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Direct Methods of Latitude and Longitude Determination by Mini-Computer

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ABSTRACT

IN CELESTIAL NAVIGATION, the advent of the hand held or mini-computer brought forth a plethora of methods for the solution of spherical triangles. The azimuths and altitudes thus derived have not only eliminated the need for bulky tables but also have speeded up the determination of the data needed for the graphic construction of intersecting lines of position from which the "fix" is derived. In this conventional type of solution, the corrected sextant altitude is compared with the computed altitude then, from the assumed position, the line of position is constructed perpendicular to the azimuth of the body observed and either towards the body or away from it depending upon whether the observed altitude is greater or less, respectively, than that which has been computed.

Extremely sophisticated systems are being developed which combine special computer systems directly with the sextant. However, the present paper relates only to the mini-computers currently on the market and their apparent ultimate utility in the celestial navigation field—the direct display of both latitude and longitude without recourse to graphic methods.

INTRODUCTION

Direct methods of latitude and longitude determination without graphic construction have been proposed, among others, by Thomas H. Sumner[1], Charles T. Dozier[2], Stjepo Kotlaric[3], Edward M. Weyer[4], and Louis A. Valier[5]. These methods, except for Valier's, seem to be either too complicated or too limited in scope.

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However, the Kotlaric method does include five excellent formulae which are free from most ambiguities and which are adaptable for mini-computers with trigonometric capabilities. The number of steps involved in the solution make the Hewlett Packard H.P.65 particularly attractive as that model can accept pre-programmed magnetic tapes and it has the capability of remembering not only all the essential input data but also the correct order for insertion of that data in the various formulae. The Sumner and Valier methods use a different approach than the others and the latter method, in spite of certain limitations, may well find its way to general acceptance because of its simplicity.

THE SUMNER PARALLEL OF EQUAL ALTITUDE METHOD

Back in 1837, Captain Thomas Sumner chanced upon a new navigation method. The time sight enabled the navigator to determine his longitude if he knew his latitude, the declination, the corrected sextant altitude and the correct time. On the occasion of his discovery, Captain Sumner was unsure of his latitude so he worked out time sights for two additional latitudes. The net result was that all three longitudes thus derived were in a straight line. The observed altitude was then seen to be identical at all points on the line at the same time. This was a "parallel of equal altitude" or, as it was soon called, a Sumner Line.

Sumner's book, which was first published in 1843, told how two such intersecting Sumner Lines would produce a fix which could be graphically determined. He also described what seems to be a rather complex mathematical time solution for the direct determination of latitude and longitude. The fact that present day methods are not compatible with those of Sumner's day probably accounts for the apparent complexity. Regardless of this, Sumner's method as described does not lend itself to mini-computer solution.

THE DOZIER-WEYER-KOTLARIC METHOD

The above name may be a misnomer but it relates it to the original method of Dozier, Weyer's independent work, and Kotlaric's formulae.

In Fig. 1, a celestial triangle is portrayed in which N is the North Pole, and A and B are two stars.

The zenith of the observer is at Z, sides NA and NB are the co-declinations of stars A and B respectively. An observation of Star A, the star to the east, is taken first and the time and altitude recorded. Then Star B, the star to the west of the zenith, is similarly observed. The Greenwich Hour Angles of each star are then determined and that of A is subtracted from that of B. The result is angle BNA. A direct function of the time between observations is the increase of this angle and also the resultant increase in the interstellar distance AB. (See Fig. 2).

Weyer, in his book, provides for the modification of this triangle (NAB) to allow for the interval between sights and he points out that the new triangle, (NAB′) although hypothetical still has integrity for trigonometric solutions.

With angle BNA and sides NA and NB we now have two sides and the included angle of triangle NBA. First we solve for side AB, the interstellar distance, using Formula I (see Appendix for this and subsequent formulas). Then, having three sides and using Formula II, we get angle NBA.

