Borrowable Limit Order Book

prevert *

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Abstract

A borrowable limit order book is a non-custodial, peer to peer, permissionless lending protocol which allows the borrowing of assets backing limit orders. This new primitive brings users multiple benefits: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, and high leverage for leveraged traders and interest-bearing limit orders for makers. In addition, the protocol is immune to the risk of bad debt and can be run with minimized governance.

Introduction

Lending protocols offer users the opportunity to lend and borrow cryptoassets in a decentralized, permissionless, and trustless manner. However, despite the numerous benefits they present, their expansion has been hindered by a common birth defect - the risk of accumulating bad debts. A bad debt appears when the price of the deposited collateral rapidly falls and its value no longer covers the outstanding loan, putting the protocol at risk of insolvency and bank run. The

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loss can be exacerbated by the lack of sufficient liquidity in the pools in which the collateral assets are exchanged to cover borrowed assets during liquidation events.[1]

To mitigate this existential risk, lending protocols impose various constraints on the borrowers side, such as high collateral-to-debt ratios and high liquidation costs. They also limit lending markets to high-quality assets. These constraints impairs borrowers' experience by making borrowing more expensive and riskier and by restricting the range of borrowable assets.

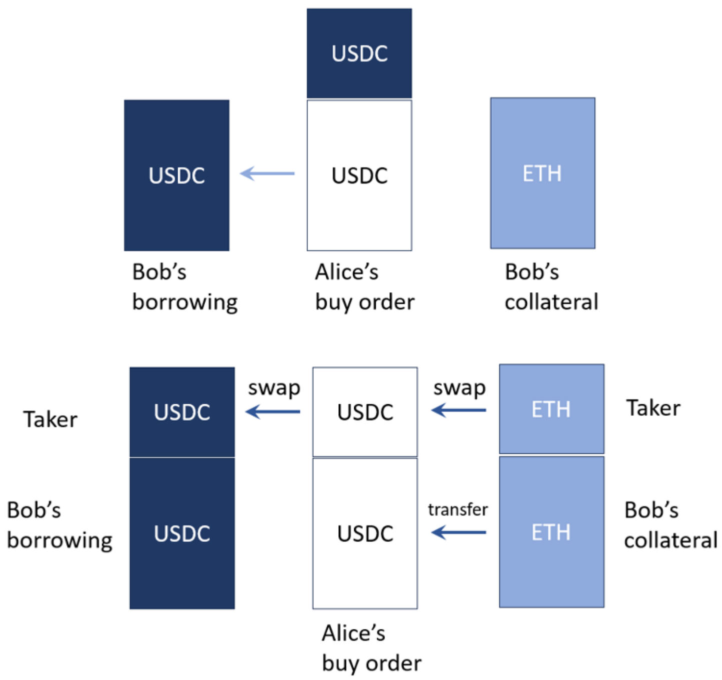
Despite many innovative features introduced in the sector since its inception in 2018, significant improvement of users experience is still awaiting a foolproof solution for the risk of bad debt. LendBook, a borrowable limit order book, is a new primitive which eliminates the insolvency risk and brings along the way many benefits for lenders and borrowers.

A borrowable limit order book is simultaneously a non-custodial, peer to peer, decentralized exchange and a fully-fledged lending protocol in which (i) the assets backing the limit orders can be borrowed and (ii) the borrowed assets of the bid side are collateralized by the assets in the ask side, and reciprocally. The markets are organized around one golden rule: any position which permissionlessly borrows from a limit order is liquidated when the limit order is filled. The strict coincidence of the two events greatly simplifies the settlement process on both sides. The benefits are multiple: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, high leverage, minimized loss ratio and programmable strategies for leveraged traders and interest-bearing limit orders for makers. On top of those benefits, the protocol gets immunity from the risk of bad debt and can be run with minimized governance.

[1] See also RiskDAO.

