Borrowable Limit Order Book

prevert *

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Abstract

A borrowable limit order book is a non-custodial, peer to peer, oracleless, permissionless lending protocol which allows the borrowing of assets backing limit orders. The coincidence of limit order fillings with borrowing liquidations greatly simplifies the settlement process on both sides. The benefits are multiple: stop loss orders with guaranteed stop price for borrowers, zero liquidation costs, high leverage and minimized loss ratio for leveraged traders, interest-bearing limit orders for makers and no risk of bad debt and minimized governance at the protocol level.

Introduction

A Borrowable limit order book is a special order book in which (*i*) the assets backing the limit orders can be borrowed and (*ii*) the borrowed assets of the bid side are collateralized by the assets in the ask side, and reciprocally.

Let us begin by illustrating how a lending operation works. Suppose Alice posts a buy order of 1 ETH for 1800 USDC while market price is 1900. To do

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so, she deposits 1800 USDC in the protocol's USDC vault. Bob is willing to borrow Alice's asset. As a collateral, he places a sell order of 1 ETH at 2000 (or whatever price) and deposits 1 ETH in the ETH vault. He can then borrow 1800 USDC from Alice's buy order. If the price increases to 2000, Bob's buy order is filled and his borrowing position is closed. If the price decreases to 1800, Bob is liquidated. He keeps the borrowed USDC and Alice is given Bob's collateral of 1 ETH. Nothing changes for Alice compared to a vanilla buy order. Importantly, Bob's assets are not swapped on the market in case of liquidation but are just transferred to Alice who is paid back in the collateral's currency.

Before reviewing the benefits of implementing a borrowable order book, let's dive into the details of its functioning.

1 Functioning

1.1 Collateral constraints

The limit order book trades the asset pair X/Y with X the base token (e.g. ETH) and Y the quote token (e.g. USDC). It is populated with buy orders (y_i, p_i) and sell orders (x_i, p_i) where y_i and x_i are the assets backing the limit orders on both sides of the book. $\hat{x}_j^i \leq x_i$ represents the assets deposited with a sell order (x_j, p_j) and possibly borrowed by a position backed by a buy order (y_i, p_i). Symmetrically, $\hat{y}_j^i \leq y_j$ is the assets deposited with a buy order (y_j, p_j) and possibly borrowed by a position backed by a sell order (x_i, p_i).

The coincidence of events is programatically enforced between the filling of limit orders and the liquidation of position borrowing from the limit orders. A position borrowing the assets of a sell order (x_i, p_i) is sufficiently collateralized by a buy order (y_j, p_j) if it has enough collateral to meet the filling of the sell order prorata the share borrowed:

$$y_j \geq p_i \hat{x}_i^j$$

If the position borrows from multiple sell orders, the solvency constraint is:

$$y_j \geq \sum_i p_i \hat{x}_i^j$$

In the case the borrower opens several buy orders to collateralize multiple borrowing positions, his solvency constraint becomes:

$$\sum_j y_j \geq \sum_i p_i \hat{x}_i^j$$

Symmetrically, a position borrowing from several buy orders (y_i, p_i) collateralized by several sell orders is sufficiently collateralized if it can meet the filling of all buy orders prorata the share borrowed:

$$\sum_j x_j \geq \sum_i \hat{y}_i^j / p_i$$

In addition, assets of a limit order can be borrowed by several positions at the same time. A buy order (y_i, p_i) can be borrowed by multiple sell orders (x_j, p_j) providing each borrowing position is well collateralized and they collectively don't borrow more than the amount deposited:

$$\sum_j \hat{y}_i^j \leq y_i$$

Likewise, multiple buy orders (y_j, p_j) can borrow from the same sell order (x_i, p_i) if they are collateralized enough and don't borrow more than the amount deposited:

$$x_i \geq \sum_j \hat{x}_i^j$$

1.2 Liquidation

When a limit order is taken, all positions which borrowed its assets are liquidated. If the position B_j borrowed $\hat{x}_0^j$ from the filled sell order $O_0 = (x_0, p_0)$, his collateral worth $p_0 \hat{x}_0^j$ is transferred to the owner of O_0 and his debt is simultaneously written off for the same amount. If the position B_j borrowed $\hat{y}_0^j$ from the filled buy order $O_0 = (y_0, p_0)$, his collateral worth $\hat{y}_0^j / p_0$ is transferred to the owner of O_0 and his debt is reduced accordingly.

1.3 Rules

The order book is organized around three additional rules.

R1. Limit orders' assets which serve as collateral for borrowing positions cannot be borrowed.

The requirement is not strictly necessary but greatly simplifies the protocol's design in the pilot version.

R2. In the case the borrower's own limit order, which assets serve as collateral, is filled, his borrowing position is automatically closed out at the time of the filling.

The rule guarantees the absence of mismatch between the type of assets serving as collateral and the type needed in case of liquidation.

R3. Cancellation of limit orders which assets are borrowed is not allowed if the associated borrowed positions cannot be successfully transferred to available orders.