Fig. 3 shows ARC NZ, the assumed co-latitude of the observer, and also the co-altitudes of stars ZA and ZB. (The altitudes of each star have already been recorded so sides AZ and BZ are each 90° minus the altitudes.) In triangle ABZ we now know all three sides so, using Formula III, we solve for angle ZBA. This angle is

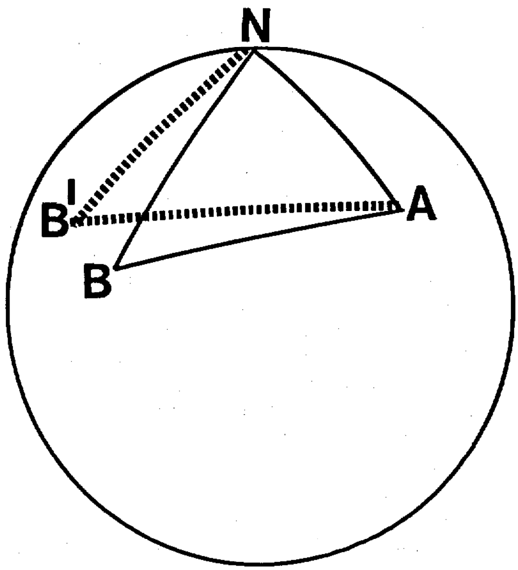
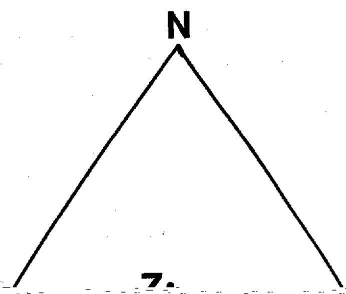
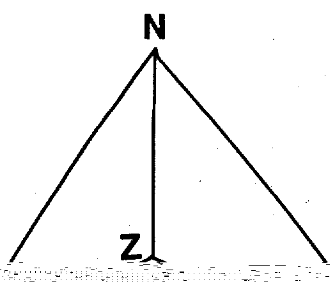


Figure 2.





subtracted from angle NBA which was determined earlier to get angle NBZ. A new triangle is now considered: triangle NBZ. The side NB (co-declination of B), side ZB (co-altitude of star B) and enclosed angle NBZ are known so Formula IV is used to get side NZ which when subtracted from 90° gives the latitude of the observer. Finally, in triangle NBZ, with all three sides known, Formula V gives angle ZNB which, when subtracted from the Greenwich Hour Angle of star B at the time of its observation, gives the longitude of the observer.

The foregoing is an idealized situation. The position of the stars may vary depending upon their declination and sidereal hour angle. Then too, the zenith may be in various positions relative to the two stars. It might be north or south of the interstellar line and also possibly either east of star A, west of star B, or between them.

For example, Fig. 4 shows a case in which the zenith is west of Star B. Here, after Angle ZNB has been determined, it must be added to the Greenwich Hour Angle of Star B to get the longitude, instead of being subtracted from it as was the case in the situation depicted in Fig. 3.

Other problems are encountered such as is shown in Fig. 5 where the zenith is south of the interstellar line. Then angle NBZ, measured clockwise from N, is greater than 180°. All of these situations can be handled by proper programming steps and after the data has been inserted in the mini-computer, it takes only a minute or so to see the latitude and longitude displayed.

Now that the position is determined, taking into consideration the correction for motion of

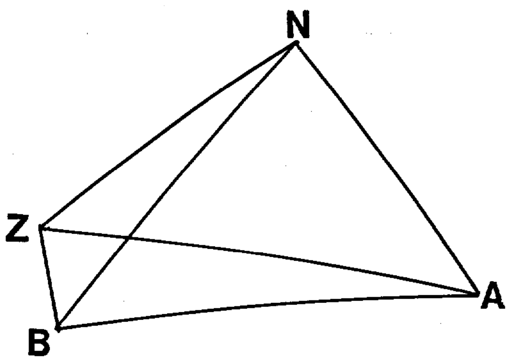


Figure 4.

the second star because of the difference between the times of observation of the first and second stars, consideration should be devoted to the essential correction for possible movement of the vessel between the two sights.

