Assignment #3

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1 Group members: Lenny Gonzalez & Ichwan Palongengi

2 Assignment 3: Linear Discriminant Analysis

3 CSc 59970 - Intro to Data Science

4 10/03/2017

In [1]:
```
# Libraries
import math
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from sklearn.preprocessing import LabelEncoder
```

In [2]:
```
#Reading in the DataSet
df = pd.read_csv("C:/Users/lenny/Documents/Data-Science-Assignment2/cars.csv",
                 sep = ';')

#removed the row that desribed the type of each column
data = df.drop(df.index[[0]])

#removed the column that described the car model
cars = data.drop("Car",axis = 1)
cars.head()
```

Out[2]:

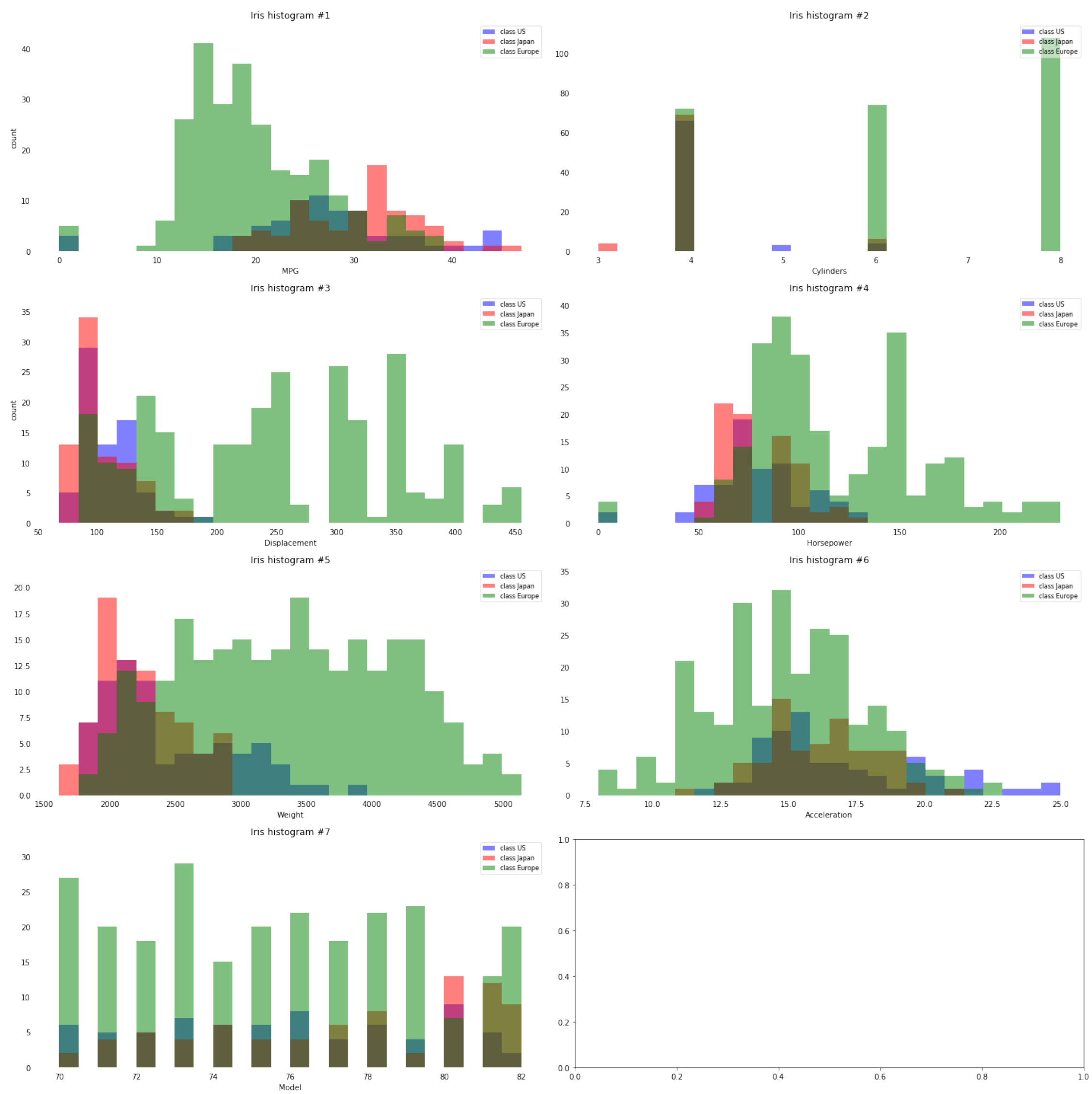
	MPG	Cylinders	Displacement	Horsepower	Weight	Acceleration	Model	Origin
1	18.0	8	307.0	130.0	3504.	12.0	70	US
2	15.0	8	350.0	165.0	3693.	11.5	70	US
3	18.0	8	318.0	150.0	3436.	11.0	70	US
4	16.0	8	304.0	150.0	3433.	12.0	70	US
5	17.0	8	302.0	140.0	3449.	10.5	70	US

```
In [3]: X = cars[["MPG","Cylinders","Displacement","Horsepower",
                  "Weight","Acceleration","Model",]].values
        y = cars["Origin"].values

        enc = LabelEncoder()
        label_encoder = enc.fit(y)
        y = label_encoder.transform(y) + 1
        X = np.asfarray(X, dtype='float')
```

