Tarea7_LeninAmanagndi

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Tarea Nº7

Métodos Númericos

Tema: Splines Cúbicos

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Enlace GitHub Tarea 7

0.1 Ejercicio 1

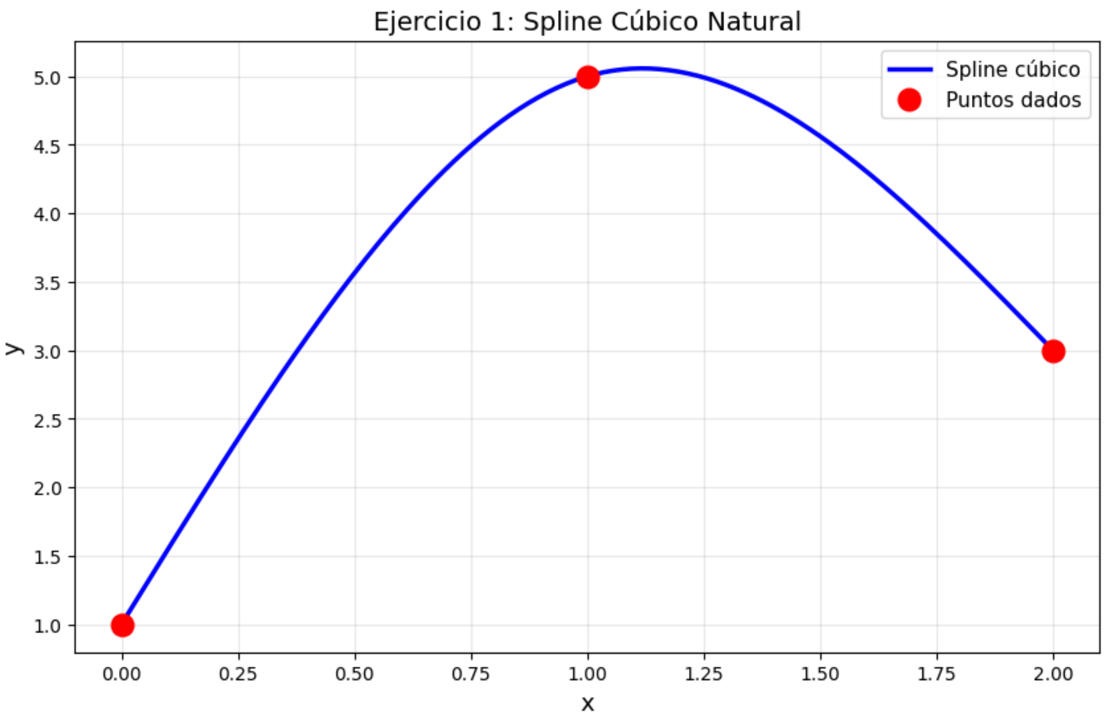
Dados los puntos (0,1) , (1,5) , (2,3) determine el spline cúbico

[7]:
```python
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

def spline_natural(x, y):
    n = len(x) - 1
    h = np.diff(x)
    A = np.zeros((n+1, n+1))
    b = np.zeros(n+1)

    A[0, 0] = 1
    for i in range(1, n):
        A[i, i-1:i+2] = [h[i-1], 2*(h[i-1]+h[i]), h[i]]
        b[i] = 3*((y[i+1]-y[i])/h[i] - (y[i]-y[i-1])/h[i-1])
    A[n, n] = 1

    c = np.linalg.solve(A, b)
    a = y[:-1]
    coef_b = np.zeros(n)
    d = np.zeros(n)

    for i in range(n):
        coef_b[i] = (y[i+1]-y[i])/h[i] - h[i]*(2*c[i]+c[i+1])/3
        d[i] = (c[i+1]-c[i])/(3*h[i])

    return a, coef_b, c[:-1], d
```

```
def evaluar_spline(x_data, coef, x_eval):
    a, b, c, d = coef
    y = np.zeros_like(x_eval)
    for i, xi in enumerate(x_eval):
        j = np.searchsorted(x_data[1:], xi)
        j = min(j, len(a)-1)
        dx = xi - x_data[j]
        y[i] = a[j] + b[j]*dx + c[j]*dx**2 + d[j]*dx**3
    return y

def crear_tabla_coeficientes(x, a, b, c, d):
    tabla = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'a': [f'{a[i]:.4f}' for i in range(len(a))],
        'b': [f'{b[i]:.4f}' for i in range(len(b))],
        'c': [f'{c[i]:.4f}' for i in range(len(c))],
        'd': [f'{d[i]:.4f}' for i in range(len(d))],
    })
    print("\nCoeficientes del Spline:")
    print(tabla.to_string(index=False))

def crear_tabla_ecuaciones(x, a, b, c, d):
    ecuaciones = []
    for i in range(len(a)):
        ecuacion = f"S_{i}(x) = {a[i]:.2f} + {b[i]:.2f}(x-{x[i]:.2f}) + {c[i]:.
 ↪2f}(x-{x[i]:.2f})² + {d[i]:.2f}(x-{x[i]:.2f})³"
        ecuaciones.append(ecuacion)

    tabla_ecuaciones = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'Ecuación': ecuaciones
    })
    print("\nEcuaciones del Spline:")
    print(tabla_ecuaciones.to_string(index=False))

x = np.array([0, 1, 2])
y = np.array([1, 5, 3])

a, b, c, d = spline_natural(x, y)
crear_tabla_coeficientes(x, a, b, c, d)
crear_tabla_ecuaciones(x, a, b, c, d)

x_plot = np.linspace(0, 2, 200)
y_plot = evaluar_spline(x, (a, b, c, d), x_plot)

plt.figure(figsize=(10, 6))
```

```python
plt.plot(x_plot, y_plot, 'b-', linewidth=2.5, label='Spline cúbico')
plt.plot(x, y, 'ro', markersize=12, label='Puntos dados')
plt.grid(True, alpha=0.3)
plt.xlabel('x', fontsize=13)
plt.ylabel('y', fontsize=13)
plt.title('Ejercicio 1: Spline Cúbico Natural', fontsize=14)
plt.legend(fontsize=11)
plt.show()
```

```
Coeficientes del Spline:
Intervalo      a      b       c       d
   [0, 1] 1.0000 5.5000  0.0000 -1.5000
   [1, 2] 5.0000 1.0000 -4.5000  1.5000

Ecuaciones del Spline:
Intervalo                                                     Ecuación
   [0, 1] S_0(x) = 1.00 + 5.50(x-0.00) + 0.00(x-0.00)² + -1.50(x-0.00)³
   [1, 2] S_1(x) = 5.00 + 1.00(x-1.00) + -4.50(x-1.00)² + 1.50(x-1.00)³
```