Note first that borrowers can close their position whenever they wish and so can lenders which assets are not borrowed. If they are borrowed, the cancellation hinges on the possibility to reallocate the borrowing positions to other borrowable same-side limit orders. Removal can be partial if not enough assets are released by the reallocation of borrowing positions.

The process of debt transfer, explained in the next section, is a central mechanism of the protocol which allows traders to cancel their orders without provoking the involuntary winding up of borrowing positions. As will be explained, the debt transfer will be successful most of the time. Otherwise, lenders will be compensated by an interest rate on their locked assets. They will have to wait that (*i*) the vault's utilization rate decreases allowing associated borrowing positions to be reallocated, or (*ii*) their limit order is filled, or (*iii*) the limit orders of the borrowers are filled.

2 Debt transfer

In case a limit order O_0 is canceled, the protocol tries to transfer all borrowing positions B_j to other same-side limit orders. Let us consider two sell orders $O_0 = (x_0, p_0)$, which assets $\hat{x}_0^j$ are borrowed by B_j and $O_1 = (x_1, p_1)$ which assets $x_1 \geq \hat{x}_0^j$ are available. Suppose the price hits p_0. Upon a taker initiating the filling of O_0, the protocol lets B_j tap into O_1 by transferring the amount $\hat{x}_0^j$ from O_1 to the taker in exchange for $p_0\hat{x}_1^j$, which is given to the owner of O_0. B_j is now borrowing from O_1.

The transfer may increase or decrease the total liability of the borrower. The previous debt, worth $p_0\hat{x}_0^j$, is now worth $p_1\hat{x}_0^j$. If the new execution price is higher than the previous one ($p_1 > p_0$), the debt is increased after the transfer. If it is lower, the debt is decreased. In the first case, the viability of the transfer is conditional on B_j having enough collateral:

$$\sum_j y_j \geq \sum_i p_i\hat{x}_i^j + (p_1 - p_0)\hat{x}_0^j$$

whereas in the second case, the transfer only depends on the existence of limit orders with enough available assets. Another difference is that in the first case, the probability of a liquidation is decreased whereas it is increased in the second case.

If two sell orders $O_1 = (x_1, p_1)$ and $O_2 = (x_2, p_2)$ have available assets, the protocol selects O_1 if its price is closest to the taken sell order: $|p_1 - p_0| \leq |p_2 - p_0|$.

Symmetrically, if the assets $\hat{y}_0^j$ of a filled buy order $O_0 = (y_0, p_0)$ are borrowed by B_j, the viability of the debt transfer to the buy order $O_1 = (x_1, p_1)$ is conditional on B_j having enough collateral in the case $p_1 < p_0$:

$$\sum_j x_j \geq \sum_i \hat{y}_i^j/p_i + \hat{y}_0^j(1/p_1 - 1/p_0)$$

If two buy orders $O_1 = (y_1, p_1)$ and $O_2 = (y_2, p_2)$ have available assets, the protocol selects the one which price is closest to the taken buy order.

3 Fee market

The lending of limit orders' assets is rewarded by an interest rate paid by borrowers. The pricing is decentralized at the limit order level. For every limit order placed on the order book, the maker indicates the execution price and the quantity he is willing to exchange, but also the interest rate required to borrow his assets.

4 Benefits of borrowing limit orders' assets

The benefits of appending a lending protocol to an order book are multiple: stop loss orders with guaranteed stop price, zero liquidation costs, high leverage, capital efficiency and minimized loss ratio. Let's review them one by one.

4.1 Stop loss orders

A stop-loss order allows a trader to close a long position by selling the asset or a short position by buying the asset.

Recall that the borrower j of a sell order (x_i, p_i) is liquidated if the price hits p_i. He keeps the loan $\hat{x}_j^i$ while his collateral worth $p_i\hat{x}_j^i$ is transferred to i. j has a stop-loss at price p_i: this is as if he's buying $\hat{x}_j^i$ when the price increases to p_i.

In traditional or crypto finance, once the stop price is met, the stop loss order becomes a market order and is executed at the next available price. The obtained price can be significantly less favorable than the specified price when markets move fast. Here the stop price is guaranteed by the filling of the sell order at price p_i.

Symmetrically, the borrower j of a buy order (y_i, p_i) is liquidated if the price decreases below p_i. He keeps the loan $\hat{y}_j^i$ while his collateral worth $\hat{y}_j^i/p_i$ is transferred to i. j has a stop-loss when the price decreases to p_i. The stop price is guaranteed by the filling of the buy order at price p_i.

4.2 Zero liquidation costs

In the introduction's example, suppose the price hits 1800 USDC and Alice's buy order of 1 ETH is filled. Bob's collateral is transferred to Alice's wallet as if Bob where Alice's counterparty to the trade. Compared to what happens in other lending protocols, the liquidation of a borrowing position does not rely on the swap of the collateral on a decentralized exchange. Alice is happy to receive the collateral as a payment. The fact that the lender accepts a repayment in kind (here in ETH) rather than in the currency lent (in USDC) has far-reaching implications.

