1 Models

1.1 SVMALD

$$\begin{bmatrix} Y_{1,t+1} - Y_{1,t} \\ Y_{2,t+1} - Y_{1,t} \\ V_{1,t+1} - V_{1,t} \\ V_{2,t+1} - V_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \Delta \\ \mu_2 \Delta \\ \kappa_1(\theta_1 - V_{1,t})\Delta \\ \kappa_2(\theta_2 - V_{2,t})\Delta \end{bmatrix} + \sqrt{\Delta}\Sigma_t^{\frac{1}{2}} \begin{bmatrix} \epsilon_{y_1,t+1} \\ \epsilon_{y_2,t+1} \\ \epsilon_{v_1,t+1} \\ \epsilon_{v_2,t+1} \end{bmatrix} + \begin{bmatrix} N_{t+1}(M_{y_1,t+1}\xi_{y_1,t+1} + M_{c,t+1}\xi_{y_1,c,t+1}) \\ N_{t+1}(M_{y_2,t+1}\xi_{y_2,t+1} + M_{c,t+1}\xi_{y_2,c,t+1}) \\ 0 \\ 0 \end{bmatrix}, \quad (1)$$

where $\Sigma_t = \begin{bmatrix} V_{1,t} & \rho_y\sqrt{V_{1,t}V_{2,t}} & \rho_1\sigma_{v,1}V_{1,t} & 0 \\ \rho_y\sqrt{V_{1,t}V_{2,t}} & V_{2,t} & 0 & \rho_2\sigma_{v,2}V_{2,t} \\ \rho_1\sigma_{v,1}V_{1,t} & 0 & \sigma_{v,1}^2 V_{1,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} \\ 0 & \rho_2\sigma_{v,2}V_{2,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} & \sigma_{v,2}^2 V_{2,t} \end{bmatrix}$,
$\epsilon_{k,t+1} \overset{iid}{\sim} N(0,1), k = \{y_1, y_2, v_1, v_2\}$, $\xi_{y_1,t+1} \overset{iid}{\sim} \mathcal{AL}_1(\mu_{y_1}, \nu_{y_1}^2)$, $\xi_{y_2,t+1} \overset{iid}{\sim} \mathcal{AL}_1(\mu_{y_2}, \nu_{y_2}^2)$, $\boldsymbol{\xi}_{c,t+1} = [\xi_{y_1,c,t+1}, \xi_{y_2,c,t+1}]' \overset{iid}{\sim} \mathcal{AL}_2(\boldsymbol{\mu}_c, \boldsymbol{\nu}_c)$, $\Big[M_{y_1,t+1}, M_{y_2,t+1}, M_{c,t+1} \sim Multinomial(1; \frac{\lambda_{y_1}}{\lambda}, \frac{\lambda_{y_2}}{\lambda}, \frac{\lambda_c}{\lambda})\Big]$, and $N_{t+1} \sim Bernoulli(\lambda\Delta)$

1.2 SVLD

$$\begin{bmatrix} Y_{1,t+1} - Y_{1,t} \\ Y_{2,t+1} - Y_{1,t} \\ V_{1,t+1} - V_{1,t} \\ V_{2,t+1} - V_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \Delta \\ \mu_2 \Delta \\ \kappa_1(\theta_1 - V_{1,t})\Delta \\ \kappa_2(\theta_2 - V_{2,t})\Delta \end{bmatrix} + \sqrt{\Delta}\Sigma_t^{\frac{1}{2}} \begin{bmatrix} \epsilon_{y_1,t+1} \\ \epsilon_{y_2,t+1} \\ \epsilon_{v_1,t+1} \\ \epsilon_{v_2,t+1} \end{bmatrix} + \begin{bmatrix} N_{t+1}(M_{y_1,t+1}\xi_{y_1,t+1} + M_{c,t+1}\xi_{y_1,c,t+1}) \\ N_{t+1}(M_{y_2,t+1}\xi_{y_2,t+1} + M_{c,t+1}\xi_{y_2,c,t+1}) \\ 0 \\ 0 \end{bmatrix}, \quad (2)$$

where $\Sigma_t = \begin{bmatrix} V_{1,t} & \rho_y\sqrt{V_{1,t}V_{2,t}} & \rho_1\sigma_{v,1}V_{1,t} & 0 \\ \rho_y\sqrt{V_{1,t}V_{2,t}} & V_{2,t} & 0 & \rho_2\sigma_{v,2}V_{2,t} \\ \rho_1\sigma_{v,1}V_{1,t} & 0 & \sigma_{v,1}^2 V_{1,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} \\ 0 & \rho_2\sigma_{v,2}V_{2,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} & \sigma_{v,2}^2 V_{2,t} \end{bmatrix}$,
$\epsilon_{k,t+1} \overset{iid}{\sim} N(0,1), k = \{y_1, y_2, v_1, v_2\}$, $\xi_{y_1,t+1} \overset{iid}{\sim} \mathcal{AL}_1(0, \nu_{y_1}^2)$, $\xi_{y_2,t+1} \overset{iid}{\sim} \mathcal{AL}_1(0, \nu_{y_2}^2)$, $\boldsymbol{\xi}_{c,t+1} = [\xi_{y_1,c,t+1}, \xi_{y_2,c,t+1}]' \overset{iid}{\sim} \mathcal{AL}_2(\mathbf{0}, \boldsymbol{\nu}_c)$, $\Big[M_{y_1,t+1}, M_{y_2,t+1}, M_{c,t+1} \sim Multinomial(1; \frac{\lambda_{y_1}}{\lambda}, \frac{\lambda_{y_2}}{\lambda}, \frac{\lambda_c}{\lambda})\Big]$, and $N_{t+1} \sim Bernoulli(\lambda\Delta)$

