



# A Fast Convoluted Story: Scaling Probabilistic Inference for Integer Arithmetic

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Do you want to reason 100 000 quicker
when dealing with uncertainty?

# Linear arithmetic can model probabilistic reasoning but is computationally expensive

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U ∼ Uniform(0, 10)
X = 0
for i in range(10):
    X = 7 · X + U
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$\mathbb{P}(X = 7)$?

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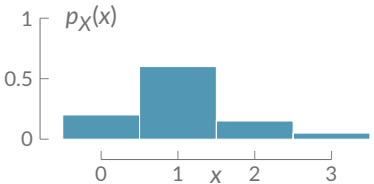
We improve scalability by orders of magnitude
through tensorised representations and the FFT

1. Tensorised representations allow acceleration of probabilistic inference
2. The FFT efficiently computes the PMF of sums of integer-valued random variables
3. PLIA$_t$ provides orders of magnitude faster inference and learning

# Tensorised representations allow acceleration of probabilistic inference




Integer-valued random variables are determined by their probability mass function (PMF)

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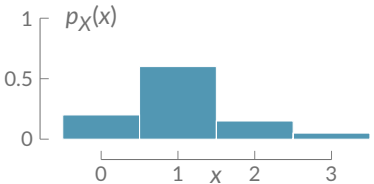


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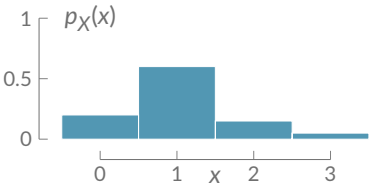


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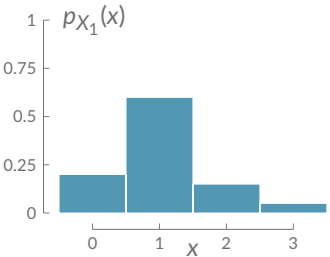
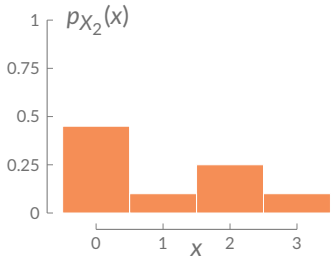
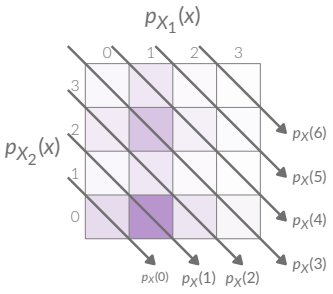
$X \equiv 7 \cdot X_1 + X_2 \mod 5$

Operations on integer-valued random variables can also be cast as tensor operations

# The FFT efficiently computes the PMF of sums of integer-valued random variables

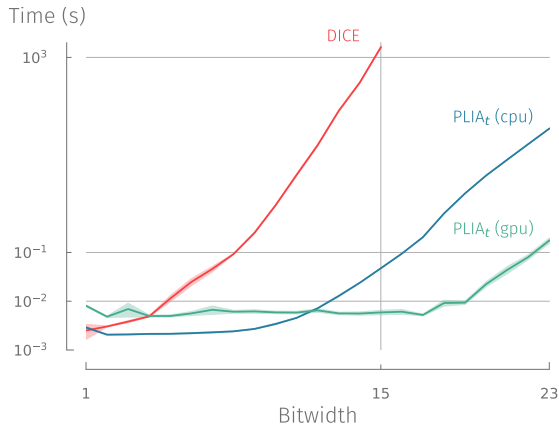



Calculating PMF of $\boldsymbol{X = X_1 + X_2}$ is usually $\mathcal{O}(N^2)$
but $\text{PLIA}_\text{t}$ does it in $\mathcal{O}(N \log N)$ with the FFT

# PLIA$_t$ provides orders of magnitude faster inference



# PLIA$_t$ provides orders of magnitude faster learning

\+

= 8 1 7 6 5

| Method | MNIST Addition (minutes) | | |
|---|---|---|---|
| | $N = 4$ | $N = 15$ | $N = 50$ |
| DeepProbLog | T/O | T/O | T/O |
| Scallop | $50.41^{+6.46}_{-0.17}$ | T/O | T/O |
| A-NeSI | $53.62^{+6.40}_{-1.76}$ | $714.55^{+27.66}_{-8.17}$ | T/O |
| PLIA$_t$ | $2.44^{+0.04}_{-0.04}$ | $7.85^{+0.64}_{-0.19}$ | $11.98^{+0.68}_{-0.04}$ |

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# PLIA$_t$ is a fast and scalable framework for probabilistic integer arithmetic

Tensors and the FFT...

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# $PLIA_t$ is a fast and scalable framework for probabilistic integer arithmetic

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...open up a new modelling paradigm
for probabilistic reasoning

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