A first major implication is the dramatic simplification and high safety of the liquidation process. Since there is no trade, there is no need to rely on an AMM pool with the risk of a sub-optimal execution. There is also no risk the liquidation creates a bad debt for the protocol if the trade size is too large relative to the pool's liquidity. Borrowing positions cannot end under water even in case of strong and rapid price action and possibly gas fee spike.

The type of extreme events that generate a bad debt for lending protocols translates into an opportunity cost borne by the owner of the limit order which execution deteriorates. In the example, if the price is rapidly falling, Alice gets her buy order executed for 1800 when the market price may actually be 1750. Or the buy order could be not executed at all if the price rapidly reverses.

As a result, there is also no need to heavily incentivize liquidation bots to perform the trades in a timely manner. If the order's assets are partially loaned out, arbitrageurs will initiate the internal transfer from the borrower to the lender by taking the part of the orders not borrowed. If the order's assets are fully borrowed, the protocol will offer external actors a moderate fee to execute the internal transfer. In both cases, the liquidation costs incurred by borrowers will be zero or close to zero.

4.3 Minimized governance

4.4 High leverage

Traders can borrow buy orders' assets and swap them to amplify their position in X. Or they can borrow sell orders' assets and swap them to short X. In the example, if Bob sells the borrowed USDC, he obtains $1800/1900 = 0.95$ ETH. His leverage is $\times 1.95$. If the price crosses 1800, Bob is liquidated and his loss is the difference between how much ETH he bought at 1900 with 1800 USDC and how much he owes Alice: $0.95p - p$. His P&L is

$$\max(0.95p - 1800, -0.05p)$$

Above 1800, his assets are in ETH and his debt in USDC. Below 1800, his debt is in ETH (denominated in USDC) with a maximum loss of 0.05 ETH.

In the general case, if a trader borrows $\hat{y}_i^j$ from a buy order (y_i, p_i) and sells the assets at price $p^s > p_i$. His P&L is:

$$\hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{1}$$

His leverage is $1 + p_i/p^s$. He can also level up his long by loop-borrowing and swapping more assets. The amount of leverage, denoted λ_n, is function of the number n of borrowing rounds:

$$\lambda_n = \sum_{t=0}^{n} \left(\frac{p_i}{p^s}\right)^t$$

His P&L with n loops scales linearly with leverage:

$$\lambda_n \hat{y}_i^j \max\left(\frac{p}{p^s} - 1, \frac{p}{p^s} - \frac{p}{p_i}\right) \tag{2}$$

Symmetrically, if a trader borrows $\hat{x}_i^j$ from a sell order (x_i, p_i) and sells the assets at price $p^s < p_i$, the amount of leverage λ_n he can obtain is function of the number n of borrowing rounds:

$$\lambda_n = \sum_{t=0}^{n} \left(\frac{p^s}{p_i}\right)^t$$

His P&L after n rounds of borrowing is:

$$\lambda_n \hat{x}_i^j \max(p^s - p, p^s - p_i)$$

Below p_i, the borrower's debt is in X (denominated in Y). Above p_i, his debt is in Y with a maximum loss of $p^s - p_i$ USDC. By infinitely iterating the borrowing process, the maximum theoretical leverage is:

$$\lambda_\infty = \frac{p^s}{p^s - p_i}$$

in the buy order market and

$$\lambda_\infty = \frac{p_i}{p_i - p^s}$$

in the sell order market.

Both leverages tend to infinity when p^s gets closer to p_i. Traders can therefore attain arbitrarily high leverage by borrowing assets from limit orders which price is as close to current price as possible. As usual, the risk of being liquidated increases with leverage when p^s approaches p_i.

4.5 Minimized loss ratio

A key performance metric for leveraged traders is how much can be lost compared to potential profit. The ratio of maximum loss (the total collateral lost in case of liquidation) to profit can be computed from the P&L formulas (1) and (2) in the sell and buy order markets. Remarkably, the two ratios tend to zero when the price at which the assets are borrowed and swapped approaches the limit price:

$$\lim_{p^s \to p_i} \frac{p^s - p_i}{p^s - p} = 0$$

and

$$\lim_{p^s \to p_i} \frac{1 - \dfrac{p^s}{p_i}}{1 - \dfrac{p^s}{p}} = 0$$

The property holds for finite price variations and any level of leverage.

5 Conclusion

A borrowable limit order book is a decentralized exchange to which is appended a minimalist yet full-fledged lending protocol which efficiently lends assets backed by limit orders. The absence of risk of bad debt is a remarkable improvement over existing lending markets which are forced to set many guardrails and constantly update their security parameters to prevent the risk. The protocol is primarily optimized for traders who will find highly appealing its features (stop-loss orders with guaranteed stop price, high leverage without liquidation costs and low loss ratio).

The protocol will also cater to lenders who will earn high interest rate paid by borrowers.