Weyer's explanation is a masterpiece and I quote it with his permission:

"This adjustment is made by ascertaining what the sextant reading on the First Star would have been if it had been taken from the same position as the Second."

"Referring to Fig. 6, highly exaggerated for the sake of clarity, assume that the vessel moves instantly from C to C′. A is the substellar point of the First Star, which does not

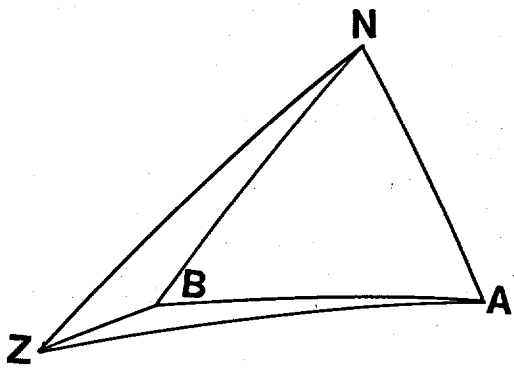


Figure 5.

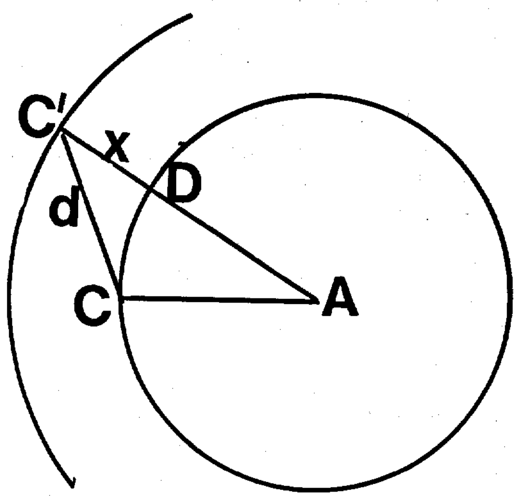


Figure 6.

change. C and C′ are on concentric circles of position. The radius of a circle of position is always equal to 90° minus the sextant reading. In this instance, the radius changes from AC to AC′, and the difference is equal to DC′. If the radius increases by this amount, the sextant reading decreases by the same amount. Of course, the vessel might move toward the star, in which case the sextant angle would be increased.

"In this way, the navigator can ascertain what the sextant reading would have been if he had taken it from the second position.

". . . The adjustment to be made in the altitude of the First Star for movement of the vessel between star sights combines speed, time interval, and direction of travel with reference to the First Star. Because nautical miles are interchangeable with minutes of angle on a great circle, the "effective" distance in miles is applied directly to the star's altitude in minutes. . . .

"The maximum allowance applies when the vessel is traveling directly toward or away from the star. When the star is abeam, no adjustment is necessary. The more the course is toward the star or away from it, the greater the adjustment.

"The amount of the adjustment varies as the cosine of the bearing of the star. This is shown in Fig. 6. In this diagram C is the observer's position at the time of his first star sight, C′ is his position after moving the distance d, A is the substellar point of the First Star, Angle ACC′ is his direction of travel relative to the star, and x is the "effective" distance, the amount by which the radius of his circle of position has been altered. This is the amount by which his sextant reading is to be adjusted."

Weyer's equation is "Effective distance = $d \times \cos ACC'$"

"This equation assumes that the angle CDC′ is a right angle. That it differs slightly from a right angle is inconsequential in the present applications."

In Fig. 6, the first star is due east of the observer (90°) at the time of its observation. The ship is on a true course of 330° at 15 knots, thus the stars relative bearing is 330° minus 90° or 240°. The cosine of 240° is −.50000. Suppose that the time difference between observations is exactly 6 minutes (.1000 of an hour) and the speed of the ship is 15 knots, the distance traveled between sights is 1.5 nautical miles. Multiply the distance of 1.5 nautical miles by the cosine of 240° (−.5000) equals −.7500 mile, and as one nautical mile equals one minute of time, −.7500 of a mile in this case is equal to −45′ of arc. Thus, from the observed altitude of the first star the correction of −45′ is applied and this is applied in the third formula. Note that when the relative bearing is between 90° and 270° the cosine is negative, making the sextant reduction negative as it should be.

