Maksym Bondarenko ‘22 analyzed how granular materials can behave similarly to liquids when oscillated. He completed this study at the Lawrenceville School during the 2019-2020 school year, with the help of Dr. Keith Voss.

Fluid-like Properties of Oscillation Driven Granular Material in Connected Vessels

Abstract

Sand does not fit nicely into the fundamental types of matter defined in science. While any given grain is solid on its own, sand as a substance technically qualifies as a fluid, because it “flows under shear stress.” (Encyclopedia Britannica) The goal of this research was to explore the similarities between granular materials and fluids and to see if, on the macro scale, the difference between the two is numerical or categorical. The experimenters focused on a particular property of fluids described by Pascal's Law, which states that fluids transmit the pressure equally in all directions. The goal was to find out if granular materials can exhibit similar behavior, and if so, under what conditions. To observe this, a setup commonly known as connected vessels was used. Under atmospheric pressure, liquid columns in connected tubes will have the same height, regardless of the shapes of the tubes. Instead of using a liquid, the experimenter used a fine-grained Soda Lime Solid Glass Microspheres with the size of 3 µm to 75 µm. (Cospheric, 2019). Initial experiments showed that, as expected, two columns of different heights will not reach equilibrium without external influence. Oscillating the sand was attempted, which, as has been observed to “liquify sand.” (Sixty Symbols, 2011) Different starting heights and varying the frequency and amplitude of the oscillations resulted in heights of the sand columns exhibiting various damped oscillations over time. This led to eventual equilibrium, or near-equilibrium, of the two columns. Due to inconsistencies in the setup and a lack of precise measurement tools, it is impossible to make definitive quantitative judgments about the effects. The observed qualitative results are sufficient to say that sand is categorically different from fluids, and the similarities exhibited under oscillations can set the groundwork for future research exploring what that difference is.

Introduction

Unresolved Sand

Although granular materials have been around for pretty much all of human history, our knowledge of them is still incomplete and is being actively expanded upon to this day. From recent findings in Emergent Surface Tension in Vibrated, Noncohesive Granular Media (James P. D. Clewett and et, 2012) to Rheological behavior of colloidal suspension with long-range interactions (S. Arietaleaniz et al., 2018), our knowledge of sand is constantly evolving and has the power to influence our theory of matter at large. My goal in this research was not to uncover the properties of sand itself but to see how these properties expand and challenge our understanding of fluids. The differences we find between granular materials and fluids help us better understand just what defines fluids. Moreover, the similarities allow us to simulate and experiment on fluid properties in a more accessible granular medium.

The Problem with Fluids

Fluid is currently defined as any liquid or gas or generally any material that cannot sustain a tangential, or shearing, force when at rest and that undergoes a continuous change in shape when subjected to such a stress (Encyclopedia Britannica).

It is important to note that this definition is not enough to differentiate fluids from other forms of matter. On their own, fluids like liquids can sustain their shape at rest just as solids do because of their surface tension. This means that "shearing force" is some non-zero force, and the upper limit of this force is therefore undefined. This leads to an ambiguity where, depending on the magnitude of the shear force and observation time, both solids and granular materials can experience flow (Ferguson, 2016). This means flow itself is not a sufficient condition to define fluids.

The global issue I aim to tackle in my research is this fluid ambiguity. Is there a way for us to define a fluid that will make it categorically different from other forms of matter? In particular, I have focused on the distinction between fluids and granular materials.

Definition of Granular Material

A granular material is a multiphase material made up of a large collection of closely packed solid particles surrounded by a gas or a liquid. Because the ratio of the volume of solid to fluid phases is very high, the particles are in very close contact with each other (Oran, et al.,J 2002).

In general, the size of particles in granular materials can vary between very fine powder of spheres a few nanometers in diameter to stones of gravel a few centimeters in diameter. This means the extent to which granular materials display fluid-like and solid-like behavior varies greatly. In general, the more fine-grained the particles are, the closer they are to fluids.

