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Do Bitcoin and other cryptocurrencies jump together?

Elie Bouri [a,*], David Roubaud [b], Syed Jawad Hussain Shahzad [b]

[a] USEK Business School, Holy Spirit University of Kaslik, Jounieh, Lebanon
[b] Montpellier Business School, Montpellier, France

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ABSTRACT

We detect the presence of jumps in the return series of 12 cryptocurrencies and find significant jump activity in all cases, especially in Ripple, Bitcoin and Litecoin. We also examine whether cryptocurrencies' returns jump together and the results of various analyses show evidence of co-jumping behaviour, except for a few cases (Ripple and Bytecoin). These results suggest that the presence of jumps in one cryptocurrency increases the probability of inducing a jump in other cryptocurrencies. However, co-jumping is joint with a jumping activity in trading volume. This later result highlights the importance of jumps in trading volume to the formation of jumps in cryptocurrencies, confirming earlier findings on the importance of trading volume to the volatility of cryptocurrencies.

1. Introduction

The jump activity represents an important aspect of asset pricing (Barndorff-Nielsen & Shephard, 2006). Bates (2000) indicates that jumps can reflect crash risk, and many studies highlight the role of jumps in capturing the empirical properties of an asset and modelling its volatility dynamics (Driessen & Maenhout, 2013; Eraker, Johannes, & Polson, 2003). Asset jumps are also important for risk management, asset allocation and derivatives pricing (see, among others, Clements & Liao, 2017; Oliva & Renò, 2018). Advanced volatility and options models now incorporate not only jumps but also co-jumps among assets (Clements & Liao, 2017).

While the jump behaviour of most conventional assets has been examined in empirical studies (Gilder, Shackleton, & Taylor, 2014; Ma, Zhang, Wahab, & Lai, 2019), less attention has been given to the presence of jumps and co-jumps in cryptocurrencies that now constitute a new (digital) asset.

Cryptocurrencies are appreciated by some investors because of their independence from sovereign authorities and their reliance on mass collaboration though innovative technology called blockchain (Shahzad, Bouri, Roubaud, Kristoufek, & Lucey, 2019). Economics and finance literature has so far focused on cryptocurrencies by examining return and volatility spillovers (Ji, Bouri, Lau, & Roubaud, 2019; Ji, Bouri, Gupta, & Roubaud, 2018; Yi, Xu, & Wang, 2018), price bubbling (Bouri, Shahzad, & Roubaud, 2019), market efficiency (Sensoy, 2018; Aggarwal, 2019), and volatility modelling via GARCH processes (Chu, Chan, Nadarajah, & Osterrieder, 2017). Importantly, cryptocurrencies exhibit enormous volatility, and the largest cryptocurrency, Bitcoin, is known for its extreme price volatility and large abrupt price variations in the form of jumps (Chaim & Laurini, 2018). Jumps can substantially impact the structure of losses and gains related to Bitcoin. However, except for Chaim and Laurini (2018) who focus on the presence of jumps in Bitcoin, there is no empirical evidence on whether other cryptocurrencies such as Ethereum, Ripple, Litecoin or Stellar exhibit jump behaviour.

In this study, we contribute to the literature by examining the presence the jump in 12 leading cryptocurrencies (Bitcoin, Bitshares, Bytecoin, Dash, Digibyte, Dogecoin, Ethereum, Litecoin, Monero, Nem, Ripple and Stellar). We also offer another extension to the related literature by assessing whether cryptocurrencies

* Corresponding author.
E-mail addresses: eliebouri@usek.edu.lb (E. Bouri), d.roubaud@montpellier-bs.com (D. Roubaud), j.syed@montpellier-bs.com (S.J.H. Shahzad).

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Table 1
Statistical properties of daily returns.

	Mean	Max.	Min.	Std. Dev.	Skewness	Kurtosis	P-value ADF test	P-value ARCH-LM test
BITCOIN	0.2017	22.5119	−20.7530	3.9576	−0.2618	7.7917	0.000	0.000
BITSHARES	0.1794	51.9989	−39.1702	7.8798	0.8441	10.2539	0.000	0.000
BYTECOIN	0.1584	159.7832	−91.0302	12.0547	3.7096	51.4836	0.000	0.000
DASH	0.2492	43.7746	−24.3225	5.9739	0.8473	9.0560	0.000	0.000
DIGIBYTE	0.3942	116.5601	−36.1404	10.2128	2.6296	27.0477	0.000	0.000
DOGECOIN	0.1895	51.8345	−49.2867	6.6989	0.9707	15.3849	0.000	0.000
ETHEREUM	0.2997	41.2337	−130.2106	7.6973	−3.3837	68.2754	0.000	0.000
LITECOIN	0.1842	51.0348	−39.5151	5.7458	1.2605	15.3050	0.000	0.000
MONERO	0.3222	58.4637	−29.3176	6.9955	1.0124	10.5013	0.000	0.000
NEM	0.4371	99.5577	−36.1450	8.7648	1.9876	20.5458	0.000	0.000
RIPPLE	0.2809	102.7356	−61.6273	7.4411	3.0172	42.6362	0.000	0.000
STELLAR	0.2736	72.3102	−36.6358	8.2234	2.0552	18.8741	0.000	0.000

Note: The sample period is August 8, 2015 - February 28, 2019.

exhibit co-jumping behaviour, i.e., jump together, and whether their jumps are joint with jumps in trading volume.

On the methodological front, the detection of the jump behaviour is conducted via the semi-parametric approach of Laurent, Lecourt, and Palm (2016), which has the ability to detect and date-stamp jumps in various conventional assets. The presence of co-jumps is examined via logistic regressions and the method of Ma et al. (2019), and further analysis on co-jumps in volatility and trading volume adds to prior studies on the volatility-volume relationship in the cryptocurrency market (e.g., Bouri, Lau, Lucey, & Roubaud, 2019).

The main results indicate jump activity in all cryptocurrencies under study. Furthermore, there is evidence of co-jumping behaviour that is generally joint with jump activity in trading volume.

For the rest of the paper, Section 2 provides the dataset. Section 3 describes the models. Section 4 presents and discusses the results. Section 5 concludes.

2. Data

Data on the daily prices of 12 main cryptocurrencies (Bitcoin, Bitshares, Bytecoin, Dash, Digibyte, Dogecoin, Ethereum, Litecoin, Monero, Nem, Ripple and Stellar) are collected from https://coinmarketcap.com. The sample is August 8, 2015 to February 28, 2019, as constrained by the price availability of leading cryptocurrencies such as Ethereum and the need to cover the largest number of cryptocurrencies from the first 50 cryptocurrencies by market value. Empirical analyses are conducted with log returns multiplied by 100, leading to 1301 daily return observations. As we require stationary return series, we apply the augmented Dickey-Fuller (ADF) test, and the corresponding results in Table 1 show that all return series are stationary at the 1% level. Table 1 also shows the four moments of the distribution of the return series. Nem has the highest mean return, followed Digibyte. Conversely, Bytecoin has the lowest mean return and highest standard deviation. However, the lowest standard deviation is for Bitcoin, which has only the fifth largest return after Bytecoin, Bitshares, Litecoin and Dogecoin. Table 1 also shows evidence of excess kurtosis. The skewness is positive, except for Bitcoin and Ethereum. Results from the ARCH-LM test of Engle (1982) show evidence of conditional heteroscedasticity for all the return series. Fig. A1 in the Appendix plots the daily returns of the 12 cryptocurrencies, whereas Table A1 presents market capitalization value and ranking.