1.3 SVMVN

$$\begin{bmatrix} Y_{1,t+1} - Y_{1,t} \\ Y_{2,t+1} - Y_{1,t} \\ V_{1,t+1} - V_{1,t} \\ V_{2,t+1} - V_{2,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \Delta \\ \mu_2 \Delta \\ \kappa_1(\theta_1 - V_{1,t})\Delta \\ \kappa_2(\theta_2 - V_{2,t})\Delta \end{bmatrix} + \sqrt{\Delta}\Sigma_t^{\frac{1}{2}} \begin{bmatrix} \epsilon_{y_1,t+1} \\ \epsilon_{y_2,t+1} \\ \epsilon_{v_1,t+1} \\ \epsilon_{v_2,t+1} \end{bmatrix} + \begin{bmatrix} N_{t+1}(M_{y_1,t+1}\xi_{y_1,t+1} + M_{c,t+1}\xi_{y_1,c,t+1}) \\ N_{t+1}(M_{y_2,t+1}\xi_{y_2,t+1} + M_{c,t+1}\xi_{y_2,c,t+1}) \\ 0 \\ 0 \end{bmatrix}, \quad (3)$$

where $\Sigma_t = \begin{bmatrix} V_{1,t} & \rho_y\sqrt{V_{1,t}V_{2,t}} & \rho_1\sigma_{v,1}V_{1,t} & 0 \\ \rho_y\sqrt{V_{1,t}V_{2,t}} & V_{2,t} & 0 & \rho_2\sigma_{v,2}V_{2,t} \\ \rho_1\sigma_{v,1}V_{1,t} & 0 & \sigma_{v,1}^2 V_{1,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} \\ 0 & \rho_2\sigma_{v,2}V_{2,t} & \rho_v\sigma_{v,1}\sigma_{v,2}\sqrt{V_{1,t}V_{2,t}} & \sigma_{v,2}^2 V_{2,t} \end{bmatrix}$,
$\epsilon_{k,t+1} \overset{iid}{\sim} N(0,1), k = \{y_1, y_2, v_1, v_2\}$, $\xi_{y_1,t+1} \overset{iid}{\sim} N(\mu_{y_1}, \nu_{y_1}^2)$, $\xi_{y_2,t+1} \overset{iid}{\sim} N(\mu_{y_2}, \nu_{y_2}^2)$, $\boldsymbol{\xi}_{c,t+1} = [\xi_{y_1,c,t+1}, \xi_{y_2,c,t+1}]' \overset{iid}{\sim} MVN(\boldsymbol{\mu}_c, \boldsymbol{\nu}_c)$, $\Big[M_{y_1,t+1}, M_{y_2,t+1}, M_{c,t+1} \sim Multinomial(1; \frac{\lambda_{y_1}}{\lambda}, \frac{\lambda_{y_2}}{\lambda}, \frac{\lambda_c}{\lambda})\Big]$, and $N_{t+1} \sim Bernoulli(\lambda\Delta)$

1.4 SVIND

$$\begin{bmatrix} Y_{t+1} - Y_t \\ V_{t+1} - V_t \end{bmatrix} = \begin{bmatrix} \mu\Delta \\ \kappa(\theta - V_t)\Delta \end{bmatrix} + \sqrt{V_t\Delta} \begin{bmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{1-\rho^2}\sigma_v \end{bmatrix} \begin{bmatrix} \epsilon_{y,t+1} \\ \epsilon_{v,t+1} \end{bmatrix} + \begin{bmatrix} N_{y,t+1}\xi_{y,t+1} \\ 0 \end{bmatrix}, \quad (4)$$

