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ME 4203 CSE

# ASSIGNMENT 1

## I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}[3 - e^{-3t} + 5\sin 2t] = F(s)$

ⓐ $3\mathcal{L}\{1\} = \frac{1}{s} = \frac{3}{s}$

ⓑ $\mathcal{L}\{e^{-3t}\} = \frac{1}{s+a} = \frac{1}{s+3}$, $a = 3$

ⓒ $5\mathcal{L}\{\sin 2t\} = \frac{\omega}{s^2+\omega^2} = 5 \cdot \frac{2}{s^2+2^2} = 5 \cdot \frac{2}{s^2+4} = \frac{10}{s^2+4}$, $\omega = 2$

$$\therefore \boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$$

2. $\mathcal{L}[3 + 12t + 42t^3 - 3e^{2t}] = F(s)$

ⓐ $3\mathcal{L}\{1\} = \frac{1}{s} = \frac{3}{s}$

ⓑ $12\mathcal{L}\{t\} = \frac{1}{s^2} = \frac{12}{s^2}$

ⓒ $42\mathcal{L}\{t^3\} = \frac{n!}{s^{n+1}} = \frac{3!}{s^{3+1}} = \frac{6}{s^4} = \frac{252}{s^4}$, $n = 3$

ⓓ $3\mathcal{L}\{e^{2t}\} = \frac{1}{s+a} = \frac{1}{s+(-2)} = \frac{3}{s-2}$, $a = -2$

$$\therefore \boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$$

3. $\mathcal{L}[(t+1)(t+2)] = F(s) \Rightarrow t^2 + 3t + 2$

ⓐ $\mathcal{L}\{t^2\} = \frac{n!}{s^{n+1}} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$, $n = 2$

ⓑ $3\mathcal{L}\{t\} = \frac{1}{s^2} = \frac{3}{s^2}$

ⓒ $2\mathcal{L}\{1\} = \frac{1}{s} = \frac{2}{s}$

$$\therefore \boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$$

## II. SOLVE FOR INVERSE LAPLACE OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left[\frac{8 - 3s + s^2}{s^3}\right] = f(t) \Rightarrow \frac{8}{s^3} - \frac{3s}{s^3} + \frac{s^2}{s^3}$

$= 8\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} - 3\mathcal{L}^{-1}\left\{\frac{s}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{s^2}{s^3}\right\}$

ⓐ $8\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{8}{s^3}$ ; $4\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$ ; $\frac{2}{s^3} = \frac{n!}{s^{n+1}}$ ; $t^n$

$= 4t^2$

ⓑ $3\mathcal{L}^{-1}\left\{\frac{s}{s^3}\right\} = 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$ ; $\frac{1}{s^2} = t$

$= 3t$

ⓒ $\mathcal{L}^{-1}\left\{\frac{s^2}{s^3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$$\therefore \boxed{f(t) = 4t^2 - 3t + 1}$$

2. $\mathcal{L}^{-1}\left[\frac{5}{s-2} - \frac{4s}{s^2+9}\right] = f(t)$

ⓐ $\mathcal{L}^{-}\left\{\frac{5}{s-2}\right\} = 5\mathcal{L}^{-}\left\{\frac{1}{s-2}\right\}$ ; $\frac{1}{s-2} = e^{at}$ where $a=2$ $= 5e^{2t}$

ⓑ $\mathcal{L}^{-}\left\{\frac{4s}{s^2+9}\right\}$ ; $\sin w = \frac{w}{s^2+w^2}$ ; where $w=3$ $= \frac{3}{s^2+3^2}$

$\left(\frac{4s}{s^2+9}\right) = \frac{4}{3}\sin 3t$ ; $\frac{d}{dt}\left(\frac{4}{3}\sin 3t\right) = 4\cos 3t$

$\therefore \boxed{f(t) = 5e^{2t} + 4\cos 3t}$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = f(t)$

$7\mathcal{L}^{-}\left\{\frac{1}{s^2+6}\right\} = \frac{7}{\sqrt{6}}\ \mathcal{L}^{-}\left\{\frac{\sqrt{6}}{s^2+6}\right\} : \sin\sqrt{6}$

$= \left(\frac{7}{\sqrt{6}}\sin\sqrt{6}\,t\right)\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$\therefore \boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}\,t}$