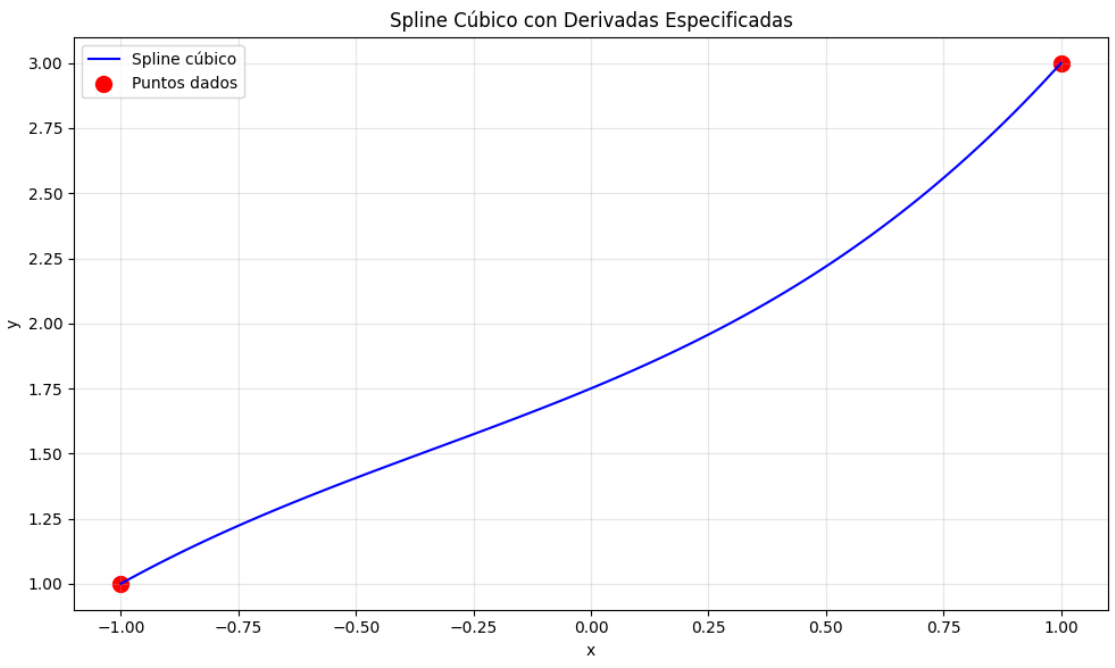


0.2 Ejercicio 2

Dado los puntos (-1, 1) y (1, 3), determina el spline cúbico sabiendo que:

- $$f'(x_0) = 1$$
- $$f'(x_n) = 2$$

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt

# Datos del problema
x = np.array([-1, 1])
y = np.array([1, 3])
derivadas = np.array([1, 2])  # f'(-1)=1, f'(1)=2

# Coeficientes obtenidos manualmente
a, b, c, d = 1, 1, -0.5, 0.25

# Definición del spline
def S(x_val):
    return a + b*(x_val + 1) + c*(x_val + 1)**2 + d*(x_val + 1)**3

# Definición de la derivada
def S_prime(x_val):
    return b + 2*c*(x_val + 1) + 3*d*(x_val + 1)**2

# Crear datos para graficar
x_vals = np.linspace(-1, 1, 100)
y_vals = S(x_vals)

# Gráfico
plt.figure(figsize=(10, 6))
plt.plot(x_vals, y_vals, 'b-', label='Spline cúbico')
plt.scatter(x, y, color='red', s=100, label='Puntos dados')

plt.title('Spline Cúbico con Derivadas Especificadas')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True, alpha=0.3)
plt.legend()
plt.tight_layout()
plt.show()


print("\nEcuación del spline cúbico:")
print(f"S(x) = {a:.2f} + {b:.2f}(x+1) + {c:.2f}(x+1)^2 + {d:.2f}(x+1)^3")
```



```
Ecuación del spline cúbico:
S(x) = 1.00 + 1.00(x+1) + -0.50(x+1)^2 + 0.25(x+1)^3
```

0.3 Ejercicio 3

Diríjase al pseudocódigo del spline cúbico con frontera natural provisto en clase, en base a ese pseudocódigo complete la siguiente función:

Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	−1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	−4.0				
7	13	6.7									
8	17	4.5	−0.67								

[15]:
```python
import sympy as sym
from IPython.display import display

def cubic_spline(xs: list[float], ys: list[float]) -> list[sym.Symbol]:
    points = sorted(zip(xs, ys), key=lambda x: x[0])
    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1
    h = [xs[i+1] - xs[i] for i in range(n)]

    alpha = [0] * (n+1)
    for i in range(1, n):
        alpha[i] = (3/h[i])*(ys[i+1]-ys[i]) - (3/h[i-1])*(ys[i]-ys[i-1])

    l = [1.0]
    mu = [0.0]
    z = [0.0]

    for i in range(1, n):
        l.append(2*(xs[i+1]-xs[i-1]) - h[i-1]*mu[i-1])
        mu.append(h[i]/l[i])
        z.append((alpha[i] - h[i-1]*z[i-1])/l[i])

    l.append(1.0)
    z.append(0.0)
    c = [0.0]*(n+1)

    for i in range(n-1, -1, -1):
        c[i] = z[i] - mu[i]*c[i+1]

    x = sym.Symbol('x')
    splines = []

    for j in range(n):
        a = ys[j]
        b = (ys[j+1]-ys[j])/h[j] - h[j]*(c[j+1]+2*c[j])/3
        d = (c[j+1]-c[j])/(3*h[j])

        S = a + b*(x-xs[j]) + c[j]*(x-xs[j])**2 + d*(x-xs[j])**3
        splines.append(S)

    return splines
```

[17]:
```
xs = [1, 2, 3]
ys = [-5, -4, 3]

splines = cubic_spline(xs=xs, ys=ys)
_ = [display(s) for s in splines]
print("______")
```

```
_ = [display(s.expand()) for s in splines]
```

$-0.5x + 1.5\left(x - 1\right)^3 - 4.5$

$4.0x - 1.5\left(x - 2\right)^3 + 4.5\left(x - 2\right)^2 - 12.0$

