TEMPERATURE CONTROL OF A SHRINK TUNNEL WITH MULTIPLE HEATING ZONES

BENCHMARK DOCUMENTATION

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ABSTRACT

This paper provides additional documentation for the Book Chapter "Temperature control of a shrink tunnel with multiple heating zones" [6]. In particular, we report the mathematical expressions for the matrices that govern the linear part of the `Heat-Temperatures` model (Section 1.2), the estimation of the parameters of the `Heat-Temperatures` and `Packs-Temperatures` models (Section 2), and a software guide for the MATLAB/Simulink benchmark available at the GitHub repository `https://github.com/Lenyor/Temperature_control_of_a_shrink_tunnel_with_multiple_heating_zones` (Section 3).

1 Review of the models

This Section reviews the `Heat-Temperatures` and `Packs-Temperatures` models described in [6], paying particular attention to the linear part of the former model. The reader is referred to [6] for their derivation.

1.1 Recap of the signals

Before reviewing the `Heat-Temperatures` and `Packs-Temperatures` models, we recap the signals of interest for the shrink tunnel described in [6], whose control scheme is reported in Figure 1. Each signal represents [6]:

- $t \in \mathbb{R}$ (in s) is the time.
- $w_1(t), w_2(t) \in [0, 1]$ are the control actions produced by the temperature controller for the first and the second heating zone respectively. In particular, $w_1(t), w_2(t)$ are the duty cycles associated with the voltage

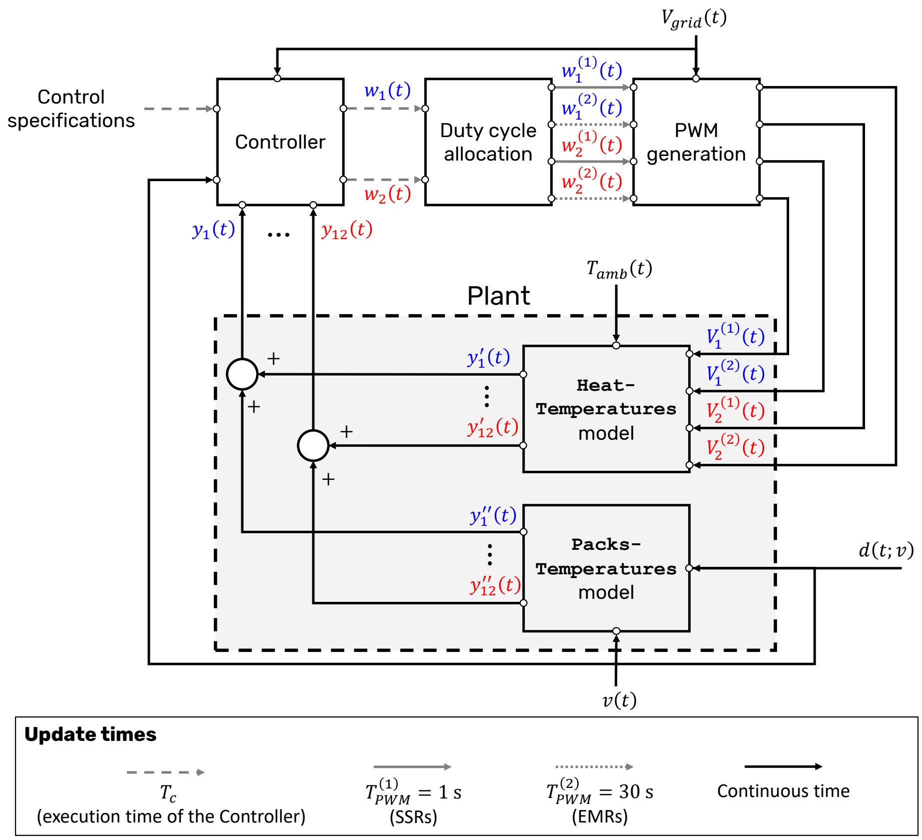

Figure 1: Block diagram of the system under study [6]. The signals in blue are associated to the first zone while those in red are related to the second zone. Instead, the signals in black are not related to any zone in particular. The different arrows highlight how often each signal is updated.