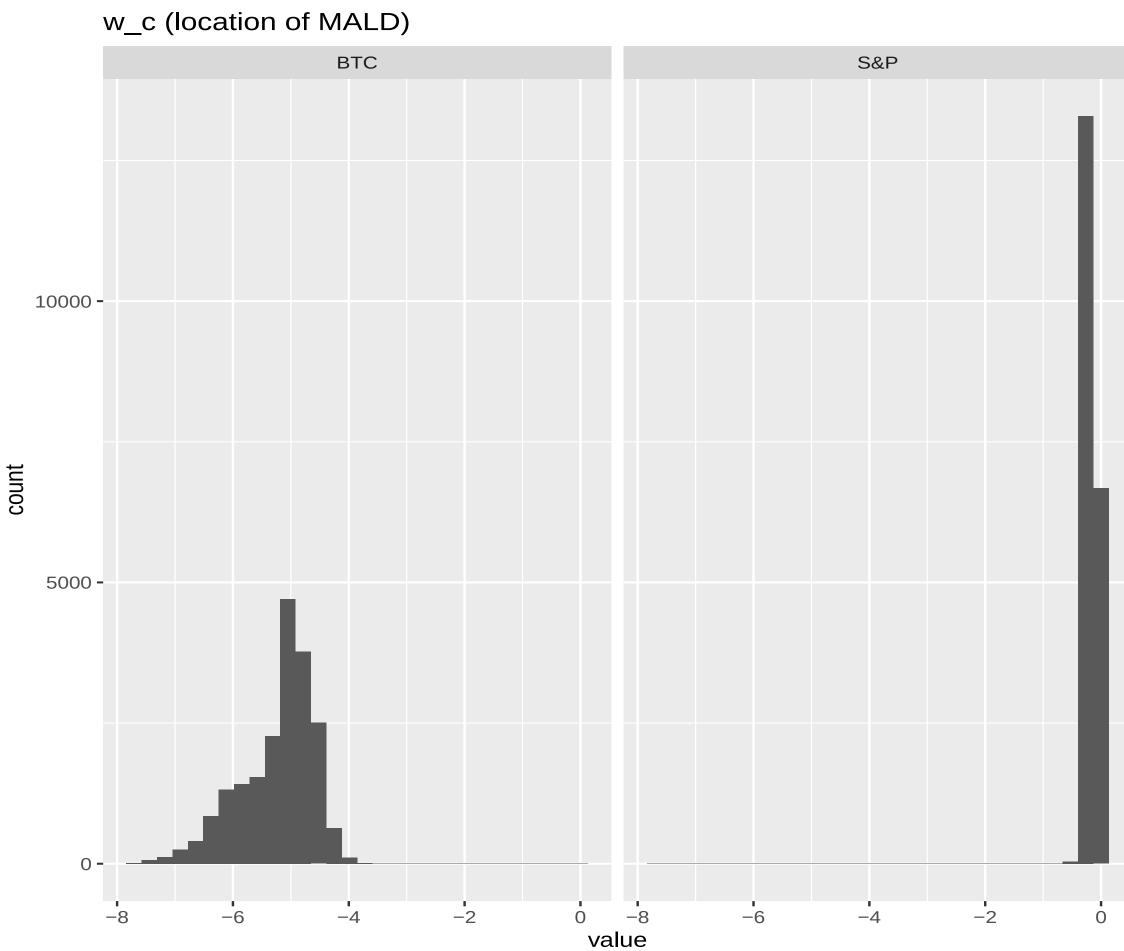
$\epsilon_{k,t+1} \overset{iid}{\sim} N(0,1), k = \{y, v\}$, $\xi_{y,t+1} \overset{iid}{\sim} \mathcal{AL}_1(\mu_y, \nu_y^2)$, $N_{y,t+1} \sim Bernoulli(\lambda\Delta)$

2 Empirical study: S&P and BTC jointly

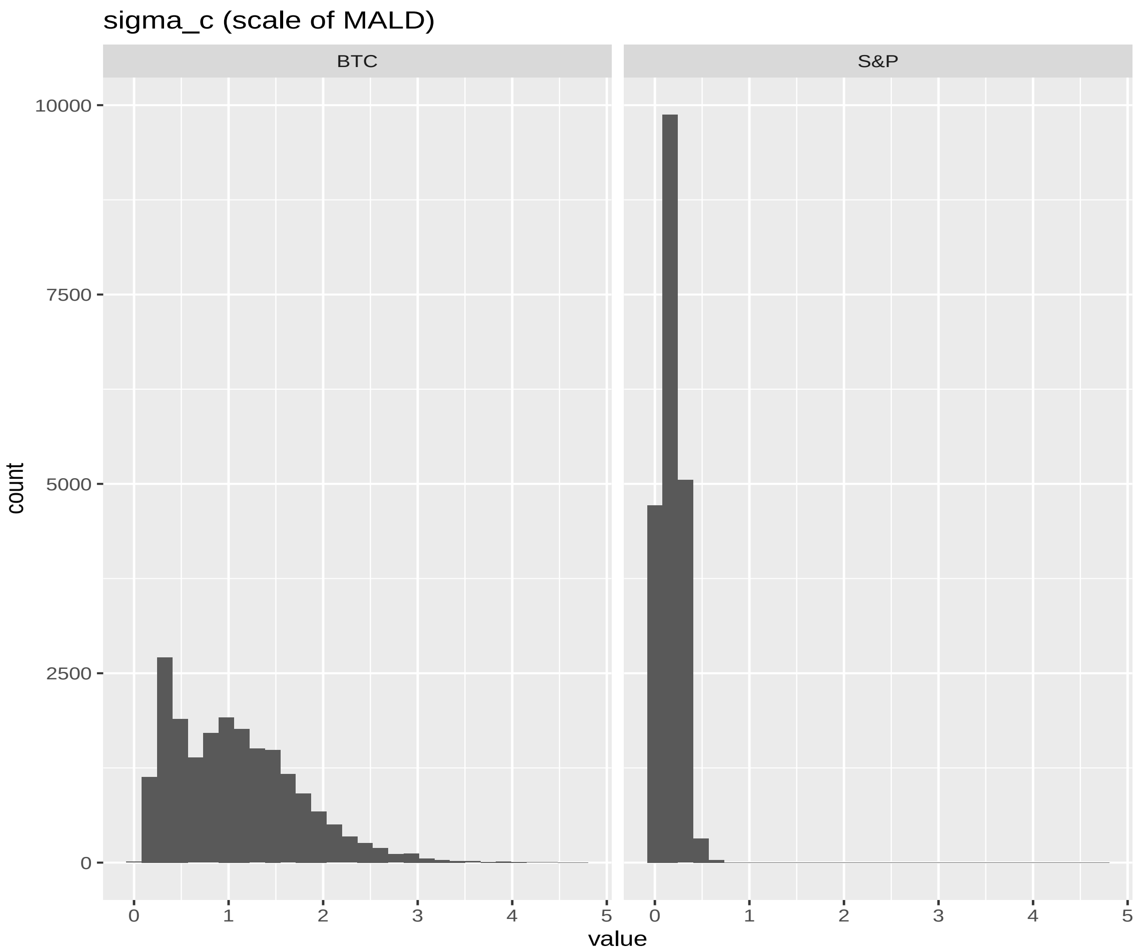
2.1 Results: Joint Jumps

```
##
## Attaching package:  'dplyr'
## The following objects are masked from 'package:stats':
##
##     filter, lag
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
## List of 13
##  $ lambda  : num [1:4] 0.097 0.0367 0.085 0.7813
##  $ sigma_v : num [1:2] 0.436 0.212
##  $ sigma_c : num [1:2] 1.074 0.174
##  $ rhoc    : num -0.104
##  $ xi_cw   : num [1:2] -5.236 -0.166
##  $ xi_y1eta: num 0.839
##  $ xi_y1w  : num 4.91
##  $ xi_y2eta: num 0.347
##  $ xi_y2w  : num -0.668
##  $ phi     : num [1:2] 0.985 0.976
##  $ theta   : num [1:2] 6.96 1.12
##  $ mu      : num [1:2] 0.165 0.0547
##  $ rho     : num [1:4] 0.00665 0.07259 0.45863 -0.70195
```

