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ME - 4203 | 20-03314

# ASSIGNMENT 2

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

if $s = 0$

$$1 = A(2) + 0$$

$$\frac{1}{2} = A$$

SUBSTITUTING

$$\left[1 = \frac{1}{2}(s^2+2s+2) + Bs^2 + Cs\right]2$$

$$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$$

$$2 = s^2(2B+1) + s(2C+2) + 2$$

$$B = -\frac{1}{2} \ ; \ C = -1$$

$$\mathcal{L}^{-1}\left\{\frac{1/2}{s} - \frac{(1/2s+1)}{s^2+2s+2}\right.$$

a. $\mathcal{L}^{-1}\left\{\frac{1}{2}/s\right\} = \frac{1}{2}$

b. $\mathcal{L}^{-1}\left\{\frac{(1/2s+1)}{s^2+2s+2}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+2}\right\}$

$$= \frac{(s+1)+1}{(s^2+2s+2)+1} \quad ; a = -1 \ , \ \omega = \sqrt{1} = 1$$

$$= \frac{(s+1)+1}{(s+1)^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}\left[\cos\omega t + \sin\omega t\right]u(t)$$

$$= \frac{1}{2}e^{-t}(\cos t + \sin t)\,u(t)$$

$$= \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)$$

$$\boxed{f(t) = \frac{1}{2}\left[1 - e^{-t}(\cos t + \sin t)\right]}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\mathcal{L}^{-1}\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$$

$$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1) + D(s)(s+1)(s+3)$$

if $s = 0$;
$10 = 3A + 0 + 0 + 0$
$A = \frac{10}{3}$

if $s = -1$
$5 = 0 + 2B + 0 + 0$
$B = \frac{5}{2}$

if $s = -3$
$-5 = 0 + 0 - 18C + 0$
$C = \frac{5}{18}$

if $s = -2$

$$5[-2+2] = \frac{10}{3}(-2+3)(-2+1) + \frac{5}{2}(-2)^2(-2+3) + \frac{5}{18}(-2)^2(-2+1) + D(-2)(-2+1)(-2+3)$$

$$0 = -\frac{10}{3} + 10 - \frac{10}{9} + 2D$$

$$0 = \frac{50}{9} + 2D$$

$$\left[2D = \frac{50}{9}\right]\frac{1}{2} \Rightarrow D = -\frac{25}{9}$$

$$\mathcal{L}^{-1}\left\{\frac{10/3}{s^2} + \frac{5/2}{s+1} + \frac{5/18}{s+3} - \frac{25/9}{s}\right\}$$

$$\boxed{f(t) = \frac{10t}{3} + \frac{5e^{-t}}{2} + \frac{5e^{-3t}}{18} - \frac{25}{9}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$\begin{array}{r|l} & s^2 + s + 2 \\ s^2+s & s^4 + 2s^3 + 3s^2 + 4s + 5 \\ & -\;s^4 + s^3 \\ & \quad s^3 + 3s^2 + 4s + 5 \\ & -\;s^3 + s^2 \\ & \quad 2s^2 + 4s + 5 \\ & -\;2s^2 + 2s \\ & \quad 2s + 5 \end{array}$$

$$\mathcal{L}^{-1}\left\{s^2 + s + 2 + \frac{2s+5}{s^2+s}\right\}$$

a. $y''$
b. $y'$
c. $2\delta(t)$

d. $\mathcal{L}^{-1}\left\{\dfrac{2s+5}{s^2+s}\right\} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$2s+5 = A(s+1) + Bs$

if $s = -1$
$3 = -B$
$B = -3$

if $s = 0$
$5 = A$

$$\mathcal{L}^{-1}\left\{\frac{5}{s} - \frac{3}{s+1}\right\} = 5(1) + \left[-3e^{-t}\right]$$
$$= 5 - 3e^{-t}$$

$$\boxed{f(t) = y'' + y' + 2\delta(t) + 5 - 3e^{-t}}$$