Table 2 presents the correlation matrix among the 12 return series. The highest positive correlation is between Bitcoin and its fork, Litecoin (0.6178) and Ripple and Stellar (0.5513). The lowest correlation is between Bytecoin and Litecoin (0.0697) and Bytecoin and Digibyte (0.0703).

3. Methods

3.1. Testing for jumps

The methodology for detecting the presence of jumps is based on the semi-parametric approach of Laurent et al. (2016)[1], which tests for additive jumps in AR-GJR-GARCH models[2] . Let us describe random returns (r_t) by an AR(1)-GJR-GARCH(1,1) model as follows:

$$r_t = \mu_t + \zeta r_{t-1} + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \sigma_t z_t \text{ and } z_t \sim i.i.d.N(0,\ 1) \tag{2}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \theta_1 D_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

where $\varepsilon_t$is the error term, $z_t$is the white noise process, and σ_t^2 is the conditional variance of $r_t \cdot D_{t-1} = 1$ if $\varepsilon_{t-i} < 0$.

Adding an independent jump component $a_t I_t$ to r_t, we can write:

$$r_t^* = r_t + a_t I_t \tag{4}$$

where r_t^* is the observed returns, I_t, is a dichotomous variable taking the value of 1 if there is a jump on day t and 0 otherwise; a_t denotes the jump size. As argued by Laurent et al. (2016), the conditional variance at t_{+1} (σ_{t+1}^2)is not affected by $a_t I_t$.

Based on the bounded innovation propagation (BIP)-AR(1) of Muler, Pena, and Yohai (2009) and the BIP-GARCH(1,1) of Muler and Yohai (2008), we obtain estimates of μ_t and r_t that are the robust to potential jumps $a_t I_t$. These are $\sim\mu_t$ and $\sim\sigma_t$.

Considering the standardized return on day t as:

$$\sim J_t = \frac{r_t^* - \sim\mu_t}{\sim\sigma_t} \tag{5}$$

The presence of jumps can be detected via testing the null hypothesis $H_0: a_t I_t = 0$, against the alternative $H_1: a_t I_t \neq 0$. H_0 is rejected if $\max_T \left|\sim J_t\right| > g_{T,\lambda}$, where $\max_T$ is the maximum of $\left|\sim J_t\right|$ for $t = 1, \ldots, T$,and $g_{T,\lambda}$ is the critical value. If H_0 is rejected, the following dummy variable is proposed:

$$\sim I_t = I\left(\left|\sim J_t\right| > k\right) \tag{6}$$

where I(.) is the indicator function, with $\sim I_t$ equals 1 if there is a jump on day t.

3.2. Co-jumping analysis

Once the jump behaviour is detected and date-stamped via the semi-parametric approach of Laurent et al. (2016), we uncover evidence of co-jumping among the return series. To this purpose,

[1] Laurent et al. (2016) argue that their test is similar to the non-parametric tests of Lee and Mykland (2007) and Andersen, Bollerslev, and Dobrev, (2007).

[2] Our estimated results are not sensitive to the choice between the AR- GJR-GARCH and AR-GARCH models.

Table 2
Correlation matrix.

	BITCOIN	BITSHARES	BYTECOIN	DASH	DIGIBYTE	DOGECOIN	ETHEREUM	LITECOIN	MONERO	NEM	RIPPLE	STELLAR
BITCOIN	1.0000											
BITSHARES	0.4287	1.0000										
BYTECOIN	0.0991	0.1295	1.0000									
DASH	0.4786	0.3839	0.0876	1.0000								
DIGIBYTE	0.3558	0.3956	0.0703	0.2992	1.0000							
DOGECOIN	0.5122	0.5332	0.1317	0.3781	0.4131	1.0000						
ETHEREUM	0.3845	0.3841	0.0939	0.3682	0.2478	0.3185	1.0000					
LITECOIN	0.6178	0.4279	0.0697	0.4503	0.2896	0.5148	0.3728	1.0000				
MONERO	0.5041	0.3817	0.0924	0.4827	0.2913	0.3694	0.3623	0.4382	1.0000			
NEM	0.3918	0.3670	0.1355	0.3383	0.3064	0.3667	0.2642	0.3867	0.3094	1.0000		
RIPPLE	0.3103	0.4530	0.1320	0.2525	0.2751	0.4233	0.2288	0.3649	0.2959	0.2987	1.0000	
STELLAR	0.3722	0.5245	0.0729	0.2998	0.3701	0.4783	0.2539	0.3898	0.3875	0.3945	0.5513	1.0000

Notes: This table presents pair-wise Pearson correlation coefficients. The sample period is August 8, 2015 - February 28, 2019.

we apply a logistic regression with the dependent variable being a dichotomous variable Y taking the value of 1 when there is a volatility jump and 0 otherwise.

$$\log\left(\frac{P(Y=1|X)}{1-P(Y=1|X)}\right) = \beta_0 + \beta_i X_{i,t} + \varepsilon_t \tag{7}$$

where, β_0 is the constant; $X_{i,t}$ is a set of 11 dichotomous variables, where $i = 1, 2. \ldots.11$; each dichotomous variable indicates the presence of jump, as shown for the dependent variable, in each of the other remaining 11 return series. The distribution of the error term (ε_t) follows the logistic regression.

We also uncover evidence of co-jumping via the co-exceedance rule used by Ma et al. (2019).

$$\sum_{i=1}^{n} I\left(\text{Jump}_{t,i} > 0\right) \begin{cases} = n \text{ cojump} \\ \leq 1\ \textit{No} \text{ cojump} \end{cases} \tag{8}$$

Where n is half the number of cryptocurrencies rounded up to the nearest one; $I(\text{Jump}_{t,i} > 0)$ is an indication function that takes the value of 1 when a jump is detected in cryptocurrency i on day t.

4. Empirical results

4.1. Results of the jump test

Table 3 presents statistics of the detected jumps, while the plots of the jumps on the return series are given in Fig. A2 in the Appendix. The least jump activity is recorded in Dash and Digibyte, where the number of jumps is 24. Conversely, the largest number of jumps is found in the daily returns of Ripple (52), Bitcoin (48) and Litecoin (47), representing 4.00%, 3.69%, and 3.61% of days. It follows that jumps do occur repeatedly in those large cryptocurrencies. Furthermore, most of the jumps occur in 2017–2018. We can interpret the jumps that occur in 2017 as being related to the boom in Bitcoin and other large cryptocurrencies. As for the jumps that occur in 2018, they can be related to big turnaround in the overall cryptocurrency market in this period, when it lost almost 80% of its market value, driven by combination of factors such as regulatory oversight and wrangling over a split in one of Bitcoin's forks, Bitcoin Cash[3] . Our findings generally indicate evidence of infrequent large shocks in leading cryptocurrencies, suggesting the importance of accounting for such large shocks in any volatility modelling involving the return series in the cryptocurrency market. This is crucial, given evidence that more frequent jumps can make the tails fatter, leading to magnified risk measures and affecting options pricing.

