sheet8

10:10

Exercise 2

1. $T_s(N) = c_1(\sqrt{N})^2 = c_1 N$

$T_p(N,p) = c_1 p \cdot \frac{\sqrt{N}}{p} + c_1\sqrt{N}\cdot\frac{\sqrt{N}}{p} + c_2(p+\sqrt{N}) = c_1\frac{N}{p} + (c_1+c_2)\sqrt{N} + c_2 p$

$c_1 p \cdot \frac{\sqrt{N}}{p}$: wait for p times to start pth process; $c_1\sqrt{N}\cdot\frac{\sqrt{N}}{p}$: one process; pth process to finish one iteration; $c_2(p+\sqrt{N})$: data transfer time between process

$W = T_s(N) = c_1 N \Rightarrow N = T_s^{-1}(W) = \frac{W}{c_1}$

$\tilde{T}_p(W,p) = T_p(T_s^{-1}(W), p) = c_1\frac{W}{c_1 p} + (c_1+c_2)\sqrt{\frac{W}{c_1}} + c_2 p$

$= \frac{W}{p} + \frac{c_1+c_2}{\sqrt{c_1}}\sqrt{W} + c_2 p$

$$\frac{W}{p\tilde{T}_p(W,p)} = \frac{W}{p\left(\frac{W}{p} + \frac{c_1+c_2}{\sqrt{c_1}}\sqrt{W} + c_2 p\right)} = \frac{W}{W + \frac{c_1+c_2}{\sqrt{c_1}}\sqrt{W}\,p + c_2 p^2} = E_0 = \frac{1}{1 + \frac{1-E_0}{E_0}}$$

$$= \frac{1}{1 + \frac{c_1+c_2}{\sqrt{c_1}} p W^{-\frac{1}{2}} + c_2 p^2 W^{-1}}$$

$$\therefore \quad \frac{c_1+c_2}{\sqrt{c_1}} p W^{-\frac{1}{2}} + c_2 p^2 W^{-1} = \frac{1-E_0}{E_0}$$

$a := \frac{c_1+c_2}{\sqrt{c_1}}$, $b := c_2$, $c := \frac{1-E_0}{E_0}$

$$W(p) = \frac{a p^2\sqrt{a^2+4bc} + a^2 x^2 + 2bcx^2}{2c^2}$$

$\therefore W(p) = O(p^2)$

Ex2.2

$T_s = k \cdot c_1(\sqrt{N})^2 = k c_1 N$ — total data transfer time

$$T_s = k \cdot c_1 (\sqrt{N})^2 = k c_1 N$$

$$T_p(N,p) = c_1 \frac{\sqrt{N}}{p} \cdot p + k \cdot c_1 \cdot \frac{\sqrt{N}}{p} \sqrt{N} + c_2 (p + k \cdot \sqrt{N}) = \frac{k c_1}{p} N + (c_1 + c_2 k) \sqrt{N} + c_2 p$$

total data transfer time

wait time for pth process to start

computation time for pth process

$$W = T_s(N) = k c_1 N \Rightarrow N = \frac{W}{k c_1}$$

$$\tilde{T}_p(W,p) = T_p(T_s^{-1}(W), p) = \frac{k c_1}{p} \frac{W}{k c_1} + (c_1 + c_2 k) \sqrt{\frac{W}{k c_1}} + c_2 p$$

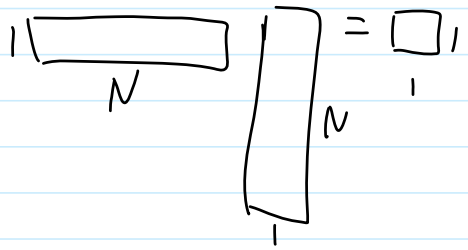
$$= \frac{W}{p} + \frac{c_1 + c_2 k}{\sqrt{k c_1}} \sqrt{W} + c_2 p$$

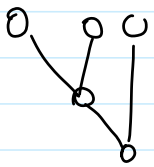
$$\frac{W}{p \tilde{T}_p(W,p)} = \frac{W}{W + \frac{p(c_1 + c_2 k)}{\sqrt{k c_1}} \sqrt{W} + c_2 p^2} = E_0 = \frac{1}{1 + \frac{1 - E_0}{E_0}}$$

$$\therefore \quad \frac{p(c_1 + c_2 k)}{\sqrt{k c_1}} W^{-\frac{1}{2}} + c_2 p^2 W^{-1} = \frac{1 - E_0}{E_0}$$

$$\Rightarrow W_{E_0}(p) = O(\sqrt{k} p^2)$$

Exercise 3.1





$$T_s(N) = c_1 N$$

$$T_p(N,p) = c_1 \cdot \frac{N}{p} \cdot p + c_2 (p-1) = c_1 N + c_2 p - c_2 = c_1 N + c_2 p - c_2$$

$$T_p(N,P) = c_1 \cdot \frac{N}{P} \cdot P + c_2(P-1) = c_1 N + c_2 P - c_2 = c_1 N + c_2 P - c_2$$

reduction time

$$W = T_s(N) = c_1 N$$

$$\Rightarrow N = \frac{W}{c_1}$$

$$\tilde{T}_p(N,P) = \tilde{T}_p(T_s^{-1}(W), P) = c_1 \cdot \frac{W}{c_1} + c_2 P = W + c_2 P - c_2$$

$$\frac{W}{P \cdot \tilde{T}_p(N,P)} = \frac{W}{WP + c_2 P^2 - c_2 P} = \frac{W}{(W - c_2)P + c_2 P^2} = E_0$$

$$W = E_0(WP - c_2 P + c_2 P^2)$$

$$1 = E_0 P - \frac{E_0}{W} c_2 P(P-1)$$

$$\frac{1 - E_0 P}{-E_0 c_2 P(P-1)} = \frac{1}{W}$$

$$\therefore W_{E_0}(P) = \frac{-E_0 c_2 P(P-1)}{1 - E_0 P}$$

$$\therefore W_{E_0}(P) = O(P)$$