Look Ma, No Sampling!

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Tired of waiting for your MCMC to run? No problem, just skip the MCMC and evaluate expectations using function approximation and numerical integration.

Bayesian Formulation

θ Hyperparameters: Hyper-prior: $[\theta]$

x Latent field: Representation and prior $[x|\theta]$

y Observed data: Likelihood: $[y|x]$

The focus of inference is the posterior distribution:

$$[x,\theta|y] = \frac{[y|x]\,[x|\theta]\,[\theta]}{[y]}$$

We assume the normalizing constant $[y]$ is finite.

Posterior Inference

We wish to compute expectations:

$$\mathrm{E}_{x,\theta|y}[f(x)] = \int f(x)[x,\theta|y]\,\mathrm{d}x\,\mathrm{d}\theta$$

Notation

We learned this notation from Alan Gelfand: read $[a]$ as "the distribution over a", and $[a|b]$ as "the distribution over a given b". We will abuse this notation to also denote the density function.

Monte Carlo Integration

$$\mathrm{E}_{x,\theta|y}[f(x)] \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

where $\{(x_i,\theta_i)\}$ is ergodic for $[x,\theta|y]$.

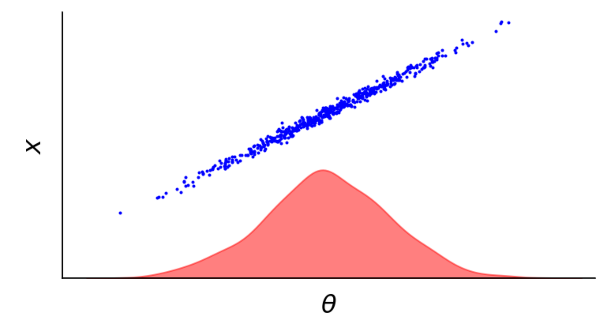
"Monte Carlo is an extremely bad method; it should be used only when all alternative methods are worse." (Alan Sokal)

Markov chain Monte Carlo

A representative MCMC scheme is the block Gibbs sampler

- Draw $\theta' \sim [\theta|x]$
- Draw $x' \sim [x|\theta', y]$

simulating a transition kernel that targets the posterior $[x,\theta|y]$.



Narrow scatter plot shows why this is slow

Better is to move in the marginal posterior over hyperparameters $[\theta|y]$
H. Rue and L. Held. *Gaussian Markov random fields : Theory and applications*. Chapman Hall, New York, 2005.

Factorize Posterior (MTC)

factorisation of the posterior density

$$[x,\theta|y] = [x|\theta,y]\,[\theta|y]$$

i.e. into the full conditional for x and the marginal posterior over θ.

Lemma Sampling $\theta' \sim [\theta|y]$ then $x' \sim [x|\theta', y]$ generates a sample from the posterior distribution, i.e.,

$$(x',\theta') \sim [x,\theta|y].$$

C. Fox and R. A. Norton. Fast sampling in a linear-Gaussian inverse problem. *SIAM/ASA Journal on Uncertainty Quantification*, 4(1):1191–1218, 2016.

Some kind of pictorial break

But not a box with text like this

Marginal Posterior over Hyperparameters

The marginal posterior distribution over hyperparameters is defined by the integral $[\theta|y] = \int_X [x,\theta|y]\,\mathrm{d}x$, as mentioned above, but this calculation is to be avoided because the integral is over the h ent field x. A cheap algebraic calculation is available when the full conditional for x

These terms simplify when $A^{-1}b = \mathcal{A}^{-1}\beta$

$$[x|\theta,y] = \frac{[y|x]\,[x|\theta]}{[y|\theta]}$$

has known form, implying that the normalising constant $[y|\theta]$ has known θ dependence, and hence one can evaluate the marginal posterior over θ

$$[\theta|y] \propto [y|\theta][\theta].$$

R. A. Norton, J. A. Christen, and C. Fox. Sampling hyperparameters in hierarchical models: improving on Gibbs for high-dimensional latent fields and large datasets. *Communications in Statistics - Simulation and Computation*, 47(9):2639–2655, 2018.

Tensor Train Representation of PDF

A surface rendering of a density in TT format
TT formula

S. Dolgov, K. Anaya-Izquierdo, C. Fox, and R. Scheichl. Approximation and sampling of multivariate probability distributions in the tensor train decomposition. *Statistics and Computing*, 30(3):603–625, 2020.

Inverse Problem of Recovering Ozone Profile

I'm thinking picture of satellite, DAG, RTE

Posterior Expectation by TT and Affine Representations

Picture of posterior inference, or timings, or some output summaries – perhaps a box below with some snappy summary or conclusions

R. A. Norton and C. Fox, Efficiency and computability of MCMC with Langevin, Hamiltonian, and other matrix-splitting proposals, arxiv:1501.03150.