Exercise 2

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2. For $L=1$:

$Z_0 = X$

$\tilde{Z}_1 = Z_0 B_0 + b_0$

$Z_1 = \tilde{Z}_1 = X B_0 + b_0$

For $L=2$:

$\tilde{Z}_2 = Z_1 B_1 + b_1$

$Z_2 = \tilde{Z}_2 = (Z_0 B_0 + b_0) B_1 + b_1$

$= X B_0 B_1 + b_0 B_1 + b_1$

Let $\tilde{B}_1 = B_0 B_1$, $\tilde{b}_1 = b_0 B_1 + b_1$

$Z_2 = X \tilde{B}_1 + \tilde{b}_1$

∴ network with depth $L=2$ is equivalent to 1-layer network (a)

For $L=n$ and $L=n+1$

$Z_n = Z_{n-1} \cdot B_n + b_n$

$Z_{n+1} = Z_n \cdot B_{n+1} + b_{n+1} = (Z_{n-1} B_n + b_n) B_{n+1} + b_{n+1}$

$= Z_{n-1} B_n B_{n+1} + b_n B_{n+1} + b_{n+1}$

Let $\tilde{B}_{n+1} = B_n B_{n+1}$, $\tilde{b}_{n+1} = b_n B_{n+1} + b_{n+1}$

$Z_{n+1} = Z_{n-1} \tilde{B}_{n+1} + \tilde{b}_{n+1}$

∴ network with depth $L=n$ is equivalent to network with depth $L=n+1$ (b)

∴ (a) and (b)

∴ Any network with depth $L>1$ is equivalent to a 1-layer