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A METHOD OF DETERMINATION OF ASTRONOMICAL LATITUDE AND LONGITUDE WHEN ONLY TIME AND HORIZONTAL ANGLES ARE OBSERVED

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J. C. Bhattacharjee, M.A.

1. Introduction

The usual methods of determination of astronomical latitudes and longitudes of geodetic standard are as follows:

(1) Both latitude and longitude: by observing stars of fixed altitude with the astrolabe.
The method is more suitable for longitude than for latitude' though the former is burdened with errors of personal equation.

(2) Longitude: by observing transits of meridian stars with the transit instrument fixed in the meridian of the place.
The results require to be computed by a lengthy process of approximations. The method is more suitable for observation in a fixed observatory than in the field due to the cumbersome size of the instrument.

(3) Latitude: by observing the differences in zenith distances of carefully selected meridian stars with the zenith telescope. This is known as "Talcott's Method".

By the method presented by the author in this paper, on the other hand, both latitude and longitude are obtained simply by determining the error in the difference of the computed azimuths of a pair of stars at the instants of observation of the horizontal angle between them. Provided the observed angle between the stars is correctly determined, the error in the difference of their computed azimuths can be due only to the error either in the chronometer time (L.S.T.) or in the assumed latitude of the place or in both. For stars near culmination a small error in the assumed latitude, or for stars near the prime vertical a small error in the chronometer time (L.S.T.), has practically no effect on the difference of their computed azimuths. Thus two simple relations are easily established, one between the error in the computed azimuths of the meridian stars and the error in the chronometer time (L.S.T.), and the other, between the error in the computed azimuths of the prime vertical stars and the error in the assumed latitude of the place. The accuracy of the method thus depends entirely on the accuracy with which the observed angle between the stars is determined.

The observed angle is improved upon in two ways, (i) by observing each pair of stars on opposite faces of the theodolite without operating the horizontal clamp, thereby avoiding any uncertainty in the observed angle due to the movement of the horizontal circle, (ii) by using theodolites of more improved pattern having proper means of correctly determining the instrumental constant and corrections, and (iii) obtaining as many intersections as possible of a star in a single setting. Thus the method promises great possibilities; for, with the advance of science, we can always hope for instruments of such high precision as to enable us to obtain the observed angle with an accuracy not thought of today.

While taking experimental observations it was found that none of the existing field theodolites, not even the Geodetic Wild, yielded satisfactory results. The main difficulty was that the theodolites were not provided with any means of determining the transit axis tilt errors which substantially affect the results. In the meantime the author became familiar with a new pattern theodolite known as the Geodetic Tavistock fitted with the shutter eye-piece devised by Dr. J. de Graaff-Hunter. The special feature of the instrument is that it is provided with a striding level to determine the transit axis tilt and dislevelment error, and a multi-graticule scale on which the positions of a star can be read at intervals of three complete seconds. The latter obviates completely the doubts of time recording so common with other methods. The instrument thus appeared to be an answer to the author's problem and on taking actual observations, it yielded results of the desired accuracy which it must be admitted would otherwise have never been possible.

It is also to be noted that though the observation of stars in the vertical plane is not one of the conditions required of the method, as is quite clear from above and also from relations (4) and (9), yet this limitation in the procedure of observation has been advantageously accepted as a matter of convenience and also to suit observations with the shutter which, as already stated, is responsible for the primary accuracy claimed to be obtained by the method in this paper.

2. Time Correction and Longitude

The longitude of a place as obtained from maps being known generally to a few seconds in time only, the chronometer time (L.S.T.) is liable to error by the same amount and the determination of this error plays an important part in astro-observations, particularly for longitude.

Selection of stars. Two fast moving stars of magnitude varying generally from 2·5 to 5, one north and the other south of the zenith, are selected for observation while near their upper culminations. The declination of the south star requires to be as small as conveniently possible. A carefully made programme showing the names, magnitudes, altitudes and times (L.S.T.) of passage for each pair of stars is always found useful.