```
------
```

$1.5x^3 - 4.5x^2 + 4.0x - 6.0$

$-1.5x^3 + 13.5x^2 - 32.0x + 18.0$

0.4 Ejercicio 4

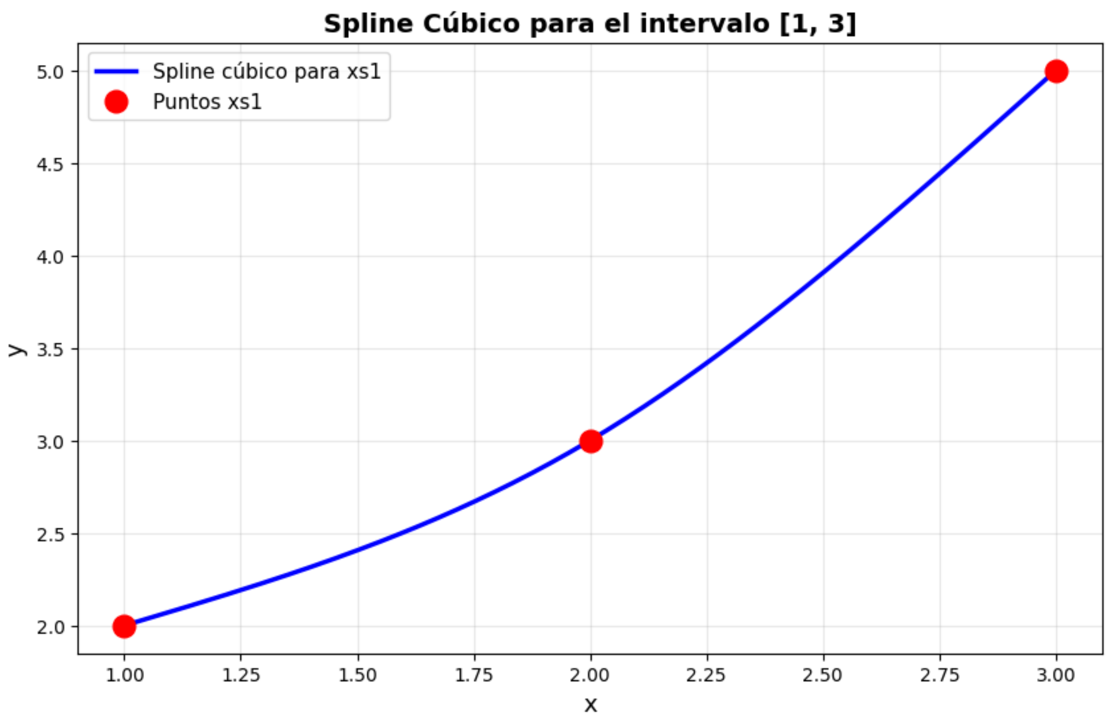
Usando la función anterior, encuentre el spline cúbico para:

- $$xs = [1, 2, 3]$$
- $$ys = [2, 3, 5]$$

```
[ ]: import numpy as np
     import sympy as sym
     import matplotlib.pyplot as plt
     import pandas as pd
     from IPython.display import display

     def cubic_spline(xs: list[float], ys: list[float]) -> list:
         points = sorted(zip(xs, ys), key=lambda x: x[0])

         xs = [x for x, _ in points]
         ys = [y for _, y in points]

         n = len(points) - 1  # number of splines

         h = [xs[i + 1] - xs[i] for i in range(n)]

         alpha = [0] * (n + 1)
         for i in range(1, n):
             alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] -␣
      ↪ys[i - 1])

         l = [1]
         u = [0]
         z = [0]

         for i in range(1, n):
             l.append(2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1])
             u.append(h[i] / l[i])
             z.append((alpha[i] - h[i - 1] * z[i - 1]) / l[i])
```

```
    l.append(1)
    z.append(0)

    c = [0] * (n + 1)

    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]

    b = [(ys[i + 1] - ys[i]) / h[i] - h[i] * (c[i + 1] + 2 * c[i]) / 3 for i in␣
 ↪range(n)]
    d = [(c[i + 1] - c[i]) / (3 * h[i]) for i in range(n)]
    a = [ys[i] for i in range(n)]

    return a, b, c[:-1], d

def evaluar_spline(x_data, coef, x_eval):
    a, b, c, d = coef
    y = np.zeros_like(x_eval)
    for i, xi in enumerate(x_eval):
        j = np.searchsorted(x_data[1:], xi)
        j = min(j, len(a)-1)
        dx = xi - x_data[j]
        y[i] = a[j] + b[j]*dx + c[j]*dx**2 + d[j]*dx**3
    return y

def crear_tabla_coeficientes(x, a, b, c, d):
    tabla = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'a': [f'{a[i]:.4f}' for i in range(len(a))],
        'b': [f'{b[i]:.4f}' for i in range(len(b))],
        'c': [f'{c[i]:.4f}' for i in range(len(c))],
        'd': [f'{d[i]:.4f}' for i in range(len(d))],
    })
    print("\nCoeficientes del Spline:")
    print(tabla.to_string(index=False))

def crear_tabla_ecuaciones(x, a, b, c, d):
    ecuaciones = []
    for i in range(len(a)):
        ecuacion = f"S_{i}(x) = {a[i]:.2f} + {b[i]:.2f}(x-{x[i]:.2f}) + {c[i]:.
 ↪2f}(x-{x[i]:.2f})² + {d[i]:.2f}(x-{x[i]:.2f})³"
        ecuaciones.append(ecuacion)

    tabla_ecuaciones = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'Ecuación': ecuaciones
    })
    print("\nEcuaciones del Spline:")
    print(tabla_ecuaciones.to_string(index=False))


xs1 = [1, 2, 3]
ys1 = [2, 3, 5]


splines1 = cubic_spline(xs=xs1, ys=ys1)

crear_tabla_coeficientes(xs1, *splines1)
crear_tabla_ecuaciones(xs1, *splines1)


x_plot = np.linspace(1, 3, 200)
y_plot1 = evaluar_spline(xs1, splines1, x_plot)

plt.figure(figsize=(10, 6))
plt.plot(x_plot, y_plot1, 'b-', linewidth=2.5, label='Spline cúbico para xs1')
plt.plot(xs1, ys1, 'ro', markersize=12, label='Puntos xs1')

plt.grid(True, alpha=0.3)
plt.xlabel('x', fontsize=13)
plt.ylabel('y', fontsize=13)
plt.title('Spline Cúbico para el intervalo [1, 3]', fontsize=14,␣
  ↪fontweight='bold')
plt.legend(fontsize=11)
plt.show()
```