In general, the consensus is that granular materials are not fluids. The main reason for that is their distinction on the particle level: where granular materials consist of granules which consist of hundreds of millions of molecules, fluids have a single molecule as their fundamental particle. This leads to the fact that particle collisions are elastic for fluids but non-elastic for granular materials, which in turn means any spontaneous movement in granular material seizes very fast without external energy addition. This leads to a few interesting effects, such as thermal collapse and force distribution, which are not observed in fluids. Commonly, sand is shaken in order to simulate prolonged fluid-like behavior. This heats up the system, keeping the kinetic energy high.

Materials and Methods

Experimental Setup

Figure 1. Experimental setup. The beads of granular material (1) around 0.001 mm in diameter are placed in a glass U-tube (2) with an internal diameter of 15 mm. The U-tube is closed on top with foam to avoid the beads escaping the tube. The U-tube is mounted on a harmonic oscillator (3), which shakes the tube with varying frequency and amplitude. The beads are Soda Lime Solid Glass Microspheres with the size of 3 μm to 75 μm.

In total 23 trials were performed with different frequencies and amplitudes, and the 12 highest quality videos were analyzed in detail. In a typical trial, the oscillator was turned on, and the movement of beads was filmed. To reset the setup, the tube was manually tilted to create a difference in height. The amount of granular material in the tube was kept constant in the trials.

Data Collection and Processing

In a trial involving the oscillation, the tube was tilted to create a difference in height then mounted on an oscillator. The camera was turned on and the oscillations were started. After the frequency was set, the setup was not changed.

The video later was processed using Tracker software. I have used the autotracker functionality to track the positions of the topmost points of both columns. The shaking of the camera was accounted for by tracking its origin. Approximately every 3 to 10 frames are labeled depending on the length of the video, which would translate to approximately 0.05 to 0.17 seconds for a timestep.

The data for time and position was later visualized and fitted using Mathematica.

Study Limitations and Potential Improvements

The study was concerned with qualitative observation, so the results are not suitable for quantitative analysis. This arises from the use of soft parts for the tube holder, lack of measurement consistency, and incomplete information about the technical specification of the oscillator. In the future, if adjustments to the setup are made, it can be used for quantitative analysis as well.

The study can be further expanded with the use of particles of different sizes, tubes of different shapes or diameters, oscillations at different angles, and using viscous liquids instead of granular materials.

Results

Observations without Shaking

It appears that when not shaken, sand does not come to equilibrium in a reasonable time frame. Stacking it forever is impossible — at some critical difference in height the sand will "jump" down to equilibrium a few centimeters. These rapid, discrete changes in height contrast with the smooth, continuous changes observed in liquids. Therefore, there is certainly some categorical difference between fluids and granular materials with no external influence.

General Observations

Next, I transitioned to oscillating the sand. 12 videos with approximately the same starting height difference and different frequencies of the oscillations have been tracked. Frequencies of the oscillations ranged from 40 to 200 Hz, with some frequencies being recorded more than others. Preliminary experiments showed that frequencies below 40 Hz made the set-up unstable, and frequencies above 200 Hz required extremely long observations, following the trend of more viscous behavior described in more detail later.

In general, when shaken sand did go to equilibrium, but it also showed an array of unexpected behavior along the way. In some cases, the system asymptomatically approached the equilibrium, while in others it overshot the equilibrium position and oscillated around it. In many cases, the U-tube also experienced horizontal oscillation in the first few seconds, which coincided with the largest drop in height difference. The data is summarized in the following table.