Observation. The theodolite is placed approximately in the meridian of the place so that, when its vertical circle is set to the altitude reading of the star, the latter may come into the field of view of the instrument at the L.S.T. of transit of the star. After taking a few intersections of the star with the vertical wire or wires of the theodolite and their horizontal readings, the instants of intersection being noted by the chronometer, which can easily be set correct to a few seconds in time (L.S.T.), the horizontal circle of the instrument is clamped and the telescope turned only in the vertical direction to observe the other star on the opposite face exactly in the same manner as in the previous case. The interval of time between any two instants of intersections, one for a north star and the other for a south star, and the horizontal angle (corrected for collimation, transit axis tilt or dislevelment errors and diurnal aberration) between intersections at those instants, determine the error in the chronometer time (L.S.T.).

As a precaution against any likely error arising from the faulty determination of collimation of the theodolite, it is advisable for the unit of observation to comprise two pairs of north and south stars instead of one on opposite faces of the instrument.

The time of observation may be noted to a tenth of a second by a stop-watch in case the accuracy of time is required within 1/5th of a second in time or to 1/100th of a second by chronograph in case the accuracy of time is required to geodetic standard.

Longitude. For difference in longitude, it is necessary to determine the difference of local time at two stations of some event which can be observed at both. The event is usually provided by some wireless rhythmic signal giving the beats of the emitting local clock which is either noted by a stop-watch or registered on a chronograph, which also records the beats of the local clock at the receiving station. Thus after eliminating the error in the clock time, it is possible to obtain the longitude difference within 1/5th of a second in time by a stop-watch or within 0·05 of a second in time by chronograph after noting the coincidences in the usual manner.

Formula. Let α denote the azimuth of a star of declination δ at hour angle t.

Then with the usual notations

$$\tan\alpha = \frac{-\sin t}{\cos\phi\,.\,\tan\delta - \cos t\,.\,\sin\phi}. \tag{1}$$

For stars near culmination, α and t being small, and with α in seconds of arc and t in seconds of time, we have,

$$\alpha'' = 15\,.\,t\cos\delta\,.\,\mathrm{cosec}(\phi-\delta). \tag{2}$$

Differentiating formula (2) with respect to α and t, we have

$$\Delta\alpha'' = 15\,.\,\Delta t\,.\cos\delta\,.\,\mathrm{cosec}(\phi-\delta). \tag{3}$$

If the suffixes n and s refer to the north and the south star respectively, then we have, since

$$\Delta t_s = \Delta t_n = \Delta t,$$

$$\Delta\alpha_s'' - \Delta\alpha_n'' = 15\,.\,\Delta t\,.\,[\cos\delta_s\,.\,\mathrm{cosec}(\phi-\delta_s) - \cos\delta_n\,.\,\mathrm{cosec}(\phi-\delta_n)].$$

But $\qquad \Delta\alpha_s'' - \Delta\alpha_n'' = \Delta(\alpha_s'' - \alpha_n'') = \Delta H'' = (\alpha_s'' - \alpha_n'') - (H_s'' - H_n'')$

where $\qquad H_s =$ corrected horizontal reading to south star,

$\qquad H_n =$ corrected horizontal reading to north star,

and $\qquad \Delta H =$ error in the horizontal angle between the two stars due to error Δt in time.

Hence we have,

$$\Delta H'' = 15 . \Delta t[\cos \delta_s . \operatorname{cosec}(\phi-\delta_s) - \cos \delta_n . \operatorname{cosec}(\phi-\delta_n)]$$

or

$$\Delta t \text{ (in seconds of time)} = \frac{\Delta H''}{15} \Big/ [\cos \delta_s . \operatorname{cosec}(\phi-\delta_s) - \cos \delta_n . \operatorname{cosec}(\phi-\delta_n)] \quad (4)$$

Since $\Delta H''$ and Δt are both small, Δt can be easily evaluated using only three or four figures for cosines and cosecants in relation (4).

Accuracy. (i) Effect on α of an error of $\Delta\phi$ in ϕ: Differentiating formula (1) with respect to ϕ and α, we have

$$\Delta\alpha'' = -\sin \alpha . \cot(\phi-\delta) . \Delta\phi''. \quad (5)$$

Since α is small, it is clear from relation (5) that the effect of an error in ϕ on α_n or α_s is inappreciable. Thus α_n and α_s can be computed to the nearest 1/10th of a second if ϕ is known correct to about half a minute. Therefore Δt which is the error in the H.A.s of the stars in time, can also be determined from relation (4) correct to about 1/100th of a second in time. Hence with the help of the wireless rhythmic signals, longitude difference can also be determined to about 1/100th of a second in time.