5 Creating histograms to understand the 7 classes, we'll be focusing on, distribution

```
In [4]: feature_dict = {i:label for i, label in zip(range(7),
                                                    ("MPG","Cylinders","Displacement","Horsepower",
                                                     "Weight","Acceleration","Model"))}


        label_dict = {1:'US', 2: 'Japan', 3: 'Europe'}



        # 4 rows #2 columns --> this spreads the data in a more organized way,
        # we could do 1 column but messy

        fig, axes = plt.subplots(nrows=4, ncols=2, figsize=(20,20)) #figsize (x, y)

        for ax,cnt in zip(axes.ravel(), range(7)):

            # set bin sizes
            min_b = math.floor(np.min(X[:,cnt]))
            max_b = math.ceil(np.max(X[:,cnt]))
            bins = np.linspace(min_b, max_b, 25)

            # plottling the histograms
            for lab,col in zip(range(1,4), ('blue', 'red', 'green')):
                ax.hist(X[y==lab, cnt],
                           color=col,
                           label='class %s' %label_dict[lab],
                           bins=bins,
                           alpha=0.5,)
            ylims = ax.get_ylim()

            # plot annotation
            leg = ax.legend(loc='upper right', fancybox=True, fontsize=8)
            leg.get_frame().set_alpha(0.5)
            ax.set_ylim([0, max(ylims)+2])
            ax.set_xlabel(feature_dict[cnt])
            ax.set_title('Iris histogram #%s' %str(cnt+1))
```

```
    # hide axis ticks
    ax.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

axes[0][0].set_ylabel('count')
axes[1][0].set_ylabel('count')

fig.tight_layout()

plt.show()
```



6 Beginning Linear Discriminant Analysis

7 Step 1: Computinig the d-dimensional mean vectors

```
In [5]: # Step 1
        np.set_printoptions(precision=4)

        mean_vectors = []
        for cl in range(1,4):
            mean_vectors.append(np.mean(X[y==cl], axis=0))
            print('Mean Vector class %s: %s\n' %(cl, mean_vectors[cl-1]))
```

```
Mean Vector class 1: [   26.7452     4.1507   109.4658    78.7808  2431.4932    16.8219
    75.7397]

Mean Vector class 2: [   30.4506     4.1013   102.7089    79.8354  2221.2278    16.1722
    77.443 ]

Mean Vector class 3: [   19.6882     6.2835   247.935    118.0118  3372.7008    14.9425    75.5
```

8 Step 2: Compute the WITHIN - Class Scatter Matrix

```
In [6]: S_W = np.zeros((7,7))
        for cl,mv in zip(range(1,4), mean_vectors):
            class_sc_mat = np.zeros((7,7))                    # scatter matrix for every class
            for row in X[y == cl]:
                row, mv = row.reshape(7,1), mv.reshape(7,1) # make column vectors
                class_sc_mat += (row-mv).dot((row-mv).T)
            S_W += class_sc_mat                               # sum class scatter matrices
        print('within-class Scatter Matrix:\n', S_W)
```

```
within-class Scatter Matrix:
 [[  2.0395e+04  -2.4347e+03  -1.5063e+05  -6.7298e+04  -1.3428e+06
    2.8236e+03   6.2775e+03]
 [ -2.4347e+03   7.4412e+02   4.0093e+04   1.5200e+04   3.0978e+05
   -7.0038e+02  -7.0368e+02]
 [ -1.5063e+05   4.0093e+04   2.5398e+06   9.8249e+05   1.9325e+07
   -4.5821e+04  -4.5180e+04]
 [ -6.7298e+04   1.5200e+04   9.8249e+05   5.2264e+05   7.8315e+06
   -2.5681e+04  -2.1764e+04]
 [ -1.3428e+06   3.0978e+05   1.9325e+07   7.8315e+06   1.8394e+08
   -2.6477e+05  -2.7961e+05]
```

```
 [  2.8236e+03  -7.0038e+02  -4.5821e+04  -2.5681e+04  -2.6477e+05
    2.9408e+03   1.1624e+03]
 [  6.2775e+03  -7.0368e+02  -4.5180e+04  -2.1764e+04  -2.7961e+05
    1.1624e+03   5.4610e+03]]
```

9 Between Class Scatter Matrix

In [7]:
```
overall_mean = np.mean(X, axis=0)

S_B = np.zeros((7,7))
for i,mean_vec in enumerate(mean_vectors):
    n = X[y==i+1,:].shape[0]
    mean_vec = mean_vec.reshape(7,1) # make column vector
    overall_mean = overall_mean.reshape(7,1) # make column vector
    S_B += n * (mean_vec - overall_mean).dot((mean_vec - overall_mean).T)

print('between-class Scatter Matrix:\n', S_B)
```

```
between-class Scatter Matrix:
 [[  8.1942e+03  -1.8507e+03  -1.2223e+05  -3.2895e+04  -9.2690e+05
    1.2256e+03   1.2005e+03]
 [ -1.8507e+03   4.4313e+02   2.9155e+04   7.9379e+03   2.1601e+05
   -3.1522e+02  -2.3411e+02]
 [ -1.2223e+05   2.9155e+04   1.9187e+06   5.2201e+05   1.4237e+07
   -2.0649e+04  -1.5626e+04]
 [ -3.2895e+04   7.9379e+03   5.2201e+05   1.4234e+05   3.8558e+06
   -5.6971e+03  -4.0701e+03]
 [ -9.2690e+05   2.1601e+05   1.4237e+07   3.8558e+06   1.0662e+08
   -1.4882e+05  -1.2597e+05]
 [  1.2256e+03  -3.1522e+02  -2.0649e+04  -5.6971e+03  -1.4882e+05
    2.4204e+02   1.2294e+02]
 [  1.2005e+03  -2.3411e+02  -1.5626e+04  -4.0701e+03  -1.2597e+05
    1.2294e+02   2.3043e+02]]
```