4.2. Results for co-jumping

We uncover evidence of co-jumping among the return series by conducting a logistic regression. Table 4 presents the estimated results from Eq. (7). It shows that the jump activity among leading cryptocurrencies is mostly positively related, suggesting that the occurrence of a jump in one cryptocurrency increases the likelihood of a jump in other cryptocurrencies. Bitshares, Ethereum and Stellar are most dependent on the presence of jumps in other cryptocurrencies. Conversely, Ripple and Bytecoin are completely independent of the jump activity in other cryptocurrencies. The jump in Digibyte is dependent on the presence of jumps in Ethereum only. In fact, the presence of jumps in Ethereum, Litecoin, Nem and Stellar increases the probability of a jump in Bitshares. The results for the second largest cryptocurrency, Ethereum, show that its jump activity increases with the presence of jumps in Bitcoin, Bitshares, Dash and Digibyte. Furthermore, the formation of jumps in Bitshares, Digibyte, Dogecoin and Ripple increases the likelihood of the occurrence of jumps in Stellar. This finding generally concords with prior evidence showing significant return and volatility linkages among leading cryptocurrencies (Ji et al., 2019; Yi et al., 2018). The presence of jump activity in Litecoin and Nem is positively affected by the probability of the occurrence of jumps in Bitcoin. The occurrence of jumps in Monero increases with the presence of jumps in Dash and Litecoin, whereas it decreases when Ethereum experiences jumps.

We search, via another approach, for co-jumping behaviour among the returns of large cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin and Stellar)[4] and smaller cryptocurrencies (Bitshares, Bytecoin, Dash, Digibyte, Dogecoin, Monero and Nem)[5] (see also Table A1 in the Appendix). Following Ma et al. (2019), we define co-jumps as contemporaneous jumps occurring among at least half of the return series, in this case three (3 = 5/2 rounded up to the nearest one) for the cryptocurrencies that have a market value above \$1 Billion and four (4 = 7/2 rounded up to the nearest one) for the cryptocurrencies that have a market value below \$1 Billion[6] . Overall, the results in Table 5 show evidence of co-jumps, although the co-jump activity is more pronounced in large cryptocurrencies.

In fact, we date-stamp 11 co-jumps among large cryptocurrencies and only 7 among smaller cryptocurrencies. Furthermore, most of the co-jumps occur during 2018. These findings are not surprising given that most of the trading activity and volume is concentrated in large cryptocurrencies, which represent almost 75% of the market value of the overall cryptocurrency market, while the

[3] https://www.cnbc.com/2018/11/23/cryptocurrencies-have-shed-almost-700-billion-since-january-peak.html.

[4] Each of these five cryptocurrencies has a market value above \$1 Billion.
[5] Each of these seven cryptocurrencies has a market value below \$1 Billion.
[6] See Table A1.

Table 3
Statistics of jumps detected via Laurent et al.'s (2016) method.

	BITCOIN	BITSHARES	BYTECOIN	DASH	DIGIBYTE	DOGECOIN	ETHEREUM	LITECOIN	MONERO	NEM	RIPPLE	STELLAR	**Sum**
Panel A: Number of jumps													
2015	2	3	3	0	1	3	5	7	4	4	6	4	**42**
2016	18	7	6	3	8	5	6	12	10	10	7	15	**107**
2017	12	9	14	13	10	15	12	17	7	8	20	14	**151**
2018	12	12	10	6	4	12	6	10	6	8	15	6	**107**
2019	4	0	0	2	1	2	3	1	1	2	4	1	**21**
Sum	**48**	**31**	**33**	**24**	**24**	**37**	**32**	**47**	**28**	**32**	**52**	**40**	**428**
Panel B: % of days with jumps													
2015	1.37%	2.05%	2.05%	0.00%	0.68%	2.05%	3.42%	4.79%	2.74%	2.74%	4.11%	2.74%	
2016	4.93%	1.92%	1.64%	0.82%	2.19%	1.37%	1.64%	3.29%	2.74%	2.74%	1.92%	4.11%	
2017	3.29%	2.47%	3.84%	3.56%	2.74%	4.11%	3.29%	4.66%	1.92%	2.19%	5.48%	3.84%	
2018	3.29%	3.29%	2.74%	1.64%	1.10%	3.29%	1.64%	2.74%	1.64%	2.19%	4.11%	1.64%	
2019	6.78%	0.00%	0.00%	3.39%	1.69%	3.39%	5.08%	1.69%	1.69%	3.39%	6.78%	1.69%	
Total %	**3.69%**	**2.38%**	**2.54%**	**1.84%**	**1.84%**	**2.84%**	**2.46%**	**3.61%**	**2.15%**	**2.46%**	**4.00%**	**3.07%**	

Notes: Panel A presents the number of jumps detected in the return series. Panel B provides the percentage (%) of days with jumps, while the corresponding % for 2015 and 2019 covers only 146 and 59 days, respectively. This means comparison between the % in 2015 and 2019 and each year 2016–2018 must be done with caution. The total % is the average % of days with jumps over the sample period.

Table 4
Results for co-jumping using logistic regression.

	BITCOIN	BITSHARES	BYTECOIN	DASH	DIGIBYTE	DOGECOIN	ETHEREUM	LITECOIN	MONERO	NEM	RIPPLE	STELLAR
BITCOIN							1.5299**	2.6430***		2.3067***		1.5448**
BITSHARES							1.9560***	2.0008***		1.5441**		
BYTECOIN												
DASH							2.7234***		2.8149***			
DIGIBYTE							1.8690***					1.5100**
DOGECOIN												1.5609***
ETHEREUM		1.8661**		2.7580***	1.8481**				−2.8412*			
LITECOIN	2.6161***	1.9440***							1.6357**			
MONERO				2.5343***				1.6144**				
NEM	2.2111***	1.5324**										
RIPPLE						1.7089***						2.1120***
STELLAR		1.6030**				1.4668**						
McFadden R^2	0.2571***	0.2559***		0.2549***	0.1389***	0.1862***	0.2718***	0.2364***	0.2037***	0.2102***		0.1894***

Notes: Estimated coefficients results are based on the logistic regression in Eq. (7). ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively.

Table 5
Results of co-jumping via Ma et al.'s (2019) method.