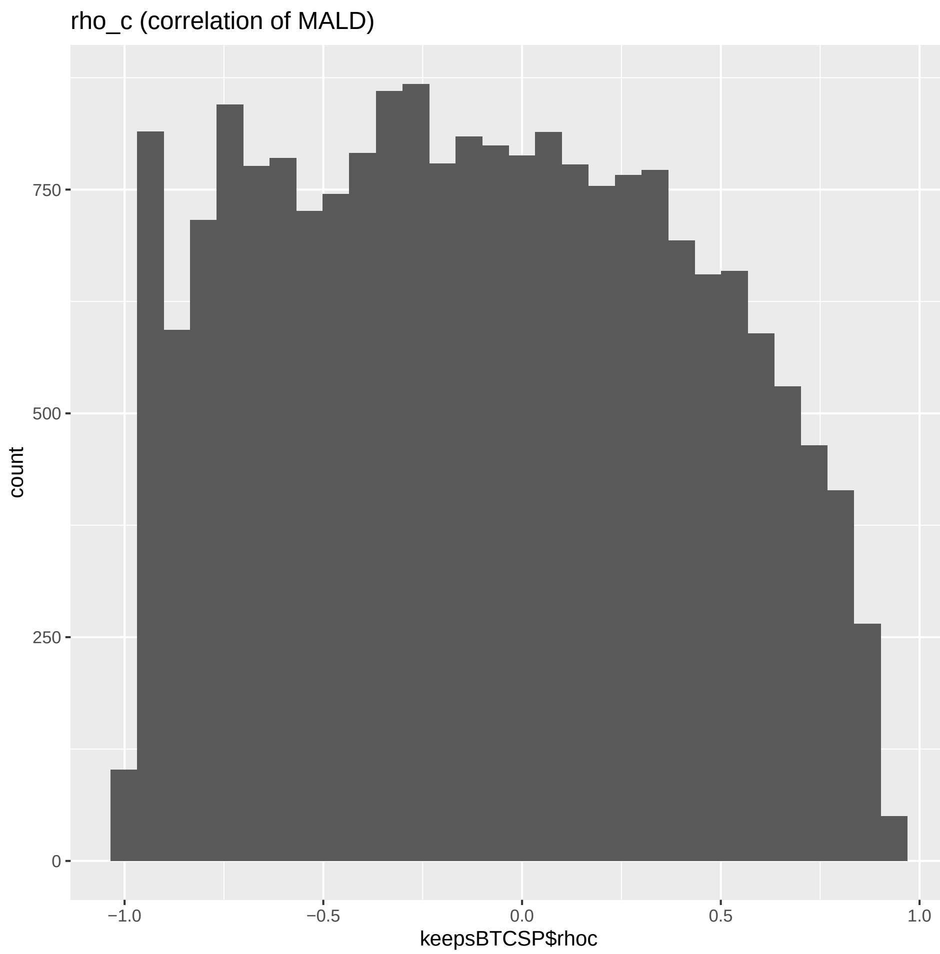
```r
#Related to joint jumps:

keepsBTCSP$xi_cw %>%
  as.data.frame() %>%
  melt() %>%
  mutate(variable = factor(variable, levels = c("V1", "V2"),labels = c("BTC", "S&P")))%>%
  ggplot() +
  geom_histogram(aes(x = value)) +
  facet_grid(~variable)+
  ggtitle("w_c (location of MALD)")

## No id variables; using all as measure variables
## `stat_bin()` using `bins = 30`.  Pick better value with `binwidth`.
```