```
Coeficientes del Spline:
Intervalo      a      b      c       d
   [1, 2] 2.0000 0.7500 0.0000  0.2500
   [2, 3] 3.0000 1.5000 0.7500 -0.2500

Ecuaciones del Spline:
Intervalo                                                      Ecuación
   [1, 2]  S_0(x) = 2.00 + 0.75(x-1.00) + 0.00(x-1.00)² + 0.25(x-1.00)³
   [2, 3] S_1(x) = 3.00 + 1.50(x-2.00) + 0.75(x-2.00)² + -0.25(x-2.00)³
```



0.5 Ejercicio 5

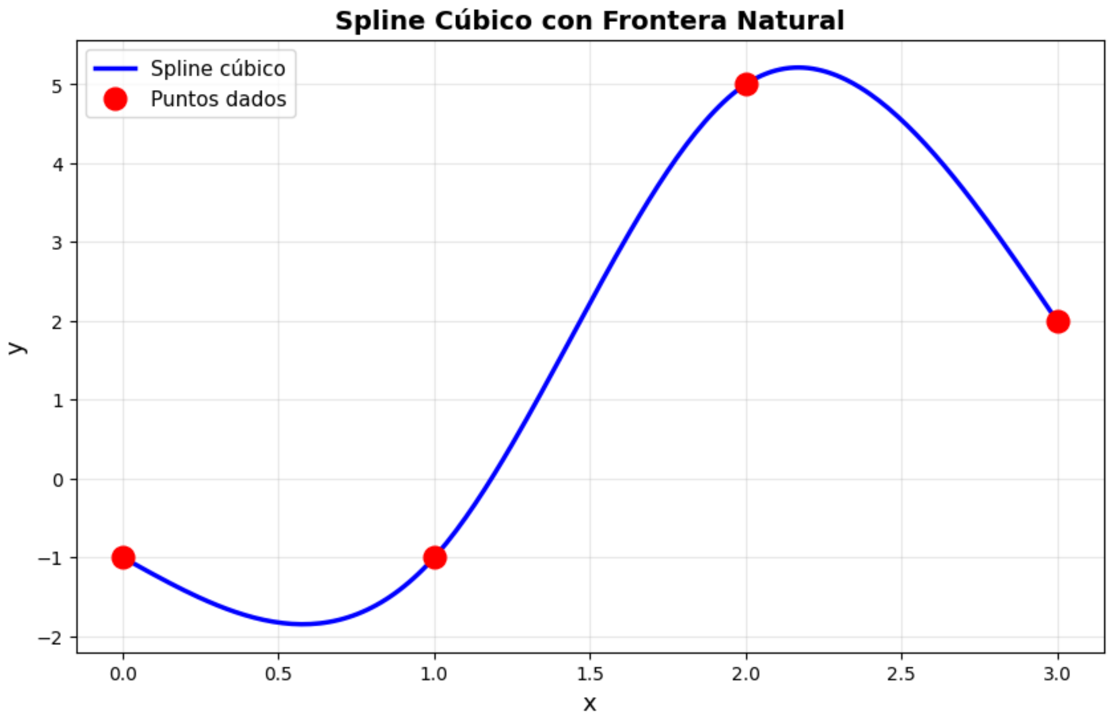
Usando la función anterior, encuentre el spline cúbico para:

- $$xs = [0, 1, 2, 3]$$
- $$ys = [-1, 1, 5, 2]$$

[ ]:
```
import numpy as np
import sympy as sym
import matplotlib.pyplot as plt
import pandas as pd
from IPython.display import display

def cubic_spline(xs: list[float], ys: list[float]) -> list:

    points = sorted(zip(xs, ys), key=lambda x: x[0])

    xs = [x for x, _ in points]
    ys = [y for _, y in points]

    n = len(points) - 1
    h = [xs[i + 1] - xs[i] for i in range(n)]

    alpha = [0] * (n + 1)
    for i in range(1, n):
        alpha[i] = 3 / h[i] * (ys[i + 1] - ys[i]) - 3 / h[i - 1] * (ys[i] -␣
 ↪ys[i - 1])

    l = [1]
    u = [0]
    z = [0]

    for i in range(1, n):
        l.append(2 * (xs[i + 1] - xs[i - 1]) - h[i - 1] * u[i - 1])
        u.append(h[i] / l[i])
        z.append((alpha[i] - h[i - 1] * z[i - 1]) / l[i])

    l.append(1)
    z.append(0)

    c = [0] * (n + 1)

    for j in range(n - 1, -1, -1):
        c[j] = z[j] - u[j] * c[j + 1]

    b = [(ys[i + 1] - ys[i]) / h[i] - h[i] * (c[i + 1] + 2 * c[i]) / 3 for i in␣
 ↪range(n)]
    d = [(c[i + 1] - c[i]) / (3 * h[i]) for i in range(n)]
    a = [ys[i] for i in range(n)]

    return a, b, c[:-1], d

def evaluar_spline(x_data, coef, x_eval):
    a, b, c, d = coef
    y = np.zeros_like(x_eval)
    for i, xi in enumerate(x_eval):
        j = np.searchsorted(x_data[1:], xi)
        j = min(j, len(a)-1)
        dx = xi - x_data[j]
        y[i] = a[j] + b[j]*dx + c[j]*dx**2 + d[j]*dx**3
    return y

def crear_tabla_coeficientes(x, a, b, c, d):
    tabla = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'a': [f'{a[i]:.4f}' for i in range(len(a))],
        'b': [f'{b[i]:.4f}' for i in range(len(b))],
        'c': [f'{c[i]:.4f}' for i in range(len(c))],
        'd': [f'{d[i]:.4f}' for i in range(len(d))],
    })
    print("\nCoeficientes del Spline:")
    print(tabla.to_string(index=False))

def crear_tabla_ecuaciones(x, a, b, c, d):
    ecuaciones = []
    for i in range(len(a)):
        ecuacion = f"S_{i}(x) = {a[i]:.2f} + {b[i]:.2f}(x-{x[i]:.2f}) + {c[i]:.
↪2f}(x-{x[i]:.2f})² + {d[i]:.2f}(x-{x[i]:.2f})³"
        ecuaciones.append(ecuacion)