(ii) Effect on α of an error of $\Delta\delta$ in δ: Differentiating the formula (1) with respect to α and δ, we have,

$$\Delta\alpha'' = \sin \alpha[\cot(\phi-\delta) - \tan \delta] . \Delta''\delta. \quad (6)$$

From relation (6) it is clear that the effect on α of an error in δ is also inappreciable.

Features of the Method.

(i) Altitude observation of stars is not necessary for actual computations, thereby avoiding the possible source of error from refraction corrections.

(ii) Intersection of stars with only the vertical wire or wires of a theodolite, makes the observational work easier and helps much in accuracy and quickness.

It may be noted here that the observation with the new Geodetic Tavistock fitted with shutter eyepiece as designed by Dr. J. de Graaff-Hunter, has simplified the method of observation still further as the personal equation in the intersection of stars and the recording of the instants of intersections is totally eliminated. Another useful feature in the instrument is that it is provided with a striding level which helps in determining more precisely the corrections for transit axis tilt and dislevelment errors to the observed horizontal readings.

(iii) The principle is too simple to need any elaborate explanation.

Both the north star and the south star move very fast and in opposite directions near their upper culminations so that there is considerable error in the difference of their azimuths due only to the error in the observed chronometer time (L.S.T.), which can easily be found out by comparing the difference of the computed azimuths against the corrected horizontal angle between the two stars. From this a very simple relation as given by formula (4), between the error in the observed time and the corresponding error in the difference of azimuths, is established.

(iv) The computational part is still easier. Only three or four figures are required for the entire calculations.

(v) The observational equipment is simple and convenient for quick movement in the field.

3. Latitude

Selection of stars. Two high altitude stars of magnitude varying generally from 2·5 to 5, one east and the other west, are selected for observation while near the prime vertical.

In this case a good programme similar to the one for time correction observation has got to be made in order to obtain a suitable number of stars for observation. Altitudes may have to be computed if no suitable star chart is available for the purpose.

Observation. The whole procedure of observation is exactly similar to that used for time correction except that the instrument in this case has got to be placed approximately in the prime vertical instead of in the meridian of the place. Finally, the interval of time between any two instants of intersections, one for an east star and the other for a west star, and the horizontal angle (corrected for collimation, transit axis tilt or dislevelment errors and diurnal aberration) between intersections at those instants, determine the error in the assumed value of ϕ.

Here also as a precaution against any likely error arising from the faulty determination of collimation of the theodolite, it is advisable to have the unit of observation comprising of two pairs of east and west stars, instead of one, on opposite faces of the instrument.

The time of observation is to be noted exactly in the same manner as in the case of time correction.

Formula. From formula (1), we have where $\alpha = 90°$,

$$\cos t = \tan\delta \,.\, \cot\phi. \tag{7}$$

This gives the hour angle t for which a star of declination δ is in the prime vertical.

Differentiating formula (1) with respect to α and ϕ, we have for $\alpha \to 90°$,

$$\Delta\alpha'' = -\cot t \,.\, \sec\phi \,.\, \Delta\phi''. \tag{8}$$

If the suffixes E and W refer to the east star and the west star respectively, then we have,

$$\Delta\alpha_W'' - \Delta\alpha_E'' = -\Delta\phi''.\sec\phi.(\cot t_W - \cot t_E).$$

But

$$\Delta\alpha_W'' - \Delta\alpha_E'' = \Delta(\alpha_W'' - \alpha_E'') = \Delta H'' = (\alpha_W'' - \alpha_E'') - (H_W'' - H_E''),$$

where H_W = corrected horizontal reading to west star,

H_E = corrected horizontal reading to east star,

and ΔH = error in the horizontal angle between the two stars due to error $\Delta\phi$ in ϕ.

