



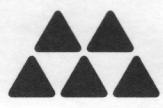
2.5.1

b)

Because there is a limited number of states, the integrals are computed by summation. The random variable  $\ell$  sets a threshold for choosing a state.

Then  $i$  is increased until the sum of the probabilities is higher than  $\ell$ .

Because it is  $i \in [0, 1]$  the sample with the cumulative probability of  $\ell$  is chosen.



### 2.5.2

$$\begin{aligned}
 \text{Show: } \tilde{c}_{ij}(\omega) &= \langle \tilde{x}_i^k(-\omega) \tilde{x}_j^k(\omega) \rangle_k = \mathcal{F}^*(\tilde{c}_{ij}) \\
 &= \left\langle \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{x}_i^k(t') \tilde{x}_j^k(t^*) e^{i(t'-t^*)\omega} dt' dt^* \right\rangle_k \\
 &= \int_{-\infty}^{\infty} \left\langle \langle \tilde{x}_i^k(t) \tilde{x}_j^k(t+\tau) e^{i\tau\omega} \rangle_k \right\rangle_t e^{i\tau\omega} d\tau \\
 &= \int_{-\infty}^{\infty} \left\langle \langle \tilde{x}_i^k(t) \tilde{x}_j^k(t+\tau) \rangle_k \right\rangle_t e^{i\tau\omega} d\tau \\
 &= \mathcal{F}^*(\tilde{c}_{ij}) \quad \square
 \end{aligned}$$

### 2.5.3

$$k = \frac{\langle \tilde{x}_i^k(\omega)^* \tilde{x}_j^k(\omega) \rangle_k}{\sqrt{|x_i^k(\omega)|^2 |x_j^k(\omega)|^2}} \quad | k=1$$

$$|k|^2 = \frac{(x_i^k(\omega)^* x_j^k(\omega)) \cdot (x_i^k(\omega) x_j^k)^*(\omega))}{(x_i^k)^*(\omega) x_i^k(\omega)) \cdot (x_j^k)^*(\omega) \cdot x_j^k(\omega))}$$

$= 1$  if process is deterministic  
 with  $x_i^k = x_i$   $\forall k \in \mathbb{N}$

2.5.4

$$x_i^k(t) = \alpha_i^k(t) + c^k(t) \quad i \in [0, 1]$$

$$\Rightarrow \tilde{x}_i^k = \tilde{\alpha}_i^k(t) + \tilde{c}^k(t)$$

$$\mathfrak{C}_{ij} = \langle \tilde{x}_i^k(t) \tilde{x}_j^k(t+\tau) \rangle_k$$

$$= \delta_{ij} \tilde{\alpha}_\alpha(\tau) + \tilde{\alpha}_c(\tau)$$

$$\tilde{C}_{ij}(\omega) = \mathcal{F}(\tilde{c}_{ij}(t))$$

$$= \int e^{-i\omega\tau} \tilde{c}_{ij}(\tau) d\tau$$

$$= \int e^{-i\omega\tau} (\delta(\tau) [\alpha r + \delta_{ij}(1-\alpha)r]) d\tau$$

$$= r (\delta_{ij}(1-\alpha) + \alpha)$$

$$= \begin{cases} \alpha r & i \neq j \\ 1 & i = j \end{cases}$$

$$\Rightarrow k_{ij} = \frac{\tilde{C}_{ij}}{\sqrt{\tilde{C}_{ii} \cdot \tilde{C}_{jj}}} = \alpha r$$