PWM signals that need to be supplied to the heat resistors. Notation-wise:

$$\boldsymbol{w}(t) = [w_1(t) \quad w_2(t)]^\top \in \mathbb{R}^2. \tag{1}$$

- $w_1^{(1)}(t)$ and $w_2^{(1)}(t)$ are the duty cycles for the voltage PWM signals across the sets of heat resistors in the first and the second heating zone that are driven by the Solid-State Relays (SSRs). Instead, $w_1^{(2)}(t)$ and $w_2^{(2)}(t)$ are the duty cycles related to the Electro-Mechanical Relays (EMRs). $w_1^{(1)}(t), w_1^{(2)}(t), w_2^{(1)}(t), w_2^{(2)}(t) \in [0,1]$ are generated from $w_1(t), w_2(t)$ based on the modes of operation described in [6].
- $V_1^{(1)}(t), V_1^{(2)}(t), V_2^{(1)}(t), V_2^{(2)}(t) \in \mathbb{R}_{>0}$ (in V) are the voltage PWM signals across the sets of heat resistors in each zone with corresponding duty cycles $w_1^{(1)}(t), w_1^{(2)}(t), w_2^{(1)}(t), w_2^{(2)}(t)$. Notation-wise:

$$\boldsymbol{V}(t) = \left[V_1^{(1)}(t) \quad V_1^{(2)}(t) \quad V_2^{(1)}(t) \quad V_2^{(2)}(t)\right]^\top \in \mathbb{R}^4, \tag{2a}$$

$$\boldsymbol{V}_{sq}(t) = \boldsymbol{V}(t) \odot \boldsymbol{V}(t). \tag{2b}$$

- $V_{grid}(t) \in \mathbb{R}_{>0}$ (in V) is the grid voltage.
- $T_{amb}(t) = \bar{T}_{amb}, \forall t \in \mathbb{R}, \bar{T}_{amb} \in \mathbb{R}$ (in °C), is the ambient temperature.
- $v(t) \in \{0, 30, 60, 90\}$ (in ppm) is the production rate of the industrial process which produces the beverages fed to the shrink tunnel.
- $d(t; v) \in \{0, 1\}$ is the reading of the infrared sensor at the entrance of the oven which detects the presence (1) or absence (0) of bottle packs.
- $y_k(t) = y_k'(t) + y_k''(t), k = 1, \ldots, 12$ (in °C), are the overall temperatures within the oven cavity measured by the twelve thermocouples positioned in different locations (see [6]). Notation-wise:

$$\boldsymbol{y}(t) = [y_1(t) \quad \ldots \quad y_{12}(t)]^\top \in \mathbb{R}^{12}. \tag{3}$$

- $y'_k(t) \in \mathbb{R}, k = 1, \dots, 12$ (in °C), are the temperatures due to the heat produced by the heat resistors and the ambient temperature. They are the outputs of the `Heat-Temperatures` model. Notation-wise:

$$\boldsymbol{y}'(t) = \begin{bmatrix} y'_1(t) & \dots & y'_{12}(t) \end{bmatrix}^\top \in \mathbb{R}^{12}. \tag{4}$$

- $y''_k(t) \in \mathbb{R}, k = 1, \dots, 12$ (in °C), are the temperature drops due to the flow of bottle packs inserted into the oven cavity. They are the outputs of the `Packs-Temperatures` model. Notation-wise:

$$\boldsymbol{y}''(t) = \begin{bmatrix} y''_1(t) & \dots & y''_{12}(t) \end{bmatrix}^\top \in \mathbb{R}^{12}. \tag{5}$$

Finally, for the sake of notation, let

$$\boldsymbol{u}(t) = \begin{bmatrix} \boldsymbol{V}_{sq}(t) \\ T_{amb}(t) \end{bmatrix} \in \mathbb{R}^5 \tag{6}$$

be the vector of inputs related to the linear part of the `Heat-Temperatures` model, which is reviewed in the next Section.

1.2 Review of the `Heat-Temperatures` model

The `Heat-Temperatures` model describes the relationship between the heat produced by the heat resistors, the ambient temperature, and the temperatures measured by the twelve thermocouples installed inside the shrink tunnel. The derived state-space model is [6]:

$$\begin{cases} \boldsymbol{V}_{sq}(t) = \boldsymbol{f}(\boldsymbol{w}(t), V_{grid}(t)) \\ \dot{\boldsymbol{x}}(t) = A \cdot \boldsymbol{x}(t) + B \cdot \boldsymbol{u}(t) \\ \boldsymbol{y}'(\boldsymbol{t} + \boldsymbol{T}'_d) = C \cdot \boldsymbol{x}(t) + D \cdot \boldsymbol{u}(t) \end{cases}. \tag{7}$$

$\boldsymbol{f}(\boldsymbol{w}(t), V_{grid}(t))$ is a nonlinear function that describes the relationship between the control actions $\boldsymbol{w}(t)$ produced by the controller, the grid voltage $V_{grid}(t)$, and the squared voltages $\boldsymbol{V}_{sq}(t)$, see [6] for its derivation. In this Section, we focus on the linear part of the system in Eq. (7), which is related to electro-equivalent thermal circuit reported in Figure 2. In particular, let $i = 1, 2, k = 1, 2, 3$, and $j = l, r$. The parameters of the electro-equivalent thermal circuit are:

- $R_i^{(k,j)} \in \mathbb{R}_{>0}$ (in $\frac{°\text{C}}{\text{J}} \cdot \text{s}$) is the thermal resistance between the oven walls of zone i, region k, side j, and the ambient;
- $R_{T_i}^{(k,j)} \in \mathbb{R}_{>0}$ (in $\frac{°\text{C}}{\text{J}} \cdot \text{s}$) is the thermal resistance related to the transfer of heat between two regions of zone i, side j;
- $R_{T_{12}}^{(j)} \in \mathbb{R}_{>0}$ (in $\frac{°\text{C}}{\text{J}} \cdot \text{s}$) is the thermal resistance related to the transfer of heat between the two zones of the oven, for what concerns side j;
- $R_{T_{rl}} \in \mathbb{R}_{>0}$ (in $\frac{°\text{C}}{\text{J}} \cdot \text{s}$) is the thermal resistance related to the transfer of heat between the left and the right side of the oven cavity;
- $C_z \in \mathbb{R}_{>0}$ (in $\frac{\text{J}}{°\text{C}}$) is the thermal capacitance of the air in each region/zone/side of the oven cavity. We consider the oven cavity (which has an overall volume of $\mathcal{V}_{oven} = 2.26\,\text{m}^3$ [6]) divided into twelve boxes of the same volume. Assuming that we are dealing with dry air at constant pressure, we have [2]:

$$\begin{aligned} C_z &= c_{air} \cdot \rho_{air} \cdot \frac{\mathcal{V}_{oven}}{12} \\ &= 1000\,\frac{\text{J}}{\text{Kg} \cdot {}^\circ\text{C}} \cdot 1.225\,\frac{\text{Kg}}{\text{m}^3} \cdot \frac{2.26\,\text{m}^3}{12} \\ &= 231\,\frac{\text{J}}{°\text{C}}, \end{aligned} \tag{8}$$

where c_{air} is the specific heat of air and ρ_{air} is the air density.

In what follows, we report the mathematical expressions for the matrices A, B, C, D that govern the linear part of the system in Eq. (7), which are related to the aforementioned parameters and the time constants τ'_1, τ'_2 (one per zone, in s) associated with the propagation of heat from the heat resistors to the air inside the oven cavity, as described in [6].

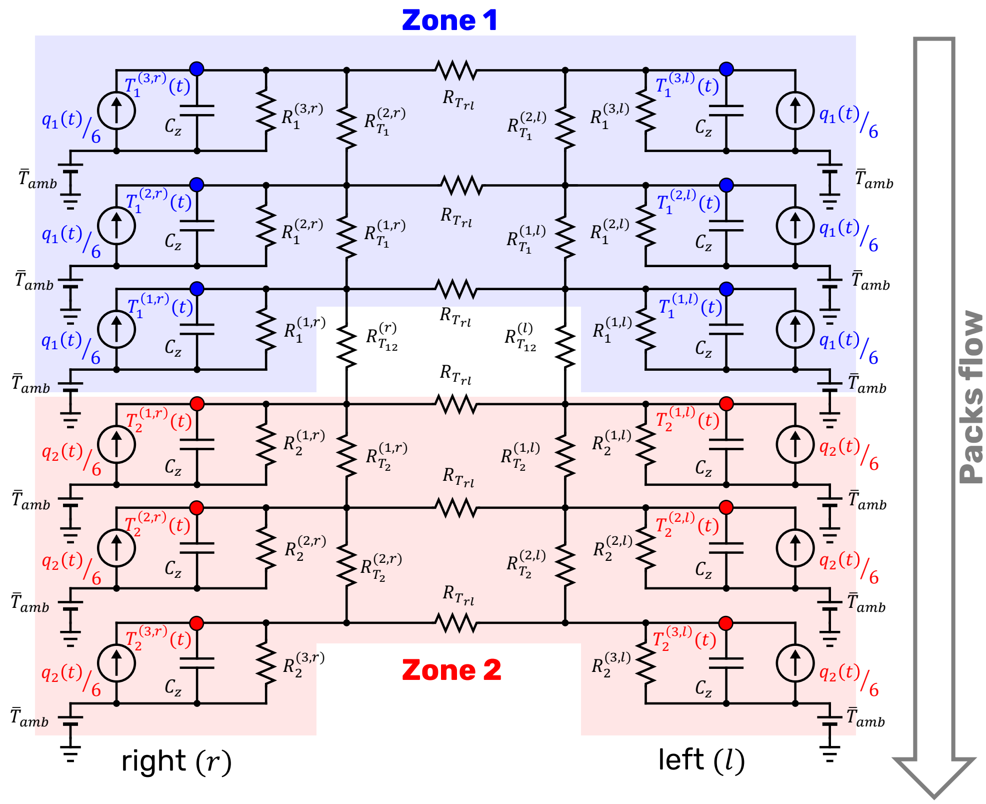

Figure 2: Electro-equivalent thermal circuit describing the relationship between the temperatures $T_i^{(k,j)}(t), i = 1,2, k = 1,2,3, j = l,r$, and the heat generated in each zone. See [6] for more details.