Example

Let us begin by illustrating how a lending operation works. Suppose Alice posts a buy order of 3 ETH at price 2000 USDC while market price is 2100. To do so, she deposits 6000 USDC in the protocol's USDC vault. Bob is willing to borrow 4000 USDC from Alice's buy order. He places a sell order of 2 ETH at 2300 and deposits 2 ETH in the ETH vault. With 2 ETH of collateral, he can then borrow $2 \times 2000 = 4000$ USDC from Alice. If the price increases to 2300, Bob's sell order is filled and his borrowing position is closed. Alternatively, if the price decreases to 2000, a taker swaps Alice's remaining USDC for ETH, which liquidates Bob's position. Bob keeps the borrowed USDC and Alice is given 2 ETH taken from Bob's collateral. Nothing changes for Alice compared to a vanilla buy order. Importantly, Bob's assets are not swapped on the market in case of liquidation but are just transferred to Alice who is paid back in the collateral's currency. The financial flows are summarized in the Diagram.



Bob borrows from Alice (upper diagram). When her buy order is taken, his collateral is transferred to her (lower diagram).

Before reviewing the benefits of implementing a borrowable order book, let's dive into the details of its formal functioning.

1 Functioning

1.1 Collateral constraints

The limit order book trades the asset pair X/Y with X the base token (e.g. ETH) and Y the quote token (e.g. USDC). It is populated with buy orders (y_i, p_i) and sell orders (x_i, p_i) where y_i and x_i are the assets backing the limit orders on both sides of the book. $\hat{x}_j^i \leq x_i$ represents the assets deposited with a sell order (x_j, p_j) and possibly borrowed by a position backed by a buy order (y_i, p_i). Symmetrically, $\hat{y}_j^i \leq y_j$ is the assets deposited with a buy order (y_j, p_j) and possibly borrowed by a position backed by a sell order (x_i, p_i).

The coincidence of events is programatically enforced between the filling of limit orders and the liquidation of position borrowing from the limit orders. A position borrowing the assets of a sell order (x_i, p_i) is sufficiently collateralized by a buy order (y_j, p_j) if it has enough collateral to meet the filling of the sell order prorata the share borrowed:

$$y_j \geq p_i \hat{x}_i^j$$

If the position borrows from multiple sell orders, the solvency constraint is:

$$y_j \geq \sum_i p_i \hat{x}_i^j$$

In the case the borrower opens several buy orders to collateralize multiple borrowing positions, his solvency constraint becomes:

$$\sum_j y_j \geq \sum_i p_i \hat{x}_i^j$$

Symmetrically, a position borrowing from several buy orders (y_i, p_i) collateralized by several sell orders is sufficiently collateralized if it can meet the filling of all buy orders prorata the share borrowed:

$$\sum_j x_j \geq \sum_i \hat{y}_i^j / p_i$$

In addition, assets of a limit order can be borrowed by several positions at the same time. A buy order (y_i, p_i) can be borrowed by multiple sell orders (x_j, p_j) providing each borrowing position is well collateralized and they collectively don't borrow more than the amount deposited:

$$\sum_j \hat{y}_i^j \leq y_i$$

Likewise, multiple buy orders (y_j, p_j) can borrow from the same sell order (x_i, p_i) if they are collateralized enough and don't borrow more than the amount deposited:

$$x_i \geq \sum_j \hat{x}_i^j$$

1.2 Liquidation

When a limit order is taken, all positions which borrowed its assets are liquidated. If the position B_j borrowed $\hat{x}_0^j$ from the filled sell order $O_0 = (x_0, p_0)$, his collateral worth $p_0 \hat{x}_0^j$ is transferred to the owner of O_0 and his debt is simultaneously written off for the same amount. If the position B_j borrowed $\hat{y}_0^j$ from the filled buy order $O_0 = (y_0, p_0)$, his collateral worth $\hat{y}_0^j / p_0$ is transferred to the owner of O_0 and his debt is reduced accordingly.

1.3 Rules

In addition to the coincidence constraint, the order book is organized around three rules.

R1. Limit orders' assets which serve as collateral for borrowing positions cannot be borrowed.

The prohibition against borrowing collateral greatly simplifies the protocol's design and strengthens its safety for lenders.