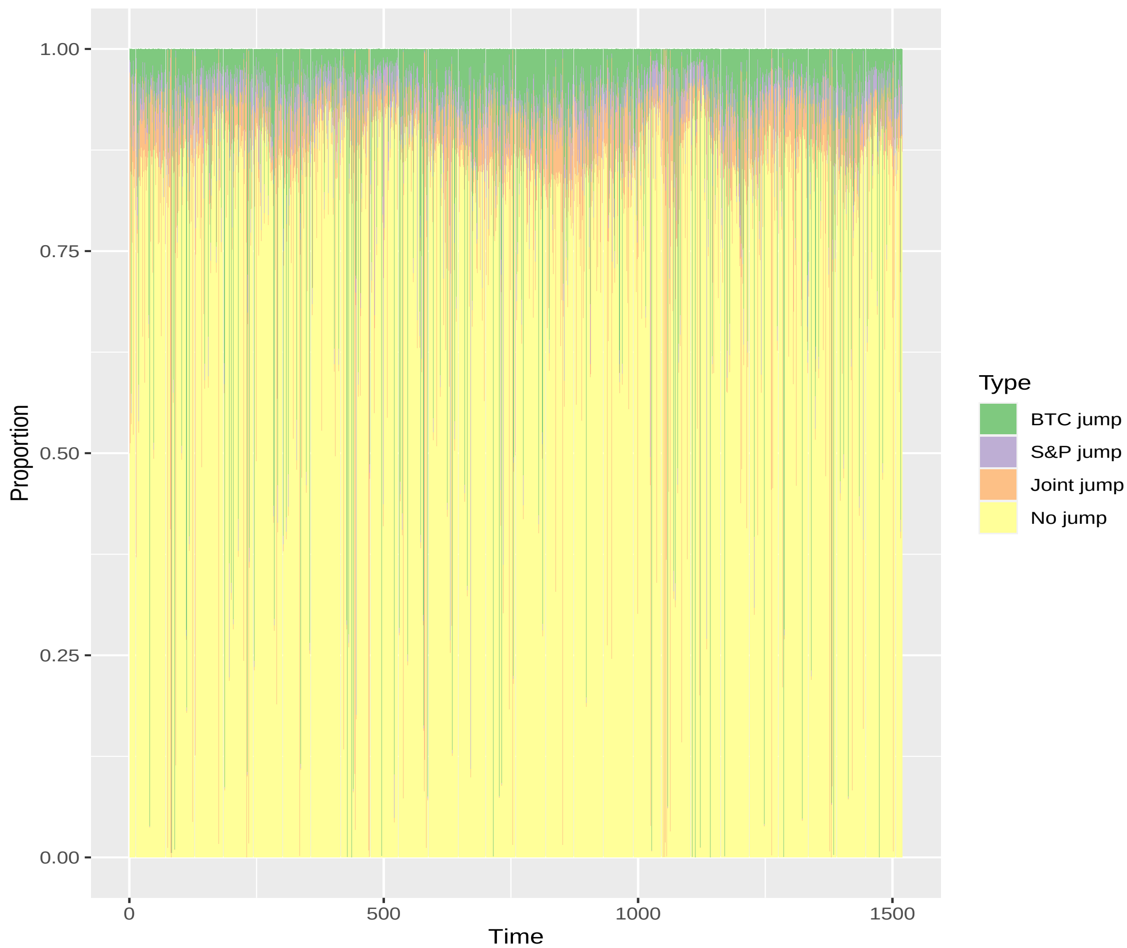
```r
keepsBTCSP$sigma_c %>%
    as.data.frame() %>%
  melt() %>%
  mutate(variable = factor(variable, levels = c("V1", "V2"),labels = c("BTC", "S&P")))%>%
  ggplot() +
  geom_histogram(aes(x = value)) +
  facet_grid(~variable) +
  ggtitle("sigma_c (scale of MALD)")

## No id variables; using all as measure variables
## `stat_bin()` using `bins = 30`.  Pick better value with `binwidth`.
```



```r
ggplot() +
  geom_histogram(aes(x = keepsBTCSP$rhoc)) +
  ggtitle("rho_c (correlation of MALD)")

## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



```
keepsBTCSP$delta %>%
  melt() %>%
  mutate(value = factor(value, levels = c(0:3),
                        labels = c("BTC jump", "S&P jump", "Joint jump", "No jump"))) %>%
  ggplot() +
  geom_bar(aes(x = Var2, fill = value), position = "fill") +
  scale_fill_brewer("Type",type = "qual") +labs(x = "Time", y = "Proportion")
```