Hence we have,

$$\Delta H'' = -\Delta\phi''.\sec\phi.(\cot t_W - \cot t_E)$$

or

$$\Delta\phi'' = -\Delta H''/[\cot t_W - \cot t_E].\sec\phi. \tag{9}$$

Accuracy. From relation (9) it is clear that the effect on ϕ of an error in ΔH is considerably reduced by taking two prime vertical stars, one east and the other west and of small hour angles so that the only variable factor $1/[\cot t_W - \cot t_E]$ becomes small enough to reduce to a great extent the effect on ϕ of the error in the measurement of ΔH.

At Dehra Dun for $t = \pm 20°$, $\Delta\phi = \frac{1}{6}.\Delta H$, *i.e.*, for an error of about 6″ in the measurement of ΔH there is an error of about one second in the determination of latitude of the place.

Differentiating formula (1) with respect to α and t, we have for $\alpha \to 90°$, with $\Delta\alpha$ in seconds of arc and Δt in seconds of time,

$$\Delta\alpha'' = 15.\sin\phi.\Delta t. \tag{10}$$

The relation (10) is a very important result. Apart from giving us the deviation of a star in azimuth from the prime vertical for a change Δt in t, this relation shows that the rate of change of azimuth of any star near the prime vertical is constant and therefore the effect of a constant error Δt in t on the difference of azimuth of the two east and west stars, is nil. Moreover, this relation opens up another avenue for research for determining the latitude of a place, knowing only the change of azimuth during a known but sufficiently long interval of time.

(ii) Effect on ϕ of an error of $\Delta\delta$ in δ: Differentiating formula (1) with respect to ϕ and δ, we have for $\alpha \to 90°$

$$\Delta\phi'' = \frac{\sin 2\phi}{\sin 2\delta}.\Delta''\delta. \tag{11}$$

Since the altitudes of the stars are high, the factor $\sin 2\phi/\sin 2\delta$ is never much greater than unity and therefore the effect of any possible error of δ on ϕ, is always less than 0″·05.

Features of the Method. (i) and (ii) are the same as for time correction.

(iii) The principle is very simple and follows the same lines as in the case of time correction.

The stars for observation for latitude have been so selected that their hour angles are small and opposite in sign—a condition which, as is evident from formula (9), considerably reduces the effect on ϕ of the error in the difference of their measured azimuths; for the difference of the measured azimuths of the two east and west stars bears a constant ratio to the error of the assumed latitude of the place of observation.

(iv) The computational part is quite easy. Only three or four figures are required for use in the calculation except for evaluating $\cos t$, where seven figures are necessary.

(v) Same as for time correction.

4. Experimental Observation

Geodetic Tavistock and Shutter Eye-piece. In 1950, some experimental observations were made with a Geodetic Tavistock, time being recorded by an ordinary stop-watch. The final results agreed quite well. The probable error in longitude was $\pm 0{\cdot}1$ seconds in time and in latitude $\pm 2''{\cdot}0$. The accuracy though not of geodetic standard, can, however, be improved a lot with sufficient practice and by recording time with a chronograph instead of with an ordinary stop-watch. But all the same the personal equation of individual observers is bound to come in in respect of the estimation of time and of intersection by the vertical wire in any case, which again is a disadvantage.

In view of that, recourse had to be taken to the Geodetic Tavistock fitted with shutter eye-piece. The shutter eye-piece device was evolved by J. de Graaff-Hunter, C.I.E., Sc.D., F.R.S. The full description of it has been given in the *Proceedings of the Royal Society*, Vol. 166, No. 925, 19th May, 1938, pp. 197-213, and in *E.S.R.*, ix, 63, 20-24, January 1947.

Method of observation with shutter eye-piece. (i) For time correction observation the theodolite requires to be placed approximately in the meridian of the place. The horizontal circle is then clamped in this position with the vertical circle set roughly to the altitude of a star, say north star, and the scale readings of a series of the star appearances are taken conveniently at regular intervals of three seconds. Once the star is picked up, the scale readings of its successive appearances at progressively different positions of the scale can be easily read and recorded, the instant of the last star appearance being noted by the observer by means of a stop-watch. From these a mean star position corresponding to the mean time of the star appearances is derived. This is expressed in scale units and, reduced to seconds of arc by means of the scale factor (determined beforehand) multiplied by the secant of altitude of the star, and combined with collimation, aberration, transit axis tilt and dislevelment corrections, gives the angle of departure of the mean star position corresponding to the mean time of the star appearances from the zero line of collimation of the theodolite. Without touching the horizontal setting, the face of the theodolite is now changed by transitting the telescope, the vertical circle being set roughly to the altitude of the south star. Then, after taking the scale readings of a series of star appearances, the angle of departure of the mean star position $+180°$ corresponding to the mean time of the star appearances from the zero-line of collimation of the theodolite, is derived exactly as in the previous case. Similarly, another pair of a north star and a south star is observed on opposite faces of the theodolite.