Panel A: Cryptocurrencies with a market value above $1 Billion

	Jumps-BITCOIN	Jumps-ETHEREUM	Jumps-LITECOIN	Jumps-RIPPLE	Jumps-STELLAR	Number of jumps ≥3
8/18/2015	1	0	1	1	1	**4**
2/4/2017	0	0	1	1	1	**3**
9/14/2017	1	1	1	1	1	**5**
10/6/2018	1	1	1	1	1	**5**
6/22/2018	1	1	1	1	1	**5**
5/9/2018	1	1	1	1	0	**4**
11/10/2018	1	1	1	1	1	**5**
11/14/2018	1	1	1	0	0	**3**
11/19/2018	1	1	1	0	0	**3**
10/1/2019	1	1	0	1	0	**3**
8/2/2019	1	1	1	1	0	**4**
					Total number of co-jumps: 11	

Panel B: Cryptocurrencies with a market value below $1 Billion

	Jumps-BITSHARES	Jumps-BYTECOIN	Jumps-DASH	Jumps-DIGIBYTE	Jumps-DOGECOIN	Jumps-MONERO	Jumps-NEM	Number of jumps ≥4
9/14/2017	1	0	1	0	1	1	1	**5**
6/22/2018	1	1	0	0	1	0	1	**4**
5/9/2018	1	0	1	1	0	0	1	**4**
11/10/2018	1	1	1	0	1	1	1	**6**
11/14/2018	1	0	1	1	0	1	0	**4**
11/19/2018	1	1	1	1	0	1	0	**5**
8/2/2019	0	0	1	1	1	1	0	**4**
							Total number of co-jumps: 7	

Notes: This table presents the number of co-jumps in cryptocurrencies having a market value above $1 Billion (Panel A) and cryptocurrencies having a market value below $1 Billion (Panel B). The estimation method is in line with the approach of Ma et al. (2019), see Eq. 8. In Panel A, 3 is the half of number of cryptocurrencies (i.e., 5/2) rounded up to the nearest one. In Panel B, 4 is the half of number of cryptocurrencies (i.e., 7/2) rounded up to the nearest one.

Table 6
Results of co-jumps between cryptocurrencies and their trading volumes.

	BITCOIN	BITSHARES	BYTECOIN	DASH	DIGIBYTE	DOGECOIN	ETHEREUM	LITECOIN	MONERO	NEM	RIPPLE	STELLAR
BITCOIN	1.6689											
BITSHARES		4.2176***										
BYTECOIN			1.9951***									
DASH				1.7055***								
DIGIBYTE					4.8473***							
DOGECOIN						3.6415***						
ETHEREUM							3.0800***					
LITECOIN								3.5937***				
MONERO									4.2556***			
NEM										3.3361***		
RIPPLE											1.4008	
STELLAR												3.5521***

Notes: Estimated coefficient results are based on the logistic regression in Eq. (7), where the jumps in a cryptocurrency are regressed on jumps in its trading volume. ***, ** and * denote statistical significance at 1%, 5% and 10%, respectively.

share of smaller cryptocurrencies is only 2%. This indicates a certain contemporaneous correlation in the jump activity among large cryptocurrencies, which may entail lower diversification benefits. Conversely, the relative disjoint in the jump activity between large and smaller cryptocurrencies implies more diversification benefits and risk management inferences.

Taken together, the results above show that the probability of the existence of jumps in one cryptocurrency affects the presence of jumps in other cryptocurrencies. Although Bitcoin is the biggest cryptocurrency in terms of market value, it is not totally isolated from the effects of jumps in other relatively smaller cryptocurrencies. This evidence points to the mounting importance of other altcoins to the co-jumps among cryptocurrencies, which partially concurs with the results of Bouri, Shahzad et al. (2019) on co-explosivity, Ji et al. (2019) on connectedness and Yi et al. (2018) on volatility spillovers among cryptocurrencies. In fact, jumps in smaller cryptocurrencies such as Bitshares seem to be important for the formation of jumps in larger cryptocurrencies such as Ethereum, whereas jumps in some middle-sized cryptocurrencies such as Ripple are disjointed from other cryptocurrencies.

These results could be related to some kind of contagion effect among leading cryptocurrencies, which can be used by crypto-traders who might seek to profit by hunting for jumps in one cryptocurrency based on the jumps in others.

4.3. Further analysis – jumps and trading volume

We extend the analyses to assess whether the jumps detected in the return series of leading cryptocurrencies potentially concur with the occurrence of jumps in trading volume[7] . This might give insight into whether the returns of cryptocurrencies and their corresponding trading volumes jump together. We therefore rerun the jump test of Laurent et al. (2016) on trading volume in each of the 12 cryptocurrencies under study. Trading volume data are collected from https://coinmarketcap.com. Fig. A3 shows the plots of jumps in the trading volume, where there is evidence of co-jumping activity among the trading volume of cryptocurrencies. Importantly, we run the logistic regression and report the results for co-jumps in Table 6. The results are significant in all cases, except for Bitcoin, Dash and Ripple. They generally reveal that the occurrence of a jump in the trading volume of one cryptocurrency increases the likelihood of a jump in the volatility of the same cryptocurrency. This finding nicely complements prior studies on the volatility-volume relationship in the cryptocurrency market (e.g., Bouri, Lau et al., 2019).

For a robustness check, we apply the method of Ma et al. (2019), and unreported results show quite similar evidence of co-jumping between each cryptocurrency and its trading volume. Co-jumping is more frequent in Ethereum, Monero, Digibyte and Dogecoin, whereas it is almost nonexistent in Bitcoin and Dash, which confirms our earlier findings from the logistic regression (see Table 6). These results for cryptocurrencies can be compared to those in earlier studies on stock market indices (e.g., Jawadi, Louhichi, Cheffou, & Randrianarivony, 2016), confirming the importance of jumps in trading volume for the formation of jumps in cryptocurrencies. Such evidence adds to prior studies (e.g., Bouri, Lau et al., 2019) highlighting the role of jumps as an element characterizing the volatility of cryptocurrencies.

5. Concluding remarks

Studying and capturing the jump behaviour of assets has attracted mounting interest among academics, practitioners and investors given the consequences for risk management, asset allocation and derivatives pricing (Ma et al., 2019, Clements & Liao, 2017; Oliva & Renò, 2018). In this research paper, we focus on the cryptocurrency market that continues to intrigue investment communities not least because of its large price spikes and slippage. Our analyses show evidence of jumps, especially in Ripple, Bitcoin and Litecoin. Accordingly, evidence of infrequent large shocks suggests the importance of accounting for such large shocks in any modelling of the return series of those cryptocurrencies, which might lead to policy implications regarding asset pricing and option and variance modelling (Clements & Liao, 2017; Driessen & Maenhout, 2013; Oliva & Renò, 2018). This is crucial given that jumps represent a sort of tail-risk (Oliva & Renò, 2018) and an input into option pricing models, helping the enhancement of model prediction. Further results show evidence of co-jumping among the leading cryptocurrencies under study, which has implications for the modelling of volatility and options with advanced techniques (Clements & Liao, 2017). In fact, our results suggest that the occurrence of a jump in one cryptocurrency increases with the presence of a jump in other cryptocurrencies, which reduces the benefits of diversification. Such results are also useful when crafting options on cryptocurrencies given that it is now standard to use jumps and co-jumps (Clements & Liao, 2017) as well as portfolio allocation with co-jump risk (Oliva & Renò, 2018). Regarding further policy implications, the findings imply the need to incorporate not only jumps but co-jumps when modelling the volatility dynamics of cryptocurrencies within multivariate models. The same is required when studying the dynamics of volatility spillovers or connectedness within the cryptocurrency market. In this regard, and within the context of conventional assets, Driessen and Maenhout (2013) highlight the importance of accounting for jump activity

[7] Jawadi et al. (2016), among others, find evidence of a significant relationship between volatility jumps and trading volume in stock market indices.

for the sake of trading volatility or jump[8], which might also entail some diversification benefits. Furthermore, evidence of co-jumping between cryptocurrencies and their trading volume may suggest the importance of accounting for this type of co-jumping when studying the volatility-volume relationship in the cryptocurrency market (e.g., Bouri, Lau et al., 2019) and when modelling volatility dynamics or making risk spillover networks (Ji et al., 2019; Yi et al., 2018).