THE VALIER TWO BODY TIME SIGHT SOLUTION

Louis A. Valier, of Honolulu, Hawaii, in his paper "Navigation by Pocket Calculator", developed his own solution of the time sight method which is a masterpiece of simplicity and is easily adapted to the mini-computer.

Valier's method involves the solution of time sights for each of two celestial bodies on each of two adjacent latitudes that bracket the dead-reckoning position. Appendix B shows the formula and the steps involved. Lines are drawn connecting the two longitudes thus determined, first AB for the body to the east and then line CD connecting the longitudes of the other body. It is preferred that the two lines intersect between the two latitudes. If they do not, different latitudes, even if closer together or further apart, or if necessary different celestial bodies should be selected. A typical intersection is shown in Fig. 7. The small arrows at right angles to the Sumner Lines AB and CD indi-

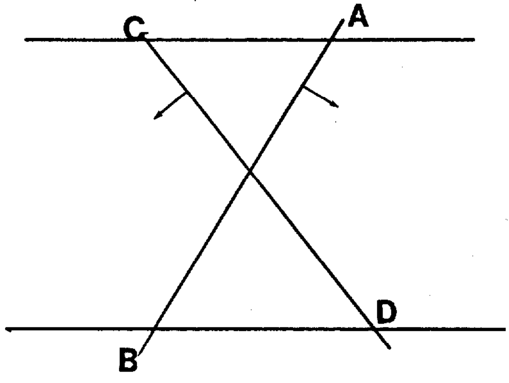


Figure 7.

cate the azimuth of the bodies used in this case. Please note that the azimuth of either body could be reversed but it is simpler if the first body observed is to the east especially, as will be seen, if the vessel is moving so that later correction for that movement is necessary.

If it were desired to get a graphic solution for the determination of the fix, a perpendicular would be dropped from G on the upper latitude through the intersection F to E on the lower latitude. (See Fig. 8)

As the difference between the upper and lower latitudes (EG) is known, EF can be proportionally determined and added to the lower latitude to get the latitude of the fix at F. Similarly, the longitude of both B and D on the lower latitude are known and E the longitude of the fix is a proportional distance between them.

This construction has pointed the way to a geometric solution of the latitude. Look again at Fig. 8 and triangles ACF and DBF. It is easy to prove that they are similar triangles with congruent angles. If two triangles are similar their corresponding altitudes have the same ratio as a pair of corresponding sides.
Thus

$$EF:FG = BD:CA$$

and also

$$EF:EF + FG = BD:BD + CA$$

simplifying and then multiplying the means by the extremes

$$EF \times (BD + CA) = BD \times EG$$

and dividing by BD + CA

$$EF = \frac{BD \times EG}{BD + CA}$$

The difference between the upper and lower latitudes, EG, is 1° in this case which further simplifies the solution. EF is then ascertained using this formula. (The longitudes A, B, C and D have already been figured by time sights thus permitting the determination of CA and BD by subtraction.)

The result, EF, is called the latitude increment and, in the Northern Hemisphere, it is added to the lower latitude to produce the latitude of the fix. It should be noted here that in the Southern Hemisphere the latitude increment thus determined is reversed and is subtracted from the lower latitude to get the fix.

The longitude of the fix is now desired and is determined in the following manner: Line FH, parallel to the upper and lower latitudes, is constructed to intersect (at H) a perpendicular DJ which is erected from D on the lower latitude. (See Fig. 9)

In this figure there are two similar triangles, DFH and DCJ. As they are similar the following ratios prevail:

$$FH:CJ = DH:DJ$$

multiplying the means by the extremes

$$FH \times DJ = CJ \times DH$$

dividing each side by DJ

$$FH = \frac{CJ \times DH}{DJ}$$

In the above formula, J is by construction at the same longitude as D so if we subtract D from C we get CJ. DH is identical to the latitude increment EF and DJ is identical to EG so