Comparing the difference of these angles of departure (with proper sign) from the zero-line of collimation against the difference of the computed azimuths of the two mean star positions, one for the north star and the other for the south star, the time correction is obtained with geodetic accuracy.

(ii) For latitude observation, the whole procedure is exactly similar to that used for the north and the south stars for time correction observation except that the theodolite in this case has got to be placed approximately in the prime vertical. Finally, comparing the difference of the angles of departure (with proper sign) from the zero-line of collimation against the difference of the computed azimuths of the two mean star positions, one for the east star and the other for the west star, the error in the assumed value of ϕ is obtained with geodetic accuracy.

Observational results. Observation with the Geodetic Tavistock theodolite fitted with shutter eye-piece had been carried out on 16th June and 17th July, 1951. Unfortunately the wireless signal could not be taken on 16th June, and therefore the longitude could not be determined on that day though the clock errors as obtained were in good agreement among themselves. On 17th July both longitude and latitude were determined and the results compared well with the accepted values, the longitude differing by $0''\cdot 3$* (after applying emission correction) and the latitude by only $0''\cdot 1$. Moreover, the value of latitude as obtained on 16th June differed from the value as determined on 17th July by only $0''\cdot 1$.

Examples of the computations of the observations of 17th July, 1951, for time correction and longitude are included here on their appropriate forms. Computations for latitude follow a somewhat similar pattern, Forms 1(a), Parts 1 and 2, being used for both computations, and Form 2 modified to agree with Formula (9).

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* The discrepancy was $0''\cdot 6$ before applying emission correction (Demi-Definitive) from *Bulletin Horaire*.

Form (1a)

ANGLE BOOK

PART ONE

(For Shutter Observation only)

Place: *Hunter Observatory.* Instrument: *Geodetic Tavistock No. 530.* Date: *17.7.1951.*

S.T. Chrono. No. *13560.*

Instrumental constants: Value of 1 div. of bubble $d = 3''$.

Value of 1 div. of scale $F = 25''\cdot 2$.

Horiz. colln. $C = +2''\cdot 8$ (+ when FL > FR).

Observation for Time Correction or Latitude	Time Correction	Time Correction
Face of instrument	FR	FL
Stars, E./W. or N./S....	ζ Drac. N.	σ Oph. S.
Latitude $= \phi$	30° 18′ 45″	30° 18′ 45″
Altitude $= h$ (approximate)	54 32 15	63 52 00
$\cos\phi$	0·863	0·863
$\sec h$	1·724	2·270
$\tan h$	1·404	2·038
Striding level reading before scale reading (1st position) ...	L 8·0 R —	L 1·0 R 4·2
Striding level reading before scale reading (2nd position after reversing)	4·5 0·7	3·1 2·0
Striding level readings after scale reading (1st position) ...	4·4 0·9	3·0 2·1
Striding level reading after scale readings (2nd position after reversing)	— —	— —
$\frac{L-R}{2n}$	3·6	−0·55
Correction for transit axis tilt and dislevelment: $\left(\frac{L-R}{2n}\right) d \cdot \tan h$	15″·2	−3″·4
Correction for collr.: $C \cdot \sec h$ (for FR, opp. sign) ...	4 ·8	−6 ·4
Correction for aberration: $0{\cdot}319 \cdot \cos\phi \cdot \cos a \cdot \sec h$ ...	−0 ·5	−0 ·6
Value of mean scale reading in arc: H' (1)	−02 ·2	180°+52″·1
H''	17 ·3	180 +41 ·7
Chrono. time corresponding to 1st star appearance ...	h m s 17 09 09·0	h m s 17 24 36·0
Chrono. time corresponding to last star appearance ...	17 09 30·0	17 25 19·0
Chrono. time corresponding to mean value of star positions on scale	17 09 19·50	17 24 59·0

(1) $H' = S.F. \sec h$. S from Part Two.
Change the sign of H' for north stars, and add 180°, 90° or 270° to H' according as the star is south, east or west respectively of zenith.