Video number	Freq	Initial diff	Equilibrium achieved?	Initial higher side	Rotation?	Oscillations?
1	40	5.765	no	left	no	no
2	60	5.778	no	left	yes, both sides	no
3	80	6.917	yes	left	yes, both sides	no
4	100	3.770	no	left	yes, counterclockwise	No
5	100	4.346	no	left	yes, clockwise	no
6	100	4.603	yes	left	no	no
7	150	7.146	no	right	no	no
8	150	6.119	yes	left	no	yes, not full
9	Unspecified	7.671	yes	left	yes, clockwise	yes, 2 full
10	200	5.417	no	left	no	no
11	200	4.943	no	left	no	no
12	Unspecified	4.162	yes	left	yes, clockwise	yes, 5 full

Table 1. Qualitative results for different frequencies. Summarizes equilibrium, rotation, oscillation, and initial conditions for 12 different trials.

In 5 out of 12 videos, two sides of the tube eventually reached equilibrium, with 3 out of that 5 experiencing post-equilibrium oscillations.

Another noticeable effect was the horizontal rotation of the tube in the first few seconds observed in 6 out of 12 videos. The presence and direction of the rotation were inconsistent across initial conditions which suggest that those oscillations may be a byproduct of inconsistencies in the setup itself.

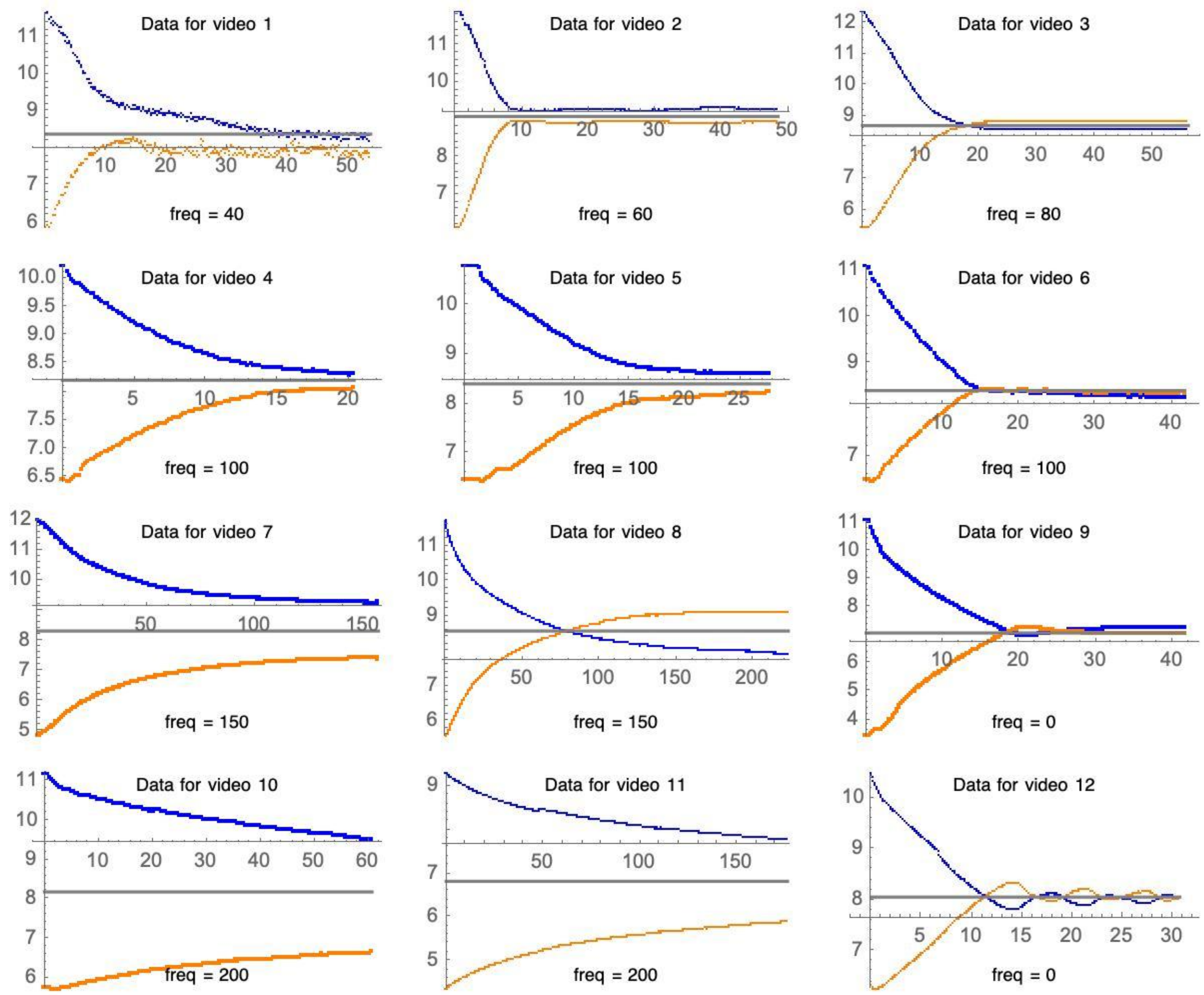

Figure 2. Height changes over time. Height in cm of the sand column on the Y-axis and time passed, in seconds, on the X-axis.

These graphs illustrate how the height of both sides of the U-tubes changed some amount of seconds after oscillations began, with the orange and blue portraying initially lower and higher sides of the tube respectively, and the grey portraying the equilibrium height. In some cases, the heights show asymptotic behavior to equilibrium, and in others it is similar to an exponential decay. In videos 9 and 12, there are also subsequent oscillations.

Modeling the Motion

The different behaviors (oscillations with decaying amplitude, asymptotic approach to equilibrium, and exponential decay to equilibrium) seem to be similar to the motion characterized by the damped oscillations.

In general, damped oscillations are governed by the equation:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$$

This is similar to how one would describe any oscillation, except for the friction factor b dx/dx, which we expect to be significant for the oscillations of sand in the tube.

Solving for x(t) we get:

$$x(t) = A_0 e^{-\frac{b}{2m}t} \cos(\omega t + \phi).$$

Fitting this equation to the height data, we get the following results:

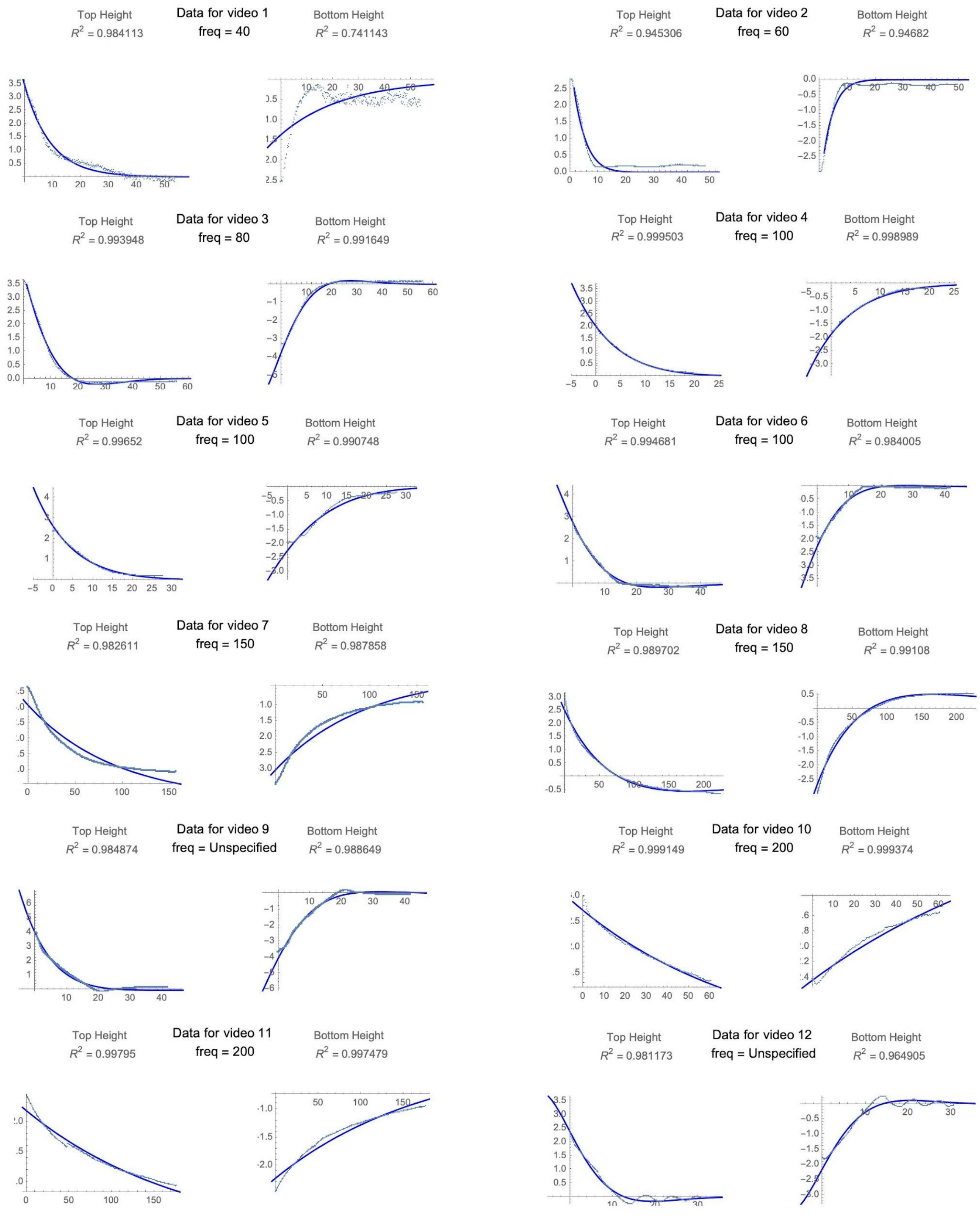

Figure 3. Damping fit for different processes. The damping equation approximates the motion well and yields high R values. In a few cases, the fitting fails to find the optimal solution such as in fitting for data from video 12 or video 7. Due to imperfections in the fitting functionality itself when dealing with huge amounts of data and when data is trimmed to a smaller dataset damping function follows the data more closely.

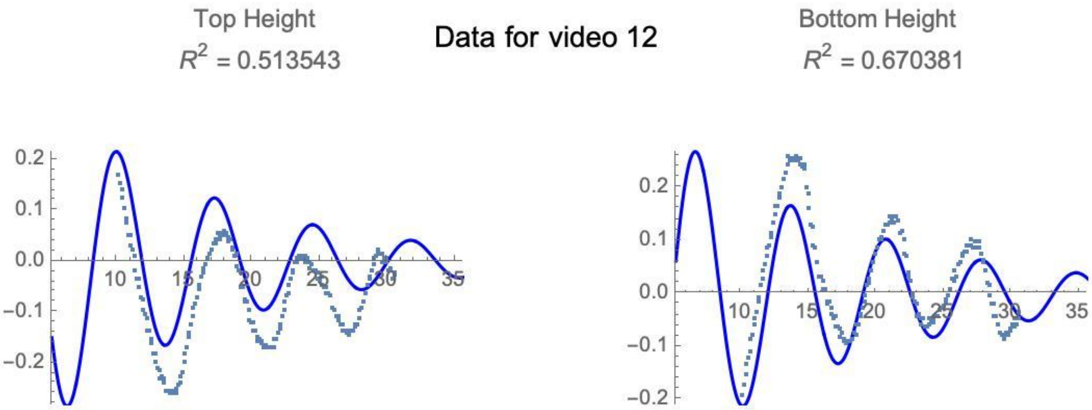

Figure 4. Cut-off fit for video 12. If the function is tasked with fitting only the data after 10 seconds it follows the true trajectory more closely.

