Test Wiskunde A 27 november 2017

1. Bereken de integraal:

$$\int \cos^2(t) - \sin^3 \cos^6 dt = \int \cos^2(t)dt - \int \sin^3 \cos^6 dt$$

$$\begin{aligned}
\int \cos^2(t)dt &= \int \frac{1+\cos(2t)}{2}dt \\
&= \frac{1}{2}\left(\int dt + \int \cos(2t)dt\right) \\
&= \frac{1}{2}\left(t + \frac{\cos(2t)}{2}\right) \\
&= \frac{t}{2} + \frac{\cos(2t)}{4}
\end{aligned}$$

$$\begin{aligned}
\int \sin^3 \cos^6 dt &= \int \sin(t)\sin^2(t) cos^6(t)dt \\
&= \int \sin(t)\Big(1-\cos^2(t)\Big) cos^6(t)dt \\
&= -\int (1-u^2)u^6 du \\
&= -\left(\int u^6 du - \int u^8 du\right) \\
&= -\int u^6 du + \int u^8 du \\
&= -\frac{u^7}{7} + \frac{u^9}{9} \\
&= -\frac{\cos^7(t)}{7} + \frac{\cos^9(t)}{9}
\end{aligned}$$

$$\begin{aligned}
\int \cos^2(t)dt - \int \sin^3 \cos^6 dt &= \frac{t}{2} + \frac{\cos(2t)}{4} - (-\frac{\cos^7(t)}{7} + \frac{\cos^9(t)}{9}) \\
&= \frac{t}{2} + \frac{\cos(2t)}{4} + \frac{\cos^7(t)}{7} - \frac{\cos^9(t)}{9}
\end{aligned}$$

2. Bepaal de 2de orde afgeleide van:

3. Geef de oplossingen van z in $a + bj$ vorm: $z^3 = -8\left(\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)^{30}$

$$r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$$

$$\theta = \arctan\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2}j = e^{j\frac{\pi}{6}}$$

$$\Rightarrow -8(e^{j\frac{\pi}{6}})^{30}$$

$$= -8e^{j\frac{30\pi}{6}}$$

$$= -8e^{j5\pi}$$

$$= -8e^{j\pi}$$

$$z^3 = -8e^{j\pi}$$

$$z = \sqrt[3]{-8e^{j\pi}}$$

$$z = -2e^{j(\frac{\pi}{3}+2k\pi)} \text{ met k} \in \{0, 1, 2\}$$

- $z_1 = -2e^{j\frac{\pi}{3}} = -\sqrt{3} - 1j$
- $z_2 = -2e^{j\frac{7\pi}{3}} = -\sqrt{3} - 1j$
- $z_3 = -2e^{j\frac{14\pi}{3}} = -\sqrt{3} - 1j$

4. Teken de rechte in poolcoördinaten: $r\sin(\theta+\frac{\pi}{3})=3$

$$
\begin{aligned}
&r\sin(\theta+\frac{\pi}{3})=3\\
&<=> r\cos\left(\frac{\pi}{2}-(\theta+\frac{\pi}{3})\right)=3\\
&<=> r\cos\left(\frac{\pi}{2}-\theta-\frac{\pi}{3}\right)=3\\
&<=> r\cos\left(\frac{\pi}{6}-\theta\right)=3\\
&<=> r\cos\left(-\theta+\frac{\pi}{6}\right)=3\\
&<=> r\cos\left(\theta-\frac{\pi}{6}\right)=3
\end{aligned}
$$

5. Bereken de limiet:

$$
\begin{aligned}
\lim_{x\to\frac{\pi}{2}} (\sin(x))^{\frac{1}{\pi-2x}} &= (\sin(\frac{\pi}{2}))^{\frac{1}{\pi-2\frac{\pi}{2}}} = 1^{\infty} \\
&= \lim_{x\to\frac{\pi}{2}} e^{\frac{\ln(\sin(x))}{\pi-2x}} \\
&= e^{\frac{\lim_{x\to\frac{\pi}{2}} \ln(\sin(x))}{\lim_{x\to\frac{\pi}{2}} \pi-2x}} = e^{\frac{0}{0}} \\
&= \lim_{x\to\frac{\pi}{2}} e^{\frac{\frac{d[\ln(\sin(x))]}{dx}}{\frac{d[\pi-2x]}{dx}}} \\
&= \lim_{x\to\frac{\pi}{2}} e^{\frac{\cot(x)}{2}} \\
&= e^{\frac{\lim_{x\to\frac{\pi}{2}} \cot(x)}{\lim_{x\to\frac{\pi}{2}} 2}} \\
&= e^{\frac{0}{2}} \\
&= e^{0} \\
&= 1
\end{aligned}
$$