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ME 4203 | 20-03314

ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING

1. $\mathcal{L}\{3-e^{-3t}+5\sin 2t\} = F(s)$

$\mathcal{L}\{3\} = 3\mathcal{L}\{1\} = 3\left(\frac{1}{s}\right) = \frac{3}{s}$

$\mathcal{L}\{e^{-3t} = \frac{1}{s+3}$; $a=3$

$\mathcal{L}\{5\sin 2t\} = 5\mathcal{L}\{\sin 2t\} = 5\left(\frac{2}{s^2+2^2}\right) = \frac{10}{s^2+4}$; $\omega=2$

$$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$$

2. $\mathcal{L}\{3+12t+42t^3-3e^{2t}\} = F(s)$

$\mathcal{L}\{3\} = 3\mathcal{L}\{1\} = \frac{3}{s}$

$\mathcal{L}\{12t\} = 12\mathcal{L}\{t\} = 12\left(\frac{1}{s^2}\right) = \frac{12}{s^2}$

$\mathcal{L}\{42t^3\} = 42\mathcal{L}\{t^3\} = 42\left(\frac{3!}{s^{3+1}}\right) = 42\left(\frac{6}{s^4}\right) = \frac{252}{s^4}$; $n=3$

$\mathcal{L}\{3e^{2t}\} = 3\mathcal{L}\{e^{2t}\} = 3\left(\frac{1}{s-2}\right) = \frac{3}{s-2}$; $a=2$

$$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$$

3. $\mathcal{L}\{(t+1)(t+2)\} = F(s)$

$\mathcal{L}\{t^2+3t+2\} = F(s)$

$\mathcal{L}\{t^2\} = n=2$; $\frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$\mathcal{L}\{2\} = 2\mathcal{L}\{1\} = 2\left(\frac{1}{s}\right) = \frac{2}{s}$

$$\boxed{F(s) = \frac{2}{s^3} + \frac{2}{s^2} + \frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFER OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left\{\frac{8-3s+s^2}{s^3}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{8}{s^3}\right\} = 4\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = 4t^2u(t)$

$\mathcal{L}^{-}\left\{\frac{3}{s^2}\right\} = 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = 3t\,u(t)$

$\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = u(t)$

$$\boxed{f(t) = (4t^2-3t+1)\,u(t)}$$

2. $\mathcal{L}^{-1}\left\{\frac{5}{s-2} - \frac{4s}{s^2+9}\right\} = f(t)$

$\mathcal{L}^{-1}\left\{\frac{5}{s-2}\right\} = 5\mathcal{L}^{-}\left\{\frac{1}{s-2}\right\} = 5e^{2t}u(t)$

$\mathcal{L}^{-1}\left\{\frac{4s}{s^2+9}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = 4\cos 3t\ u(t)$

$$\boxed{f(t) = (5e^{2t} - 4\cos 3t)\,u(t)}$$

3. $\mathcal{L}^{-1}\left\{\frac{7}{s^2+6}\right\} = f(t)$

$7\mathcal{L}^{-1}\left\{\frac{1}{s^2+6}\right\} = \frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left\{\frac{\sqrt{6}}{s^2+6}\right\} = \left[\frac{7}{\sqrt{6}}(\sin\sqrt{6})\,u(t)\right]\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$$\boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}\ u(t)}$$

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ASSIGNMENT 2

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$1 = A(s^2+2s+2) + s(Bs+C)$

if $s=0$

$1 = A(2) + 0$

$\frac{1}{2} = A$

SUBSTITUTING

$\left[1 = \frac{1}{2}(s^2+2s+2) + Bs^2 + Cs\right]2$

$2 = s^2+2s+2+2Bs^2+2Cs$

$2 = s^2(2B+1) + s(2C+2) + 2$

$B = -\frac{1}{2}$; $C = -1$

$$\mathcal{L}^{-1}\left\{\frac{1/2}{s} - \frac{(1/2s+1)}{s^2+2s+2}\right.$$

a. $\mathcal{L}^{-1}\{\frac{1}{2}/s\} = \frac{1}{2}$

b. $\mathcal{L}^{-1}\left\{\frac{(1/2s+1)}{s^2+2s+2}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+2}\right\}$

$= \dfrac{(s+1)+1}{(s^2+2s+2)+1}$; $a=-1$, $\omega = \sqrt{1} = 1$

$= \dfrac{(s+1)+1}{(s+1)^2+1}$

$$\mathcal{L}^{-1}\left\{\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right\} = e^{-at}[\cos\omega t + \sin\omega t]\,u(t)$$

$= \frac{1}{2}e^{-t}(\cos t + \sin t)\,u(t)$

$= \frac{1}{2} - \frac{1}{2}e^{-t}(\cos t + \sin t)$

$$\boxed{f(t) = \frac{1}{2}\left[1 - e^{-t}(\cos t + \sin t)\right]}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\mathcal{L}^{-1}\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\} = \frac{A}{s^2} + \frac{B}{s+1} + \frac{C}{s+3} + \frac{D}{s}$$

$5(s+2) = A(s+1)(s+3) + B(s^2)(s+3) + C(s^2)(s+1) + D(s)(s+1)(s+3)$

if $s = 0$;
$10 = 3A + 0 + 0 + 0$
$A = \frac{10}{3}$

if $s = -1$
$5 = 0 + 2B + 0 + 0$
$B = \frac{5}{2}$

if $s = -3$
$-5 = 0 + 0 - 18C + 0$
$C = \frac{5}{18}$

if $s = -2$

$5[-2+2] = \frac{10}{3}(-2+3)(-2+1) + \frac{5}{2}(-2)^2(-2+3) + \frac{5}{18}(-2)^2(-2+1) +$
$D(-2)(-2+1)(-2+3)$

$0 = -\frac{10}{3} + 10 - \frac{10}{9} + 2D$

$0 = \frac{50}{9} + 2D$

$\left[2D = \frac{50}{9}\right]\frac{1}{2} \Rightarrow D = -\frac{25}{9}$

$$\mathcal{L}^{-1}\left\{\frac{10/3}{s^2} + \frac{5/2}{s+1} + \frac{5/18}{s+3} - \frac{25/9}{s}\right\}$$

$$\boxed{f(t) = \frac{10t}{3} + \frac{5e^{-t}}{2} + \frac{5e^{-3t}}{18} - \frac{25}{9}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$\begin{array}{r|l} & s^2 + s + 2 \\ s^2+s & s^4 + 2s^3 + 3s^2 + 4s + 5 \\ & -\,s^4 + s^3 \\ & \quad s^3 + 3s^2 + 4s + 5 \\ & -\,s^3 + s^2 \\ & \quad 2s^2 + 4s + 5 \\ & -\,2s^2 + 2s \\ & \quad 2s + 5 \end{array}$$

$$\mathcal{L}^{-1}\left\{s^2 + s + 2 + \frac{2s+5}{s^2+s}\right\}$$

a. y''
b. y'
c. $2\delta(t)$

d. $\mathcal{L}^{-1}\left\{\dfrac{2s+5}{s^2+s}\right\} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$2s+5 = A(s+1) + Bs$

if $s = -1$
$3 = -B$
$B = -3$

if $s = 0$
$5 = A$

$$\mathcal{L}^{-1}\left\{\frac{5}{s} - \frac{3}{s+1}\right\} = 5(1) + \left[-3e^{-t}\right]$$
$$= 5 - 3e^{-t}$$

$$\boxed{f(t) = y'' + y' + 2\delta(t) + 5 - 3e^{-t}}$$