Let $a, b \in \mathbb{N}$. Notation-wise, we denote the a-dimensional column vectors of ones and zeros as $\mathbf{0}_a$ and $\mathbf{1}_a$ respectively, the $a \times b$ zero matrix as $0_{a\times b}$, and the identity matrix of size a as I_a. The matrices of the linear part of the `Heat-Temperatures` state-space model in Eq. (7) are:

$$A = \begin{bmatrix} \operatorname{diag}\left\{-\frac{1}{\tau_1'}, -\frac{1}{\tau_1'}, -\frac{1}{\tau_2'}, -\frac{1}{\tau_2'}\right\} & 0_{4\times 6} & 0_{4\times 6} \\ \tilde{A}_1 & A_{11} & A_{12} \\ \tilde{A}_2 & A_{21} & A_{22} \end{bmatrix}, \text{ where:} \tag{9a}$$

$$\tilde{A}_1 = \begin{bmatrix} \frac{1}{3\cdot C_z}\cdot \mathbf{1}_6 & \frac{1}{6\cdot C_z}\cdot \mathbf{1}_6 & \mathbf{0}_6 & \mathbf{0}_6 \end{bmatrix},$$

$$\tilde{A}_2 = \begin{bmatrix} \mathbf{0}_6 & \mathbf{0}_6 & \frac{1}{6\cdot C_z}\cdot \mathbf{1}_6 & \frac{1}{6\cdot C_z}\cdot \mathbf{1}_6 \end{bmatrix}.$$

Instead, the (i, j)-th entries of the matrices $A_{11}, A_{12}, A_{21}, A_{22}$ are:

$$A_{11}^{(i,j)} = \begin{cases} -\frac{1}{C_z} \cdot \left(\frac{1}{R_1^{(3,r)}} + \frac{1}{R_{T_{rl}}} + \frac{1}{R_{T_1}^{(2,r)}} \right) & \text{if } i=1, j=1 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=1, j=2 \\ \frac{1}{C_z \cdot R_{T_1}^{(2,r)}} & \text{if } i=1, j=3 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=2, j=1 \\ -\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_{rl}}} + \frac{1}{R_1^{(3,l)}} + \frac{1}{R_{T_1}^{(2,l)}} \right) & \text{if } i=2, j=2 \\ \frac{1}{C_z \cdot R_{T_1}^{(2,l)}} & \text{if } i=2, j=4 \\ \frac{1}{C_z \cdot R_{T_1}^{(2,r)}} & \text{if } i=3, j=1 \\ -\frac{1}{C_z} \cdot \left(\frac{1}{R_1^{(2,r)}} + \frac{1}{R_{T_1}^{(1,r)}} + \frac{1}{R_{T_1}^{(2,r)}} + \frac{1}{R_{T_{rl}}} \right) & \text{if } i=3, j=3 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=3, j=4 \\ \frac{1}{C_z \cdot R_{T_1}^{(1,r)}} & \text{if } i=3, j=5 \\ \frac{1}{C_z \cdot R_{T_1}^{(2,l)}} & \text{if } i=4, j=2 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=4, j=3 \\ -\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_1}^{(2,l)}} + \frac{1}{R_{T_{rl}}} + \frac{1}{R_1^{(2,l)}} + \frac{1}{R_{T_1}^{(1,l)}} \right) & \text{if } i=4, j=4 \\ \frac{1}{C_z \cdot R_{T_1}^{(1,l)}} & \text{if } i=4, j=6 \\ \frac{1}{C_z \cdot R_{T_1}^{(1,r)}} & \text{if } i=5, j=3 \\ -\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_1}^{(1,r)}} + \frac{1}{R_1^{(1,r)}} + \frac{1}{R_{T_{rl}}} + \frac{1}{R_{T_{12}}^{(r)}} \right) & \text{if } i=5, j=5 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=5, j=6 \\ \frac{1}{C_z \cdot R_{T_1}^{(1,l)}} & \text{if } i=6, j=4 \\ \frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=6, j=5 \\ -\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_{rl}}} + \frac{1}{R_{T_1}^{(1,l)}} + \frac{1}{R_1^{(1,l)}} + \frac{1}{R_{T_{12}}^{(l)}} \right) & \text{if } i=6, j=6 \\ 0 & \text{otherwise} \end{cases},$$

$$A_{12}^{(i,j)} = \begin{cases} \frac{1}{C_z \cdot R_{T_{12}}^{(r)}} & \text{if } i=5, j=1 \\ \frac{1}{C_z \cdot R_{T_{12}}^{(l)}} & \text{if } i=6, j=2 \\ 0 & \text{otherwise} \end{cases},$$