10 Step 3 solving for the generalized eigenvalue for the matrix

In [8]:
```
eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(S_W).dot(S_B))

for i in range(len(eig_vals)):
    eigvec_sc = eig_vecs[:,i].reshape(7,1)
    print('\nEigenvector {}: \n{}'.format(i+1, eigvec_sc.real))
    print('Eigenvalue {:}: {:.2e}'.format(i+1, eig_vals[i].real))
```

```
Eigenvector 1:
```

```
[[ 0.2526]
 [ 0.8895]
 [-0.0927]
 [ 0.0823]
 [ 0.0017]
 [-0.0856]
 [-0.3497]]
Eigenvalue 1: 1.04e+00

Eigenvector 2:
[[-0.1316]
 [-0.8964]
 [-0.0236]
 [-0.0516]
 [ 0.0053]
 [ 0.0306]
 [-0.4183]]
Eigenvalue 2: 7.64e-02

Eigenvector 3:
[[ 0.1113]
 [ 0.9479]
 [-0.0494]
 [ 0.1107]
 [ 0.0015]
 [ 0.1311]
 [-0.239 ]]
Eigenvalue 3: -1.62e-15

Eigenvector 4:
[[-0.069 ]
 [-0.9185]
 [ 0.022 ]
 [ 0.0739]
 [-0.0042]
 [ 0.2921]
 [-0.2456]]
Eigenvalue 4: 1.07e-15

Eigenvector 5:
[[-0.1274]
 [ 0.5848]
 [-0.0104]
 [-0.0215]
 [-0.0014]
 [-0.7733]
 [-0.2078]]
Eigenvalue 5: 1.00e-16
```

```
Eigenvector 6:
[[-0.4958]
 [ 0.1495]
 [-0.0101]
 [ 0.0187]
 [-0.0029]
 [ 0.1441]
 [ 0.7307]]
Eigenvalue 6: -1.35e-16

Eigenvector 7:
[[-0.4958]
 [ 0.1495]
 [-0.0101]
 [ 0.0187]
 [-0.0029]
 [ 0.1441]
 [ 0.7307]]
Eigenvalue 7: -1.35e-16
```

In [9]:
```
# Checks the eigenvector - eigenvalue calculation
```

In [10]:
```
for i in range(len(eig_vals)):
    eigv = eig_vecs[:,i].reshape(7,1)
    np.testing.assert_array_almost_equal(np.linalg.inv(S_W).dot(S_B).dot(eigv),
                                         eig_vals[i] * eigv,
                                         decimal=6, err_msg='', verbose=True)
print('ok')
```

```
ok
```

11 Step 4: Selecting linear discriminants for the new feature space

In [11]:
```
# Make a list of (eigenvalue, eigenvector) tuples
eig_pairs = [(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len(eig_vals))]

# Sort the (eigenvalue, eigenvector) tuples from high to low
eig_pairs = sorted(eig_pairs, key=lambda k: k[0], reverse=True)

# Visually confirm that the list is correctly sorted by decreasing eigenvalues

print('Eigenvalues in decreasing order:\n')
for i in eig_pairs:
    print(i[0])
```

```
Eigenvalues in decreasing order:

1.03958087968
0.0764048551857
1.61892677173e-15
1.07363841714e-15
1.44894393138e-16
1.44894393138e-16
1.00119058818e-16
```

In [12]:
```python
# We will use the 2 eigenvalues closest to zero
```

In [13]:
```python
print('Variance explained:\n')
eigv_sum = sum(eig_vals)
for i,j in enumerate(eig_pairs):
    print('eigenvalue {0:}: {1:.2%}'.format(i+1, (j[0]/eigv_sum).real))
```

```
Variance explained:

eigenvalue 1: 93.15%
eigenvalue 2: 6.85%
eigenvalue 3: 0.00%
eigenvalue 4: 0.00%
eigenvalue 5: 0.00%
eigenvalue 6: 0.00%
eigenvalue 7: 0.00%
```

In [14]:
```python
# Constructing the K x D dimensional eigenvector matrix W
```

In [15]:
```python
eig_pairs
```

Out[15]:
```
[(1.0395808796799615,
  array([ 0.2526+0.j,  0.8895+0.j, -0.0927+0.j,  0.0823+0.j,  0.0017+0.j,
         -0.0856+0.j, -0.3497+0.j])),
 (0.076404855185684378,
  array([-0.1316+0.j, -0.8964+0.j, -0.0236+0.j, -0.0516+0.j,  0.0053+0.j,
          0.0306+0.j, -0.4183+0.j])),
 (1.6189267717312139e-15,
  array([ 0.1113+0.j,  0.9479+0.j, -0.0494+0.j,  0.1107+0.j,  0.0015+0.j,
          0.1311+0.j, -0.2390+0.j])),
 (1.0736384171379932e-15,
  array([-0.0690+0.j, -0.9185+0.j,  0.0220+0.j,  0.0739+0.j, -0.0042+0.j,
          0.2921+0.j, -0.2456+0.j])),
 (1.4489439313822653e-16,
  array([-0.4958+0.0144j,  0.1495-0.4106j, -0.0101+0.0015j,  0.0187+0.0013j,
         -0.0029+0.0006j,  0.1441-0.0886j,  0.7307+0.j    ])),
 (1.4489439313822653e-16,
```

```
   array([-0.4958-0.0144j,  0.1495+0.4106j, -0.0101-0.0015j,  0.0187-0.0013j,
          -0.0029-0.0006j,  0.1441+0.0886j,  0.7307-0.j    ])),
  (1.0011905881828998e-16,
   array([-0.1274+0.j,  0.5848+0.j, -0.0104+0.j, -0.0215+0.j, -0.0014+0.j,
          -0.7733+0.j, -0.2078+0.j]))]
```

```
In [16]: # Eig_pairs is a list of tuples that consists of 1 value and 1 array
         # we focus on the first two tuples, the remaining tuples are unecessary
         # information.
```

```
In [17]: # So here we pass the 1 tuple and the 2 tuple and
         # calculate W, which will reduce the dimensions of the data
         W = np.hstack((eig_pairs[0][1].reshape(7,1), eig_pairs[1][1].reshape(7,1)))
         print('Matrix W:\n', W.real)
```

