Repaso prueba 2

martes, 6 de enero de 2026 21:03

Encontrar la inversa.

$$A_1 = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 4 & 2 & 1 \end{pmatrix}$$ Matriz aumentada:

$$\left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \overset{-2F_1+F_2\to F_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\overset{-4F_1+F_3\to F_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & -10 & -15 & -4 & 0 & 1 \end{array}\right) \overset{-2F_2+F_3\to F_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & -5 & -5 & -2 & 1 & 0 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array}\right)$$

$-F_3+F_2\to F_2;\ 5F_1\to F_1$

$$\sim \left(\begin{array}{ccc|ccc} 5 & 15 & 20 & 5 & 0 & 0 \\ 0 & -5 & 0 & -2 & 3 & -1 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array}\right) \overset{4F_3+F_1\to F_1}{\sim} \left(\begin{array}{ccc|ccc} 5 & 15 & 0 & 5 & -8 & 4 \\ 0 & -5 & 0 & -2 & 3 & -1 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array}\right)$$

$3F_2+F_1\to F_1$

$$\sim \left(\begin{array}{ccc|ccc} 5 & 0 & 0 & -1 & 1 & 1 \\ 0 & -5 & 0 & -2 & 3 & -1 \\ 0 & 0 & -5 & 0 & -2 & 1 \end{array}\right) \overset{\substack{-F_2\to F_2 \\ -F_3\to F_3}}{\sim} \left(\begin{array}{ccc|ccc} 5 & 0 & 0 & -1 & 1 & 1 \\ 0 & 5 & 0 & 2 & -3 & 1 \\ 0 & 0 & 5 & 0 & 2 & -1 \end{array}\right)$$

$$A^{-1} = \frac{1}{5}\cdot\begin{pmatrix} -1 & 1 & 1 \\ 2 & -3 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

$A = LU$

$F_2 \to F_2 - 2F_1$

$F_3 \to F_3 - 4F_1$

$F_3 \to F_3 - 2F_2$

$$U = \begin{pmatrix} 1 & 3 & 4 \\ 0 & -5 & -5 \\ 0 & 0 & -5 \end{pmatrix} \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$$ Matriz aumentada.

$-5F_1+F_3\to F_3$

15 6 07 Matriz aumentada.

$$-5F_1+F_3\to F_3$$

$$A_2|I=\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&4&0&1&0\\5&6&0&0&0&1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&4&0&1&0\\0&-4&-15&-5&0&1\end{array}\right)$$

$$+4F_2+F_3\to F_3 \qquad\qquad -4F_3+F_2\to F_2$$

$$\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&4&0&1&0\\0&0&1&-5&4&1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&0&20&-15&-4\\0&0&1&-5&4&1\end{array}\right)$$

$$3F_3+F_1\to F_1 \qquad\qquad -2F_2+F_1\to F_1$$

$$\sim\left(\begin{array}{ccc|ccc}1&2&0&16&-12&-3\\0&1&0&20&-15&-4\\0&0&1&-5&4&1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}1&0&0&-24&18&5\\0&1&0&20&-15&-4\\0&0&1&-5&4&1\end{array}\right)$$

$$A_2^{-1}=\begin{pmatrix}-24&18&5\\20&-15&-4\\-5&4&1\end{pmatrix}$$

$A=LU$

$F_3\to F_3-5F_1$
$F_3\to F_3+4F_2$

$$U=\begin{pmatrix}1&2&3\\0&1&4\\0&0&1\end{pmatrix},\quad L=\begin{pmatrix}1&0&0\\0&1&0\\5&-4&1\end{pmatrix}$$

$$A_3=\begin{pmatrix}4&2&1\\2&1&3\\4&3&4\end{pmatrix}$$

$$-\tfrac{1}{2}F_1+F_2\to F_2$$
$$-\tfrac{1}{4}F_1+F_3\to F_3$$

$$A|I=\left(\begin{array}{ccc|ccc}4&2&1&1&0&0\\2&1&3&0&1&0\\4&3&4&0&0&1\end{array}\right)\sim\left(\begin{array}{ccc|ccc}4&2&1&1&0&0\\0&0&2{,}5&-0{,}5&1&0\\0&2{,}5&3{,}75&-0{,}25&0&1\end{array}\right)$$

$$\tfrac{1}{2{,}5}F_3\to F_3 \qquad F_2\leftrightarrow F_3 \qquad -3{,}75F_3+F_2\to F_2 \qquad -F_3+F_1\to F_1 \qquad \tfrac{1}{2{,}5}F_2\to F_2$$

$$\sim\left(\begin{array}{ccc|ccc}4&2&1&1&0&0\\0&2{,}5&3{,}75&-0{,}25&0&1\\0&0&1&-0{,}2&0{,}4&0\end{array}\right)\sim\left(\begin{array}{ccc|ccc}4&2&0&1{,}2&-0{,}4&0\\0&1&0&0{,}2&-0{,}6&0{,}4\\0&0&1&-0{,}2&0{,}4&0\end{array}\right)$$

$$\tfrac{1}{4}F_1\to F_1$$
$$-2F_2+F_1\to F_1$$

$$\sim\left(\begin{array}{ccc|ccc}4&2&0&1{,}2&-0{,}4&0\\0&1&0&0{,}2&-0{,}6&0{,}4\end{array}\right)\quad\left(\begin{array}{ccc|ccc}1&0&0&0{,}2&0{,}2&-0{,}2\\0&1&0&0{,}2&-0{,}6&0{,}4\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|ccc} 4 & 2 & 0 & 1,2 & -0,4 & 0 \\ 0 & 1 & 0 & 0,2 & -0,6 & 0,4 \\ 0 & 0 & 1 & -0,2 & 0,4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0,2 & 0,2 & -0,2 \\ 0 & 1 & 0 & 0,2 & -0,6 & 0,4 \\ 0 & 0 & 1 & -0,2 & 0,4 & 0 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 0,2 & 0,2 & -0,2 \\ 0,2 & -0,6 & 0,4 \\ -0,2 & 0,4 & 6 \end{pmatrix}$$

