$y_{ij} \sim \text{Bernoulli}(\pi_{ij})$

$\pi_{ij} = \Phi(h_j(\underset{\sim}{x}_i))$

$\Phi^{-1}(\pi_{ij}) = h_j(\underset{\sim}{x}_i) = \sum_{t=1}^{T} g(x_i ; T_t, \underset{\sim}{M}_t^{(j)})$

$= \sum_{t=1}^{T} \sum_{\ell \in \mathcal{L}_t} \theta_{t\ell}^{(j)} I(\underset{\sim}{x}_i \rightsquigarrow (t,\ell))$

— Data Augmentation

$z_{ij} \mid T, M^{(j)} \sim N(h_j(x_i), 1)$

$y_{ij} = I(z_{ij} > 0)$

$R_{ijk} = z_{ij} - \sum_{t \neq k} g(x_i ; T_t, \underset{\sim}{M}_t^{(j)})$

$f(z_{ij} \mid T, M^{(j)})$
$\equiv$
$f(R_{ijk} \mid T, \theta_{k\ell}^{(j)})$

need: $f(T_t \mid \underset{\sim}{Z}) = \Lambda(T_t)$

$\propto f(\underset{\sim}{Z} \mid T_t) f(T_t)$

$= f(T_t) \int\int \left[\prod_{i=1}^{n} f(z_{i1} \mid h_1(\underset{\sim}{x}_i)) f(z_{i2} \mid h_2(\underset{\sim}{x}_i)) \right]$

$\times \prod_{\ell \in \mathcal{L}_t} p(\theta_{1t\ell}) p(\underset{\sim}{\theta}_{2t\ell}) \, d\theta_{1t\ell} \, d\theta_{2t\ell}$

$$= f(T_t) \prod_{\ell \in \mathcal{L}_t} \underbrace{\left[\prod_{j=1}^{2} \underbrace{\int \prod_{i : x_i \rightsquigarrow (t,\ell)} N(z_{ij} \mid \theta_{0j} + h_j(\underset{\sim}{x}_i)) N(\theta_{jt\ell} \mid 0, \sigma_\theta^2) \, d\theta_{jt\ell}}_{\mathcal{L}_{\theta_j}(t\ell)} \right]}_{\mathcal{L}_t(t,\ell) = \mathcal{L}_{\theta_1}(t,\ell) \mathcal{L}_{\theta_2}(t,\ell)}$$

$= \int f(\{R_i\}_{i : x_i \rightsquigarrow (t,\ell)} \mid \theta_{jt\ell}, T) f(\theta_{jt\ell}) d\theta$

$= f(\{Z_{ij}\}_{i : \underset{\sim}{x}_i \rightsquigarrow (t,\ell)} \mid T_t)$

$\mathcal{L}_t(t,\ell) = \mathcal{L}_{\theta_1}(t,\ell) \mathcal{L}_{\theta_2}(t,\ell)$

* modification from Linero

$$\mathcal{L}_{\theta_j}(t,\ell) = (\sqrt{2\pi})^{-n_\ell} \sqrt{\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + n_\ell}} \; e^{-\frac{SSE_\ell}{2} - \frac{n_\ell \sigma_\theta^{-2} \bar{R}_\ell^2}{2(n_\ell + \sigma_\theta^{-2})}}$$

where

$n_\ell = \left| \{ i : X_i \rightsquigarrow (t,\ell) \} \right| \qquad SSE_\ell = \sum_{i : X_i \rightsquigarrow (t,\ell)} (R_{ij} - \bar{R}_{j\ell})^2$

$\bar{R}_{j\ell} = \frac{1}{n_\ell} \sum_{i : X_i \rightsquigarrow (t,\ell)} R_{ij} \qquad R_{ij} = Z_{ij} - \theta_0 - \sum_{k \neq t} g(\underset{\sim}{x}_i ; T_t, M_{0,t}^{(j)})$

(Kapelner + Bleich 2016)