R2. In the case the borrower's own limit order, which assets serve as collateral, is filled, his borrowing position is automatically closed out at the time of the filling.

The rule guarantees the absence of mismatch between the type of assets serving as collateral and the type needed in case of liquidation.

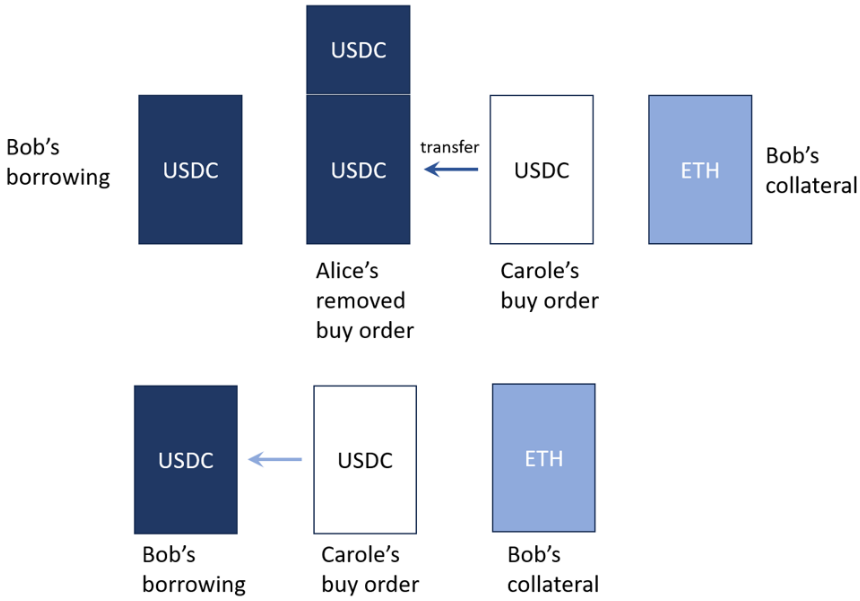
R3. Removal of limit orders' assets is conditional on borrowing positions successfully transferred to other limit orders with available assets.

Note first that borrowers can close their position whenever they wish and so can lenders which assets are not borrowed. If they are borrowed, the removal hinges on the possibility to relocate the borrowing positions to other borrowable same-side limit orders. Removal can be partial if not enough assets are released by the relocation of borrowing positions.

The process of debt transfer, explained in the next section, is a central mechanism of the protocol which allows traders to cancel their orders without provoking the involuntary winding up of borrowing positions. The transfer is expected to be successful most of the time. Otherwise, lenders will be compensated by an interest rate on the temporarily locked assets. They will have to wait that (i) the vault's utilization rate decreases allowing borrowing positions to be relocated, or (ii) their limit order is filled, or (iii) the limit orders of the borrowers are filled.

2 Debt transfer

In case a limit order O_0 is canceled, the protocol tries to transfer the borrowing positions B_j to other same-side limit orders. In previous example, suppose Alice removes her buy order and takes her USDC back. The protocol scans the order book and finds Carole's buy order of 2 ETH at 2000 with 4000 USDC deposited. Bob's borrowing position is transferred to Carole's buy order as shown in the upper diagram. After the removal is completed, Bob is borrowing from Carole (lower diagram).



When Alice removes her buy order, Carole's assets are transferred to Alice (upper diagram). After the removal, Bob is borrowing from Carole (lower diagram).

Let us consider two sell orders $O_0 = (x_0, p_0)$, which assets $\hat{x}_0^j$ are borrowed by B_j and $O_1 = (x_1, p_1)$ which assets $x_1 \geq \hat{x}_0^j$ are available. Suppose the price hits p_0. Upon a taker initiating the filling of O_0, the protocol lets B_j tap into O_1 by transferring the amount $\hat{x}_0^j$ from O_1 to the taker in exchange for $p_0\hat{x}_1^j$, which is given to the owner of O_0. B_j is now borrowing from O_1.