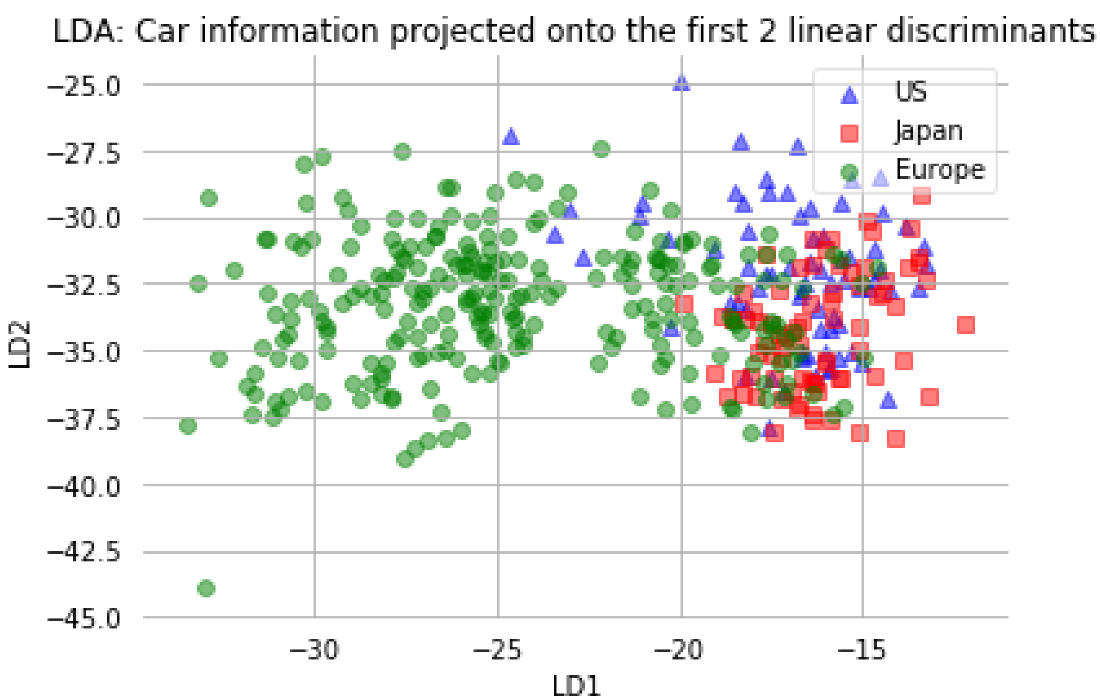
```
Matrix W:
 [[ 0.2526 -0.1316]
 [ 0.8895 -0.8964]
 [-0.0927 -0.0236]
 [ 0.0823 -0.0516]
 [ 0.0017  0.0053]
 [-0.0856  0.0306]
 [-0.3497 -0.4183]]
```

12 Step 5: transforming the samples onto the new subspace

```
In [18]: # In continuation we have Y = X * W
         # X is our original data
         # W is 7 x 2 dimensional matrix that'll transform our samples onto the new subpsace
         X_lda = X.dot(W)
         assert X_lda.shape == (406,2), "The matrix is not 150x2 dimensional."
```

```
In [19]: from matplotlib import pyplot as plt

         def plot_step_lda():

             ax = plt.subplot(111)
             for label,marker,color in zip(
                 range(1,7),('^', 's', 'o'),('blue', 'red', 'green')):

                 plt.scatter(x=X_lda[:,0].real[y == label],
                         y=X_lda[:,1].real[y == label],
                         marker=marker,
                         color=color,
                         alpha=0.5,
                         label=label_dict[label]
                         )
```

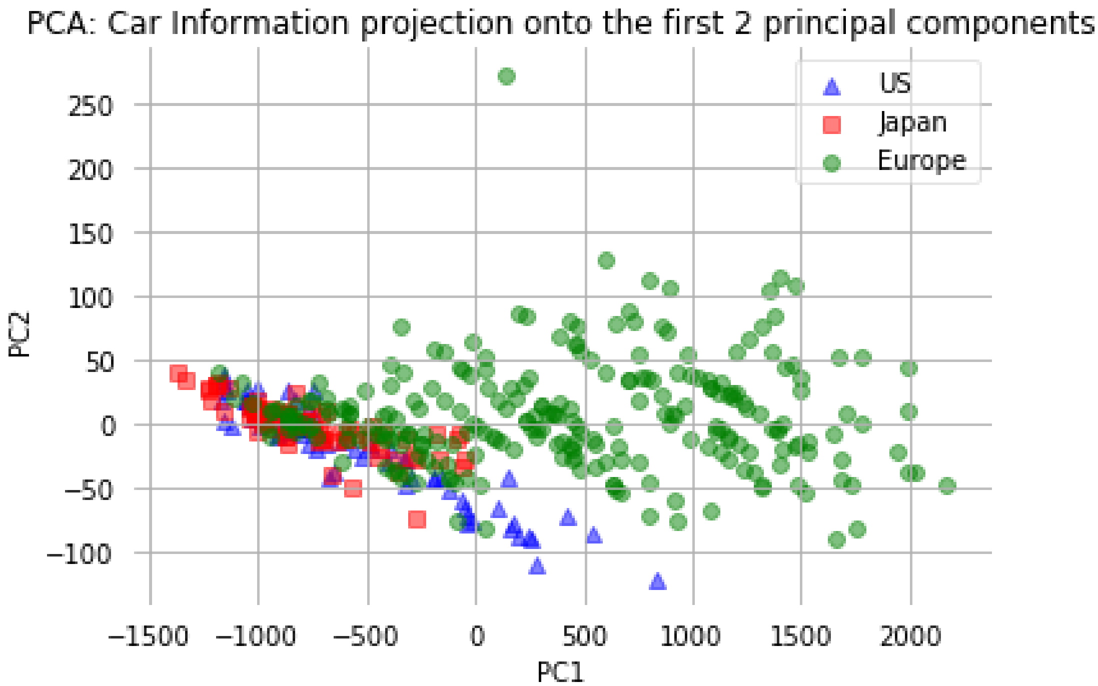
```
    plt.xlabel('LD1')
    plt.ylabel('LD2')

    leg = plt.legend(loc='upper right', fancybox=True)
    leg.get_frame().set_alpha(0.5)
    plt.title('LDA: Car information projected onto the first 2 linear discriminants')

    # hide axis ticks
    plt.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

    plt.grid()
    plt.tight_layout
    plt.show()

plot_step_lda()
```