    tabla_ecuaciones = pd.DataFrame({
        'Intervalo': [f'[{x[i]}, {x[i+1]}]' for i in range(len(a))],
        'Ecuación': ecuaciones
    })
    print("\nEcuaciones del Spline:")
    print(tabla_ecuaciones.to_string(index=False))


xs = [0, 1, 2, 3]
ys = [-1, -1, 5, 2]

splines = cubic_spline(xs=xs, ys=ys)

crear_tabla_coeficientes(xs, *splines)
crear_tabla_ecuaciones(xs, *splines)

x_plot = np.linspace(0, 3, 200)
y_plot = evaluar_spline(xs, splines, x_plot)

plt.figure(figsize=(10, 6))
plt.plot(x_plot, y_plot, 'b-', linewidth=2.5, label='Spline cúbico')
plt.plot(xs, ys, 'ro', markersize=12, label='Puntos dados')
plt.grid(True, alpha=0.3)
plt.xlabel('x', fontsize=13)
plt.ylabel('y', fontsize=13)
plt.title('Spline Cúbico con Frontera Natural', fontsize=14, fontweight='bold')
plt.legend(fontsize=11)
plt.show()
```

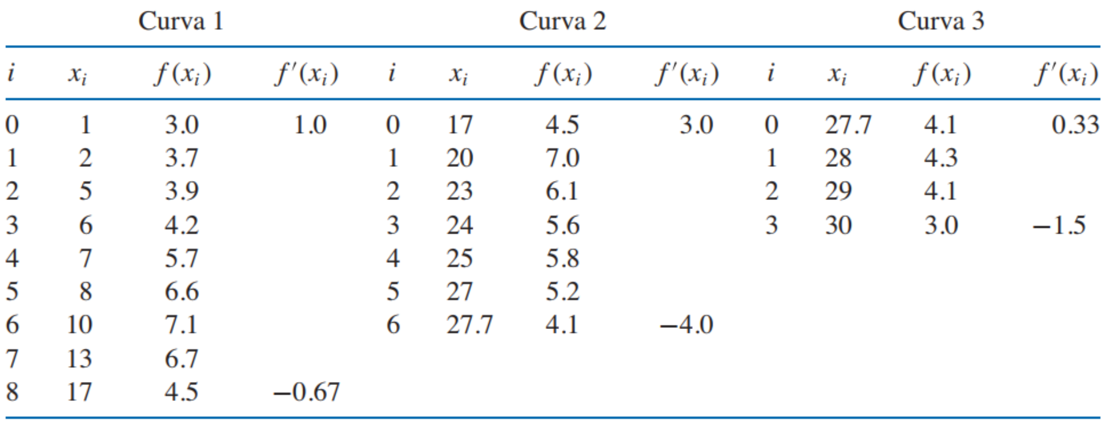
```
Coeficientes del Spline:
Intervalo       a       b       c       d
   [0, 1] -1.0000 -2.2000  0.0000  2.2000
   [1, 2] -1.0000  4.4000  6.6000 -5.0000
   [2, 3]  5.0000  2.6000 -8.4000  2.8000
```

