}$ of assets when they make a limit order.

Borrowing asset X reduces lender's excess collateral in X as less assets are available for others to borrow. It also reduces borrower's excess collateral in Y as more assets are required to secure the loan. It follows that borrowable assets in sell order (x_i, p_i) are limited by:

- assets X deposited in the order
- user's excess collateral in X
- borrower's excess collateral in Y

Formally, the maximum borrowable asset is:

$$\min(0, x_i - \bar{x} - \sum_{n=1}^{N} \hat{x}_i^k, \mathrm{EC}_{i,Y})$$

where:

- $x_i - \bar{x} - \sum_{n=1}^{N} \hat{x}_i^k$ are order's available assets, net of borrowed amounts and minimum deposit
- $\mathrm{EC}_{i,X}$ is owner of order i's excess collateral

In addition, borrower's excess collateral must also be positive: $\mathrm{EC}_{j,Y} > 0$.

Appendix B: Formal analysis of leverage

In the general case, if a trader borrows $\hat{y}_i^j$ from a buy order (y_i, p_i) and sells the assets at price $p^s > p_i$. His P&L is:

$$\hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{1}$$

His leverage is $1 + p_i/p^s$. He can also level up his long by loop-borrowing and swapping more assets. The total amount of leverage, denoted λ_n, is function of the LTV $= p_i/p^s$ and the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p_i}{p^s}\right)^{t-1}$$

His P&L with n loops scales linearly with leverage:

$$\lambda_n \hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{2}$$

Symmetrically, if a trader borrows $\hat{x}_i^j$ from a sell order (x_i, p_i) and sells the assets at price $p^s < p_i$, the amount of leverage λ_n he can obtain is function of the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p^s}{p_i}\right)^{t-1}$$

His P&L after n rounds of borrowing is:

$$\lambda_n \hat{x}_i^j \max(p^s - p, p^s - p_i)$$

Below p_i, the borrower's debt is in X (denominated in Y). Above p_i, his debt is in Y with a maximum loss of $p^s - p_i$. By infinitely iterating the borrowing process, the maximum theoretical leverage is:

$$\lambda_\infty = \frac{p^s}{p^s - p_i}$$

in the buy order market and

$$\lambda_\infty = \frac{p_i}{p_i - p^s}$$

in the sell order market.

The two leverage factors tend to infinity when p^s gets closer to p_i. Traders can therefore attain arbitrarily high leverage by borrowing assets from limit orders which price is as close to current price as possible.