

ASSIGNMENT 2

$$1.) F(s) = \frac{1}{s(s^2 + 2s + 2)}$$

$$\left[\frac{1}{s(s^2 + 2s + 2)} \right] = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 2}$$

$$1 = A(s^2 + 2s + 2) + s(Bs + C)$$

$$1 = A(s^2 + 2s + 2) + Bs^2 + Cs$$

$$\text{IF } s=0; \quad 1 = 2A$$

$$A = \frac{1}{2}$$

SUBSTITUTING A:

$$\left[1 = \frac{s^2 + 2s + 2}{2} + Bs^2 + Cs \right] \cdot 2$$

$$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$$

$$2 = s^2 + (2B+1)s + (2C+2) + 2$$

$$2B+1 = 0 \quad 2C+2 = 0$$

$$2B = -1 \quad 2C = -2$$

$$B = -\frac{1}{2} \quad C = -1$$

$$\therefore \mathcal{L}^{-1} \frac{1/2}{s} - \frac{1/2 s + 1}{s^2 + 2s + 2}$$

$$\frac{1}{2} \mathcal{L}^{-1} \frac{1}{s} - \frac{1}{2} \mathcal{L}^{-1} \frac{s+2}{s^2 + 2s + 2} \Rightarrow \frac{1}{2} \mathcal{L}^{-1} \frac{1}{s} - \frac{1}{2} \mathcal{L}^{-1} \frac{s+2}{s^2 + 2s + 1 + 1}$$

$$\frac{1}{2} \mathcal{L}^{-1} \frac{1}{s} - \frac{1}{2} \mathcal{L}^{-1} \frac{(s+1)+1}{(s+1)^2 + 1}; \text{ WHERE } a=1 \text{ AND } w=1$$

$$f(t) = \frac{1}{2} - \frac{1}{2} e^{-t} \cdot \cos t + \sin t$$

$$f(t) = \frac{1}{2} - \frac{1}{2} e^{-t} (\cos t + \sin t)$$

$$2.) F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}$$

$$5 \mathcal{L}^{-1} \frac{s+2}{s^2(s+1)(s+3)}$$

$$\left[\frac{s+2}{s^2(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+3} \right] s^2(s+1)(s+3)$$

$$s+2 = As^3(s+3) + Bs^2(s+1) + Cs(s+1)(s+3) + D(s+1)(s+3)$$

$$\text{IF } s=3 \quad \text{IF } s=-1 \quad \text{IF } s=0$$

$$-1 = B(-3)^2(-3+1)$$

$$-1 = 9B(-2)$$

$$-1 = -18B$$

$$1 = A(-1)^2(-1+3)$$

$$1 = 2A$$

$$A = \frac{1}{2}$$

$$2 = D(0+1)(0+3)$$

$$2 = 3D$$

$$D = \frac{2}{3}$$

$$\text{IF } s=1$$

$$3 = 4A + 2B + C(2)(4) + D(2)(4)$$

$$3 = 2 + \frac{1}{9} + BC + \frac{16}{3}$$

$$C = -\frac{5}{9}$$

$$5 \left[\mathcal{L}^{-1} \left\{ \mathcal{L}^{-1} \frac{1/2}{s+1} + \frac{1/18}{s+3} + \frac{-5/9}{s} + \frac{2/3}{s^2} \right\} \right]$$

$$5 \left[\frac{1}{2} \mathcal{L}^{-1} \frac{1}{s+1} + \frac{1}{18} \mathcal{L}^{-1} \frac{1}{s+3} - \frac{5}{9} \mathcal{L}^{-1} \frac{1}{s} + \frac{2}{3} \mathcal{L}^{-1} \frac{1}{s^2} \right]$$

WHERE $a=1$ AND $a=3$

$$5 \left(\frac{1}{2} e^{-t} + \frac{1}{18} e^{-3t} - \frac{5}{9} e^{-t} + \frac{5}{18} e^{-3t} \right)$$

$$f(t) = \frac{10}{3}t - \frac{25}{9} + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}$$

$$3.) F(s) = \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)} \Rightarrow s^2 + 9 \frac{s^2 + s + 2}{s^4 + 2s^3 + 3s^2 + 4s + 5}$$

$$\mathcal{L}^{-1} s^2 + 9 + 2 \frac{s+5}{s^2+9}$$

$$\frac{d^2f}{dt^2} + \frac{df}{dt} + 2g(t) : \text{EQUATION 1}$$

$$\mathcal{L}^{-1} \frac{2s+5}{s^2+9} \Rightarrow \mathcal{L}^{-1} \frac{2s+5}{s(s+1)}$$

$$\mathcal{L}^{-1} \frac{(2s+2)+3}{s(s+1)}$$

$$2\mathcal{L}^{-1} \frac{(s+1)}{s(s+1)} + 3\mathcal{L}^{-1} \frac{1}{s(s+1)}$$

$$2\mathcal{L}^{-1} \frac{1}{s} + 3\mathcal{L}^{-1} \frac{1}{s(s+1)}$$

$$\left[\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} \right] s(s+1)$$

$$1 = A(s+1) + Bs$$

$$|F s = -1 \quad |F s = 0$$

$$1 = -1B \quad 1 = 1A$$

$$B = -1 \quad A = 1$$

$$3 \left[\mathcal{L}^{-1} \frac{1}{s} - \mathcal{L}^{-1} \frac{1}{s+1} \right] \text{ WHERE } a=1$$

$$3 - 3e^{-t} \Rightarrow 2 + 3 - 3e^{-t}$$

$$5 - 3e^{-t} : \text{EQUATION 2}$$

SUBSTITUTING EQ. 1 AND EQ. 2

$$f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2g(t) + 5 - 3e^{-t}$$