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ME 4203 CSE

# ASSIGNMENT 2

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

IF $s=0$,

$$1 = A(2)+0$$

$$\frac{1}{2} = \frac{2A}{2} \Rightarrow A = \frac{1}{2}$$

SUBSTITUTE A:

$$\left[1 = \frac{1}{2}(s^2+2s+2) + Bs^2 + Cs\right]2$$

$$2 = s^2+2s+2+2Bs^2+2Cs$$

$$2 = s^2(2B+1) + s(2C+2)+2$$

$$B = -\frac{1}{2}, \quad C = -1$$

ⓐ $\mathcal{L}^{-1}\left\{\dfrac{\frac{1}{2}}{s}\right\} = \dfrac{1}{2}$

ⓑ $\mathcal{L}^{-1}\left\{\dfrac{\frac{1}{2}s+1}{s^2+2s+2}\right\}; \dfrac{1}{2}\mathcal{L}^{-1}\left\{\dfrac{s+2}{s^2+2s+2}\right\}$

$$= \frac{(s+1)+1}{(s^2+2s+2)+1} = \frac{(s+1)+1}{(s+1)^2+1} \quad ; a=1, \ \omega=\sqrt{1}$$

$$= e^{-at}\cos\omega t + \sin\omega t$$

$$= \frac{1}{2}e^{-t}\cos t + \sin t$$

$$\therefore \boxed{f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\cos t + \sin t}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\mathcal{L}^{-1}\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$$

$$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1) + D(s)(s+1)(s+3)$$

if $s=0$: $\dfrac{10}{3} = \dfrac{3A}{3}$, $A = \dfrac{10}{3}$

if $s=-1$: $\dfrac{5}{2} = \dfrac{2B}{2}$, $B = \dfrac{5}{2}$

if $s=-3$: $\dfrac{-5}{-18} = \dfrac{-18C}{-18}$, $C = \dfrac{5}{18}$

if $s=-2$

$$D = -\frac{10}{3} + 10 - \frac{10}{9} + 20 \Rightarrow D = -\frac{25}{9}$$

$$\mathcal{L}^{-1}\left\{\frac{\frac{10}{3}}{s^2} + \frac{\frac{5}{2}}{s+1} + \frac{\frac{5}{18}}{s+3} - \frac{\frac{25}{9}}{s}\right\}$$

$$\therefore \boxed{f(t) = \frac{10t}{3} + \frac{5e^{-t}}{2} + \frac{5e^{-3t}}{18} - \frac{25}{9}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$\begin{array}{r} s^2 + s + 2 \\ s^2 + s \,\overline{)\, s^4 + 2s^3 + 3s^2 + 4s + 5} \\ \underline{s^4 + s^3} \\ s^3 + 3s^2 \\ \underline{s^3 + s^2} \\ 2s^2 + 4s \\ \underline{2s^2 + 2s} \\ 2s + 5 \end{array}$$

$$\mathcal{L}^{-}\left\{ s^2 + s + 2 + \frac{2s+5}{s^2+s} \right\}$$

ⓐ $\mathcal{L}^{-}\{s^2\} = y''$

ⓑ $\mathcal{L}^{-}\{s\} = y'$

ⓒ $\mathcal{L}^{-}\{2\} = 2\delta t$

ⓓ $\mathcal{L}^{-}\left\{\frac{2s+5}{s^2+s}\right\} = \frac{A}{s} + \frac{B}{s+1}$

$2s+5 = A(s+1) + Bs$

if $s = 0$, $A = 5$

if $s = -1$, $B = -3$

$$\mathcal{L}^{-}\left\{\frac{5}{s} - \frac{3}{s+1}\right\} = 5(1) + \left[-3e^{-t}\right] = 5 - 3e^{-t}$$

$$\therefore \boxed{f(t) = y'' + y' + 2\delta(t) + 5 - 3e^{-t}}$$