Indexing

Indexing Units:

N_0 = # training households
N_1 = # testing households (to be ranked)
K = # communities

Indexing Information:

A = # of response variables collected at community-level
M = # of response variables collected at household-level
R = # of rankers of the N_1 test units/households
P = # of covariates

Inputs:

$$y^{comm} = \left[y_1^{comm}, \ldots, y_A^{comm}\right]_{K \times A}$$ community-level

$$y^{micro} = \left[y_1^{micro}, \ldots, y_M^{micro}\right]_{N_0 \times M}$$ training units

$$T = \left[T(\underset{\sim}{z}_1), \ldots, T(\underset{\sim}{z}_R)\right]_{N_1 \times R}$$ testing units

X_0^{micro} is a $N_0 \times P$ matrix of covariates training units

X_1^{micro} is a $N_1 \times P$ matrix of covariates testing units

X^{comm} is a $K \times P$ matrix of covariates community level

→ Q: X^{comm} is just a mean-aggregation of $\begin{bmatrix} X_0 \\ X_1 \end{bmatrix}$, right?

Re-structuring of inputs:

Take the vec() function on a $R \times C$ matrix $X = [\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_C]$ to be column-wise stacking.

$$\text{vec}(X) = (\underset{\sim}{x}_1^T, \underset{\sim}{x}_2^T, \ldots, \underset{\sim}{x}_C^T)^T$$ ← $(RC) \times 1$ vector

$$u = \begin{pmatrix} \text{vec}(y^{comm}) \\ \text{vec}(y^{micro}) \\ \text{vec}(Z) \end{pmatrix}$$ is a $(AK + MN_0 + RN_1) \times 1$ vector

$$X = \begin{bmatrix} I_{A\times 1} \otimes X^{comm} \\ I_{M \times 1} \otimes X_0^{micro} \\ I_{R \times 1} \otimes X_1^{micro} \end{bmatrix}$$ is a $(AK + MN_0 + RN_1) \times P$ matrix

Model

This is a 1st stage model - we will have to build upon it later (e.g. random effects, etc...)

For $a = 1, \ldots, A$; $k = 1, \ldots, K$: $y_{a,k}^{comm} \sim N(\alpha + (\underset{\sim}{X}_k^{comm})^T \underset{\sim}{\beta}, (w_a^{comm})^{-1})$

For $m = 1, \ldots, M$; $i = 1, \ldots, N_0$: $y_{m,i}^{micro} \sim N(\alpha, (X_{0i}^{micro})^T \underset{\sim}{\beta}, (w_k^{micro})^{-1})$

For $r = 1, \ldots, R$; $j = 1, \ldots, N_1$: $z_{r,j} \sim N(\alpha, (X_{1j}^{micro})^T \underset{\sim}{\beta}, (w_r^{rank})^{-1})$

→ equivalently,

$$\underset{\sim}{u} \sim MVN(\alpha \underset{\sim}{1} + X \underset{\sim}{\beta}, \Sigma)$$

where

$$\Sigma^{-1} = \text{diag}\left((\underset{\sim}{w}^{comm} \otimes \underset{\sim}{1})^T, (\underset{\sim}{w}^{micro} \otimes \underset{\sim}{1}), (\underset{\sim}{w}^{rank} \otimes \underset{\sim}{1})\right)$$

Priors:

$\alpha \sim N(0, \sigma_\alpha^2)$ fix at first

$\underset{\sim}{\beta} \sim MVN(0, \sigma_\beta^2 I_{P\times P})$

$$w_a^{comm} = \begin{cases} 0.5 & \text{w.p. } 1/3 \\ 1 & \text{w.p. } 1/3 \\ 2 & \text{w.p. } 1/3 \end{cases}$$

$$w_m^{micro} = \begin{cases} 0.5 & \text{w.p. } 1/3 \\ 1 & \text{w.p. } 1/3 \\ 2 & \text{w.p. } 1/3 \end{cases}$$

$$w_r^{rank} = \begin{cases} 0.5 & \text{w.p. } 1/3 \\ 1 & \text{w.p. } 1/3 \\ 2 & \text{w.p. } 1/3 \end{cases}$$

larger w ⟺ that measure has "stronger" relationship w/ X covariates. not quite right.

smaller w ⟺ that measure isn't described by $\alpha + X\beta$ as the other measures are. → less reliable?