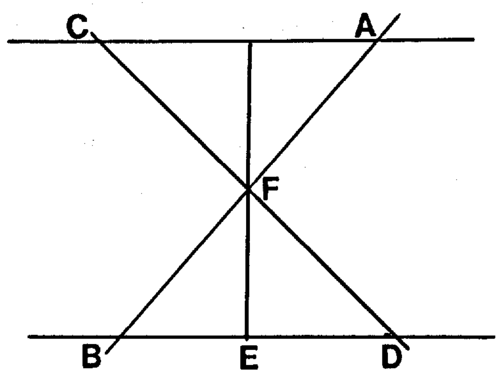


Figure 8.

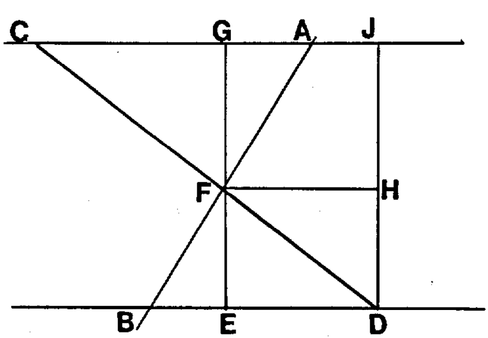


Figure 9.

FH or the longitude increment can be stated to be

$$FH = \frac{(C - D) \times EF}{EG}$$

EF is the latitude increment obtained earlier, (C − D) is the longitude of C minus that of D and EG is the difference between the upper and lower latitudes. If this is 1° exactly the longitude increment is determined simply by getting product of (C − D) and EF which is then added in Western Hemisphere to the longitude of D and subtracted from that longitude if D is in East longitude. All of this is readily programmed on a mini-computer.

We saw in the Dozier-Kotlaric Method how Weyer advanced or retarded a line of position by making the appropriate change in the altitude of the first star. The same method can be applied here in the Valier Method and the corrected altitude applied to both time sight solutions of the first star.

The Valier Two Body Time Sight solution is simple enough to be solved step by step by a mini-computer, as shown by the formula as stated in Appendix B.

The solution as the name implies may be used with stars, planets, the moon or sun. Using this method with two sun sights taken only 1 hour apart the author has derived a reasonably good fix.

This paper shows that the mini-computer has more utility than might be imagined. It is hoped that its use will provide not only rapid and accurate solutions but also give satisfaction, pleasure and a new sense of achievement to the navigator.

APPENDIX A

Kotlaric's Two Star Formulae as adapted by C. Wight:

Formula I

$$\text{Interstellar Distance AB} = 2 \times \left[\text{ARC Sin} \sqrt{\text{SIN}^2 \frac{\text{ANB}}{2} \times \text{Cos} \overset{(B)}{\text{Dec}} \times \text{Cos} \overset{(A)}{\text{Dec}} + \text{SIN}^2 \left(\frac{\overset{(B)}{\text{Dec}} - \overset{(A)}{\text{Dec}}}{2}\right)}\right]$$

Formula II

$$\text{Angle NBA} = 2 \times \left[\text{ARC SIN} \sqrt{\text{SIN}(\text{E} - \overset{(A)}{\text{Dec}}) \times \text{Cos E} \times \frac{1}{\text{cos Dec (B)}} \times \frac{1}{\text{SIN AB}}}\right]$$

Where

$$\text{E} = \frac{\overset{(A)}{\text{Dec}} + \text{AB} + \overset{(B)}{\text{Dec}}}{2}$$

Formula III

$$\text{Angle ZBA} = 2 \times \left[\text{ARC SIN} \sqrt{\text{SIN}(\text{F} - \overset{(A)}{\text{HO}}) \times \text{Cos F} \times \frac{1}{\text{Cos HO (B)}} \times \frac{1}{\text{SIN AB}}}\right]$$

Where

$$\text{F} = \frac{\overset{(A)}{\text{HO}} + \text{AB} + \overset{(B)}{\text{HO}}}{2}$$

