Exercise 1

1. $T_s(N) = 4C_1(\sqrt{N}-1)^2$

$$T_p(N,P) = 4C_1\frac{(\sqrt{N}-1)^2}{P} + 2C_2(\sqrt{N}-1)$$

$$W = T_s(N) \Rightarrow N = T_s^{-1}(W) = \left(1+\sqrt{\frac{W}{4C_1}}\right)^2$$

$$\tilde{T}_p(N,P) = T_p(T_s^{-1}(W), P) = \frac{W}{P} + \frac{C_2}{\sqrt{C_1}}\sqrt{W}.$$

$$\frac{W}{P\,\tilde{T}_p(N,P)} = \frac{W}{W+\frac{C_2}{\sqrt{C_1}}P\sqrt{W}} = E_0 = \frac{1}{1+\frac{1-E_0}{E_0}}$$

$$\Rightarrow \frac{C_2}{\sqrt{C_1}}P\cdot\frac{1}{\sqrt{W}} = \frac{1-E_0}{E_0} \Rightarrow W(P) = O(P^2)$$

2. $T_s(N) = 4C_1(\sqrt{N}-1)^2$ $\quad T_p(N,P) = 4C_1\frac{(\sqrt{N}-1)^2}{P} + 4C_2\frac{\sqrt{N}-1}{\sqrt{P}}$

$$N = T_s^{-1}(W) = \left(1+\sqrt{\frac{W}{4C_1}}\right)^2$$

$$\tilde{T}_p(N,P) = T_p(T_s^{-1}(W),P) = \frac{W}{P} + 2\frac{C_2}{\sqrt{C_1}}\sqrt{\frac{W}{P}}$$

$$\frac{W}{P\tilde{T}_p(N,P)} = \frac{1}{1+2\frac{C_2}{\sqrt{C_1}}\sqrt{\frac{P}{W}}} = E_0 = \frac{1}{1+\frac{1-E_0}{E_0}}.$$

$$\Rightarrow W = \frac{4C_2^2}{C_1}P\left(\frac{E_0}{1-E_0}\right)^2 = O(P)$$

Exercise 3.

2. $T_s(N) = 3C_1 N$

$$T_p(N,p) = \frac{3C_1N}{p} + C_2\left(p + \frac{p}{2} + \cdots + 1\right) = C_2(2p-1) + \frac{3C_1N}{p}$$

Here, we assume $p = 2^k$, $k \in \mathbb{Z}^+$

$$\cancel{T_p(N,p) \longrightarrow \frac{3C_1N}{p} + 2C_2p}$$

$$W = T_s(N) \Rightarrow N = T_s^{-1}(W) = \frac{W}{3C_1}$$

$$\frac{W}{p\tilde{T}(N,p)} = \frac{W}{W + C_2(2p^2 - p)}$$

$$\Rightarrow \frac{C_2}{W}(2p^2 - p) = \frac{1-E_0}{E_0}$$

$$\Rightarrow C_2(2p^2-p) \cdot \frac{E_0}{1-E_0} = W$$

$$= W = O(p^2)$$