```
Ecuaciones del Spline:
Intervalo                                                                Ecuación
   [0, 1] S_0(x) = -1.00 + -2.20(x-0.00) + 0.00(x-0.00)² + 2.20(x-0.00)³
   [1, 2] S_1(x) = -1.00 + 4.40(x-1.00) + 6.60(x-1.00)² + -5.00(x-1.00)³
   [2, 3]  S_2(x) = 5.00 + 2.60(x-2.00) + -8.40(x-2.00)² + 2.80(x-2.00)³
```



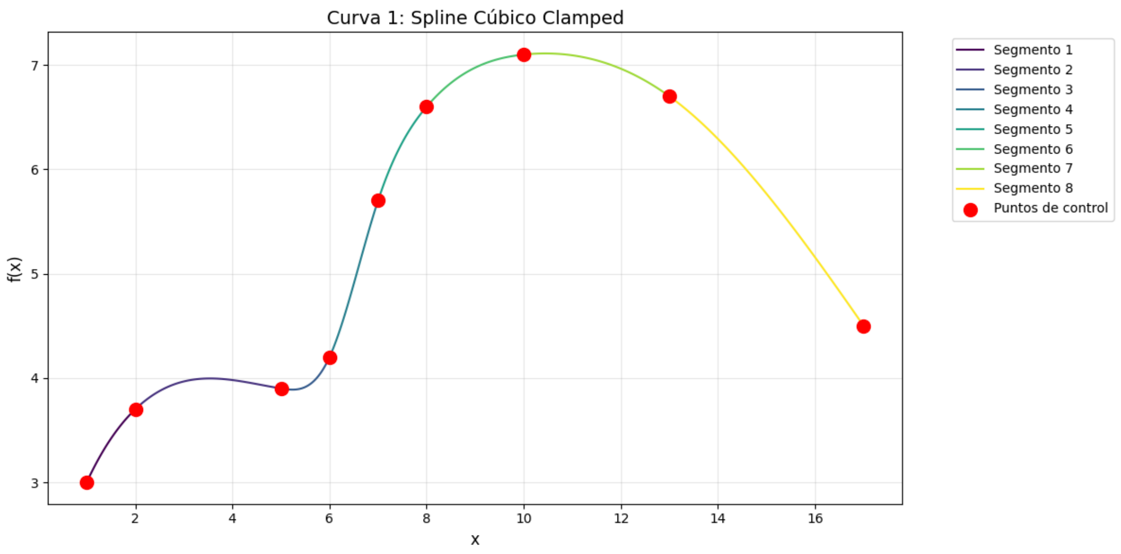
0.6 Ejercicio 6

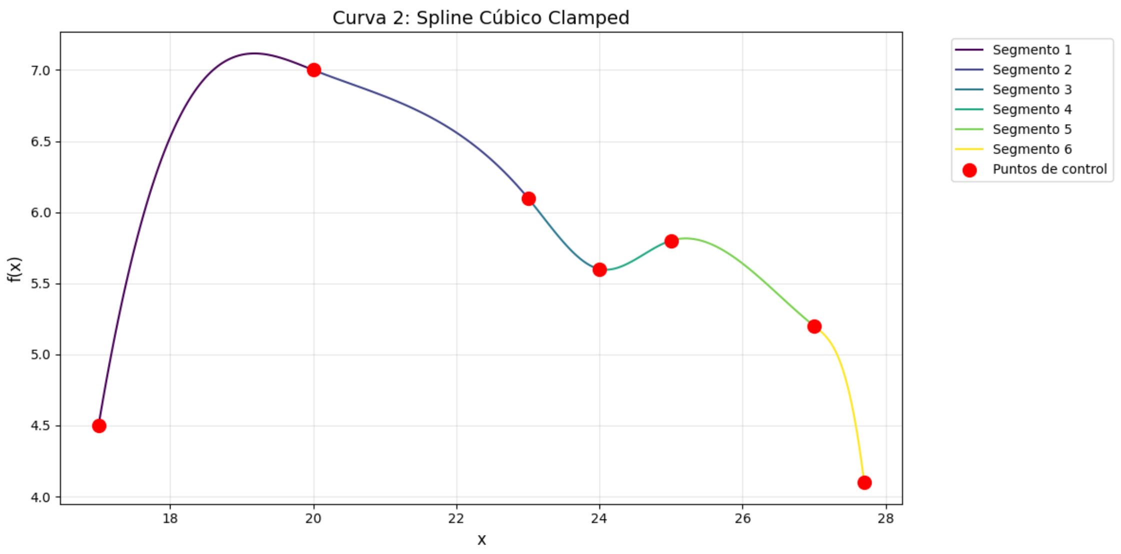
Use la función cubic_spline_clamped, provista en el enlace de Github, para graficar los datos de la siguiente tabla.

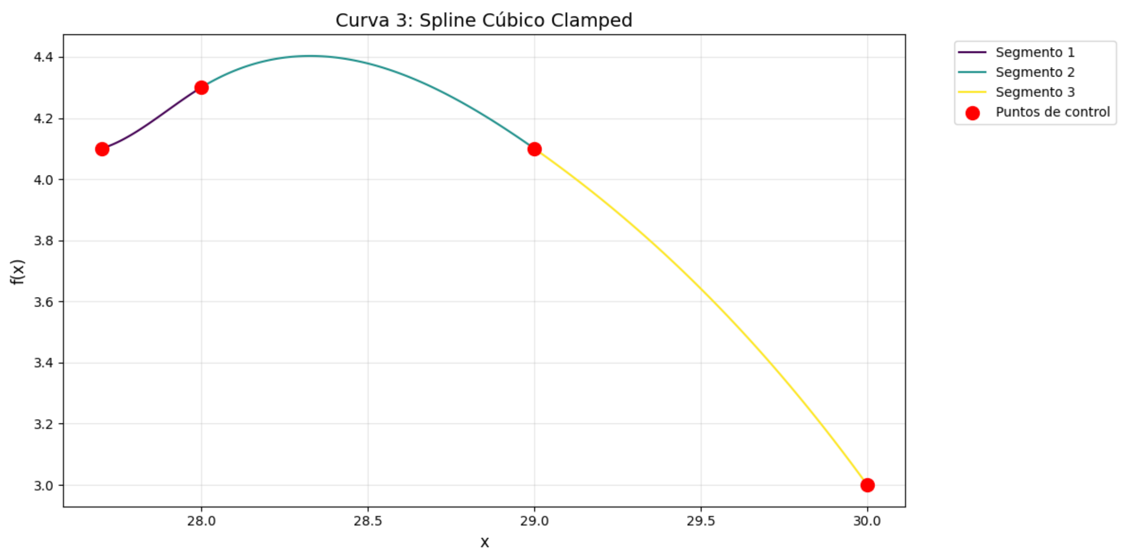


Curva 1				Curva 2				Curva 3			
i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$	i	x_i	$f(x_i)$	$f'(x_i)$
0	1	3.0	1.0	0	17	4.5	3.0	0	27.7	4.1	0.33
1	2	3.7		1	20	7.0		1	28	4.3	
2	5	3.9		2	23	6.1		2	29	4.1	
3	6	4.2		3	24	5.6		3	30	3.0	−1.5
4	7	5.7		4	25	5.8					
5	8	6.6		5	27	5.2					
6	10	7.1		6	27.7	4.1	−4.0				
7	13	6.7									
8	17	4.5	−0.67								

[24]:
```python
import sympy as sym
from IPython.display import display, Math
import matplotlib.pyplot as plt
import numpy as np

def cubic_spline_clamped(xs: list[float], ys: list[float], B0: float, B1:␣
 ↪float) -> list[sym.Symbol]:
    points = sorted(zip(xs, ys), key=lambda x: x[0])
    xs = [x for x, _ in points]
    ys = [y for _, y in points]
    n = len(points) - 1
    h = [xs[i+1] - xs[i] for i in range(n)]

    alpha = [0] * (n+1)
    alpha[0] = 3/h[0]*(ys[1]-ys[0]) - 3*B0
    alpha[-1] = 3*B1 - 3/h[n-1]*(ys[n]-ys[n-1])
    for i in range(1, n):
        alpha[i] = 3/h[i]*(ys[i+1]-ys[i]) - 3/h[i-1]*(ys[i]-ys[i-1])

    l = [2*h[0]]
    mu = [0.5]
    z = [alpha[0]/l[0]]
    for i in range(1, n):
        l.append(2*(xs[i+1]-xs[i-1]) - h[i-1]*mu[i-1])
        mu.append(h[i]/l[i])
        z.append((alpha[i] - h[i-1]*z[i-1])/l[i])
    l.append(h[n-1]*(2 - mu[n-1]))
    z.append((alpha[n] - h[n-1]*z[n-1])/l[n])

    c = [0]*(n+1)
    c[-1] = z[-1]

    x = sym.Symbol('x')
    splines = []
    for j in range(n-1, -1, -1):
        c[j] = z[j] - mu[j]*c[j+1]
        b = (ys[j+1]-ys[j])/h[j] - h[j]*(c[j+1]+2*c[j])/3
        d = (c[j+1]-c[j])/(3*h[j])
        a = ys[j]
        S = a + b*(x-xs[j]) + c[j]*(x-xs[j])**2 + d*(x-xs[j])**3
        splines.append(S)
    splines.reverse()
    return splines