Factorizar

$$A_3 = \begin{pmatrix} 4 & 2 & 1 \\ 2 & 1 & 3 \\ 4 & 3 & 4 \end{pmatrix} \underset{-\frac{1}{2}F_1 + F_2 \to F_2}{\sim} \begin{pmatrix} 4 & 2 & 1 \\ 0 & 0 & 2,5 \\ 1 & 3 & 4 \end{pmatrix}$$

no hay factorizacion directa LU por que hay intercambio de filas

$$A_4 = \begin{bmatrix} 2 & 4 & 6 & 1 \\ 4 & 7 & 5 & -6 \\ 2 & 5 & 18 & 10 \\ 6 & 12 & 38 & 16 \end{bmatrix}$$

$$\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 4 & 7 & 5 & -6 & 0 & 1 & 0 & 0 \\ 2 & 5 & 18 & 10 & 0 & 0 & 1 & 0 \\ 6 & 12 & 38 & 16 & 0 & 0 & 0 & 1 \end{array}\right)$$

$F_2 \to F_2 - 2F_1$
$F_3 \to F_3 - 1F_1$
$F_4 \to F_4 - 3F_1$

$$\sim\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -7 & -8 & -2 & 1 & 0 & 0 \\ 0 & 1 & 12 & 9 & -1 & 0 & 1 & 0 \\ 0 & 0 & 20 & 13 & -3 & 0 & 0 & 1 \end{array}\right)$$

$F_3 \to F_3 + F_2$
$F_4 \to F_4 - 4F_3$

$$\sim\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -7 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 5 & 1 & -3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 9 & 9 & -4 & -4 & 1 \end{array}\right)$$

$\frac{1}{9}F_4 \to F_4$

$$\sim\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -7 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 5 & 1 & -3 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -4/9 & -4/9 & 1/9 \end{array}\right)$$

$F_3 \to F_3 - F_4$
$F_2 \to F_2 + 8F_4$
$F_1 \to F_1 - F_4$

$$\sim\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 0 & 0 & 4/9 & 4/9 & -1/9 \\ 0 & -1 & -7 & 0 & 6 & -23/9 & -32/9 & 8/9 \\ 0 & 0 & 5 & 0 & -4 & 13/9 & 13/9 & -1/9 \\ 0 & 0 & 0 & 1 & 1 & -4/9 & -4/9 & 1/9 \end{array}\right)$$

$F_2 \to F_2 + 7F_3$
$F_1 \to F_1 - 6F_3$
$F_1 \to F_1 - 4F_2$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 286 & -154 & -334 & 133 \\ 0 & 1 & 0 & 0 & -36 & 48 & 138 & -66 \\ 0 & 0 & 1 & 0 & -72 & 26 & 26 & -2 \\ 0 & 0 & 0 & 1 & 90 & -40 & -40 & 10 \end{array}\right)$$

$$A_4^{-1} = \begin{pmatrix} 286 & -154 & -334 & 133 \\ -36 & 48 & 138 & -66 \\ -72 & 26 & 26 & -2 \\ 90 & -40 & -40 & 10 \end{pmatrix}$$

$\frac{1}{2} C_1 \to C_1$

$$A = \begin{pmatrix} 2 & 4 & 6 & 1 \\ 4 & 7 & 5 & -6 \\ 2 & 5 & 18 & 10 \\ 6 & 12 & 38 & 16 \end{pmatrix} \underset{\substack{F_2 \to F_2 - 2F_1 \\ F_3 \to F_3 - F_1 \\ F_4 \to F_4 - 3F_1}}{\sim} \begin{pmatrix} 1 & -4 & 6 & 1 \\ 0 & 1 & -7 & -8 \\ 0 & -1 & 12 & 9 \\ 0 & 0 & 20 & 13 \end{pmatrix}$$

$F_3 \to F_3 + F_2$
$F_4 \to F_4 - 4F_3$

$$\sim \begin{pmatrix} 2 & 4 & 6 & 1 \\ 0 & -1 & -7 & -8 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 9 \end{pmatrix} = U$$

$LU = A$
$LU^{-1}U = AU^{-1}$

$$\left(\begin{array}{cccc|cccc} 2 & 4 & 6 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -7 & -8 & 0 & 1 & 0 & 0 \\ 0 & 0 & 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0{,}5 & 2 & 11/5 & -49/90 \\ 0 & 1 & 0 & 0 & 0 & -1 & -7/5 & 4/15 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/5 & -1/45 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1/9 \end{array}\right)$$

$$U^{-1} = \begin{pmatrix} 0{,}5 & 2 & 11/5 & -49/90 \\ 0 & -1 & -7/5 & 4/15 \\ 0 & 0 & 1/5 & -1/45 \\ 0 & 0 & 0 & 1/9 \end{pmatrix}$$

$$A \cdot U^{-1} = L$$

$$\begin{pmatrix} 2 & 4 & 6 & 1 \\ 4 & 7 & 5 & -6 \\ 2 & 5 & 18 & 10 \\ 6 & 12 & 38 & 16 \end{pmatrix} \times \begin{pmatrix} 0{,}5 & 2 & 11/5 & -49/90 \\ 0 & -1 & -7/5 & 4/15 \\ 0 & 0 & 1/5 & -1/45 \\ 0 & 0 & 0 & 1/9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 3 & 0 & 4 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 3 & 0 & 4 & 1 \end{pmatrix}$$