Vectores Parte 1

1(b-f-c) – 2(a-c) – 3(b) – 8

1. Sean los vectores $u = 2i - 3j + 4k$, $v = -2i - 3j + 5k$, $w = i - 7j + 3k$, $t = 3i + 4j + 5k$.
Calcular:

$b)\ t + 3w - v$

$3i + 4j + 5k + 3(i - 7j + 3k) - (-2i - 3j + 5k)$

$3i + 4j + 5k + 3i - 21j + 9k + 2i + 3j - 5k$

$8i - 16j + 9k$

$f)\ 2u - 7w + 5v$

$2(2i - 3j + 4k) - 7(i - 7j + 3k) + 5(-2i - 3j + 5k)$

$4i - 6j + 8k - 7i + 49j - 21k - 10i - 15j + 25k$

$-13i + 28j + 12k$

$c)\ 2v + 7t - w$

$2(-2i - 3j + 5k) + 7(3i + 4j + 5k) - (i - 7j + 3k)$

$-2i - 6j + 10k + 21i + 28j - 35k - i + 7j - 3k$

$18i + 29j - 28k$

2. Calcular la norma de los siguientes vectores:

$a)\ u = (-2, 1, 2)$

$\|u\| = \sqrt{(-2)^2 + 1^2 + 2^2}$

$\|u\| = \sqrt{4 + 1 + 4}$

$\|u\| = \sqrt{9}$

$\|u\| = 3$

$c)\ w = (3, 0, -4)$

$\|w\| = \sqrt{3^2 + 0^2 + (-4)^2}$

$\|w\| = \sqrt{9 + 16}$

$\|w\| = \sqrt{25}$

$\|w\| = 5$

3. **b**. expresar el vector $v = (1, 1, -4)$ como una combinación lineal de los vectores $u = (5, 0, 1)$, $w = (3, 3, 2)$ y $z = (0, 2, 4)$.

$\vec{v} = a \cdot \vec{u} + b \cdot \vec{w} + c \cdot \vec{z}$

$(1, 1, -4) = a \cdot (5, 0, 1) + b \cdot (3, 3, 2) + c \cdot (0, 2, 4)$

$$\begin{cases} 5a + 3b = 1 \\ 3b + 2c = 1 \\ a + 2b + 4c = -4 \end{cases}$$

$$\left(\begin{array}{ccc|c} 5 & 3 & 0 & 1 \\ 0 & 3 & 2 & 1 \\ 1 & 2 & 4 & -4 \end{array}\right) \mapsto \left(\begin{array}{ccc|c} 1 & 2 & 4 & -4 \\ 0 & 3 & 2 & 1 \\ 5 & 3 & 0 & 1 \end{array}\right) \mapsto \left(\begin{array}{ccc|c} 1 & 2 & 4 & -4 \\ 0 & 3 & 2 & 1 \\ 0 & -7 & -20 & 21 \end{array}\right) \mapsto \left(\begin{array}{ccc|c} 1 & 2 & 2 & -4 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & -\frac{46}{3} & \frac{70}{3} \end{array}\right)$$

$$f_1 \leftrightarrow f_3 \qquad f_3 - 5f_1 \to f_3 \qquad f_3 + \frac{7}{3} f_2 \to f_3$$

$$\begin{cases} a + 2b + 4c = -4 \\ 3b + 2c = 1 \\ -\frac{46}{3}c = \frac{70}{3} \end{cases}$$

$$-\frac{46}{3}c = \frac{70}{3}$$

$$\boxed{c = -\frac{35}{23}}$$

$$3b - \frac{70}{23} = 1$$

$$3b = \frac{93}{23}$$

$$\boxed{b = \frac{31}{23}}$$

$$a + \frac{62}{23} - \frac{140}{23} = -4$$

$$\boxed{a = -\frac{14}{23}}$$

$$\vec{v} = -\frac{14}{53}\cdot\vec{u} + \frac{41}{53}\cdot\vec{w} - \frac{70}{53}\cdot\vec{z}$$

$$(1,1,-4) = -\frac{14}{23}\cdot(5,0,1) + \frac{31}{23}\cdot(3,3,2) - \frac{35}{53}\cdot(0,2,4)$$

8. Comprobar si los siguientes vectores son linealmente dependiente o independientes (para comprobarlo utilizar el método de cálculo de determinaste de la matriz.

a) (1, 2, 0), (2, 3, 1), (1, 1, 1)
b) (−2, 1, 1), (1, 0, 1), (0, 1, 2)
c) (5, 0, 4), (0, 2, 0), (−6, 0, −5)

$$a)\begin{cases} a + 2b + c = 0 \\ 2a + 3b + c = 0 \\ b + c = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\Delta = 0$$

Son linealmete dependientes

$$b)\begin{cases} -2a + b = 0 \\ a + c = 0 \\ a + b + 2c = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$\Delta = 1$$

Son linealmete independientes

$$c)\begin{cases} 5a - 6c = 0 \\ 2b = 0 \\ 4a + 2b - 5c = 0 \end{cases}$$

$$\Delta = \begin{vmatrix} 5 & 0 & -6 \\ 0 & 2 & 0 \\ 4 & 2 & -5 \end{vmatrix}$$

$$\Delta = -2$$

Son linealmete independientes