# -------------------- FUNCIÓN DE GRÁFICA --------------------
def plot_splines(xs, ys, splines, title, figsize=(12, 6)):
    plt.figure(figsize=figsize)
    colors = plt.cm.viridis(np.linspace(0, 1, len(splines)))
    x_sym = sym.Symbol('x')

    for i in range(len(splines)):
        x_segment = np.linspace(xs[i], xs[i+1], 100)
        y_segment = [splines[i].subs(x_sym, val) for val in x_segment]
        plt.plot(x_segment, y_segment, color=colors[i], label=f'Segmento {i+1}')

    plt.scatter(xs, ys, color='red', s=100, zorder=5, label='Puntos de control')
    plt.title(title, fontsize=14)
    plt.xlabel('x', fontsize=12)
    plt.ylabel('f(x)', fontsize=12)
    plt.grid(True, alpha=0.3)
    plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left')
    plt.tight_layout()
    plt.show()

# -------------------- MOSTRAR ECUACIONES --------------------
def display_equations(splines, xs, curve_name):
    print(f"\n{'-'*50}")
    print(f"Ecuaciones para {curve_name}:")
    print(f"{'-'*50}")
    for i, s in enumerate(splines):
        display(Math(f"S_{i}(x) = {sym.latex(s.simplify())} \\quad \\text{{para␣
 ↪}} x \\in [{xs[i]}, {xs[i+1]}]"))
        print(f"Forma expandida: {sym.latex(s.expand())}\n")

# -------------------- VERIFICAR FRONTERAS --------------------
def verify_boundary_conditions(splines, xs, ys, B0, B1):
    x = sym.Symbol('x')
    print("\nVerificación de condiciones de frontera:")
    S0_prime = sym.diff(splines[0], x)
    Sn_prime = sym.diff(splines[-1], x)
    calculated_B0 = S0_prime.subs(x, xs[0])
    calculated_B1 = Sn_prime.subs(x, xs[-1])
    print(f"f'({xs[0]}) calculado: {calculated_B0.evalf()}, esperado: {B0}")
    print(f"f'({xs[-1]}) calculado: {calculated_B1.evalf()}, esperado: {B1}")

# -------------------- VERIFICAR CONTINUIDAD --------------------
def check_continuity(splines, xs):
    x = sym.Symbol('x')
    print("\nVerificación de continuidad en puntos intermedios:")
    for i in range(1, len(splines)):
        left = splines[i-1]
        right = splines[i]
        point = xs[i]
        f_cont = sym.simplify(left.subs(x, point) - right.subs(x, point))
        f_prime_cont = sym.simplify(sym.diff(left, x).subs(x, point) - sym.
↪diff(right, x).subs(x, point))
        f_double_prime_cont = sym.simplify(sym.diff(left, x, 2).subs(x, point)␣
↪- sym.diff(right, x, 2).subs(x, point))
        print(f"x = {point}: f = {f_cont}, f' = {f_prime_cont}, f'' =␣
↪{f_double_prime_cont}")

# -------------------- DATOS DE LAS CURVAS --------------------
# Curva 1
xs1 = [1, 2, 5, 6, 7, 8, 10, 13, 17]
ys1 = [3.0, 3.7, 3.9, 4.2, 5.7, 6.6, 7.1, 6.7, 4.5]
B0_1 = 1.0
B1_1 = -0.67

# Curva 2
xs2 = [17, 20, 23, 24, 25, 27, 27.7]
ys2 = [4.5, 7.0, 6.1, 5.6, 5.8, 5.2, 4.1]
B0_2 = 3.0
B1_2 = -4.0

# Curva 3
xs3 = [27.7, 28, 29, 30]
ys3 = [4.1, 4.3, 4.1, 3.0]
B0_3 = 0.33
B1_3 = -1.5

# -------------------- CALCULAR SPLINES --------------------
splines1 = cubic_spline_clamped(xs=xs1, ys=ys1, B0=B0_1, B1=B1_1)
splines2 = cubic_spline_clamped(xs=xs2, ys=ys2, B0=B0_2, B1=B1_2)
splines3 = cubic_spline_clamped(xs=xs3, ys=ys3, B0=B0_3, B1=B1_3)

# -------------------- GRAFICAR CURVAS --------------------
plot_splines(xs1, ys1, splines1, 'Curva 1: Spline Cúbico Clamped')
plot_splines(xs2, ys2, splines2, 'Curva 2: Spline Cúbico Clamped')
plot_splines(xs3, ys3, splines3, 'Curva 3: Spline Cúbico Clamped')

# -------------------- MOSTRAR ECUACIONES --------------------
display_equations(splines1, xs1, "Curva 1")
display_equations(splines2, xs2, "Curva 2")
display_equations(splines3, xs3, "Curva 3")

# # -------------------- VERIFICAR FRONTERAS Y CONTINUIDAD --------------------
# verify_boundary_conditions(splines1, xs1, ys1, B0_1, B1_1)
# verify_boundary_conditions(splines2, xs2, ys2, B0_2, B1_2)
# verify_boundary_conditions(splines3, xs3, ys3, B0_3, B1_3)

check_continuity(splines1, xs1)
check_continuity(splines2, xs2)
check_continuity(splines3, xs3)
```







```
--------------------------------------------------
Ecuaciones para Curva 1:
--------------------------------------------------
```

$S_0(x) = 0.0468099653460708x^3 - 0.487239861384283x^2 + 1.83404982673035x + 1.60638006930786 \quad \text{para } x \in [1, 2]$

```
Forma expandida: 0.0468099653460708 x^{3} - 0.487239861384283 x^{2} +
1.83404982673035 x + 1.60638006930786
```

$S_1(x) = 0.0265552121382411x^3 - 0.365711342137305x^2 + 1.5909927882364x + 1.7684180949705 \quad \text{para } x \in [2, 5]$

```
Forma expandida: 0.0265552121382411 x^{3} - 0.365711342137305 x^{2} +
1.5909927882364 x + 1.7684180949705
```

$S_2(x) = 0.341862882832256x^3 - 5.09532640254753x^2 + 25.2390680902875x - 37.6450407417814 \quad \text{para } x \in [5, 6]$

```
Forma expandida: 0.341862882832256 x^{3} - 5.09532640254753 x^{2} +
25.2390680902875 x - 37.6450407417814
```

$S_3(x) = -0.574548094033905x^3 + 11.4000711810434x^2 - 73.7333174112578x + 160.299730261309 \quad \text{para } x \in [6, 7]$

```
Forma expandida: - 0.574548094033905 x^{3} + 11.4000711810434 x^{2} -
73.7333174112578 x + 160.299730261309
```

$S_4(x) = 0.156329493303363x^3 - 3.94835815303925x^2 + 33.7056879273205x - 90.3912821953733$ para $x \in [7, 8]$