The transfer may increase or decrease the total liability of the borrower. The previous debt, worth $p_0\hat{x}_0^j$, is now worth $p_1\hat{x}_0^j$. If the new limit price is higher than the previous one ($p_1 > p_0$), the debt is increased after the transfer. If it is lower, the debt is decreased. In the first case, the viability of the transfer is conditional on B_j having enough collateral:

$$\sum_j y_j \geq \sum_i p_i\hat{x}_i^j + (p_1 - p_0)\hat{x}_0^j$$

whereas in the second case, the transfer only depends on the existence of limit orders with enough available assets. Another difference is that in the first case, the probability of a liquidation is decreased whereas it is increased in the second case.

If two sell orders $O_1 = (x_1, p_1)$ and $O_2 = (x_2, p_2)$ have available assets, the protocol selects O_1 if its price is closest to the taken sell order: $|p_1 - p_0| \leq |p_2 - p_0|$.

Symmetrically, if the assets $\hat{y}_0^j$ of a filled buy order $O_0 = (y_0, p_0)$ are borrowed by B_j, the viability of the debt transfer to the buy order $O_1 = (x_1, p_1)$ is conditional on B_j having enough collateral in the case $p_1 < p_0$:

$$\sum_j x_j \geq \sum_i \hat{y}_i^j / p_i + \hat{y}_0^j (1/p_1 - 1/p_0)$$

If two buy orders $O_1 = (y_1, p_1)$ and $O_2 = (y_2, p_2)$ have available assets, the protocol selects the one which price is closest to the taken buy order.

3 Interest rate

The lending of limit orders' assets is rewarded by an interest rate paid by borrowers. The pricing is decentralized at the limit order level. The same way limit order books give users the ability to define their own price, the protocol will let makers post the interest rate they require to borrow their assets.

Given the inconvenience brought by the removal of assets for borrowers which position must be relocated on the order book, a penalty $\tau \in (0, 1)$ reducing the interest rate is introduced which rate starts at e.g. 50% then decreases over time to zero.

4 Benefits of borrowing limit orders' assets

The benefits of appending a lending protocol to an order book are multiple: stop loss orders with guaranteed stop price, zero liquidation costs, high leverage, programmability of leverage strategies and minimized governance. Let's review them one by one.

4.1 Stop loss orders

A stop-loss order allows a trader to close a long position by selling the asset or a short position by buying the asset.

In the introductory example, Bob keeps the 4000 USDC and give up his 2 ETH if the price decreases to 2000. This is as if he benefits from a stop loss at the guaranteed price of 2000. His stop price is both the price at which his borrowing position is liquidated and Alice's limit price.

More formally, Recall that the borrower j of a sell order (x_i, p_i) is liquidated if the price hits p_i. He keeps the loan $\hat{x}^i_j$ while his collateral worth $p_i \hat{x}^i_j$ is transferred to i. j has a stop-loss at price p_i: this is as if he's buying $\hat{x}^i_j$ when the price increases to p_i.

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when markets move fast. Here the stop price is guaranteed by the filling of the sell order at price p_i.

Symmetrically, the borrower j of a buy order (y_i, p_i) is liquidated if the price decreases below p_i. He keeps the loan $\hat{y}^i_j$ while his collateral worth $\hat{y}^i_j / p_i$ is transferred to i. j has a stop-loss when the price decreases to p_i. The stop price is guaranteed by the filling of the buy order at price p_i.

4.2 Zero liquidation costs

In the introduction's example, suppose the price hits 1800 USDC and Alice's buy order of 1 ETH is filled. Bob's collateral is transferred to Alice's wallet as if Bob where Alice's counterparty to the trade. Compared to what happens in other lending protocols, the liquidation of a borrowing position does not rely on the swap of the collateral on a decentralized exchange. Alice is happy to receive the collateral as a payment. The fact that the lender accepts a repayment in kind (here in ETH) rather than in the currency lent (in USDC) has far-reaching implications.

A first major implication is the dramatic simplification and high safety of the liquidation process. Since no trade is executed, the liquidation doesn't rely on an AMM pool with the risk of a sub-optimal execution. The liquidation cannot create a bad debt for the protocol if the trade size is too large relative to the pool's liquidity. Borrowing positions cannot end under water even in case of strong and rapid price action, gas fee spike, or blockchain congestion/downtime.