Future research can extend our analyses by decomposing the jumps into positive and negative components (Da Fonseca & Ignatieva, 2019), or considering the impact of news and economic events on the cryptocurrency market (Al-Khazali, Bouri, & Roubaud, 2018).

Appendix A.

Table A1
Market capitalization of the 12 cryptocurrencies under study.

Ranking	Name	Market Cap
1st	Bitcoin	71,534,370,136
2nd	Ethereum	15,112,839,523
3rd	Ripple	13,341,320,084
4th	Litecoin	3,760,335,428
8th	Stellar	2,124,180,913
13	Monero	932,105,239
15	Dash	808,442,715
20	Nem	456,094,403
27	Dogecoin	246,750,908
37	DigiByte	169,901,990
43	Bytecoin	150,407,572
46	BitShares	139,972,815

Note: The ranking is based on the rank of the 12 cryptocurrencies under study within the first largest 50 cryptocurrencies as in https://coinmarketcap.com.

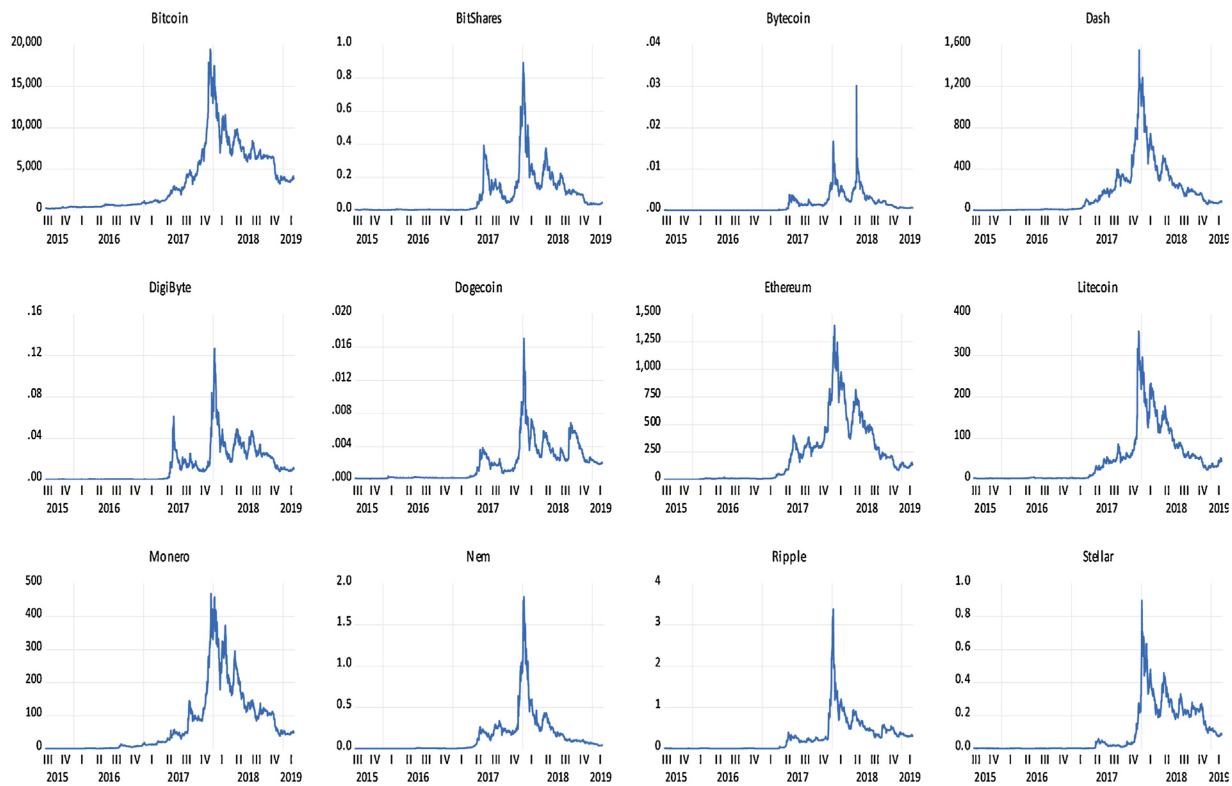


Fig. A1. Plots of price levels of cryptocurrencies under study.

[8] The authors indicate that this can be done by trading options or investing in hedge funds.

