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ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING

1. $\mathcal{L}\{3 - e^{-3t} + 5\sin 2t\} = F(s)$

$\mathcal{L}\{3\} = 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{e^{-3t} = \frac{1}{s+3}$; $a = 3$

$\mathcal{L}\{5\sin 2t\} = 5\,\mathcal{L}\{\sin 2t\} = 5\left(\frac{2}{s^2+2^2}\right) = \frac{10}{s^2+4}$; $\omega = 2$

$$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$$

2. $\mathcal{L}\{3 + 12t + 42t^3 - 3e^{2t}\} = F(s)$

$\mathcal{L}\{3\} = 3\{1\} = \frac{3}{s}$

$\mathcal{L}\{12t\} = 12\mathcal{L}\{t\} = 12\left(\frac{1}{s^2}\right) = \frac{12}{s^2}$

$\mathcal{L}\{42t^3\} = 42\,\mathcal{L}\{t^3\} = 42\left(\frac{3!}{s^{3+1}}\right) = 42\left(\frac{6}{s^4}\right) = \frac{252}{s^4}$; $n = 3$

$\mathcal{L}\{3e^{2t}\} = 3\mathcal{L}\{e^{2t}\} = 3\left(\frac{1}{s-2}\right) = \frac{3}{s-2}$; $a = 2$

$$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$$

3. $\mathcal{L}\{(t+1)(t+2)\} = F(s)$

$\mathcal{L}\{t^2 + 3t + 2\} = F(s)$

$\mathcal{L}\{t^2\} = n = 2$; $\frac{2!}{s^{2+1}} = \frac{2}{s^2}$

$\mathcal{L}\{2\} = 2\mathcal{L}\{1\} = 2\left(\frac{1}{s}\right) = \frac{2}{s}$

$$\boxed{F(s) = \frac{2}{s^3} + \frac{2}{s^2} + \frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFER OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left\{\frac{8 - 3s + s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3}\right\} = 4\,\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = 4t^2 u(t)$

$\mathcal{L}^{-1}\left\{\frac{3}{s^2}\right\} = 3\,\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 3t\,u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$

$$\boxed{f(t) = (4t^2 - 3t + 1)\,u(t)}$$

2. $\mathcal{L}^{-1}\left\{\frac{5}{s-2} - \frac{4s}{s^2+9}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{5}{s-2}\right\} = 5\mathcal{L}^{-}\left\{\frac{1}{s-2}\right\} = 5e^{2t}u(t)$

$\mathcal{L}^{-1}\left\{\frac{4s}{s^2+9}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = 4\cos 3t\ u(t)$

$$\boxed{f(t) = (5e^{2t} - 4\cos 3t)\,u(t)}$$

3. $\mathcal{L}^{-1}\left\{\frac{7}{s^2+6}\right\} = f(t)$

$7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\} = \frac{7}{\sqrt{6}}\ \mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+6}\right\} = \left[\frac{7}{\sqrt{6}}(\sin\sqrt{6})\,u(t)\right]\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$$\boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}\ u(t)}$$