Formula IV

$$\text{Lat} = (90 - \text{NZ}) = 2 \times \left[\text{ARC SIN} \sqrt{\text{SIN}^2 \frac{\text{NBZ}}{2} \times \text{COS} \overset{(B)}{\text{HO}} \times \text{Cos} \overset{(B)}{\text{Dec}} + \text{SIN}^2 \frac{\overset{(B)}{\text{HO}} - \overset{(B)}{\text{Dec}}}{2}}\right]$$

Formula V

$$\text{Angle ZNB} = 2 \times \left[\text{ARC SIN} \sqrt{\text{SIN}(\text{G} - \overset{(B)}{\text{HO}}) \times \text{Cos G} \times \frac{1}{\text{cos Dec (B)}} \times \frac{1}{\text{Cos Lat}}}\right]$$

Where

$$G = \frac{\text{Lat} + \overset{(B)}{\text{HO}} + (90 - \overset{(B)}{\text{Dec}})}{2}$$

Directions

Get difference between Angles NBA and ZBA to get Angle NBZ which is used in Formula IV to get latitude.

With this data entered in Formula V, get Angle ZNB which is applied to the GHA of Star B to get the longitude.

APPENDIX B

The Valier Two Body Time Sight Solution

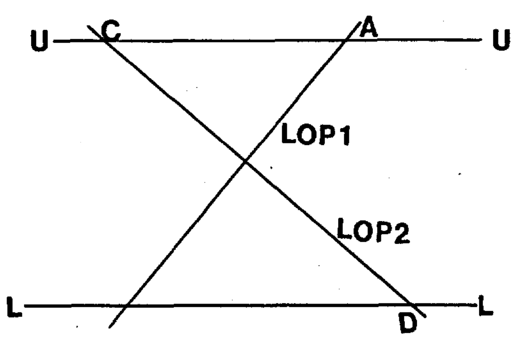


Figure B1.

In the subsequent computations, convert minutes and seconds to decimal fractions of a degree. Also enter south latitudes and east longitudes as negative figures where applicable.

Determine the longitudes of A,B,C, and D as follows:

1. (a) Using the corrected observed altitude (Ho) and the declination (Dec.) mf the first star, determine the meridian angle of that star on both the upper latitude (U) and the lower patitude (L) using the formula which follows:

   Arc Cos Meridian angle

   $$= \frac{\text{Sin Ho} - (\text{Sin Lat.} \times \text{Sin Dec})}{\text{Cos Lat.} \times \text{Cos Dec.}}$$

   (b) Similarly determine the meridian angle of the second star on both latitudes.

   (c) *If Body East of Observer*
   Add the meridian angle thus determined to the Greenwich Hour Angle of the body at the time of observation.

   (d) *If Body West of Observer*
   Subtract the meridian angle thus determined from the Greenwhich Hour Angle of the body at the time of observation.

   (e) *West Longitude*
   In both c and d above, if the result is over 360° subtract 360 to get the longitude.

   (f) *East Longitude*
   In both c and d above, if the result is a positive figure, subtract 360 repetitively until the result is a negative figure (between −0 and −180).

2. (a) Determine the latitude increment (Lat. Inc.) by the following formula:

   Lat. Inc.

   $$= \frac{(\text{Lon. B minus Lon. D}) \times (\text{U minus L})}{(\text{Lon. B minus Lon. D}) + (\text{Lon. C minus Lon. A})}$$

   (b) Add the latitude increment thus determined to the lower latitude (L) to get the latitude of the fix. (Convert the resultant decimal fraction to minutes and seconds).

3. (a) Determine the longitude increment (Lon. Inc.) by the following formula:

   Lon. Inc.

   $$= \frac{(\text{Lon. C minus Lon. D}) \times \text{Lat. Inc.}}{(\text{U minus L})}$$

   (b) Add the longitude increment thus determined to the longitude of D to get the longitude of the fix. (Convert the decimal fraction to minutes and seconds.)

4. For movement of the vessel between observations, multiply the cosine of the relative bearing of the body by the distance travelled and apply the correction in step 1(a) to the observed altitude of the first star on both upper and lower latitudes.

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