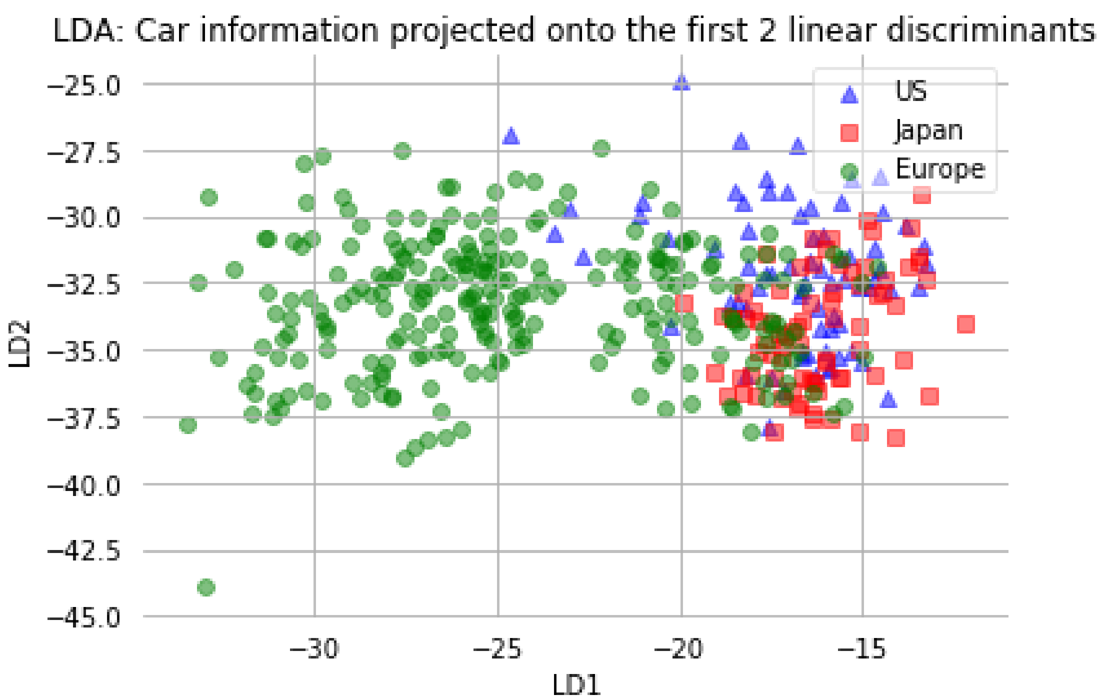


13 Comparison of PCA and LDA

In [20]:
```
from sklearn.decomposition import PCA as sklearnPCA

sklearn_pca = sklearnPCA(n_components=2)
X_pca = sklearn_pca.fit_transform(X)

def plot_pca():

    ax = plt.subplot(111)

    for label,marker,color in zip(
        range(1,4),('^', 's', 'o'),('blue', 'red', 'green')):

        plt.scatter(x=X_pca[:,0][y == label],
                y=X_pca[:,1][y == label],
                marker=marker,
                color=color,
                alpha=0.5,
                label=label_dict[label]
                )

    plt.xlabel('PC1')
    plt.ylabel('PC2')

    leg = plt.legend(loc='upper right', fancybox=True)
    leg.get_frame().set_alpha(0.5)
    plt.title('PCA: Car Information projection onto the first 2 principal components')

    # hide axis ticks
    plt.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

    plt.tight_layout
    plt.grid()

    plt.show()
```

In [21]:
```
    plot_pca()
    plot_step_lda()
```


PCA: Car Information projection onto the first 2 principal components
US
Japan
Europe
PC2
PC1
250
200
150
100
50
0
−50
−100
−1500
−1000
−500
0
500
1000
1500
2000

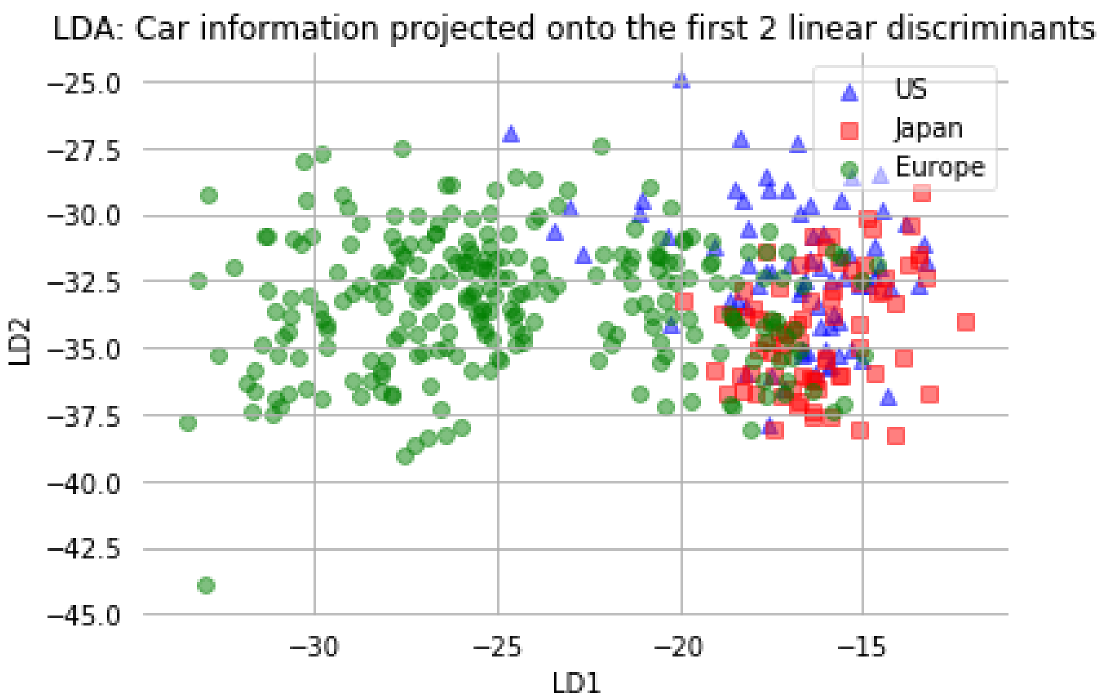

LDA: Car information projected onto the first 2 linear discriminants
US
Japan
Europe
LD2
LD1
−25.0
−27.5
−30.0
−32.5
−35.0
−37.5
−40.0
−42.5
−45.0
−30
−25
−20
−15

