2.5.1

b)

Because there is a limited number of states, the integrals are computed by summation. The random variable l sets a threshhold for choosing a state. Then i is increased until the sum of the probabilities is higher than l. Because ~~it is l~~ $i \in [0,1)$ the sample with the cumulative probability of l is chosen.

2.5.2

Show: $\tilde{C}_{ij}(\omega) = \langle \tilde{x}^k_i(-\omega)\ \tilde{x}^k_j(\omega) \rangle_k = \mathcal{F}^*(\tilde{c}_{ij})$

$$= \langle \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{x}^k_i(t')\ \tilde{x}^k_j(t^*)\ e^{i(t'-t^*)\omega}\ dt'dt^* \rangle_k$$

$t'-t^* = \tau$

$t' = t$

$$= \int_{-\infty}^{\infty} \langle \tilde{x}^k_i(t)\ \tilde{x}^k_j(t+\tau)\ e^{i\tau\omega} \rangle_{k,t}\ d\tau$$

$$= \int_{-\infty}^{\infty} \langle \langle \tilde{x}^k_i(t)\ \tilde{x}^k_j(t+\tau) \rangle_k \rangle_t\ e^{i\tau\omega}\ d\tau$$

$$= \mathcal{F}^*(\tilde{c}_{ij}) \quad \square$$

2.5.3

$$k = \frac{\langle \tilde{x}^k_i(\omega)^*\ \tilde{x}^k_j(\omega) \rangle_k}{\sqrt{|x^k_i(\omega)|^2\ |x^k_j(\omega)|^2}} \qquad \Big|\ k=1$$

$$|k|^2 = \frac{(x^k_i(\omega)^*\ x^k_j(\omega)) \cdot\ x^k_i(\omega)\ x^{k*}_j(\omega))}{(x^{k*}_i(\omega)\ x^k_i(\omega)) \cdot (x^{k*}_j(\omega) \cdot x^k_j(\omega))}$$

$= 1$ if process is deterministic with $x^k_i = x_i\ \forall k \in \mathbb{N}$

2.5.4

$$x_i^k(t) = d_i^k(t) + c^k(t) \qquad i \in [0,1]$$

$$\Rightarrow \quad \tilde{x}_i^k = \tilde{\alpha}_i^k(t) + \tilde{c}^k(t)$$

$$\tilde{c}_{ij} = \langle\langle \tilde{x}_i^k(t) \; \tilde{x}_j^k(t+\tau) \rangle_k \rangle_t$$

$$= \delta_{ij} \, \tilde{\alpha}_d(\tau) + \tilde{\alpha}_c(\tau)$$

$$\tilde{C}_{ij}(\omega) = \mathcal{F}(\tilde{c}_{ij}(\tau))$$

$$= \int e^{-i\omega\tau} \; \tilde{c}_{ij}(\tau) \, d\tau$$

$$= \int e^{-i\omega\tau} \left(\delta(\tau) \left[\alpha r + \delta_{ij}(1-\alpha) r \right] \right) d\tau$$

$$= r(\delta_{ij}(1-\alpha) + \alpha)$$

$$= \begin{cases} \alpha r & i \neq j \\ 1 & i = j \end{cases}$$

$$\Rightarrow k_{ij} = \frac{\tilde{C}_{ij}}{\sqrt{\underbrace{|\tilde{C}_{ii}|^2}_{1} \cdot \underbrace{|\tilde{C}_{jj}|^2}_{1}}} = \alpha r$$