# Borrowable Limit Order Book

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**Abstract**

A borrowable limit order book is a non-custodial, peer to peer, oracleless, permissionless lending protocol which allows the borrowing of assets backing limit orders. The coincidence of limit order executions with borrowing liquidation greatly simplifies the settlement process on both sides. The benefits are multiple: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, high leverage and minimized loss ratio for leveraged traders, interest-bearing limit orders for makers and no risk of bad debt and minimized governance at the protocol level.

## Introduction

A Borrowable limit order book is a special order book in which $(i)$ the assets backing the limit orders can be borrowed and $(ii)$ the borrowed assets of the bid side are collateralized by the assets in the ask side, and reciprocally.

Let us begin by illustrating how a lending operation works. Suppose Alice posts a buy order of 1 ETH for 1800 USDC while market price is 1900. To do so,

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she deposits 1800 USDC in the protocol's USDC vault. Bob is willing to borrow Alice's asset. He places a sell order of 1 ETH at 2000 (or whatever price) and deposits 1 ETH in the ETH vault. With the ETH collateral, he can borrow 1800 USDC from Alice's buy order. If the price increases to 2000, Bob's buy order is filled and his borrowing position is closed. If the price decreases to 1800, Bob is liquidated. He keeps the borrowed USDC and Alice is given Bob's collateral of 1 ETH. Nothing changes for Alice compared to a vanilla buy order. Importantly, Bob's assets are not swapped on the market in case of liquidation but are just transferred to Alice who is paid back in the collateral's currency.

Before reviewing the benefits of implementing a borrowable order book, let's dive into the details of its functioning.

# 1 Functioning

The limit order book trades the asset pair X/Y with X the base token (e.g. ETH) and Y the quote token (e.g. USDC). It is populated with buy orders $(y_i, p_i)$ and sell orders $(x_i, p_i)$ where $y_i$ and $x_i$ are the assets backing the limit orders on both sides of the book. $\hat{x}_j^i \leq x_i$ represents the assets deposited with a sell order $(x_j, p_j)$ and possibly borrowed by a position backed by a buy order $(y_i, p_i)$. Symmetrically, $\hat{y}_j^i \leq y_j$ is the assets deposited with a buy order $(y_j, p_j)$ and possibly borrowed by a position backed by a sell order $(x_i, p_i)$.

A position borrowing the assets of the sell order $(x_i, p_i)$ is sufficiently collateralized by a buy order $(y_i, p_i)$ if it has enough collateral to meet the execution of the sell order prorata the share borrowed:

$$y_j \geq p_i \hat{x}_i^j$$

If the position borrows from multiple sell orders, the solvency constraint is:

$$y_j \geq \sum_i p_i \hat{x}_i^j$$

In the case the borrower opens several buy orders to collateralize multiple borrowing positions, his solvency constraint becomes:

$$\sum_j y_j \geq \sum_i p_i \hat{x}_i^j$$

When a borrowed limit order is filled, the borrower can still be solvent if he has enough collateral. Suppose the price hits $p_0$ and the sell order $(x_0, p_0)$ which assets $\hat{x}_0^j$ are borrowed is filled. The protocol may let the borrower tap into another sell order $(x_1, p_1)$ which assets $x_1 \geq \hat{x}_0^j$ are available. Upon a taker initiating the filling of the sell order $(x_0, p_0)$, the protocol transfers the amount $\hat{x}_0^j$ from order 1 to the taker in exchange for $p_0\hat{x}_1^j$ given to the owner of order 0. $j$ is now borrowing from sell order 1. However, since $p_1 > p_0$ (as $x_0$ is filled at $p_0$), the reallocation is conditional on his position being still solvent:

$$y_j \geq \sum_i p_i \hat{x}_i^j + (p_1 - p_0)\hat{x}_0^j$$

If not enough assets are available in the book, or if the borrower's collateral is insufficient to secure the increased borrowing position, the borrower's position is liquidated. The borrower's collateral which value is equivalent to $p_1\hat{x}_1^j$ is then transferred to the lender which order is executed.

The borrower of a filled order is therefore still solvent if the borrower has enough collateral *and/or* the order book is sufficiently liquid (enough assets are available to be borrowed).

Symmetrically, the owner of a sell order $(x_i, p_i)$ can borrow assets Y from one or several buy orders $(y_j, p_j)$ providing he has enough collateral to meet their execution:

$$x_i \geq \sum_j \hat{y}_j^i / p_j$$

Assets of a limit order can be borrowed by several positions. A buy order $(y_i, p_i)$ can be borrowed by multiple sell orders $(x_j, p_j)$ providing each borrowing position is well collateralized and they collectively don't borrow more than the amount deposited:

$$y_i \geq \sum_j \hat{y}_i^j$$

In addition, limit orders must satisfy three rules.

R1. Limit orders' assets which serve as collateral for borrowing positions cannot be borrowed.