14 LDA using Scikit-learn

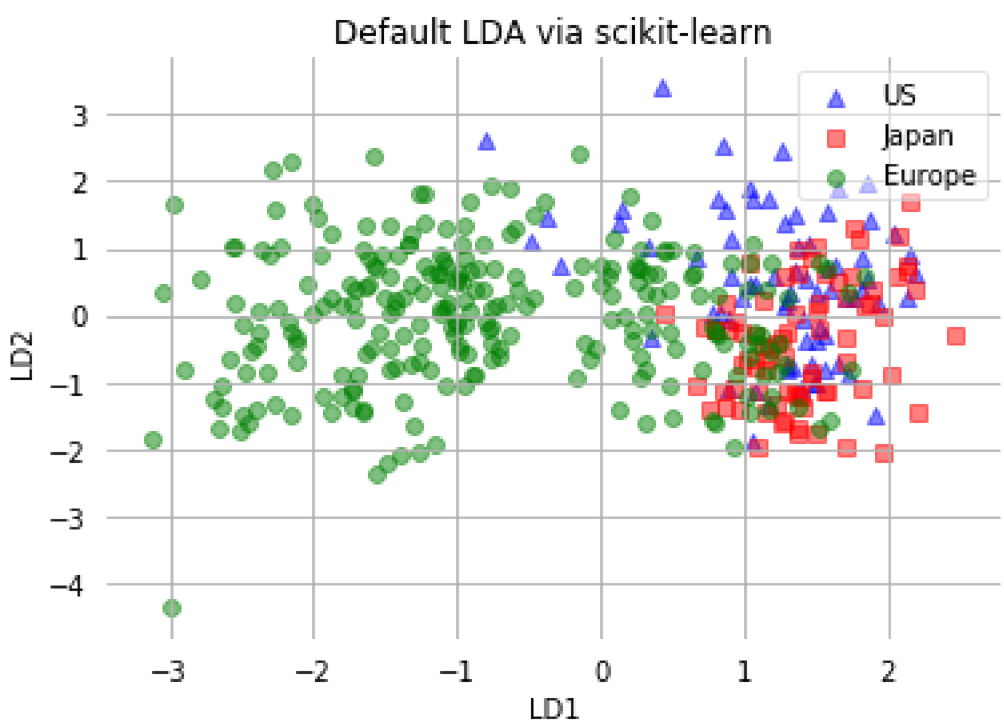
In [22]:
```
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA

# LDA
sklearn_lda = LDA(n_components=7)
X_lda_sklearn = sklearn_lda.fit_transform(X, y)
```

In [23]:
```
def plot_scikit_lda(X, title):

    ax = plt.subplot(111)
    for label,marker,color in zip(
        range(1,4),('^', 's', 'o'),('blue', 'red', 'green')):

        plt.scatter(x=X[:,0][y == label] * -1,
                    y=X[:,1][y == label] * 1,
                    marker=marker,
                    color=color,
                    alpha=0.5,
                    label=label_dict[label])

    plt.xlabel('LD1')
    plt.ylabel('LD2')

    leg = plt.legend(loc='upper right', fancybox=True)
    leg.get_frame().set_alpha(0.5)
    plt.title(title)

    # hide axis ticks
    plt.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

    plt.grid()
    plt.tight_layout
    plt.show()
```

In [24]:
```
plot_step_lda()
plot_scikit_lda(X_lda_sklearn, title='Default LDA via scikit-learn')
```



LDA: Car information projected onto the first 2 linear discriminants
US
Japan
Europe
−25.0
−27.5
−30.0
−32.5
−35.0
−37.5
−40.0
−42.5
−45.0
−30
−25
−20
−15
LD1
LD2



Default LDA via scikit-learn
US
Japan
Europe
3
2
1
0
−1
−2
−3
−4
−3
−2
−1
0
1
2
LD1
LD2

15 Standardization (centering and scaling)

```
In [25]: X = cars[["MPG","Cylinders","Displacement","Horsepower",
                   "Weight","Acceleration","Model",]].values
         y = cars["Origin"].values

         enc = LabelEncoder()
         label_encoder = enc.fit(y)
         y = label_encoder.transform(y) + 1
         X = np.asfarray(X, dtype='float')
```

```
In [26]: X_cent = X - X.mean(axis=0)
         X_std = X_cent / X.std(axis=0)
```

```
In [27]: import numpy as np

         def comp_mean_vectors(X, y):
             class_labels = np.unique(y)
             n_classes = class_labels.shape[0]
             mean_vectors = []
             for cl in class_labels:
                 mean_vectors.append(np.mean(X[y==cl], axis=0))
             return mean_vectors

         def scatter_within(X, y):
             class_labels = np.unique(y)
             n_classes = class_labels.shape[0]
             n_features = X.shape[1]
             mean_vectors = comp_mean_vectors(X, y)
             S_W = np.zeros((n_features, n_features))
             for cl, mv in zip(class_labels, mean_vectors):
                 class_sc_mat = np.zeros((n_features, n_features))
                 for row in X[y == cl]:
                     row, mv = row.reshape(n_features, 1), mv.reshape(n_features, 1)
                     class_sc_mat += (row-mv).dot((row-mv).T)
                 S_W += class_sc_mat
             return S_W

         def scatter_between(X, y):
             overall_mean = np.mean(X, axis=0)
             n_features = X.shape[1]
             mean_vectors = comp_mean_vectors(X, y)
             S_B = np.zeros((n_features, n_features))
             for i, mean_vec in enumerate(mean_vectors):
                 n = X[y==i+1,:].shape[0]
                 mean_vec = mean_vec.reshape(n_features, 1)
                 overall_mean = overall_mean.reshape(n_features, 1)
                 S_B += n * (mean_vec - overall_mean).dot((mean_vec - overall_mean).T)
```

```
        return S_B

    def get_components(eig_vals, eig_vecs, n_comp=2):
        n_features = X.shape[1]
        eig_pairs = [(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len(eig_vals))]
        eig_pairs = sorted(eig_pairs, key=lambda k: k[0], reverse=True)
        W = np.hstack([eig_pairs[i][1].reshape(7, 1) for i in range(0, n_comp)])
        return W
```

