2.5.1

a) For $n = 10^5$, the following sample values are created

	Sample	Theoretical
$\mu(z_0)$	0.5985	0.6
$\mu(z_1)$	0.6035	0.6
$\tilde{C}_{00}$	0.2403	0.24
$\tilde{C}_{11}$	0.2393	0.24
$\tilde{C}_{01}$	0.0402	0.04

(Code appended)

b) Since there is a finite number states one can calculate the cummulative probability simply by summing. If one now draws $p \in [0,1)$ from a uniform distribution, it follows:

$$P(p \in [P_{cum}(X_{i-1}), P_{cum}(X_i))) = P(X_i) \quad , \quad X_i \in \Omega$$

Therefore we can use draws from $[0,1)$ to generate random[1] samples from our Probability distribution.

[1] (pseudorandom)

2.5.4

$$x_i^k(t) = d_i^k(t) + c^k(t)$$

$$\Rightarrow \tilde{x}_i^k(t) = \tilde{d}_i^k(t) + \tilde{c}^k(t), \quad \text{with expected values linear and } d_i, d_j, c \text{ indepent, follows}$$

$$\Rightarrow \tilde{C}_{ij}(\tau) = \delta_{ij} \tilde{a}_d(\tau) + \tilde{a}_c(\tau)$$

$$= \delta(\tau)\left[\alpha \nu + \delta_{ij}(1-\alpha)\nu\right]$$

$$\Rightarrow \tilde{C}_{ij}(\omega) = \mathcal{F}[\tilde{C}_{ij}(\tau)](\omega)$$

$$= e^{-i0}\left(\alpha\nu + \delta_{ij}(1-\alpha)\nu\right)$$

$$= \left(\alpha + \delta_{ij}(1-\alpha)\right)\nu$$

$$= \begin{cases} \alpha & i \neq j \\ 1 & i = j \end{cases}$$

$$\Rightarrow k'_{ij} = \frac{\tilde{C}_{ij}(\omega)}{\sqrt{\tilde{C}_{ii}(\omega)\tilde{C}_{jj}(\omega)}} \quad \text{for } i \neq j$$

$$= \alpha$$

2.5.2

z.z. $\langle \tilde{X}_i^k(-\omega)\, \tilde{X}_j^k(\omega)\rangle_k = \mathcal{F}[\tilde{C}_{ij}(\tau)](\omega)$

Bew. $\langle \tilde{X}_i^k(-\omega)\tilde{X}_j^k(\omega)\rangle_k = \langle \int dt\, d\tilde{t}\; e^{i(t-\tilde{t})\omega}\, \tilde{x}_i^k(t)\, \tilde{x}_j^k(\tilde{t})\rangle_k$

$$= \langle \int dt\, d\tau\; e^{-i\tau\omega}\, \tilde{x}_i^k(t)\, \tilde{x}_j^k(t+\tau)\rangle_k$$

$$= \int d\tau\; e^{-i\tau\omega} \langle\langle \tilde{x}_i^k(t)\, \tilde{x}_j^k(t+\tau)\rangle_k\rangle_t$$

$$= \mathcal{F}[\langle\langle \tilde{x}_i^k(t)\, \tilde{x}_j^k(t+\tau)\rangle_k\rangle_t](\omega)$$

$$= \mathcal{F}[\langle \tilde{C}_{ij}(t, t+\tau)\rangle_t](\omega) = \mathcal{F}[\tilde{C}_{ij}(\tau)](\omega) \quad \square$$

2.5.3

a) $$K'(\omega) = \frac{\langle \tilde{X}_i^k(\omega)^* \tilde{X}_j^k(\omega) \rangle_k}{\sqrt{\langle |\tilde{X}_i^k(\omega)|^2 \rangle_k \langle |\tilde{X}_j^k(\omega)|^2 \rangle_k}}$$

$$K^*(\omega)^2 = |K'(\omega)|^2 = \frac{\langle \tilde{X}_i^k(\omega)^* \tilde{X}_j^k(\omega) \rangle_k \langle \tilde{X}_i^k(\omega)^{**} \tilde{X}_j^k(\omega)^* \rangle_k}{\langle \tilde{X}_i^k(\omega) \tilde{X}_i^k(\omega)^* \rangle_k \langle \tilde{X}_j^k(\omega) \tilde{X}_j^k(\omega)^* \rangle_k}$$

For a deterministic Process:

$$\langle z^k \rangle_k = z^k = z$$

$$\Rightarrow \quad K(\omega)^2 = \frac{\tilde{X}_i(\omega)^* \tilde{X}_j(\omega) \; \tilde{X}_i(\omega) \; \tilde{X}_j(\omega)^*}{\tilde{X}_i(\omega) \tilde{X}_i(\omega)^* \tilde{X}_j(\omega) \tilde{X}_j(\omega)^*}$$

$$= 1$$