Recorded by *P.P.C.* Date *17.7.51.* Computed by *J.C.B.* Date *21.7.51.* Checked by *J.B.M.* Date *29.7.51.* Observed by *J.C.B.*

Form 1(a)

ANGLE BOOK

PART TWO

(For Shutter Observation only)

Place : *Hunter Observatory.* Instrument : *Geodetic Tavistock No. 530.* Date : *17.7.1951.*
S.T. Chrono. No. 13560.

Star : ζ Drac. N.						Star : σ Oph. S.					
Observed scale readings*		Scale readings corrected for Groups		Correction for break of chrono. if any	Mean of each pair of readings after applying corrections from Col. 5	Observed scale readings*		Scale readings corrected for Groups		Correction for break of chrono. if any	Mean of each pair of readings after applying corrections from Col. 5
1st half before crossing central wire Value : −ve	2nd half after crossing central wire Value : +ve	1st half arranged in reverse order	2nd half			1st half before crossing central wire Value : −ve	2nd half after crossing central wire Value : +ve	1st half arranged in reverse order	2nd half		
1	2	3	4	5	6	1	2	3	4	5	6
2·5	0·6	−0·4	+0·6		+0·10	2·0	...	...	...		...
3·1	1·0	−1·0	+1·0		+0·00	...	3·9	− 2·1	+ 3·9		+0·90
4·0	1·9	−1·9	+1·9		+0·00	1·0	0·8	− 4·0	+ 5·8		+0·90
4·6	2·7	−2·5	+2·7		+0·10	2·7	2·2	− 6·0	+ 7·2		+0·60†
						4·0	4·0	− 7·3	+ 9·0		+0·85
						1·0	1·0	− 9·0	+11·0		+1·00
						2·9	2·8	(−11·0)	+12·8		+0·90
						4·7		−13·0			
Mean value of star positions on scale = mean of Col. (6) = S					+0·05	Mean value of star position on scale = mean of Col. (6) = S					+0·910

* Scale reading at central wire is *zero.* † Not taken into account when working out mean.

Observed by : *J.C.B.*

Recorded by : *P.P.C.* Computed by : *J.C.B.* Checked by : *J.B.M.*

Form 1(b).

WIRELESS RECEPTION AND G.S.T. ERROR

Place: *Hunter Observatory.* *S.T. Chrono. No. 13560.* Date: *17.7.1951.*

1. Emitting Station	Rugby	Rugby
2. Date	17.7.51	17.7.51
3. Beginning of signal	h m s 10 51 41·04	h m s 18 53 00·31
4. First coincidence	—	18 53 27·0
5. Second coincidence	—	54 39·0
6. Third coincidence	—	55 51·0
7. Fourth coincidence	10 55 30·0	18 57 03·0
8. Fifth coincidence	10 56 41·0	—
9. Mean coincidence reduced in terms of first coincidence	10 51 54·0	18 53 27·0
10. Coincidence interval from table*...	12·822	26·630
11. Beginning of signal reduced from mean coincidence ...	10 51 41·178	18 53 00·370
12. Corrections for wireless position	00·003	00.003
13. Corrections for assumed chrono. error	47·000	47·000
14. Correction for propagation of signal	00·022	00·022
15. Observed L.S.T. of reception of signal	10 50 54·153	18 52 13·345
16. G.M.T. of emission...	10 01 00·00	18 01 00·000
17. S.T. at G.M.M.	19 36 02·609	19 36 02·609
18. Acceleration on G.M.T. interval from G.M.M.	01 38·729	02 57·581
19. G.S.T. of reception	5 38 41·338	13 40 00·190
20. Line 15–Line 19 = G.S.T. error of chrono.	5 12 12·815	5 12 13·155
21. Rate of G.S.T. error of chrono. per hour	s 0·0425	—

* Wireless Telegraph reception table.