16 Step 1) print the eigenvalues, eigenvectors, transformation matrix

```
In [28]: S_W, S_B = scatter_within(X, y), scatter_between(X, y)
         eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(S_W).dot(S_B))
         W = get_components(eig_vals, eig_vecs, n_comp=2)
         print('EigVals: %s\n\nEigVecs: %s' % (eig_vals, eig_vecs))
         print('\nW: %s' % W)
```

```
EigVals: [  1.0396e+00 +0.0000e+00j   7.6405e-02 +0.0000e+00j
  -1.6189e-15 +0.0000e+00j   1.0736e-15 +0.0000e+00j
   1.0012e-16 +0.0000e+00j  -1.3546e-16 +5.1418e-17j
  -1.3546e-16 -5.1418e-17j]

EigVecs: [[ 0.2526+0.j     -0.1316+0.j      0.1113+0.j     -0.0690+0.j     -0.1274+0.j
  -0.4958+0.0144j -0.4958-0.0144j]
 [ 0.8895+0.j     -0.8964+0.j      0.9479+0.j     -0.9185+0.j      0.5848+0.j
   0.1495-0.4106j  0.1495+0.4106j]
 [-0.0927+0.j     -0.0236+0.j     -0.0494+0.j      0.0220+0.j     -0.0104+0.j
  -0.0101+0.0015j -0.0101-0.0015j]
 [ 0.0823+0.j     -0.0516+0.j      0.1107+0.j      0.0739+0.j     -0.0215+0.j
   0.0187+0.0013j  0.0187-0.0013j]
 [ 0.0017+0.j      0.0053+0.j      0.0015+0.j     -0.0042+0.j     -0.0014+0.j
  -0.0029+0.0006j -0.0029-0.0006j]
 [-0.0856+0.j      0.0306+0.j      0.1311+0.j      0.2921+0.j     -0.7733+0.j
   0.1441-0.0886j  0.1441+0.0886j]
 [-0.3497+0.j     -0.4183+0.j     -0.2390+0.j     -0.2456+0.j     -0.2078+0.j
   0.7307+0.j      0.7307-0.j    ]]

W: [[ 0.2526+0.j -0.1316+0.j]
 [ 0.8895+0.j -0.8964+0.j]
 [-0.0927+0.j -0.0236+0.j]
 [ 0.0823+0.j -0.0516+0.j]
 [ 0.0017+0.j  0.0053+0.j]
 [-0.0856+0.j  0.0306+0.j]
 [-0.3497+0.j -0.4183+0.j]]
```

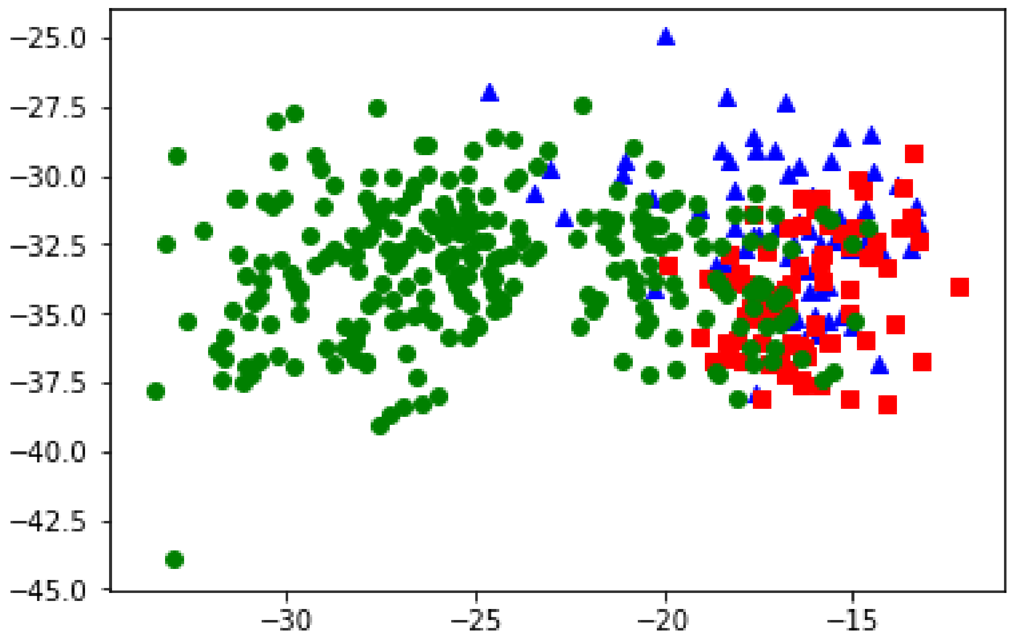
```
In [29]: X_lda = X.dot(W)
         for label,marker,color in zip(
```

```
                np.unique(y),('^', 's', 'o'),('blue', 'red', 'green')):
            plt.scatter(X_lda[y==label, 0], X_lda[y==label, 1],
                        color=color, marker=marker)
```

```
C:\Users\lenny\Anaconda3\lib\site-packages\numpy\core\numeric.py:583: ComplexWarning: Casting co
  return array(a, dtype, copy=False, order=order, subok=True)
```

In [30]:
```
plt.show()
```

```
C:\Users\lenny\Anaconda3\lib\site-packages\numpy\core\numeric.py:583: ComplexWarning: Casting co
  return array(a, dtype, copy=False, order=order, subok=True)
```



In [31]:
```
S_W, S_B = scatter_within(X_std, y), scatter_between(X_std, y)
eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(S_W).dot(S_B))
W_std = get_components(eig_vals, eig_vecs, n_comp=2)
print('EigVals: %s\n\nEigVecs: %s' % (eig_vals, eig_vecs))
print('\nW: %s' % W_std)
```

```
EigVals: [  1.0396e+00 +0.0000e+00j   7.6405e-02 +0.0000e+00j
  -8.4767e-16 +0.0000e+00j   6.8554e-16 +0.0000e+00j
  -5.9526e-17 +6.7740e-17j  -5.9526e-17 -6.7740e-17j
   1.6053e-16 +0.0000e+00j]

EigVecs: [[-0.1967+0.j      0.1820+0.j     -0.0074+0.j      0.2379+0.j
```