R2. In the case the borrower's own limit order, which assets serve as collateral, is filled, his borrowing position is automatically closed out at the time of the filling.

R3. Cancellation of limit orders which assets are borrowed is not allowed if the associated borrowed positions cannot be reallocated.

The first requirement is not strictly necessary but greatly simplifies the protocol's design in the pilot version. The second rule guarantees the absence of mismatch between the type of assets serving as collateral and the type needed in case of liquidation.

As for the last constraint, note that borrowers can close their position whenever they wish and so do lenders which assets are not borrowed or if they are borrowed, the borrowing positions can be reallocated. The possibility of reallocation will be the most frequent situations. Otherwise, lenders will be compensated by a high interest rate on their locked assets. They will have to wait that (*i*) the vault's utilization rate decreases allowing associated borrowing positions to be reallocated, or (*ii*) their limit order is filled, or (*iii*) the limit orders of the borrowers are filled.

# 2 Benefits of borrowing limit orders' assets

The benefits of appending a lending protocol to an order book are multiple: stop loss orders with guaranteed stop price, zero liquidation costs, high leverage, capital efficiency and minimized loss ratio. Let's review them one by one.

## 2.1 Stop loss orders

A stop-loss order allows a trader to close a long position by selling the asset or a short position by buying the asset.

Recall that the borrower $j$ of a sell order $(x_i, p_i)$ is liquidated if the price hits $p_i$. He keeps the loan $\hat{x}_j^i$ while his collateral worth $p_i \hat{x}_j^i$ is transferred to $i$. $j$ has a stop-loss at price $p_i$: this is as if he's buying $\hat{x}_j^i$ when the price increases to $p_i$.

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when markets move fast. Here the stop price is guaranteed by the filling of the sell order at price $p_i$.

Symmetrically, the borrower $j$ of a buy order $(y_i, p_i)$ is liquidated if the price decreases below $p_i$. He keeps the loan $\hat{y}_j^i$ while his collateral worth $\hat{y}_j^i / p_i$ is transferred to $i$. $j$ has a stop-loss when the price decreases to $p_i$. The stop price is guaranteed by the filling of the buy order at price $p_i$.

## 2.2 Zero liquidation costs

In the introduction's example, suppose the price hits 1800 USDC and Alice's buy order of 1 ETH is filled. Also suppose that Bob's collateral is not enough for his position to be reallocated elsewhere in the order book. His liquidation necessitates the transfer of his collateral to Alice's wallet. Compared to what happens in other lending protocols, the liquidation of a borrowing position does not rely on the swap of the collateral into the asset lent. Alice is happy to receive the collateral as a payment. The fact that the lender accepts a repayment in kind (here in ETH) rather than in the currency lent (in USDC) has far-reaching implications.

A first major implication is the dramatic simplification and high safety of the liquidation process. Since there is no trade, there is no need to rely on an AMM pool with the risk of a sub-optimal execution and the creation of a bad debt for the protocol if the trade size is large relative to the pool's liquidity. There isalso no need to set guardrails such as margin and leverage limits to secure the protocol's solvency. In fact, we will see that the leverage offered by the protocol can be quite high.

In case of strong and rapid price movement and possibly gas fee spike, there is no risk that some positions end under water. The type of extreme events that generate a bad debt for the protocol translates into an opportunity cost borne by the owner of the limit order which performance deteriorates. In the example, if the price is rapidly falling, Alice gets her buy order executed for 1800 when the market price may actually be 1750. Or the buy order could be not executed at all if the price rapidly reverses.

As a result, there is also no need to heavily incentivize liquidation bots to perform the trades in a timely manner. If the order's assets are partially loaned out, arbitrageurs will initiate the internal transfer from the borrower to the lender by taking the part of the orders not borrowed. If the order's assets are fully borrowed, the protocol will offer external actors a moderate fee to execute the internal transfer. In both cases, the liquidation costs incurred by borrowers will be zero or close to zero.

## 2.3 Minimized governance

## 2.4 High leverage

Traders can borrow buy orders' assets and swap them to amplify their position in X. Or they can borrow sell orders' assets and swap them to short X. In the example, if Bob sells the borrowed USDC, he obtains $1800/1900 = 0.95$ ETH. His leverage is $\times 1.95$. If the price crosses 1800, Bob is liquidated and his loss is the difference between how much ETH he bought at 1900 with 1800 USDC and how much he owes Alice: $0.95p - p$. His P&L is

$$\max(0.95p - 1800, -0.05p)$$

Above 1800, his assets are in ETH and his debt in USDC. Below 1800, his debt is in ETH (denominated in USDC) with a maximum loss of 0.05 ETH.