$$A_{21}^{(i,j)} = \begin{cases} \frac{1}{C_z \cdot R_{T_{12}}^{(r)}} & \text{if } i=1, j=5 \\ \frac{1}{C_z \cdot R_{T_{12}}^{(l)}} & \text{if } i=2, j=6 \\ 0 & \text{otherwise} \end{cases},$$

$$
A_{22}^{(i,j)} = \begin{cases}
-\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_{12}}^{(r)}} + \frac{1}{R_2^{(1,r)}} + \frac{1}{R_{T_2}^{(1,r)}} + \frac{1}{R_{T_{rl}}} \right) & \text{if } i=1, j=1 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=1, j=2 \\
\frac{1}{C_z \cdot R_{T_2}^{(1,r)}} & \text{if } i=1, j=3 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=2, j=1 \\
-\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_{rl}}} + \frac{1}{R_{T_2}^{(1,l)}} + \frac{1}{R_2^{(1,l)}} + \frac{1}{R_{T_{12}}^{(l)}} \right) & \text{if } i=2, j=2 \\
\frac{1}{C_z \cdot R_{T_2}^{(1,l)}} & \text{if } i=2, j=4 \\
\frac{1}{C_z \cdot R_{T_2}^{(1,r)}} & \text{if } i=3, j=1 \\
-\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_2}^{(1,r)}} + \frac{1}{R_{T_2}^{(2,r)}} + \frac{1}{R_2^{(2,r)}} + \frac{1}{R_{T_{rl}}} \right) & \text{if } i=3, j=3 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=3, j=4 \\
\frac{1}{C_z \cdot R_{T_2}^{(2,r)}} & \text{if } i=3, j=5 \\
\frac{1}{C_z \cdot R_{T_2}^{(1,l)}} & \text{if } i=4, j=2 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=4, j=3 \\
-\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_2}^{(1,l)}} + \frac{1}{R_{T_{rl}}} + \frac{1}{R_2^{(2,l)}} + \frac{1}{R_{T_2}^{(2,l)}} \right) & \text{if } i=4, j=4 \\
\frac{1}{C_z \cdot R_{T_2}^{(2,l)}} & \text{if } i=4, j=6 \\
\frac{1}{C_z \cdot R_{T_2}^{(2,r)}} & \text{if } i=5, j=3 \\
-\frac{1}{C_z} \cdot \left(\frac{1}{R_{T_2}^{(2,r)}} + \frac{1}{R_2^{(3,r)}} + \frac{1}{R_{T_{rl}}} \right) & \text{if } i=5, j=5 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=5, j=6 \\
\frac{1}{C_z \cdot R_{T_2}^{(2,l)}} & \text{if } i=6, j=4 \\
\frac{1}{C_z \cdot R_{T_{rl}}} & \text{if } i=6, j=5 \\
-\frac{1}{C_z} \cdot \left(\frac{1}{R_2^{(3,l)}} + \frac{1}{R_{T_{rl}}} + \frac{1}{R_{T_2}^{(2,l)}} \right) & \text{if } i=6, j=6 \\
0 & \text{otherwise}
\end{cases} .
$$

Finally, the remaining matrices of the linear part of the system in Eq. (7) are:

$$
B = \begin{bmatrix}
\frac{1}{\tau_1' \cdot R_{heat}} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\tau_1' \cdot R_{heat}} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\tau_2' \cdot R_{heat}} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\tau_2' \cdot R_{heat}} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(3,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(3,l)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(2,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(2,l)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(1,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_1^{(1,l)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(1,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(1,l)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(2,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(2,l)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(3,r)}} \\
0 & 0 & 0 & 0 & \frac{1}{C_z \cdot R_2^{(3,l)}}
\end{bmatrix}, \tag{9b}
$$

$$
C = [0_{12\times 4} \quad I_{12}], \tag{9c}
$$

$$
D = 0_{12\times 5}. \tag{9d}
$$

1.3 Review of the `Packs-Temperatures` model

The `Packs-Temperatures` model describes the relationship between the flow of bottle packs inserted into the oven and the temperatures measured by the twelve thermocouples installed inside the shrink tunnel. In [6], we model $d\,(t; v)$ using a Bernoulli distribution [7] with a success rate that depends on the production rate:

$$
d\,(t; v) \sim \text{Bernoulli}\,(p_d\,(v))\,. \tag{10}
$$

Instead, the relationship between $d\,(t; v)$ and the k-th output temperature $y''_k\,(t)\,, k = 1, \ldots, 12$, is approximated by a First Order Lag Plus time Delay (FOLPD) transfer function [6]:

$$
H_k\,(s; v) = \frac{Y''_k\,(s)}{D\,(s; v)} = \frac{\mu''_k\,(v)}{s \cdot \tau''_k\,(v) + 1} \cdot \exp\left\{-s \cdot T''_{d_k}\,(v)\right\}, \tag{11}
$$

where $\mu''_k\,(v) \in \mathbb{R}_{<0}$ (in °C) is the static gain, $\tau''_k\,(v) \in \mathbb{R}_{>0}$ (in s) is the time constant, and $T''_{d_k}\,(v) \in \mathbb{R}_{>0}$ (in s) is the time delay.