```
#it's worth noting the MVN plot looks almost identical - same patterns
```

	SVMALD	SVLD	SVMVN	SVIND
Diffuse priors	9813.312	**9796.879**	9818.368	10953.52
Fulop-style priors	9827.731	9854.88	**9714.333**	10884.23

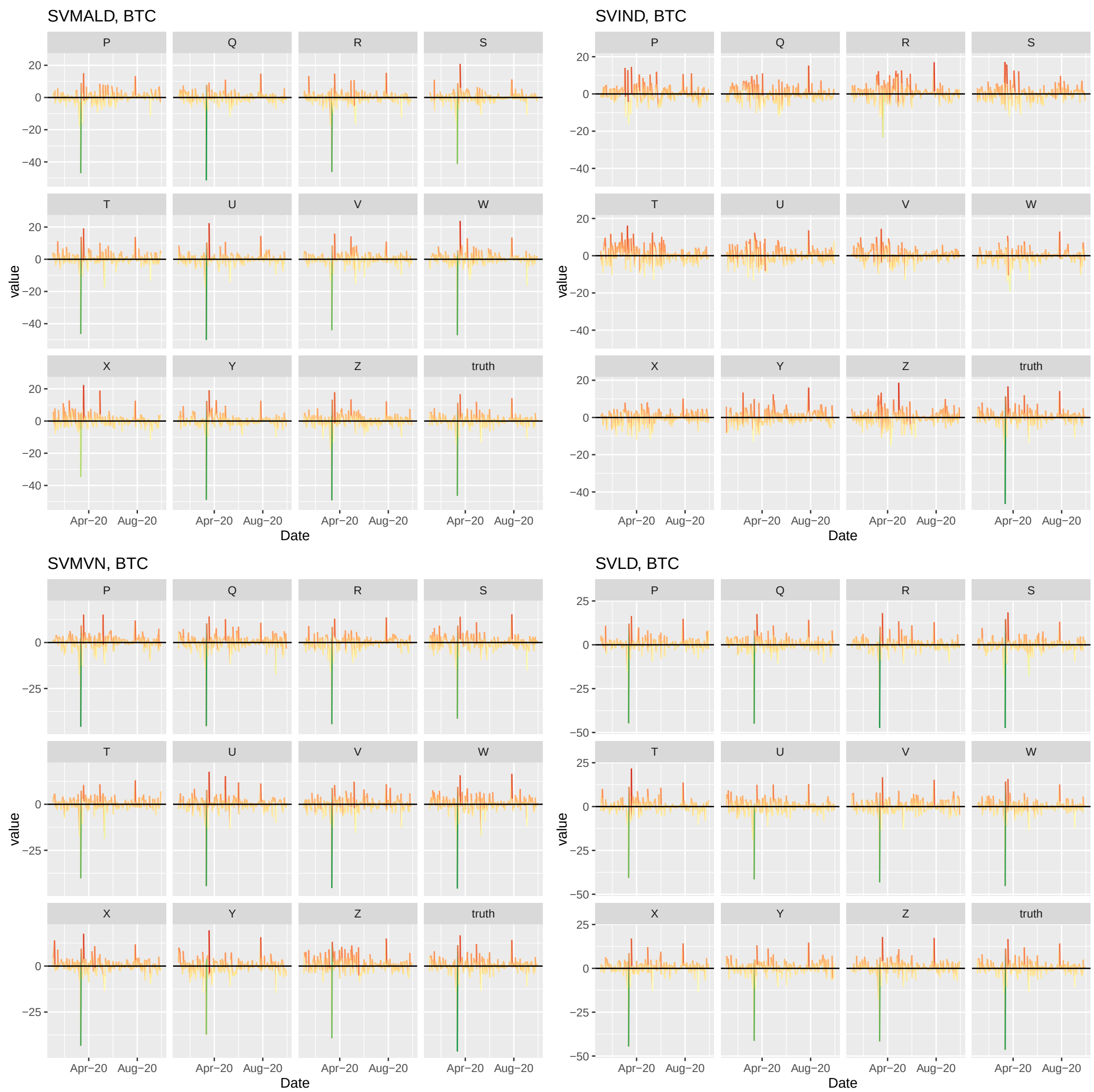
Table 1: Resulting DIC from runs with diffuse priors and runs with more informative priors.

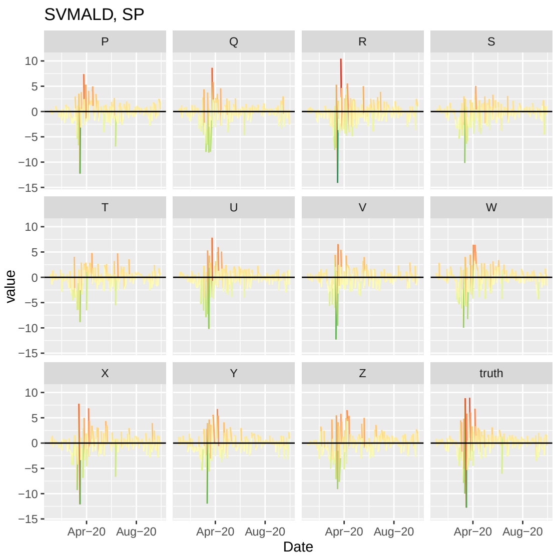
2.2 DIC

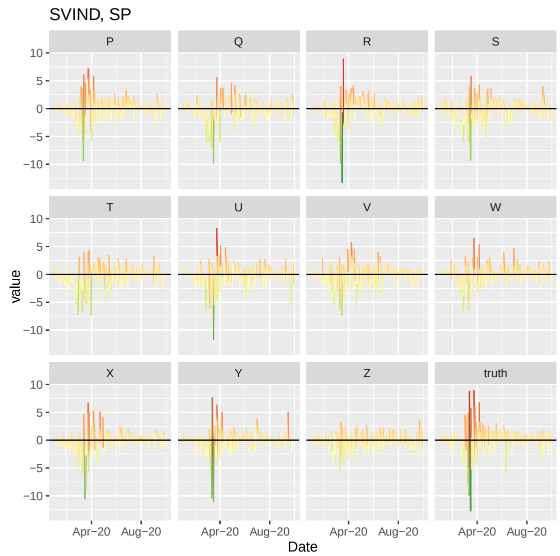
$$DIC_7 = -4E_{\theta,\boldsymbol{z}|\boldsymbol{y}}(ln(p(\boldsymbol{y}|\theta,\boldsymbol{z}))) + 2ln(p(\boldsymbol{y}|\hat{\theta},\hat{\boldsymbol{z}})).$$