In the general case, if a trader borrows $\hat{y}_i^j$ from a buy order $(y_i, p_i)$ and sells the assets at price $p^s > p_i$. His P&L is:

$$\hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{1}$$

His leverage is $1 + p_i/p^s$. He can also level up his long by loop-borrowing and swapping more assets. The amount of leverage, denoted $\lambda_n$, is function of the number $n$ of borrowing rounds:

$$\lambda_n = \sum_{t=0}^{n} \left(\frac{p_i}{p^s}\right)^t$$

His P&L with $n$ loops scales linearly with leverage:

$$\lambda_n \hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{2}$$

Symmetrically, if a trader borrows $\hat{x}_i^j$ from a sell order $(x_i, p_i)$ and sells the assets at price $p^s < p_i$, the amount of leverage $\lambda_n$ he can obtain is function of the number $n$ of borrowing rounds:

$$\lambda_n = \sum_{t=0}^{n} \left(\frac{p^s}{p_i}\right)^t$$

His P&L after $n$ rounds of borrowing is:

$$\lambda_n \hat{x}_i^j \max(p^s - p, p^s - p_i)$$

Below $p_i$, the borrower's debt is in X (denominated in Y). Above $p_i$, his debt is in Y with a maximum loss of $p^s - p_i$ USDC. By infinitely iterating the borrowing process, the maximum theoretical leverage is:

$$\lambda_\infty = \frac{p^s}{p^s - p_i}$$

in the buy order market and

$$\lambda_\infty = \frac{p_i}{p_i - p^s}$$

in the sell order market.

Both leverages tend to infinity when $p^s$ gets closer to $p_i$. Traders can therefore attain arbitrarily high leverage by borrowing assets from limit orders which price is as close to current price as possible. As usual, the risk of being liquidated increases with leverage when $p^s$ approaches $p_i$.

## 2.5 Capital efficiency

From the P&L formulas (1) and (2), we can see that the same P&L can be obtained by decreasing the loan size ($\hat{y}_i^j$ or $\hat{x}_i^j$) and simultaneously increasing the leverage factor $\lambda_n$. Downsizing the loan allows to minimize the assets deposited as collateral. It follows that, abstracting from gas and swap costs, it is always capital efficient to increase the leverage and decreasing the assets at stake. A leverage $\times 100$ allows to put 1% of one's capital in the order book and use the 99% elsewhere. There is no additional costs or risks compared to put 100% of one's capital without leverage.

In finance, leverage allows capital efficiency but comes with additional costs and risks. In traditional lending protocols and perpetuals, more leverage means a higher probability of liquidation as the price at which liquidation is triggered gets closer to current price. Here, more leverage can be obtained with the same liquidation price providing the order book is liquid enough. The probability of liquidation doesn't increase, only the share of the borrower's capital put at risk.

Another way to understand why maximum leverage is capital efficient is to observe that part of the capital deposited as collateral is idle if leverage is not maximal. In the example, Bob deposits 1 ETH to borrow 1800 USDC from Alice but his loss is limited to 0.05 ETH, meaning that only 5% of his capital is at work. He could alternatively deposit 0.11 ETH, loop borrow 10 times to attain a leverage of $\times 8.5$ and still borrow 1 ETH and get the *same* P&L.

We can check that 100% of the borrower's collateral is lost in case of liquidation when leverage is maximum. Borrower's maximum loss after an infinite rounds of borrowing is:

$$\lambda_\infty \hat{x}_i^j (p^s - p_i) = \frac{p_i}{p_i - p^s} \hat{x}_i^j (p^s - p_i) = -p_i \hat{x}_i^j$$

in the market for sell orders' assets and

$$\lambda_\infty \hat{y}_i^j \left( \frac{p}{p^s} - \frac{p}{p_i} \right) = \frac{p^s}{p^s - p_i} \hat{y}_i^j \left( \frac{p}{p^s} - \frac{p}{p_i} \right) = -p \frac{\hat{y}_i^j}{p_i}$$

in the market for buy orders' assets.

### 2.6 Minimized loss ratio

A key performance metric for leveraged traders is how much can be lost compared to potential profit. The ratio of maximum loss (the total collateral lost in case of liquidation) to profit can be computed from the P&L formulas (1) and (2) in the sell and buy order markets. Remarkably, the two ratios tend to zero when the price at which the assets are borrowed and swapped approaches the limit price:

$$\lim_{p^s \to p_i} \frac{p^s - p_i}{p^s - p} = 0$$

and

$$\lim_{p^s \to p_i} \frac{1 - \frac{p^s}{p_i}}{1 - \frac{p^s}{p}} = 0$$

The property holds for finite price variations and any level of leverage.

## 3 Conclusion

A borrowable limit order book is a decentralized exchange to which is appended a minimalist yet full-fledged lending protocol which efficiently lends assets backed by limit orders. The absence of risk of bad debt is a remarkable improvement over existing lending markets which are forced to set many guardrails and constantly update their security parameters to prevent the risk. The protocol is primarily optimized for traders who will find highly appealing its features (stop-loss orders with guaranteed stop price, high leverage without liquidation costs and low loss ratio).

The protocol will also cater to lenders who will earn high interest rate paid by borrowers.