2 Parameter estimation

In this Section, we estimate the parameters of the models reviewed in Section 1 based on data obtained from an experimental campaign carried out on the shrink tunnel under study.

2.1 Estimation of the parameters of the `Heat-Temperatures` model

First, we consider the `Heat-Temperatures` model reviewed in Section 1.2. To estimate its parameters, we rely on data coming from both open-loop and closed-loop experiments performed on the shrink tunnel under study. In both cases, relevant data has been acquired with a sampling time of $T_s = 0.01$ s. For what concerns the open-loop experiments, we rely on two step response tests: in the former test, only the heat resistors of the first zone are actuated, while the opposite is true for the second step response. Instead, in the closed-loop experiment, the temperature controller is composed of two Proportional-Integral (PI) regulators, a decoupler, and an anti-windup [1]. Only two temperatures are actually controlled in closed-loop, namely $T_1^{(1,r)}\,(t)$ and $T_2^{(2,l)}\,(t)$, which are in the central regions of the oven (see [6]). In particular, during the closed-loop trial, the shrink tunnel switches between different operating points. In all the experiments considered in this Section, the production rate is $v\,(t) = 0, \forall t \in \mathbb{R}$, i.e. there is no flow of bottle packs being inserted into the oven.

We are only interested in identifying the parameters that govern the linear part of the system in Eq. (7). That is because *the nonlinear part of the system does not depend on any unknown parameters*. Therefore, we only have to estimate the thermal resistances, the resistance of the heat resistors R_{heat}, the time constants τ_1', τ_2', and the time delays $T'_{d_k}, k = 1, \ldots, 12$ (see Section 1.2).

First, we estimate the time delays from the open-loop step responses in a heuristic fashion by means of a threshold [5]. Notation-wise, we denote the estimated time delays as $\hat{T}'_{d_k}, k = 1, \ldots, 12$. Then, we shift the output signals $y_k'(t), k = 1, \ldots, 12$, of the available closed-loop experiment backwards by their corresponding estimated time delays $\hat{T}'_{d_k}$, obtaining $\tilde{y}_k'(t)$. By proceeding this way, and recalling that there are no unknown parameters that govern the nonlinear part of the system, we only have to identify the following *delay-free linear state-space model* (cf. Eq. (7)):

$$\begin{cases} \dot{\boldsymbol{x}}(t) = A(\boldsymbol{\theta}) \cdot \boldsymbol{x}(t) + B(\boldsymbol{\theta}) \cdot \boldsymbol{u}(t) \\ \tilde{\boldsymbol{y}}'(t) = C \cdot \boldsymbol{x}(t) + D \cdot \boldsymbol{u}(t) \end{cases} . \tag{12}$$

In Eq. (12), we have made explicit the dependency of the matrices A and B on the unknown parameters $\boldsymbol{\theta} \in \mathbb{R}^{n_{\boldsymbol{\theta}}}$, which are the thermal resistances, R_{heat}, τ_1' and τ_2' (see Eq. (9)). In particular, $A(\boldsymbol{\theta})$ and $B(\boldsymbol{\theta})$ depend on $n_{\boldsymbol{\theta}} = 26$ parameters in total. To estimate them, we follow the output-error estimation methodology [8], i.e. we minimize the cost function:

$$J(\boldsymbol{\theta}) = \frac{1}{N} \cdot \sum_{\tilde{t}=0}^{N-1} \left\| \tilde{\boldsymbol{y}}'\left(\tilde{t} \cdot T_s\right) - \hat{\tilde{\boldsymbol{y}}}'\left(\tilde{t} \cdot T_s; \boldsymbol{\theta}\right) \right\|_2^2 . \tag{13}$$

In Eq. (13), $N \in \mathbb{N}$ is the number of samples acquired during the closed-loop experiment, $\tilde{t} \in \mathbb{N} \cup \{0\}$ is the sample index, and $\hat{\tilde{\boldsymbol{y}}}'\left(\tilde{t} \cdot T_s; \boldsymbol{\theta}\right)$ is the simulated output of the system in Eq. (12) for a given parametrization $\boldsymbol{\theta}$ and using the same input sequence $\boldsymbol{u}\left(\tilde{t} \cdot T_s\right)$ that produced $\tilde{\boldsymbol{y}}'\left(\tilde{t} \cdot T_s\right)$. In particular, we use MATLAB's `greyest` method [3] to minimize the cost function in Eq. (13). Notation-wise, we denote the estimate of the parameters as $\hat{\boldsymbol{\theta}}$. Table 1 reports the identified parameters for the `Heat-Temperatures` model. Then, Table 2 reports the goodness of fit of each simulated output[1] $\hat{y}_k'\left(\tilde{t} \cdot T_s; \hat{\boldsymbol{\theta}}\right), k = 1, \ldots, 12$, computed as:

$$\text{Fit}_k = \left(1 - \frac{\sqrt{\frac{1}{N} \cdot \sum_{\tilde{t}=0}^{N-1} \left| y_k'\left(\tilde{t} \cdot T_s\right) - \hat{y}_k'\left(\tilde{t} \cdot T_s; \hat{\boldsymbol{\theta}}\right) \right|^2}}{\max_{\tilde{t} \in \{0,\ldots,N-1\}} y_k'\left(\tilde{t} \cdot T_s\right) - \min_{\tilde{t} \in \{0,\ldots,N-1\}} y_k'\left(\tilde{t} \cdot T_s\right)}\right) \cdot 100 \quad [\%] . \tag{14}$$

2.2 Estimation of the parameters of the `Packs-Temperatures` model

Now, we consider the `Packs-Temperatures` models reviewed in Section 1.3. In this case, we need to estimate the success rate of the Bernoulli distribution in Eq. (10) and the parameters of the FOLPD transfer functions in Eq. (11), which all depend on the production rate. For identification purposes, we have carried out three experiments on the shrink tunnel under study in the following fashion. The oven is controlled in open-loop, with constant duty cycles $w_1(t), w_2(t)$. Initially, we let the temperatures inside the oven cavity reach an equilibrium point, denoted as $\bar{\boldsymbol{y}}'$, without inserting any bottle packs into the shrink tunnel. Then, we started the production line at a production rate v of 30 ppm and waited for the temperatures to (roughly) settle. We repeated the same procedure for $v = 60$ ppm and $v = 90$ ppm. By proceeding this way, we can easily derive the effect of the bottle packs on the temperatures by subtracting the previously reached equilibrium temperatures due to the constant control actions (and ambient temperature), i.e.

$$\boldsymbol{y}''(t) = \boldsymbol{y}(t) - \bar{\boldsymbol{y}}'.$$

For each production rate, we estimate the parameters of interest as follows. The success rates $p_d(v)$ in Eq. (10) are found by taking the sample mean of the signals $d(t; v)$ for the experiments with $v = 30, 60, 90$ ppm. In particular, we obtained:

$$\hat{p}_d(30) = 0.26,$$
$$\hat{p}_d(60) = 0.48,$$
$$\hat{p}_d(90) = 0.74.$$

[1]Differently from $\hat{\tilde{y}}_k'\left(\tilde{t} \cdot T_s; \boldsymbol{\theta}\right)$, $\hat{y}_k'\left(\tilde{t} \cdot T_s; \boldsymbol{\theta}\right)$ also takes into account of the nonlinearities in Eq. (7) and the previously identified time delays.

Table 1: Estimated parameters for the `Heat-Temperatures` model reviewed in Section 1.2. C_z is known, see Eq. (8).

$\hat{R}_1^{(3,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_1^{(3,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_1^{(2,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_1^{(2,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_1^{(1,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_1^{(1,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_1}^{(2,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_1}^{(2,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_1}^{(1,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_1}^{(1,l)}$ $\left[\frac{°C}{J}\cdot s\right]$
3.28	2.01	3.99	19.45	5.92	9.20	10.00	0.12	0.65	0.86

$\hat{R}_2^{(3,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_2^{(3,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_2^{(2,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_2^{(2,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_2^{(1,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_2^{(1,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_2}^{(2,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_2}^{(2,l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_2}^{(1,r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_2}^{(1,l)}$ $\left[\frac{°C}{J}\cdot s\right]$
2.17	2.45	84.89	100.00	2.65	2.52	0.88	1.08	0.37	1.73

$\hat{R}_{T_{12}}^{(r)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_{12}}^{(l)}$ $\left[\frac{°C}{J}\cdot s\right]$	$\hat{R}_{T_{rl}}$ $\left[\frac{°C}{J}\cdot s\right]$	C_z $\left[\frac{J}{°C}\right]$	$\hat{R}_{heat}$ $[\Omega]$	$\hat{\tau}_1'$ [s]	$\hat{\tau}_2'$ [s]
1.85	1.04	1.00	231	1239	184.50	35.67

$\hat{T}_{d_1}'$ [s]	$\hat{T}_{d_2}'$ [s]	$\hat{T}_{d_3}'$ [s]	$\hat{T}_{d_4}'$ [s]	$\hat{T}_{d_5}'$ [s]	$\hat{T}_{d_6}'$ [s]	$\hat{T}_{d_7}'$ [s]	$\hat{T}_{d_8}'$ [s]	$\hat{T}_{d_9}'$ [s]	$\hat{T}_{d_{10}}'$ [s]	$\hat{T}_{d_{11}}'$ [s]	$\hat{T}_{d_{12}}'$ [s]
66.17	75.18	62.90	67.37	58.27	60.78	50.38	53.25	36.77	42.95	66.73	63.34

Table 2: Goodness of fit of each simulated output of the `Heat-Temperatures` model.

