1. Bias and Variance of Ridge Regression

Ridge regression solves a regularized least squares problem by introducing a regularization parameter $\tau \geq 0$. The regularization balances accuracy and simplicity.

Assuming a true model $y = X\beta^* + \varepsilon$ with zero-mean Gaussian noise $\varepsilon \sim N(0, \sigma^2)$ and centered features, we can derive the expressions for the expectation and covariance matrix of the regularized solution.

Using the singular value decomposition (SVD) of X, we decompose X as $X = U\Sigma V^T$, where U and V are orthogonal matrices, and Σ is a diagonal matrix with singular values.

1. Expectation of the Regularized Solution:

   The expectation $E[\beta_{b\tau}]$ is given by $V\Sigma^+U^Ty$, where Σ^+ is the pseudo-inverse of Σ.

   So, $E[\beta_{b\tau}] = S_\tau^{-1}S\beta^*$.

2. Covariance Matrix of the Regularized Solution:

   The covariance matrix $\mathrm{Cov}[\beta_{b\tau}]$ is obtained by $V\mathrm{Cov}[\Sigma^+]V^T$, where $\mathrm{Cov}[\Sigma^+] = S_\tau^{-1}S^{-1}\sigma^2$.

   Thus, the expectation and covariance matrix of the regularized solution in ridge regression using the SVD of X are:

   $E[\beta_{b\tau}] = S_\tau^{-1}S\beta^*$ and $\mathrm{Cov}[\beta_{b\tau}] = S_\tau^{-1}S^{-1}\sigma^2$.