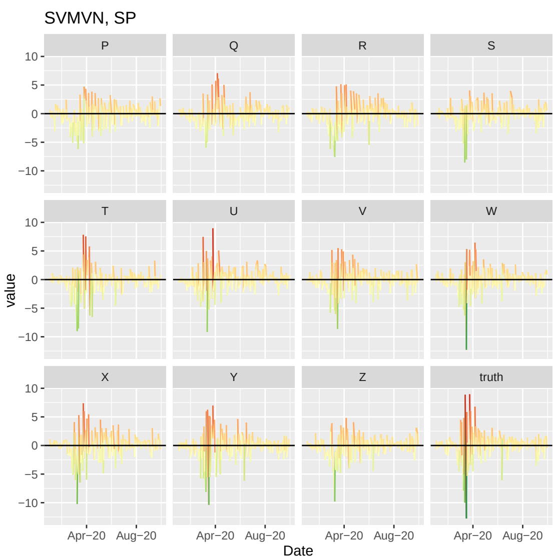
2.3 Posterior predictive p-values

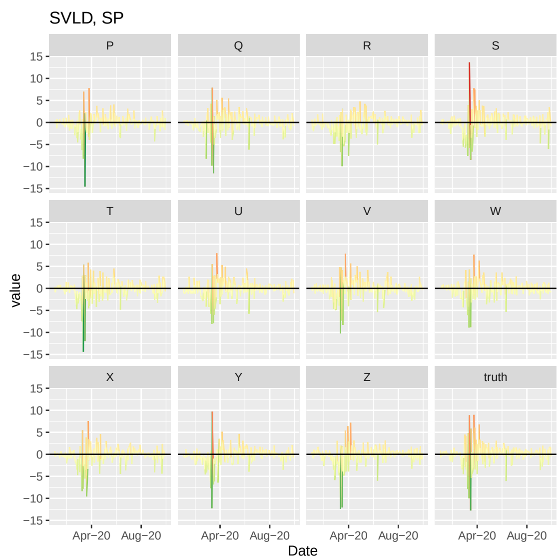
First, a lineup to motivate. Posterior predictive p-values operate on the assumption that $p(y^* \mid Y) = \int p(y^* \mid \theta)p(\theta \mid Y)d\theta$ should be consistent with what we observed in Y.

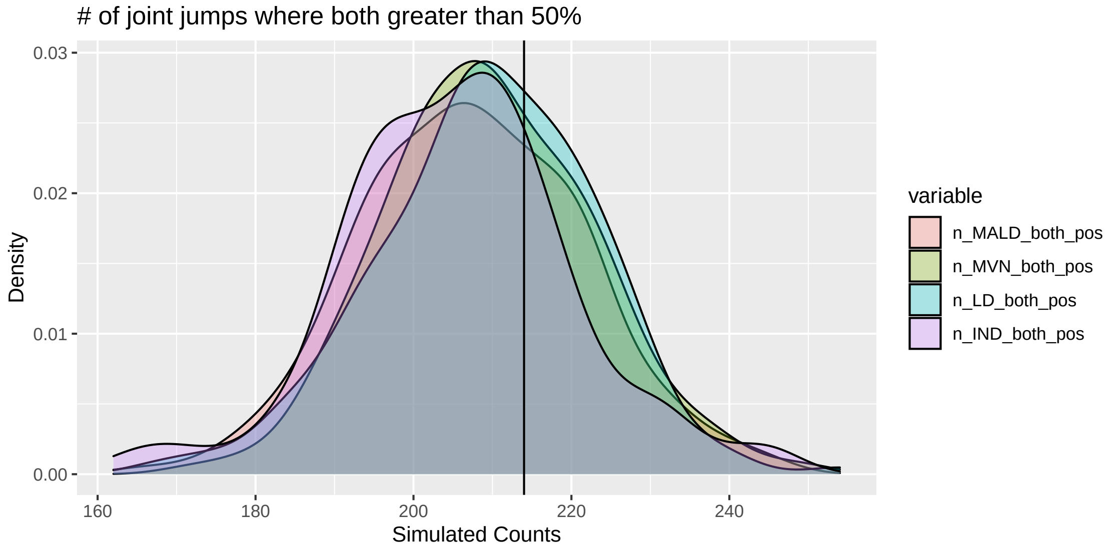



SVMALD, SP
P
Q
R
S
T
U
V
W
X
Y
Z
truth
value
Apr−20
Aug−20
Date

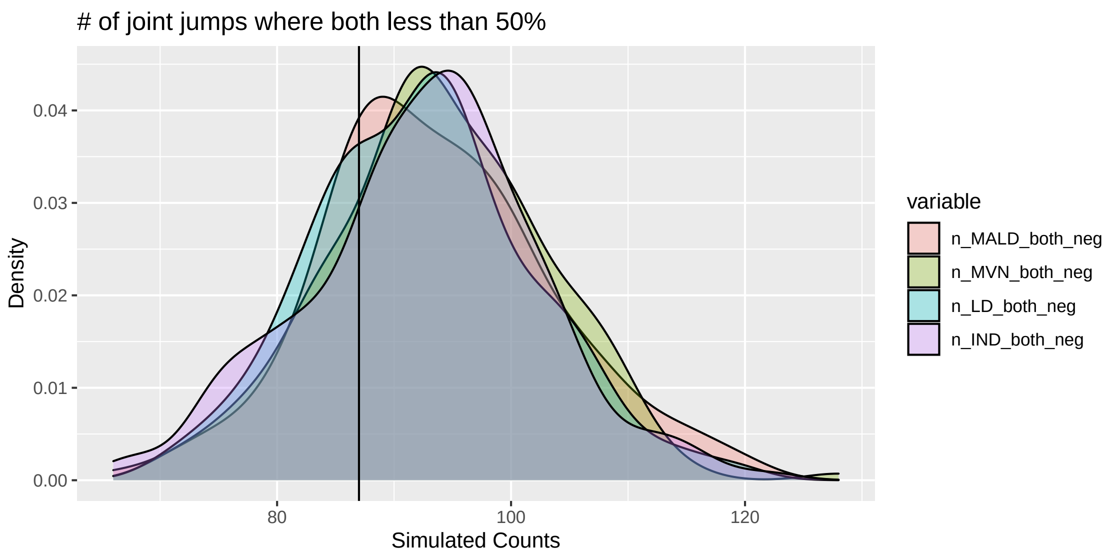

SVIND, SP
P
Q
R
S
T
U
V
W
X
Y
Z
truth
value
Apr−20
Aug−20
Date


SVMVN, SP
