$$\prod_{j=1}^{M} \prod_{i=1}^{N_0} N\left(z_{ij} \,\middle|\, \alpha_i + x_i^0 \underset{\sim}{\theta},\ \omega_j^{-1}\right)$$

latent scores for 'testing'

include column of 1s in x!

$$\prod_{i=1}^{N_1} N\left(y_i \,\middle|\, \cancel{\mu_\alpha}\ x_i^1 \underset{\sim}{\theta},\ \omega_{J+1}^{-1}\right)$$

observed ~~scores e.g. consumption~~ on training

$$\alpha_i \sim N(\theta_1, \sigma_\alpha^2)$$

$$\underset{\sim}{Z}^* = \begin{pmatrix} \underset{\sim}{Z}_1 \\ \vdots \\ \underset{\sim}{Z}_M \\ \underset{\sim}{y} \end{pmatrix} \to f(\underset{\sim}{\theta} \mid \cdot) \propto e^{-\frac{1}{2}\left(\underset{\sim}{z}^* - \binom{\underset{\sim}{\alpha}}{\underset{\sim}{0}} + \underset{\sim}{X}^* \underset{\sim}{\theta}\right)^T W^{-1} \left(z^* - \binom{\alpha}{0} + X^* \underset{\sim}{\theta}\right)}$$

$$W = \mathrm{diag}(\mathrm{rep}(\underset{\sim}{\omega}, \text{each} = N_0, N_0, \ldots, N_1))$$

$$M+1 \times 1$$

big matrices

$$\times\ e^{-\frac{1}{2} \underset{\sim}{\theta}^T \Sigma_\theta^{-1} \underset{\sim}{\theta}}$$

$$(\Sigma_\theta \approx \sigma_\theta^2 I_p)$$

easier;

$$f(\underset{\sim}{\theta} \mid \cdot) \propto \prod_{j=1}^{M} \prod_{i=1}^{N_0} N\left(\boxed{z_{ij} - \alpha_i}^{\,z_{ij}^*} \,\middle|\, x_i^{0T} \underset{\sim}{\theta},\ \omega_j^{-1}\right)$$

$$\times \prod_{i=1}^{N_1} N\left(y_i \,\middle|\, x_i^{1T} \underset{\sim}{\theta},\ \omega_{M+1}^{-1}\right) \times N\left(\underset{\sim}{\theta} \,\middle|\, \underset{\sim}{0}, \Sigma_\theta\right)$$

$$\propto \exp\left[\frac{1}{2\sigma_\theta^2} \underset{\sim}{\theta}^T \underset{\sim}{\theta}\right] \exp\left[-\frac{1}{2}\left\{\sum_{j=1}^{M} \frac{1}{\omega_j} \sum_{i=1}^{N_0} (z_{ij}^* - x_i^{0T} \underset{\sim}{\theta})^2 + \frac{1}{\omega_M} \sum_{i=1}^{N_1} (y_i - x_i^{1T} \underset{\sim}{\theta})^2\right\}\right]$$