Lab session 11: Probabilistic Reasoning

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1 The Bayesian monkey and the probabilistic banana

We consider a variant of the monkey and banana problem, similar to (but different from) the variant introduced in Lab 1, section 6. In this variant, the monkey has partial observability: it knows its position (which is the same as the position of the box), but does not know the position of the banana. However, it can perceive it:

- If on top of the box, the monkey can see whether the banana is here;
- If on the floor, it can smell it, but in a non accurate way. Indeed, when in x and with a banana located in b, the monkey will smell its presence with probability

  $$P(Smell|b, x) = \alpha \exp(-\beta|x - b|)$$

  with $\alpha \in [0, 1]$ and $\beta > 0$.

Question 1. Give an interpretation of parameters α and β. What is their role in this probabilistic model?

In this setting, a strategy of the monkey would be to try climbing on the box at each position. With this strategy, it is certain that it will eventually see the banana.

However, the monkey is a Bayesian, which involves in particular that it is so busy computing posteriors that it hasn't gone to the gym for years, if not decades. Therefore, the climbing action is very costly for it, and it would rather use probabilistic reasoning to get a clearer idea of the position of the banana before climbing.

We represent by B the random variable corresponding to the position of the banana. Then, $P(B = i)$ denotes the probability that the banana is located in i.

Initially, the monkey has uniform belief over the position of the banana, which means that for all i:

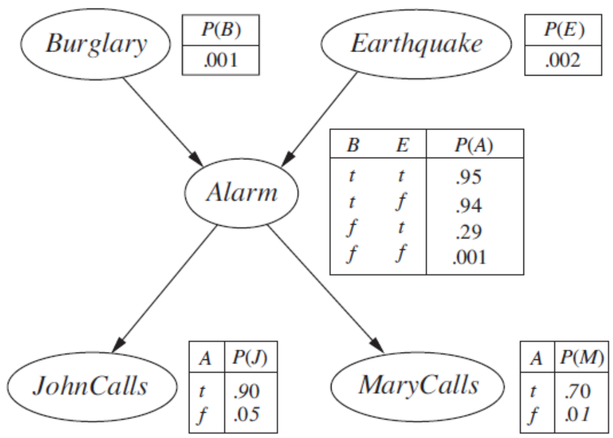
$$P(B = i) = \frac{1}{L}$$

where L is the size of the environment.

Question 2. For $L = 5$, and parameters $\alpha = 0.8$ and $\beta = 1$, compute the posterior probability $P(B|Smell, X = 2)$.

2 Inference in Bayesian Networks

In this exercise, we consider the burglary/earthquake Bayesian Network presented in class.



Question 3. Compute the posterior probability of a burglary knowing that John called but not Mary: $P(Burglary|JohnCalls, \neg MaryCalls)$. For this, propose a formula which you can compute, but you don't have to compute the numerical value (but feel free to compute it and to check that it is equal to about 0.76...).

This method you used is called **inference by enumeration**. Even though it gives the exact value of the posterior, it does not scale well to big networks and, consequently, is not usable in most cases.

Instead, we use **approximate inference**, in particular based on sampling. Sampling consists in simulating observations from a probability distribution. For instance, random number generators are sampling from uniform distributions.

Question 4. Based on a random number generator providing a number uniformely at random in $[0, 1]$ (in Python, you can use either `random.random()` or Numpy `np.random.rand()`), propose an implementation of a function sampling a random variable X taking value 1 with probability 0.8 and 0 with probability 0.2. Sample X with your function 100 times and observe the frequency to obtain 0s and 1s. Is this coherent with the distribution of X?[1]

Question 5. Consider the country BN presented in the lecture. We have two variables: C indicating the country, and taking values in $\{Norway, NotNorway\}$, and L indicating the language you hear in the street, taking two values $\{Norwegian, NotNorwegian\}$. We have $P(C = Norway) = 0.3$, $P(L = Norwegian|C = Norway) = 0.85$ and $P(L = Norwegian|C = NotNorway) = 0.05$. Implement a function `sampleNorwayBN(N)` which outputs a list of N dictionaries {`'country': ` c, `'language': ` l} where c is the country and l is the language, and (c, l) are distributed according to the joint probability of the BN. To do so, start by generating c, and then generate l based on the value of c.

Function `sampleBurglaryBN` proposes a sampling of the burglary Bayesian network. We will now use this function to perform **rejection sampling**. Rejection sampling is used to estimate the posterior $P(Q|E_1, \ldots, E_k)$. The principle is the following:

[1] If not, sorry to tell you, but your implementation is incorrect!

- Sample N times the full BN;
- In the generated samples, keep only those for which $E_1, \ldots, E_k$ are satisfied;
- Among the remaining samples, compute the proportion of those for which Q is satisfied.

Question 6. Implement rejection sampling for the burglary BN and use your implementation to compute an estimation of $P(Burglary|JohnCalls, \neg MaryCalls)$. Compare the obtained result with the exact posterior computed in Question 3.

Question 7 (Bonus, not graded)**.** Implement rejection sampling for the Wumpus world scenario seen during the lecture. Compare the obtained estimation to the exact value of the posterior computed in class.