P
Q
R
S
T
U
V
W
X
Y
Z
truth
value
Apr−20
Aug−20
Date

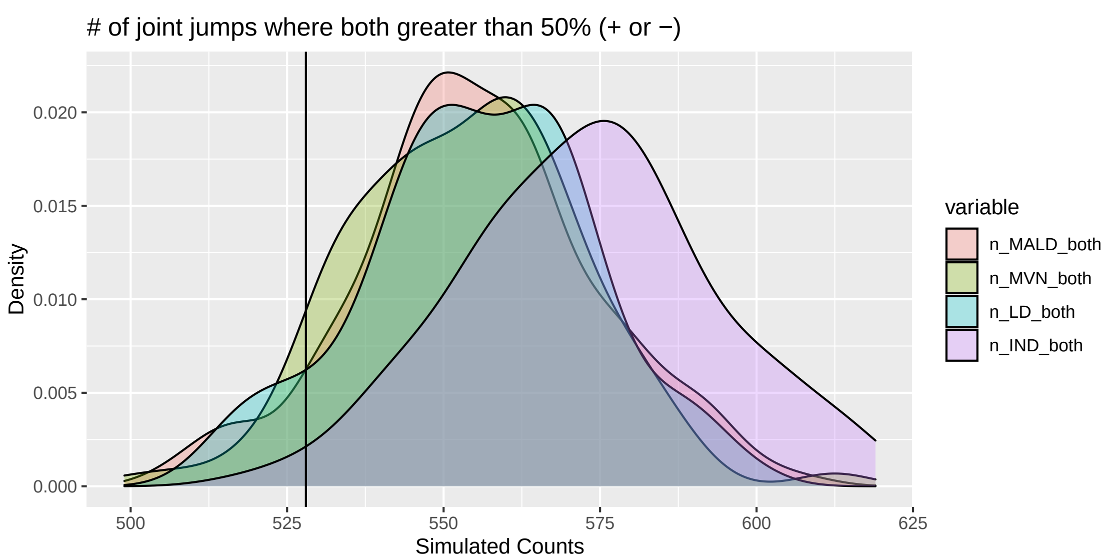

SVLD, SP
P
Q
R
S
T
U
V
W
X
Y
Z
truth
value
Apr−20
Aug−20
Date



# of joint jumps where both greater than 50%
Density
Simulated Counts
variable
n_MALD_both_pos
n_MVN_both_pos
n_LD_both_pos
n_IND_both_pos



# of joint jumps where both less than 50%
Density
Simulated Counts
variable
n_MALD_both_neg
n_MVN_both_neg
n_LD_both_neg
n_IND_both_neg



# of joint jumps where both greater than 50% (+ or −)
Density
Simulated Counts
variable
n_MALD_both
n_MVN_both
n_LD_both
n_IND_both