```
Forma expandida: 0.156329493303363 x^{3} - 3.94835815303925 x^{2} +
33.7056879273205 x - 90.3912821953733
```

$S_5(x) = 0.0239201086447503x^3 - 0.770532921232554x^2 + 8.28308607286689x - 22.5976772501638$ para $x \in [8, 10]$

```
Forma expandida: 0.0239201086447503 x^{3} - 0.770532921232554 x^{2} +
8.28308607286689 x - 22.5976772501638
```

$S_6(x) = -0.00255606547823463x^3 + 0.023752302456995x^2 + 0.340233835971401x + 3.87849687282113$ para $x \in [10, 13]$

```
Forma expandida: - 0.00255606547823463 x^{3} + 0.023752302456995 x^{2} +
0.340233835971401 x + 3.87849687282113
```

$S_7(x) = 0.00574178139926946x^3 - 0.299863725765665x^2 + 4.54724220286598x - 14.3518727170554$ para $x \in [13, 17]$

```
Forma expandida: 0.00574178139926946 x^{3} - 0.299863725765665 x^{2} +
4.54724220286598 x - 14.3518727170554


--------------------------------------------------
Ecuaciones para Curva 2:
--------------------------------------------------
```

$S_0(x) = 0.12616207628025x^3 - 7.53497434135573x^2 + 149.806607471118x - 984.439023122068$ para $x \in [17, 20]$

```
Forma expandida: 0.12616207628025 x^{3} - 7.53497434135573 x^{2} +
149.806607471118 x - 984.439023122068
```

$S_1(x) = -0.022930673285195x^3 + 1.41059063257098x^2 - 29.1046920074162x + 208.302973401493$ para $x \in [20, 23]$

```
Forma expandida: - 0.022930673285195 x^{3} + 1.41059063257098 x^{2} -
29.1046920074162 x + 208.302973401493
```

$S_2(x) = 0.280127236863149x^3 - 19.5004051676648x^2 + 451.848211398006x - 3479.00261937341$ para $x \in [23, 24]$

```
Forma expandida: 0.280127236863149 x^{3} - 19.5004051676648 x^{2} +
451.848211398006 x - 3479.00261937341
```

$S_3(x) = -0.357384536100794x^3 + 26.4004424857391x^2 - 649.772132283688x + 5333.96013008014$ para $x \in [24, 25]$

```
Forma expandida: - 0.357384536100794 x^{3} + 26.4004424857391 x^{2} -
649.772132283688 x + 5333.96013008014
```

$S_4(x) = 0.0882021573401092x^3 - 7.0185595223286x^2 + 185.702917918006x - 1628.33195493397$ para $x \in [25, 27]$

```
Forma expandida: 0.0882021573401092 x^{3} - 7.0185595223286 x^{2} +
185.702917918006 x - 1628.33195493397
```

$S_5(x) = -2.56800212665878x^3 + 208.133987481581x^2 - 5623.41585118756x + 50653.7369670161$ para $x \in [27, 27.7]$

```
Forma expandida: - 2.56800212665878 x^{3} + 208.133987481581 x^{2} -
5623.41585118756 x + 50653.7369670161


--------------------------------------------------
Ecuaciones para Curva 3:
--------------------------------------------------
```

$S_0(x) = -3.79941327466078x^3 + 317.993289328931x^2 - 8870.74279427938x + 82483.079611294$ para $x \in [27.7, 28]$

```
Forma expandida: - 3.79941327466078 x^{3} + 317.993289328931 x^{2} -
8870.74279427938 x + 82483.079611294
```

$S_1(x) = 0.296039603960395x^3 - 26.0247524752475x^2 + 761.762376237622x - 7420.30198019801$ para $x \in [28, 29]$

```
Forma expandida: 0.296039603960395 x^{3} - 26.0247524752475 x^{2} +
761.762376237622 x - 7420.30198019801
```

$S_2(x) = -0.0653465346534656x^3 + 5.41584158415843x^2 - 150.014851485149x + 1393.54455445545$ para $x \in [29, 30]$

```
Forma expandida: - 0.0653465346534656 x^{3} + 5.41584158415843 x^{2} -
150.014851485149 x + 1393.54455445545


Verificación de continuidad en puntos intermedios:
x = 2: f = 4.44089209850063E-16, f' = 1.11022302462516E-16, f'' =
-5.55111512312578E-17
x = 5: f = 8.88178419700125E-16, f' = 0, f'' = 7.77156117237610E-16
x = 6: f = -8.88178419700125E-16, f' = 0, f'' = 0
x = 7: f = 8.88178419700125E-16, f' = 4.44089209850063E-16, f'' =
-1.77635683940025E-15
x = 8: f = 8.88178419700125E-16, f' = 8.88178419700125E-16, f'' =
4.44089209850063E-16
x = 10: f = 0, f' = 2.22044604925031E-16, f'' = 2.77555756156289E-17
```

```
x = 13: f = 0, f' = -2.22044604925031E-16, f'' = -2.77555756156289E-17

Verificación de continuidad en puntos intermedios:
x = 20: f = -8.88178419700125E-16, f' = -5.32907051820075E-15, f'' =
-4.44089209850063E-16
x = 23: f = 0, f' = 4.44089209850063E-16, f'' = 4.44089209850063E-16
x = 24: f = -1.77635683940025E-15, f' = -1.77635683940025E-15, f'' = 0
x = 25: f = 0, f' = 8.88178419700125E-16, f'' = -1.77635683940025E-15
x = 27: f = 8.88178419700125E-16, f' = 8.88178419700125E-16, f'' =
1.06581410364015E-14

Verificación de continuidad en puntos intermedios:
x = 28: f = -8.88178419700125E-16, f' = -8.88178419700125E-16, f'' =
7.10542735760100E-14
x = 29: f = -8.88178419700125E-16, f' = -8.88178419700125E-15, f'' =
7.10542735760100E-15
x = 25: f = 0, f' = 8.88178419700125E-16, f'' = -1.77635683940025E-15
x = 27: f = 8.88178419700125E-16, f' = 8.88178419700125E-16, f'' =
1.06581410364015E-14

Verificación de continuidad en puntos intermedios:
x = 28: f = -8.88178419700125E-16, f' = -8.88178419700125E-16, f'' =
7.10542735760100E-14
x = 29: f = -8.88178419700125E-16, f' = -8.88178419700125E-15, f'' =
7.10542735760100E-15
```