Vectores Parte 2

1 – 2A – 3 – 4D – 8A -9D – 11D

1) Sean los vectores $\overrightarrow{X} = k\overrightarrow{U} - 2\overrightarrow{V} + 4\overrightarrow{W}$, $\overrightarrow{V} = -\overrightarrow{U} + k\overrightarrow{V} + \overrightarrow{W}$ $\overrightarrow{X} = (k;-2;4)$ y $\overrightarrow{V}(-1;k;1)$

a) Sean //
$$\overrightarrow{X} \cdot \overrightarrow{V} = 0$$
$$-k - 2k + 4 = 0$$
$$-3k = -4$$
$$k = \frac{4}{3}$$

b) sean $\perp$
$$\frac{k}{-1} = \frac{-2}{k} = \frac{4}{1}$$
$$\frac{k}{-1} = 4 \quad \wedge \quad \frac{-2}{k} = \frac{4}{1}$$
$$k = -4 \qquad k = -\frac{1}{2}$$
$$-4 \neq -\frac{1}{2}$$

Absurdo, por lo tanto no existirte valor de k para que $\vec{X} \perp \vec{V}$.

2)

$a)$ $\vec{u} = (1;1;-3)$ y $\vec{v} = (0;2;4)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ 0 & 2 & 4 \end{vmatrix}$$
$$\vec{u} \times \vec{v} = \begin{vmatrix} 1 & -3 \\ 2 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & -3 \\ 0 & 4 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} \hat{k}$$
$$\vec{u} \times \vec{v} = 10\hat{i} - 4j + 2\hat{k}$$
$$\vec{w} = 10\hat{i} - 4j + 2\hat{k}$$

$$\hat{w} = \frac{\vec{w}}{\|w\|}$$
$$\vec{w} = (10,-4,2)$$
$$\|w\| = \sqrt{10^2 + (-4)^2 + 2^2}$$
$$\|w\| = \sqrt{120}$$
$$\|w\| = 2\sqrt{30}$$

$$\hat{w} = \frac{\vec{w}}{\|w\|}$$
$$\hat{w} = \left(\frac{10}{2\sqrt{30}}, \frac{-4}{2\sqrt{30}}, \frac{2}{2\sqrt{30}}\right)$$
$$\hat{w} = \left(\frac{5}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$$

3) $\vec{w} = 2\hat{i} - 3\hat{j}$ y $\vec{v} = 4j + 3k$

$$\vec{w} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 0 \\ 0 & 4 & 3 \end{vmatrix}$$
$$\vec{w} \times \vec{v} = -9\hat{i} - 6\hat{j} + 8\hat{k}$$
$$\vec{u} = -9\hat{i} - 6\hat{j} + 8\hat{k}$$
$$\vec{s} = t \cdot \vec{u} \ \ con\, t = -2 (Ejemplo)$$
$$\vec{s} = 18\hat{i} + 12\hat{j} - 16\hat{k}$$

$$\hat{u} = \frac{\vec{u}}{\|u\|}$$
$$\vec{u} = (-9,-6,8)$$
$$\|u\| = \sqrt{(-9)^2 + (-6)^2 + 8^2}$$
$$\|u\| = \sqrt{181}$$
$$\|w\| = 2\sqrt{30}$$

$$\hat{u} = \frac{\vec{u}}{\|u\|}$$
$$\hat{u} = \left(\frac{-9}{\sqrt{181}}, \frac{-6}{\sqrt{181}}, \frac{8}{\sqrt{181}}\right)$$

$$\hat{t} = \frac{\vec{t}}{\|t\|}$$

$$\vec{t} = (18,12,-16)$$

$$\|u\| = \sqrt{18^2 + 12^2 + (-16)^2}$$

$$\|u\| = \sqrt{724}$$

$$\|w\| = 2\sqrt{181}$$

$$\hat{t} = \frac{\vec{t}}{\|t\|}$$

$$\vec{t} = \left(\frac{18}{2\sqrt{181}}, \frac{12}{2\sqrt{181}}, \frac{-16}{2\sqrt{181}}\right)$$

$$\hat{u} = \left(\frac{9}{\sqrt{181}}, \frac{6}{\sqrt{181}}, \frac{-8}{\sqrt{181}}\right)$$

4D $\vec{u} = 4\hat{i} + 5\hat{j}$ $\qquad \vec{v} = 5\hat{i} - 4\hat{j}$.

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\alpha$$

$$\|\vec{u}\| = \sqrt{4^2 + 5^2}$$

$$\|\vec{u}\| = \sqrt{41}$$

$$\|\vec{v}\| = \sqrt{5^2 + (-4)^2}$$

$$\|\vec{v}\| = \sqrt{41}$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\alpha$$

$$\cos\alpha = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} \Rightarrow \cos\alpha = \frac{(4;5)\cdot(5;-4)}{\|\vec{u}\| \cdot \|\vec{v}\|}$$

$$\cos\alpha = \frac{20-20}{\sqrt{41}\cdot\sqrt{41}} \Rightarrow \cos\alpha = 0$$

$$\alpha = arc\cos 0$$

$$\alpha = 90° = \frac{\pi}{2}$$

8A

$a)$ $\vec{u} = (-2;0;-2)$ y $\vec{v} = (3;1;-1)$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -2 \\ 3 & 1 & -1 \end{vmatrix}$$

$$\vec{u} \times \vec{v} = 2\hat{i} - 8\hat{j} - 2\hat{k}$$

$$Área = \|\vec{u} \times \vec{v}\|$$

$$Área = \sqrt{2^2 + (-8)^2 + (-2)^2}$$

$$Área = \sqrt{72}$$

$$Área = 6\sqrt{2}\, u^2$$

9D

$A(5;2;-1);\ B(3;1;-1);\ C(-1;4;-4)$

$\overrightarrow{AB}=(3-5;1-2;-1-(-1))$ $\quad\overrightarrow{AC}(-1-3;4-1;-4-(-1))$

$\overrightarrow{AB}=(-2;-1;0)$ $\quad\overrightarrow{AC}(-4;3;-5)$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & -1 & 0\\ -4 & 3 & -5\end{vmatrix}$$

$\vec{u}\times\vec{v}=5\hat{i}-10\hat{j}-10\hat{k}$

$Área\,de\,triangulo=\dfrac{1}{2}\|\vec{u}\times\vec{v}\|$

$Área=\dfrac{1}{2}\sqrt{5^2+(-10)^2+(-10)^2}$

$Área=\dfrac{1}{2}\sqrt{225}$

$Área=\dfrac{15}{2}u^2$

11D

$d)\ \vec{r}=(1;2;3)\ ;\ \vec{s}=(-2;1;-1)\ y\ \ \vec{t}=(3;-1;-1)$

$\vec{r}\cdot(\vec{s}\times\vec{t})$

$Volumen=|\vec{r}\cdot(\vec{s}\times\vec{t})|$

$$\vec{r}\cdot(\vec{s}\times\vec{t})=\begin{vmatrix}1 & 2 & 3\\ -2 & 1 & -1\\ 3 & -1 & -1\end{vmatrix} \Rightarrow$$

$\vec{r}\cdot(\vec{s}\times\vec{t})=-15$

$Volumen=|\vec{r}\cdot(\vec{s}\times\vec{t})|$

$Volumen=|-15|$

$Volumen=15u^3$