Frequency of Oscillations and Its Impact on Motion

The range of frequencies gives us an ability to analyze its impact on motion. In the following graph, the drop in height for the top side of the tube in the first 5 seconds is graphed against the frequencies of the oscillations in the trials. This change is representative of the rate at which the system approaches equilibrium.

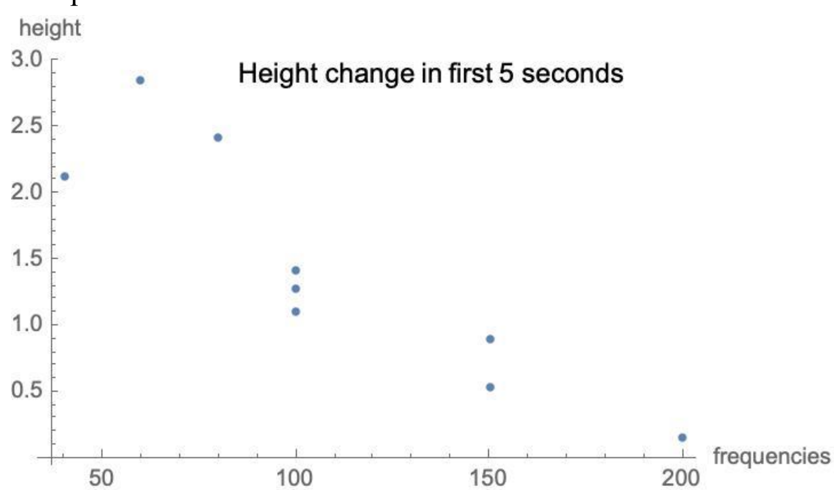

Figure 5. Height changes in the first five seconds. The rate at which the system approaches the equilibrium peaks for some frequency in the range of 40 and 80 Hz. The frequencies lower than

that display less of a drop and frequencies higher than that experience a drop in height.

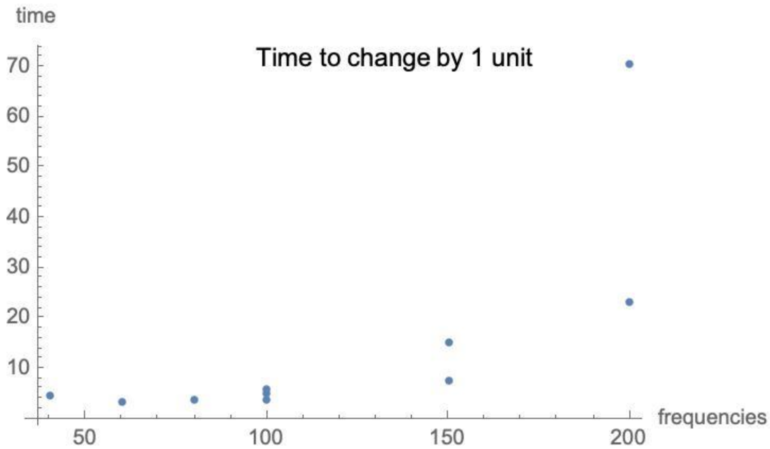

Figure 6. Time it takes for the height of the sand column to decrease by 1 cm. With the increase in frequency, it takes exponentially more time for the height to drop by the same amount. The same trend about peak equilibrium rate can also be seen here as well — it is evident the peak is somewhere between 40 and 80 Hz.

Discussion

The initial question posed by the research was to explore the qualitative differences or similarities between classical fluids and granular materials. The experiments have demonstrated that when sand is not shaken, it displays qualitatively different behavior. The system of columns of granular material does not approach the equilibrium smoothly, like fluids do, but rather discreetly, in a series of sharp jumps. In addition, when unshaken, the sand never reaches an equilibrium, even though with the increase of the initial height difference the system does tend to go to it.

