Basin Modes for Alexandre

lennard.miller

July 2024

1 Introduction

The idea for the last part of the internship is to project the flow field onto the basin modes. We write

$$\psi(x,y) = \Sigma_{m,n=0}^{\infty}\hat{\psi}(m,n)\phi_{mn}(x,y) \ + c.c. \tag{1}$$

where $\hat{\psi}$ is the basin mode coefficient (it is complex, and has an amplitude and a phase), and $\Re$ denotes the real part. ϕ is the (complex formulation of the) basin mode at m, n. $c.c.$ denotes the complext conjugate of the expression. The basin mode ψ is written as

$$\begin{aligned} \phi_{mn} &= e^{i\left(\frac{\beta x}{2\sigma}\right)} sin\left(\frac{\pi m x}{L}\right) sin\left(\frac{\pi n y}{L}\right) \\ \sigma &= -\frac{\beta L}{2\pi\sqrt{m^2+n^2}} \end{aligned} \tag{2}$$

σ is the frequency of the basin modes. (If you write the real part of this, it should give you the expression you find in the Pedlosky book).

They satisfy two important orthogonality constrains. The first one relates to the energy:

$$\int -\phi_{m'n'}\nabla\phi_{mn} dA = \frac{\pi^2(m^2+n^2)}{4}\delta_{m-m',n-n'} \tag{3}$$

From this we can construct a Parseval's equality.

$$E = \frac{1}{L^2}\int \frac{u^2+v^2}{2} \ dA = \frac{1}{L^2}\Sigma\frac{\pi^2(m^2+n^2)}{4}|\hat{\psi}|^2 \tag{4}$$

Second, we can show an orthogonality relation for the beta-term.

$$\int \phi^{(n,m)}\frac{\partial \phi^{(n',m')}}{\partial x} dA = -i\frac{\pi L\sqrt{m^2+n^2}}{8}\delta_{n-n',m-m'} \tag{5}$$

This can be used to obtain a similar framework as for the fourier basis: we take the initial equation and project it onto the basin modes. The real part will give us the energy equation, while the complex part will yield the dispersion relation. We'll not go into the details of this, though. Instead, the aim will only be to use the basin mode projection to diagnose whether the modes follow the dispersion relation or not (see notebook).

We expect that only small n and m will behave wave-like, so we will only run diags for n and m such that $2\pi n/L \sim k_R h$ (or maybe a little larger). The hope would be to see that basin modes smaller than the Rhines scale are no longer moving as waves. For this the steps are:

- Obtain time series of $\hat{\psi}$ for a given n, m
- plot the "circles"
- find the right criterion to dissect waves vs. turbulence (do the basin modes follow the dispersion relation? Or are they doppler-shifted?)
- plot binary map of where the waves are found in n, m space.

For all this, we need to find out whether we first have to subtract the mean flow to obtain meaningful results (decomposition de Reynolds).

Once we have this map, we can calculate the energy transfer terms associated to the basin modes.