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ME - 4203

ASSIGNMENT 2

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$1 = A(s^2+2s+2) + s(Bs+C)$

if $s = 0$

$1 = (A)(2) + 0$

$\dfrac{1}{2} = A$

Value of A:

$[1 = 1/2(s^2+2s+2) + Bs^2 + Cs]2$

$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$

$2 = s^2(2B+1) + s(2C+2) + 2$

$B = -\dfrac{1}{2}$

$C = -1$

$$\mathcal{L}^{-1}\left\{\frac{1/2}{s} - \frac{(1/2\,s+1)}{s^2+2s+2}\right\}$$

ⓐ $\mathcal{L}^{-}\left\{\dfrac{1/2}{s}\right\} = \dfrac{1}{2}$

ⓑ $\mathcal{L}^{-}\left\{\dfrac{1/2s+1}{s^2+2s+2}\right\};\ \dfrac{1}{2}\mathcal{L}^{-}\left\{\dfrac{s+2}{s^2+2s+2}\right\}$

$= \dfrac{(s+1)+1}{(s^2+2s+2)+1} = \dfrac{(s+1)+1}{(s+1)^2+1};\quad a = 1,\ w = \sqrt{1}$

$= e^{-at}\cos wt + \sin wt$

$= \dfrac{1}{2}e^{-t}\cos t + \sin t$

$$\boxed{f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\cos t + \sin t}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\mathcal{L}^{-}\left\{\frac{5(s+2)}{s^2\ (s+1)(s+3}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$$

$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1)$
$+ D(s)(s+1)(s+3)$

if $s=0$

$10 = 3A$

$\frac{10}{3} = A$

if $s=-3$

$-5 = -18C$

$\frac{5}{18} = C$

if $s=-1$

$5 = 2B$

$\frac{5}{2} = B$

if $s=-2$, substitute for $D = ?$ (values of A, B & C)

$0 = -\frac{10}{3} + 10 - \frac{10}{9} + 2D$

$D = -\frac{25}{9}$

$$\mathcal{L}^{-}\left\{\frac{\frac{10}{3}}{s^2} + \frac{\frac{5}{2}}{s+1} + \frac{\frac{5}{18}}{s+3} - \frac{\frac{25}{9}}{s}\right\}$$

$$\boxed{f(t) = \frac{10t}{3} + \frac{5e^{-t}}{2} + \frac{5e^{-3t}}{18} - \frac{25}{9}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$
\begin{array}{r|l}
 & s^2 + s + 2 \\
\hline
s^2 + s & s^4 + 2s^3 + 3s^2 + 4s + 5 \\
 & \underline{s^4 + s^3} \\
 & \quad s^3 + 3s^2 \\
 & \quad \underline{s^3 + s^2} \\
 & \qquad 2s^2 + 4s \\
 & \qquad \underline{2s^2 + 2s} \\
 & \qquad\quad 2s + 5
\end{array}
$$

$\mathcal{L}^{-}\left\{ s^2 + s + 2 + \dfrac{2s+5}{s^2+s} \right\}$

a) $\mathcal{L}^{-}\{s^2\} = y''$

b) $\mathcal{L}^{-}\{s\} = y'$

c) $\mathcal{L}^{-}\{2\} = 2\delta t$

d) $\mathcal{L}^{-}\left\{ \dfrac{2s+5}{s^2+s} \right\} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$2s + 5 = A(s+1) + B(s)$

if $s = 0$

$5 = A$

if $s = -1$

$3 = -B$

$-3 = B$

$\mathcal{L}^{-}\left\{ \dfrac{5}{s} - \dfrac{3}{s+1} \right\} = 5(1) + [-3e^{-t}] = 5 - 3e^{-t}$

$$\boxed{f(t) = y'' + y' + 2\delta(t) + 5 - 3e^{-t}}$$