Fit$_k$ [%]					
y_1'—$T_1^{(3,r)}$	y_2'—$T_1^{(3,l)}$	y_3'—$T_1^{(2,r)}$	y_4'—$T_1^{(2,l)}$	y_5'—$T_1^{(1,r)}$	y_6'—$T_1^{(1,l)}$
97.8%	97.5%	97.5%	97.2%	97.5%	95.9%

Fit$_k$ [%]					
y_7'—$T_2^{(1,r)}$	y_8'—$T_2^{(1,l)}$	y_9'—$T_2^{(2,r)}$	y_{10}'—$T_2^{(2,l)}$	y_{11}'—$T_2^{(3,r)}$	y_{12}'—$T_2^{(3,l)}$
97.5%	97.1%	97.0%	96.4%	97.6%	95.8%

Instead, the parameters $\mu_k''(v), \tau_k''(v), T_{d_k}''(v), k = 1, \dots, 12$, of the FOLPD models $H_k(s; v)$ in Eq. (11) are estimated from the signals $d(t; v)$ and $\boldsymbol{y}''(t)$ by means of MATLAB's `process` method [4]. Table 3 reports their estimates and the corresponding goodness of fit, which is computed analogously to Eq. (14) using $y_k''(\tilde{t}\cdot T_s)$ instead of $y_k'(\tilde{t}\cdot T_s)$ and similarly for the simulated outputs.

Table 3: Estimated parameters and fits for the `Packs-Temperatures` model reviewed in Section 1.3.

	$v = 30\,\text{ppm}$				$v = 60\,\text{ppm}$				$v = 90\,\text{ppm}$			
	$\hat{\mu}''_k(v)$	$\hat{\tau}''_k(v)$	$\hat{T}''_{d_k}(v)$	Fit$_k$	$\hat{\mu}''_k(v)$	$\hat{\tau}''_k(v)$	$\hat{T}''_{d_k}(v)$	Fit$_k$	$\hat{\mu}''_k(v)$	$\hat{\tau}''_k(v)$	$\hat{T}''_{d_k}(v)$	Fit$_k$
k	[°C]	[s]	[s]	[%]	[°C]	[s]	[s]	[%]	[°C]	[s]	[s]	[%]
1—$T_1^{(3,r)}$	-61.60	216.96	6.19	97.7%	-78.33	312.33	0.24	95.1%	-46.74	178.92	28.16	97.7%
2—$T_1^{(3,l)}$	-59.31	422.56	14.82	95.8%	-61.62	395.55	5.96	96.0%	-41.28	229.47	1.48	95.8%
3—$T_1^{(2,r)}$	-36.07	373.52	0.46	98.7%	-39.68	487.53	18.11	97.0%	-30.78	490.73	0.00	98.7%
4—$T_1^{(2,l)}$	-46.04	568.67	14.94	97.6%	-41.93	478.38	17.06	96.9%	-30.54	351.33	4.15	97.7%
5—$T_1^{(1,r)}$	-13.05	569.49	34.82	97.0%	-23.13	491.36	23.08	95.1%	-26.44	623.44	56.00	97.0%
6—$T_1^{(1,l)}$	-45.15	742.20	11.28	98.2%	-37.64	544.92	8.40	97.1%	-28.64	439.11	0.00	98.2%
7—$T_2^{(1,r)}$	-17.94	587.892	2.25	97.7%	-21.68	520.13	12.03	96.1%	-22.17	601.79	30.30	97.7%
8—$T_2^{(1,l)}$	-26.06	1057.52	28.46	98.3%	-23.08	589.10	2.80	96.4%	-19.42	497.82	0.00	98.3%
9—$T_2^{(2,r)}$	-21.76	537.59	15.62	97.6%	-24.30	404.65	1.01	95.6%	-21.19	511.00	0.00	97.6%
10—$T_2^{(2,l)}$	-28.67	666.59	0.00	98.2%	-22.89	484.28	32.33	96.7%	-17.35	465.74	0.00	98.2%
11—$T_2^{(3,r)}$	-42.79	319.95	15.19	95.1%	-39.76	209.30	19.22	93.9%	-27.72	533.19	0.00	95.1%
12—$T_2^{(3,l)}$	-24.34	1053.91	26.96	95.0%	-14.40	695.26	44.27	91.7%	-12.07	670.96	40.00	95.0%

3 Software documentation

Work in progress.

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