$$Y^{comm} = [y_1^{comm}, \dots, y_A^{comm}]_{K \times A}$$

$$Y^{micro} = [y_1^{micro}, \dots, y_M^{micro}]_{N_0 \times M}$$

$$vec(\underset{A \times B}{Y}) = (\underset{\sim}{y_1}^T, \underset{\sim}{y_2}^T, \dots, y_B^T)^T_{(AB \times 1)}$$

Similarly,

$$Z = [z_1, \dots, z_R]_{N_1 \times R}$$

comm matrix? agg'd

comm ← agg'd x

for $a = 1, \dots, A$ $\quad \underset{K \times 1}{y_a} \sim MVN(\alpha\underset{\sim}{1} + \underset{K \times D}{X_0}\underset{\sim}{\theta}, (\underset{\sim}{w}_a^{comm})^{-1} I)$

for $m = 1, \dots, M$ $\quad y_m^{micro} \overset{ind}{\sim} MVN(\alpha \underset{\sim}{1} + X_0 \underset{\sim}{\theta}, (w_m^{micro})^{-1} I)$

$$Z_r \overset{ind}{\sim} MVN\left(\alpha \underset{\sim}{1} + X_1 \underset{\sim}{\theta}_1, \left(\tilde{\omega}_r^{rank}\right)^{-1}\right)$$

$$\begin{pmatrix} vec(Y^{comm}) \\ vec(Y^{micro}) \\ vec(Z) \end{pmatrix} \sim MVN\left(\alpha \underset{\sim}{1} + X \underset{\sim}{\theta} + \Sigma\right)$$

$\leftarrow (AK + MN_0 + RN_1) \times 1$

$$X = \begin{bmatrix} \underset{\sim}{1}_{A \times 1} \otimes X_0^{comm} \\ \underset{\sim}{1}_{M \times 1} \otimes X_0 \\ \underset{\sim}{1}_{R \times 1} \otimes X_1 \end{bmatrix}$$

$$\Sigma^{-1} = diag\left(\left(\omega^{comm} \otimes \underset{\sim}{1}\right)^T, \left(\omega^{micro} \otimes \underset{\sim}{1}\right), \left(\omega^{rank} \otimes \underset{\sim}{1}\right)\right)$$