On the other hand, once shaken, the sand behaves more like a liquid, continuously approaching the equilibrium in a fashion that is well-described by the damping oscillation. Similar to how a viscous fluid will take a long time to equalize the heights of the columns, the sand shaken with varying frequencies will approach equilibrium at different rates. The difference in the rate at which the system approaches equilibrium across a range of frequencies suggests that there is some sort of “granular viscosity” which opposes the motion. The fact that the system changes continuously when shaken means that shaking removes the qualitative difference observed earlier.

Possible Explanation

The key difference between granular material and liquids is the presence of Brownian motion in the liquids. Brownian motion is the chaotic motion of the particles which we observe in liquids. In the fluids, temperature is proportional to average kinetic energy. As long as the temperature of the fluid does not change, there is going to be a Brownian motion with a near-constant average velocity.

The main reason we do not observe such a motion in granular materials is thermal collapse due to the particles being grains of sand. The collisions are inelastic and the kinetic energy dissipates into heat, which is absorbed by the air surrounding the grains. In fluids, this does not happen — all the collisions are elastic, and therefore what one particle lost in velocity, the other particle gained. As soon as some chaotic motion emerges, it immediately burns out and the kinetic energy of the sand particles turns into the kinetic energy of the surrounding air. This is in fact a reason why the granular particles are motionless when at rest and why they exhibit internal friction (viscosity) even though there is no significant inter-particle cohesion. If there is no external source of energy, the micro-collisions between grains will immediately dissipate the excess kinetic energy and result in a motionless state. Granular material behaves similarly to a liquid with an exceptionally high viscosity, keeping it at rest.

The reason this motion is important for the case of Pascal's law and pressure transmission is that Brownian motion is essential for long-range transmission of excess energy. Fundamentally, a state where each particle has a similar amount of energy is the state with the highest entropy, and the system will tend to it, if possible. In the case of fluids, Brownian motion is that mechanism that makes this state achievable. In a tube with two columns of liquids of an unequal height, articles on the higher side will have on average higher total energy, because of the higher potential energy. Once the particles start to fall down, this excess energy will transform into higher momentum. Because the collisions between molecules are elastic, this higher momentum of particles that are higher in the column will be conserved and transmitted to the particles lower in the column. Together with this momentum, the excess energy which the particles on the higher side initially possessed will travel across the fluid, finally arriving at the particles which initially had the lowest total energy — particles on the lower side. This increase in energy manifests itself in the eventual rise in height of the particles because in the long run, they can not just move any faster: that would create different temperatures across the fluid. This free transport of energy is not immediate though; it does take time for the liquid for example to come to this equilibrium. The reason for that lies in internal friction between the particles, caused by intermolecular forces, as well as heating up of the environment. The cumulative effect of this friction is what we call viscosity; in general, the fluids with higher viscosity take longer to equilibrate in the U-tube experiment than fluids with lower viscosity.

Such free transport of energy is normally impossible in sand because of the aforementioned effects of thermal collapse. Extrapolating the fluid understanding of viscosity to sand, it has a very high viscosity, which makes the equilibration take an enormous amount of time. Introducing external oscillation, or, shaking to the system, however, solves this problem by mitigating the dissipation of energy with the introduction of a new one in the form of the oscillation. It significantly decreases sand's viscosity, which allows for the system to move to the state with the lowest entropy —the state where the average total energy of the particles is nearly equal in equilibrium.

Conclusion

Such an explanation brings forth the question about fluid viscosity that scientists have yet to answer. If the granular viscosity is decreased by the introduction of external energy, will the same hold true for liquids? Will heating up a liquid decrease its viscosity? Will a shaken liquid have a lower factual viscosity? Answering those questions may allow a more comprehensive understanding of viscosity expanding beyond granular dynamics.

In the case of granular dynamics, the research demonstrates a necessity and validity for the introduction of the term "granular viscosity." The damped oscillation-like behavior observed in the experiments can be a great framework to examine this, especially since the effects of viscosity or friction are already incorporated in the solution. Experiments varying particle size, height, particle number, and angles of oscillation may provide valuable knowledge to uncover this question.

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