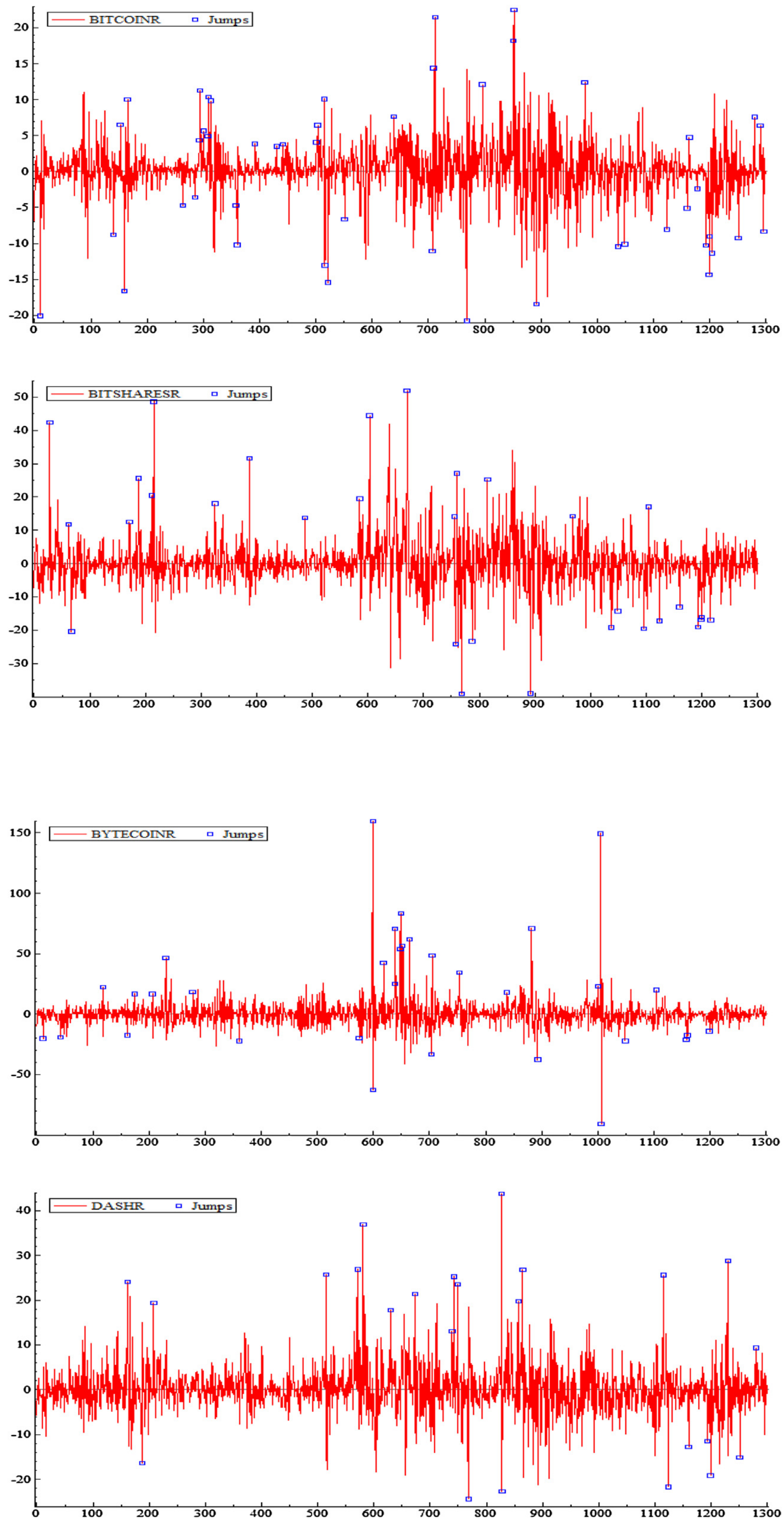
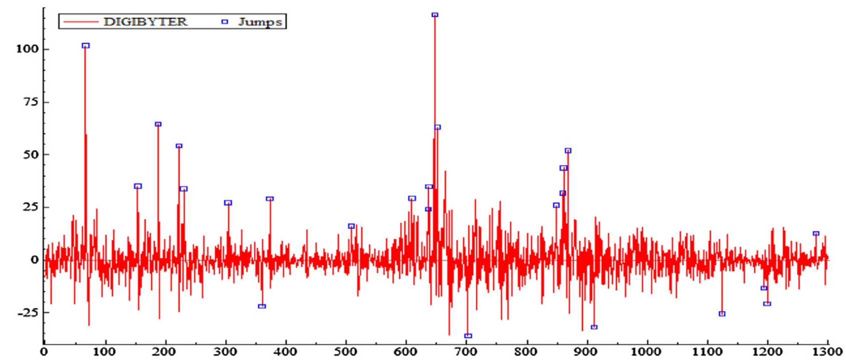
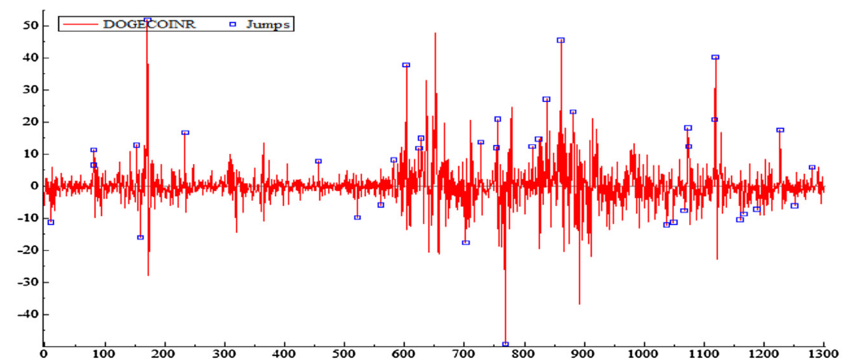
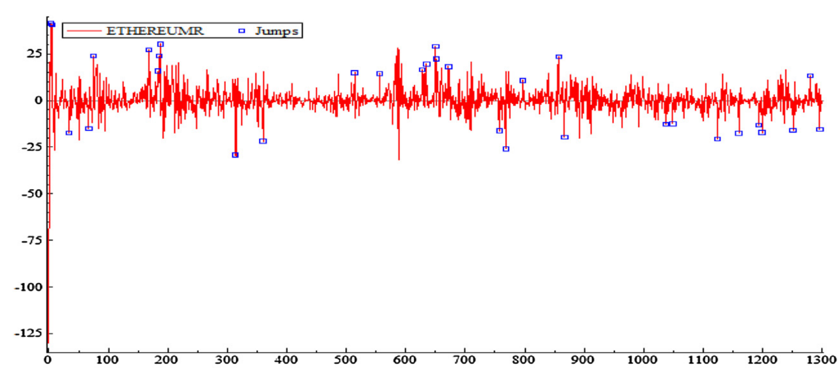


Fig. A2. Plots of jumps on the return series of cryptocurrencies under study.







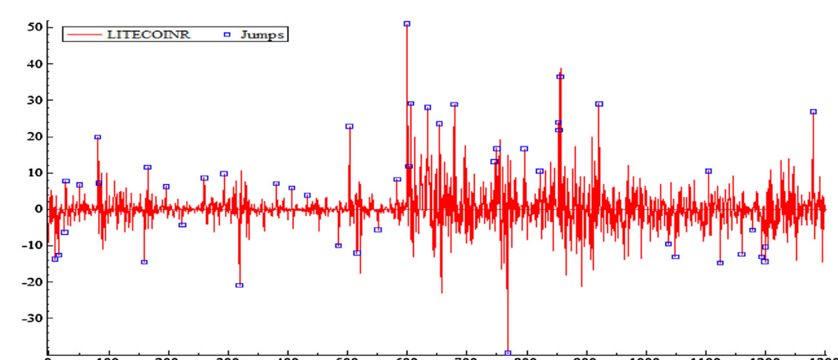


Fig. A2. (*Continued*)