Time signal taken by *P.P.C.* Computed by *J.C.B.*

Checked by *J.B.M.*

Form 2. COMPUTATION OF TIME CORRECTION AND LONGITUDE

Observer : *J.C.B.* Instrument : *Geodetic Tavistock No. 530.* Date : *17.7.1951.*
S.T. Chrono. No. 13560.

Formula : (i) $\alpha'' = 15 \,.\, t \,.\, \cos\delta \operatorname{cosec}(\phi-\delta)$, t in seconds in time.

(ii) $\Delta t = \dfrac{\Delta H''}{15} \Big/ [\cos\delta_s \,.\, \operatorname{cosec}(\phi-\delta_s) - \cos\delta_n \,.\, \operatorname{cosec}(\phi-\delta_n)]$, Δt in seconds in time.

1. Station of observation	Hunter Obsy.	Hunter Obsy.	Hunter Obsy.
2. Face of instrument	FR	FL	FL
3. Stars, N. or S.	ζ Drac. N.	σ Oph. S.	α Oph. S.
4. Observed chrono. time [from Form 1(a)]*	17 09 19·50	h m s 17 24 59·00	h m s 17 33 31·50
5. Relay lag, if any†	+ 0·14	+ 0·14	+ 0·14
6. Assumed error of chrono. time ...	+47·00	+47·00	+47·00
7. Line 4+Line 5−Line 6 = L.S.T. of observation	17 08 32·64	17 24 12·14	17 32 44·64
8. R.A. of star	17 08 41·39	17 24 07·99	17 32 42·54
9. Line 7−Line 8 = hour angle t ...	− 8·75	+ 4·15	+ 2·10
10. Latitude ϕ (1)	30° 18′ 45″	30° 18′ 45″	30° 18′ 45″
11. Declination δ (1)	65 46 30	4 10 45	12 35 30
12. 90°−(ϕ−δ) (1)	125 27 45	63 52 00	72 16 45
13. cos δ (2)	0·4103	0·9973	0·9759
14. cosec (ϕ−δ) (2)	−1·724	2·270	3·285
15. α'' [from Formula (2)] (3)	+92″·8	180°+140″·9	180°+101″·0
16. H'' [from Form (1a)]	+ 17 ·3	180 + 41 ·7	180 − 06 ·5
17. $\alpha_s''-\alpha_n''$ (4)		180 + 48 ·1	180 + 08 ·2
18. $H_s''-H_n''$ (4)		180 + 24 ·4	180 − 23 ·8
19. Line 17−Line 18 = $\Delta H''$		+ 23 ·7	+ 32 ·0
20. cos δ . cosec (ϕ−δ) . 15 (2)	−10·61	+33·96	+48·09
21. [cos δ_s . cosec ($\phi-\delta_s$)−cos δ_n . cosec ($\phi-\delta_n$)] 15		+44·57	+58·70
22. Δt [from Formula (4)] (5)		−0·532	−0·543
23. Correction for clock rate at signal time		−0·063	−0·056
24. Correction for theodolite position ...		+0·018	+0·018
25. Observed L.S.T. of reception of time signal (6)		h m s 18 52 13·345	h m s 18 52 13·345
26. Correct L.S.T. of reception of time signal		18 52 12·768	18 52 12·764
27. G.S.T. of emission of time signal ...		13 40 00·190	13 40 00·190
28. Line 26−Line 27 = Longitude in time		5 12 12·578	5 12 12·574
29. Mean Longitude in time		5 12 11·73 §	

(1) Correct to a quarter minute. (2) Correct to four figures. (3) Add 180° to α'' if the star is south of zenith. (4) Suffixes s and n refer to south star and north star respectively. (5) With sign changed. (6) Corrected for assumed chrono. error.

* Corresponding to mean star position.

† Relay lag arising only due to time signal being recorded by chronograph method.

Computed by : *J.C.B.* Checked by : *J.B.M.*

§ This is mean of FL and FR observations. Mean of FR observations from page 2 of Form 2 (not reproduced here) was $5^h\ 12^m\ 10^s{\cdot}876$.