The type of extreme events that generate a bad debt for lending protocols translates into a risk borne by the maker of the limit order which execution may deteriorate. In the example, if the price is rapidly falling, Alice gets her buy order executed for 1800 when the market price may actually be 1780. Or the buy order could be not executed at all if the price rapidly reverses. This creates an opportunity cost for the maker, common to all limit order books, but for which the maker is now compensated by an interest rate.

Another major consequence is that the protocol doesn't need to heavily incentivize bots to liquidate unhealthy positions in a timely manner. If the order's assets are partially loaned out, the takers will initiate the internal transfer from the borrower to the lender by filing the part of the orders not borrowed. If the order's assets are fully borrowed, the protocol will offer external actors a moderate fee to execute the internal transfer. In both cases, the liquidation costs incurred by borrowers will be zero or close to zero.

4.3 High leverage

As in other lending protocols, traders can leverage their positions. They can borrow asset Y from buy orders and swap them to amplify their position in X. Or they can borrow asset X from sell orders and swap them to short X. The absence of insolvency risk allows however leverage positions a magnitude higher.

4.3.1 Example continued

To continue the introductory example, after Bob borrowed 4000 USDC from Alice, he converts the USDC for 4000/2100 = 1.9 ETH. His leverage factor is 3.9/2 = 1.95. He can increase further his leverage by doing a second borrowing loop. By depositing the 1.9 ETH in his sell order, he can borrow 1.9 $\times$ 2000 = 3809 additional USDC from the same or another buy order with (assuming) the same limit price of 2000 USDC. After exchanging at market price the 3809 USDC for 3809/2100 = 1.81 additional ETH, his total leverage is now (2 + 1.9 + 1.81)/2 = 2.86.

4.3.2 High loan-to-value

A widely used risk parameter in lending protocols is the Maximum Loan to Value (MLTV) ratio, which defines the maximum amount of assets that can be borrowed with a specific collateral. For example, a MLTV of 80% in the ETH/USDC market means that users can borrow at most 0.80 USDC worth of ETH for every USDC deposited as collateral. Once a borrow occurs, the actual LTV evolves with the ETH/USDC price.

In the example, for every USDC Bob borrows from a buy order which limit price is 2000 USDC, he has to deposit at least 1/2000 ETH which, at current price, is worth 2100/2000 = 1,05 USDC. His MLTV is as high as 2000/2100 = 95%. The actual LTV doesn't fluctuate with the ETH price but rather depends on the limit price of the order from which the user borrows. His MLTV would be $2090/2100 = 99.5\%$, had Bob borrowed from a buy order which limit price is 2090 USDC.

4.3.3 High leverage

After exchanging 4000 USDC for 1.9 ETH, Bob's leverage factor is 1 + LTV, abstracting away from gas and swap costs. After a second loop of borrowing, his leverage would increase to $1 + \text{LTV} + \text{LTV}^2$. Assuming he could infinitely loop, the maximum leverage would be:

$$\frac{1}{1 - \text{LTV}}$$

which is 21 in Bob's situation.

Had Bob borrowed from a buy order which limit price is 2090 USDC, his max leverage would increase to 210. As the limit price gets closer and closer to the market price, the LTV tends to 1 and the max leverage to infinity.

4.3.4 Formal analysis

In the general case, if a trader borrows $\hat{y}_i^j$ from a buy order (y_i, p_i) and sells the assets at price $p^s > p_i$. His P&L is:

$$\hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{1}$$

His leverage is $1 + p_i/p^s$. He can also level up his long by loop-borrowing and swapping more assets. The total amount of leverage, denoted λ_n, is function of the $\text{LTV} = p_i/p^s$ and the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p_i}{p^s}\right)^{t-1}$$