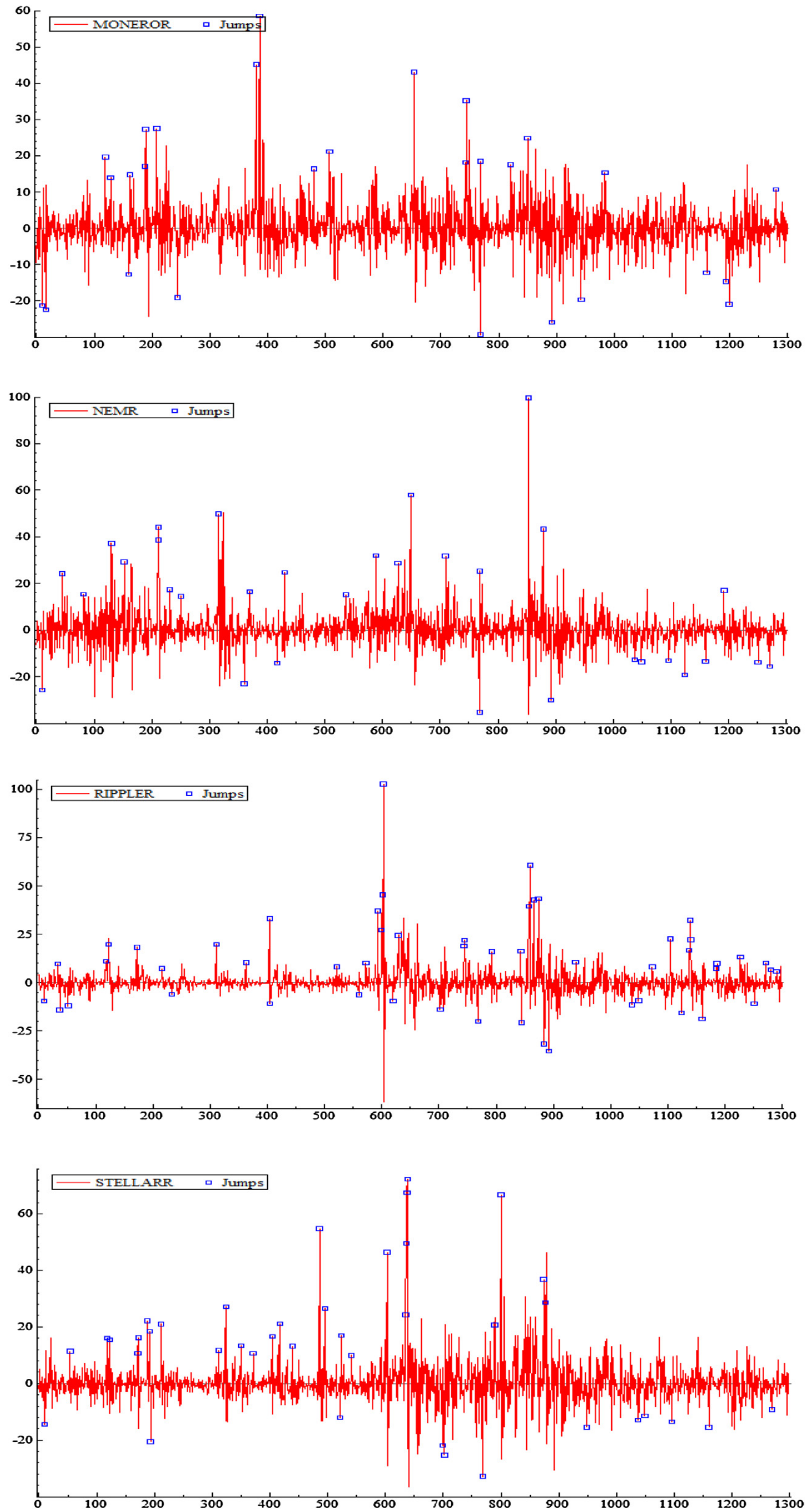


Fig. A2. (*Continued*)

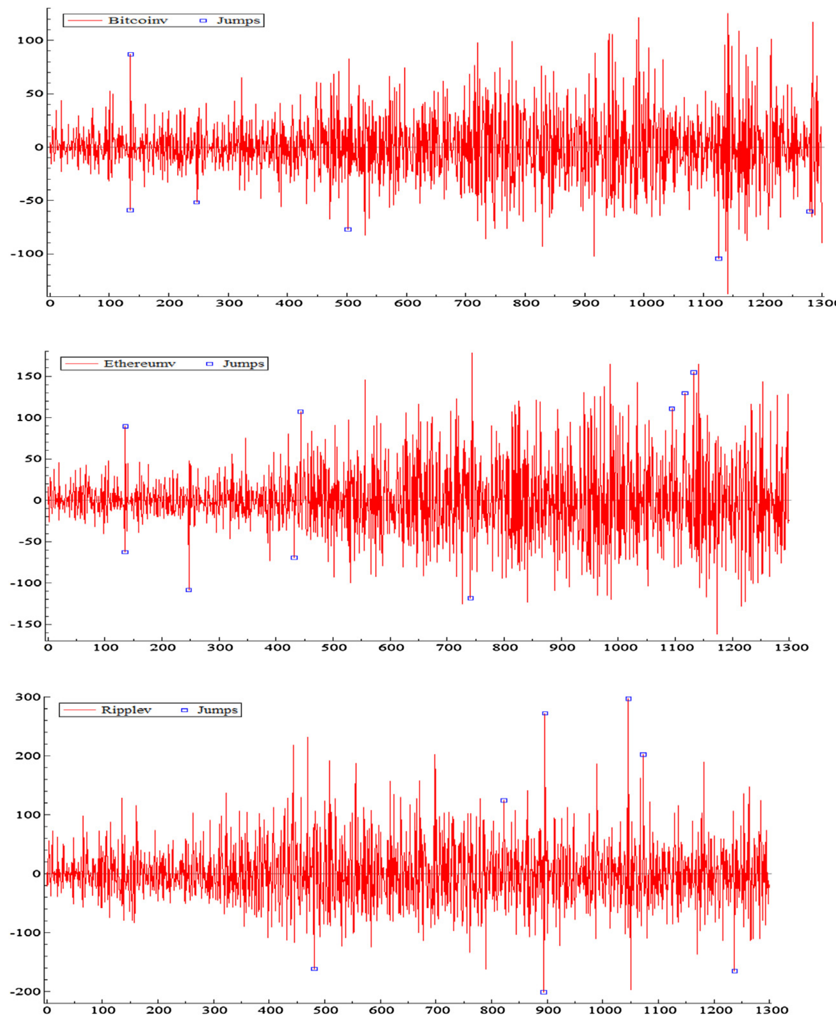
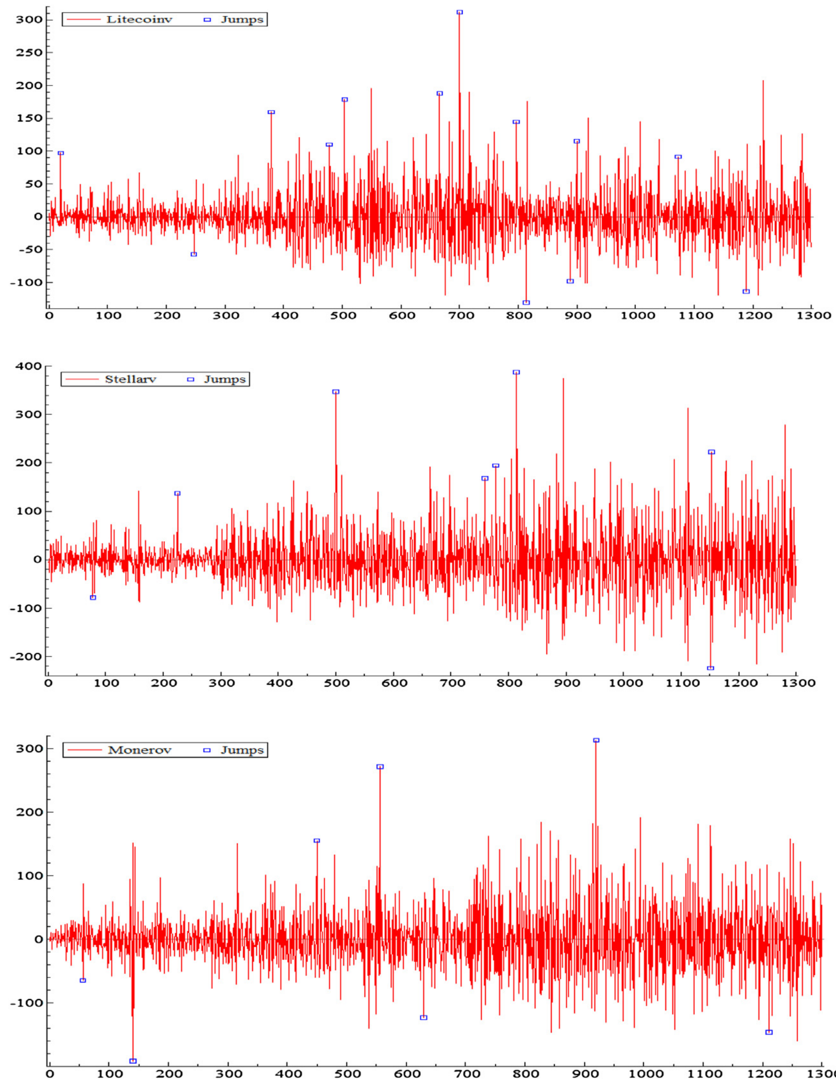


Fig. A3. Plots of jumps on the trading volume of cryptocurrencies under study.



