a) Single neuron dynamics

a)

$$\tau_m \frac{dV}{dt} = -V + R\,I_{ext}$$

Voltage stationary:

$$\frac{dV}{dt} \overset{!}{=} 0 \Rightarrow R\,I_{ext} = V$$

$$\Rightarrow R \cdot I_{rheo} = V_{thr} = V_\Theta$$

$$\Rightarrow \cancel{\frac{V_{thr}}{\tau_m} = \frac{\tau_m}{C}} \quad V_\Theta = \frac{\tau_m}{C_m} I_{rheo}$$

$$\cancel{= \frac{10 \cdot 10^{-3}\,s}{250 \cdot}}$$

$$\Rightarrow I_{rheo} = \frac{V_\Theta\, C_m}{\tau_m} = \frac{15 \cdot 10^{-3}\,mV}{10 \cdot 10^{-3}\,s} \cdot$$

$$\cdot 250\,pF$$

$$= 375\,pA$$

b)

$$V_\Theta = R\,I_{syn}$$

$$= \tau_m \sum_i J\, \delta(t - t_i)$$

$$\Rightarrow h(t) = \tau_m J\, \delta(t) \quad \text{so that} \quad R\,I_{syn} = \sum_i h(t - t_i)$$

Using Campbell:

$$\mu = \nu_{syn} \int h(t)\, dt = \nu_{syn}\, \tau_m\, J$$

b) Solve diff. equation for h

$$\tau_m \frac{dh}{dt} = -h + \tau_m J \delta(t-t_0)$$

Solution: $h(t) = \Theta(t-t_0) \cdot J \cdot e^{-\frac{t-t_0}{\tau_m}}$

Proof: $$\frac{dh}{dt} = \delta(t-t_0) J \cdot e^{-\frac{t_0-t_0}{\tau_m}}$$
$$+ \Theta(t-t_0) \cdot J \cdot \left(-\frac{1}{\tau_m}\right) \cdot e^{-\frac{(t-t_0)}{\tau_m}}$$
$$= \delta(t-t_0) J - \frac{h(t)}{\tau_m} \qquad \square$$

$$\mu = r_{ext} \int h(t)\, dt = r_{ext} \int_{-\infty}^{\infty} \Theta(t-t_0) J \cdot e^{-\frac{(t-t_0)}{\tau_m}} dt$$

$$= r_{ext} \int_{t_0}^{\infty} J \cdot e^{-\frac{(t-t_0)}{\tau_m}} dt \qquad t' = t-t_0$$

$$= r_{ext} \int_{0}^{\infty} J \cdot e^{-\frac{t'}{\tau_m}} dt'$$

$$= r_{ext} \cdot J \cdot \tau_m$$

$$\Rightarrow V_\Theta = \mu = r_{ext} \cdot J \cdot \tau_m$$

$$\Rightarrow r_{ext} = \frac{V_\Theta}{J \cdot \tau_m}$$

$$= \frac{15\,mV}{0{,}1\,mV \cdot 10\,ms} = \frac{1}{10\,ms} = \cancel{100}\ 15000\ Hz$$

c) constant current : sub-threshold : no spiking

~~th~~ " : spiking at fixed rate

super- " : no change in spiking freq

spike trains:

Spiking for sub-thr. free membrane potential because the mean value of the poisson-process has a variance $\neq 0$, so that the neuron potential can cross the threshold voltage.

1.1.3

With increasing excitation rate, the spike rate of the excitatory and inhibitory neurons is nearly the same and increasing linearly.

In an unconnected network, the spiking ~~rate~~ rate is higher, because the inhibitory effect of a part of the network is missing.