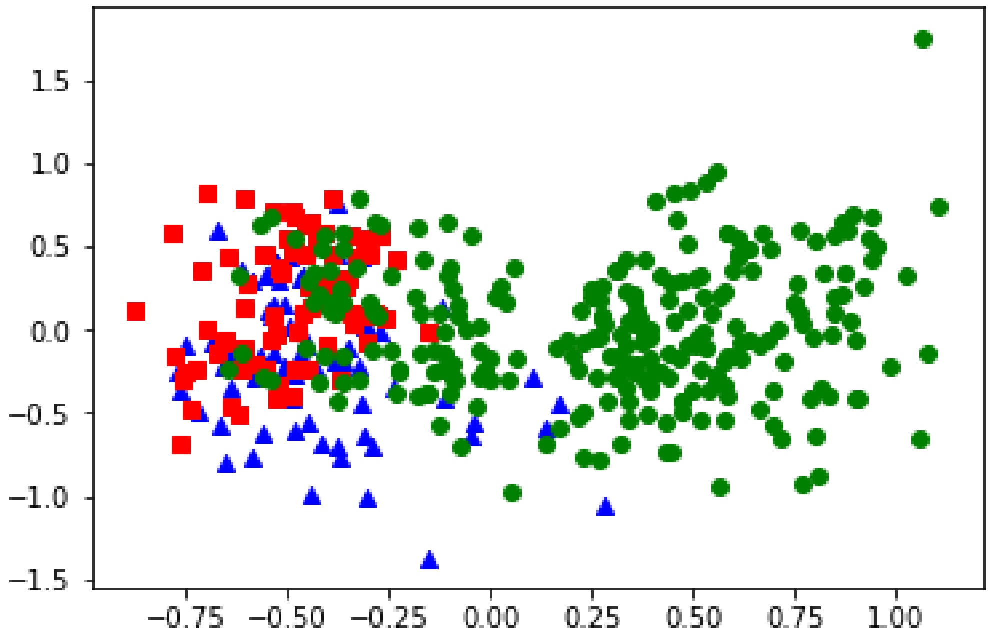
```
  -0.3751+0.1271j -0.3751-0.1271j -0.3067+0.j    ]
 [-0.1412+0.j      0.2525+0.j      0.4750+0.j     -0.6758+0.j     -0.5633+0.j
  -0.5633-0.j     -0.2396+0.j    ]
 [ 0.9013+0.j      0.4077+0.j     -0.7996+0.j      0.3151+0.j
   0.2640-0.0503j  0.2640+0.0503j  0.0716+0.j    ]
 [-0.3090+0.j      0.3437+0.j      0.3397+0.j      0.2229+0.j
   0.5040+0.0848j  0.5040-0.0848j -0.6540+0.j    ]
 [-0.1351+0.j     -0.7429+0.j      0.1354+0.j      0.5135+0.j
  -0.3558+0.0285j -0.3558-0.0285j  0.4130+0.j    ]
 [ 0.0222+0.j     -0.0141+0.j      0.0332+0.j      0.2250+0.j
   0.0642-0.0737j  0.0642+0.0737j -0.2220+0.j    ]
 [ 0.1215+0.j      0.2581+0.j     -0.0045+0.j      0.1534+0.j
   0.1752-0.1574j  0.1752+0.1574j  0.4425+0.j    ]]

W: [[-0.1967+0.j  0.1820+0.j]
 [-0.1412+0.j  0.2525+0.j]
 [ 0.9013+0.j  0.4077+0.j]
 [-0.3090+0.j  0.3437+0.j]
 [-0.1351+0.j -0.7429+0.j]
 [ 0.0222+0.j -0.0141+0.j]
 [ 0.1215+0.j  0.2581+0.j]]
```

In [32]:
```
X_std_lda = X_std.dot(W_std)
X_std_lda[:, 1] = X_std_lda[:, 1]
for label,marker,color in zip(
        np.unique(y),('^', 's', 'o'),('blue', 'red', 'green')):
    plt.scatter(X_std_lda[y==label, 0], X_std_lda[y==label, 1],
                color=color, marker=marker)
```

```
C:\Users\lenny\Anaconda3\lib\site-packages\numpy\core\numeric.py:583: ComplexWarning: Casting co
  return array(a, dtype, copy=False, order=order, subok=True)
```

In [33]:
```
plt.show()
```

```
C:\Users\lenny\Anaconda3\lib\site-packages\numpy\core\numeric.py:583: ComplexWarning: Casting co
  return array(a, dtype, copy=False, order=order, subok=True)
```



17 Comparison between PCA and LDA

PCA and LDA can reduce dimensionality in the dataset, however each are used for different cases

17.0.1 PCA:

We use PCA when we want to find components that account for maximum variance in the data, this includes error and within-variable variance. Unlike LDA, it does not take into account class membership (because it's a unsupervised method). And is used when such information is not available. So, by reducing dimensions by focusing on the variables with the most variation we can plot the data on a simple X-Y graph.

Importantly, both LDA and PCA do not require any prior notion of how the variable are related among themselves, and the resulting components can not be interpreted in terms of an underlying construct. However, there are cases where we're not interested in the variables with the most variation. Instead, we're interested in maximizing the seperability between the two groups so we can make the best decisions. Linear Discriminant Analysis focuses on maximizing the separability among the known categories.

17.0.2 LDA:

This method identifies components (linear combination of the observed variables) that maximize class seperation (between-class variance). An example would be when we having a training set containing a variable specifying the class of each observation.

PCA is not optimal for classification, we don't take class labels into account. Keeping the dimensions of largest variance is a good idea, but sometimes it is not enough. However, from a classification point of view it is not wise. PCA is a good method for finding a suitable subset of variables.