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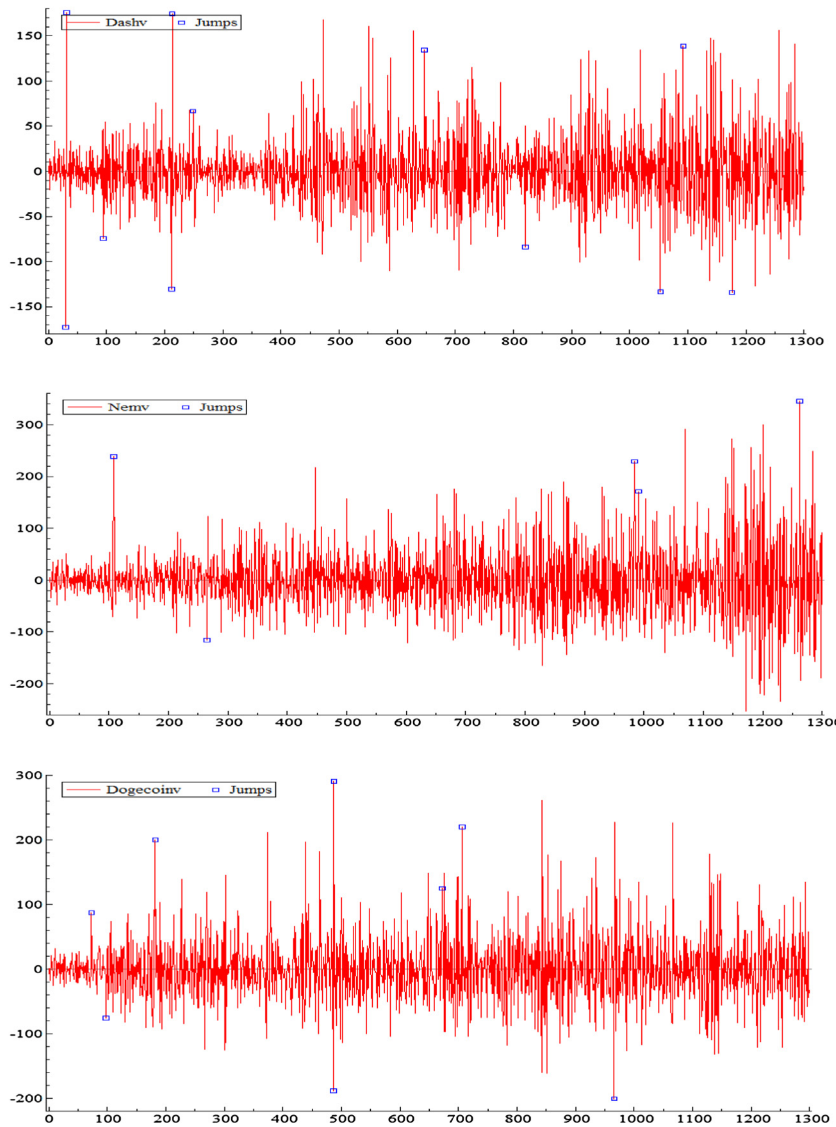

Fig. A3. (*Continued*)

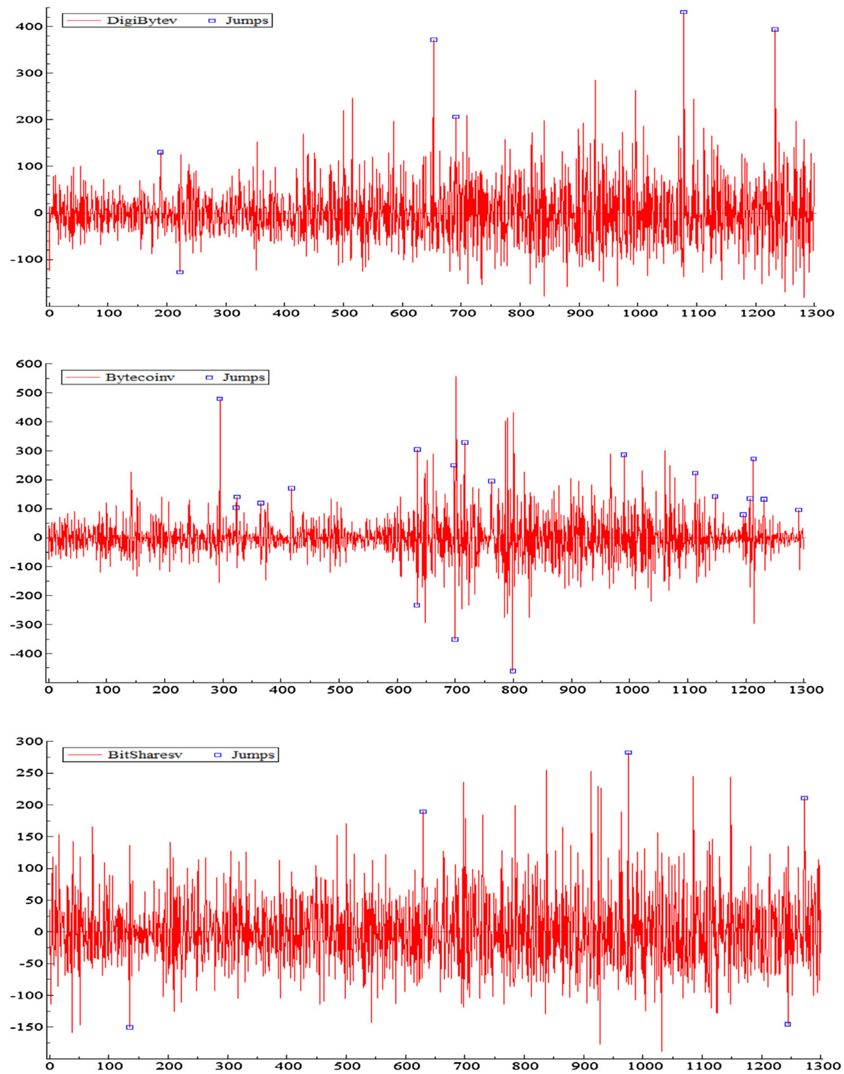

Fig. A3. (*Continued*)

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