His P&L with n loops scales linearly with leverage:

$$\lambda_n \hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{2}$$

Symmetrically, if a trader borrows $\hat{x}_i^j$ from a sell order (x_i, p_i) and sells the assets at price $p^s < p_i$, the amount of leverage λ_n he can obtain is function of the number n of borrowing rounds:

$$\lambda_n = \sum_{t=1}^{n} \left(\frac{p^s}{p_i}\right)^{t-1}$$

His P&L after n rounds of borrowing is:

$$\lambda_n \hat{x}_i^j \max(p^s - p, p^s - p_i)$$

Below p_i, the borrower's debt is in X (denominated in Y). Above p_i, his debt is in Y with a maximum loss of $p^s - p_i$ USDC. By infinitely iterating the borrowing process, the maximum theoretical leverage is:

$$\lambda_\infty = \frac{p^s}{p^s - p_i}$$

in the buy order market and

$$\lambda_\infty = \frac{p_i}{p_i - p^s}$$

in the sell order market.

The two leverage factors tend to infinity when p^s gets closer to p_i. Traders can therefore attain arbitrarily high leverage by borrowing assets from limit orders which price is as close to current price as possible. Higher leverage is not without risk however. The liquidation risk also increases when p^s approaches p_i.

4.4 Programmability of borrowing strategies

The combination of an order book with a lending protocol unlocks an infinite set of strategies that borrowers can fine-tune and program in advance.

Borrowers can easily split their positions between several orders with different limit prices. This way, their stop orders can be gradually executed as the price reaches well-specified thresholds. Or they can manage their liquidation risk by selecting various limit prices at which their borrowing is progressively liquidated.

In Bob's example, he could borrow 1 ETH from Alice's sell order which limit price is 2000 and 1 ETH from Carol's sell order which limit price is 1900. The two limit prices would diversify his liquidation risk.

Also, by choosing the limit price of their collateral orders, borrowers can program at which price their leverage will be closed with a profit.

In the example, Bob places a sell order of 2 ETH at 2300. After exchanging 4000 USDC for 1.95 ETH, if the price hits 2300, his sell order is filled and his borrowing position is simultaneously closed out. The protocol pays back Bob's debt of 4000 USDC with the 2300×2 ETH = 4600 USDC of his sell order. The residual amount of 600 USDC is kept by Bob as a profit. Borrowers can also spread their collateral between several limit orders to gradually take their profit.

Another strategy for borrowers consists in replacing their borrowed amounts on the order book to gradually raise their leverage as the price changes. For example, with Alice's 4000 USDC, Bob could spread his bet between a buy order at 2150, another one at 2200 and a last one at 2250.

4.5 Minimized governance

The governance activity of lending protocols has considerably grown over time. Managing pools' risks has been progressively delegated to experts which job is to keep in check the pools' risk and update their risk parameters. This involves assessing multiple risk factors like assets' onchain liquidity, price volatility and market capitalization.

Despite committing considerable resources and expertise to risk management, protocols' solvency is still at risk of a lack of due diligence or governance failures. Lending protocols have also implemented and funded financial buffers to absorb shortfall events and protect lenders from bad debt but at the expense of token holders.

In contrast, lending markets in LendBook can function smoothly without the need of a safety fund or borrowing restrictions like supply caps, borrowing caps or liquidation thresholds. This will allow the protocol to limit the governance scope to strategic decisions and to reach a higher level of decentralization.

5 Conclusion

A borrowable limit order book is a decentralized exchange to which is appended a minimalist yet fully-fledged lending protocol which efficiently lends assets backed by limit orders. The immunity of the protocol to the risk of bad debt is a remarkable improvement over existing lending markets which are forced to set many guardrails and constantly update the pools' safety parameters. The protocol is optimized for traders who will find highly appealing its features (stop-loss orders with guaranteed stop price, high leverage without liquidation costs).

The protocol also caters to lenders who are paid for posting limit orders. Two distinct lending strategies are possible. In the first type, lenders act as market makers. They are active traders and liquidity providers in the book and post limit orders closed to current price. They constantly replace their filled order on the other side of the market to earn the spread in addition to the lending return rate. In the second type, lenders follow single-sided Aave-style strategies. They earn a return for their deposited assets but minimize the risk of conversion by posting limit orders relatively far from current price.