Binocular Stereopsis

How multiple views enable one to reconstruct depth in the world

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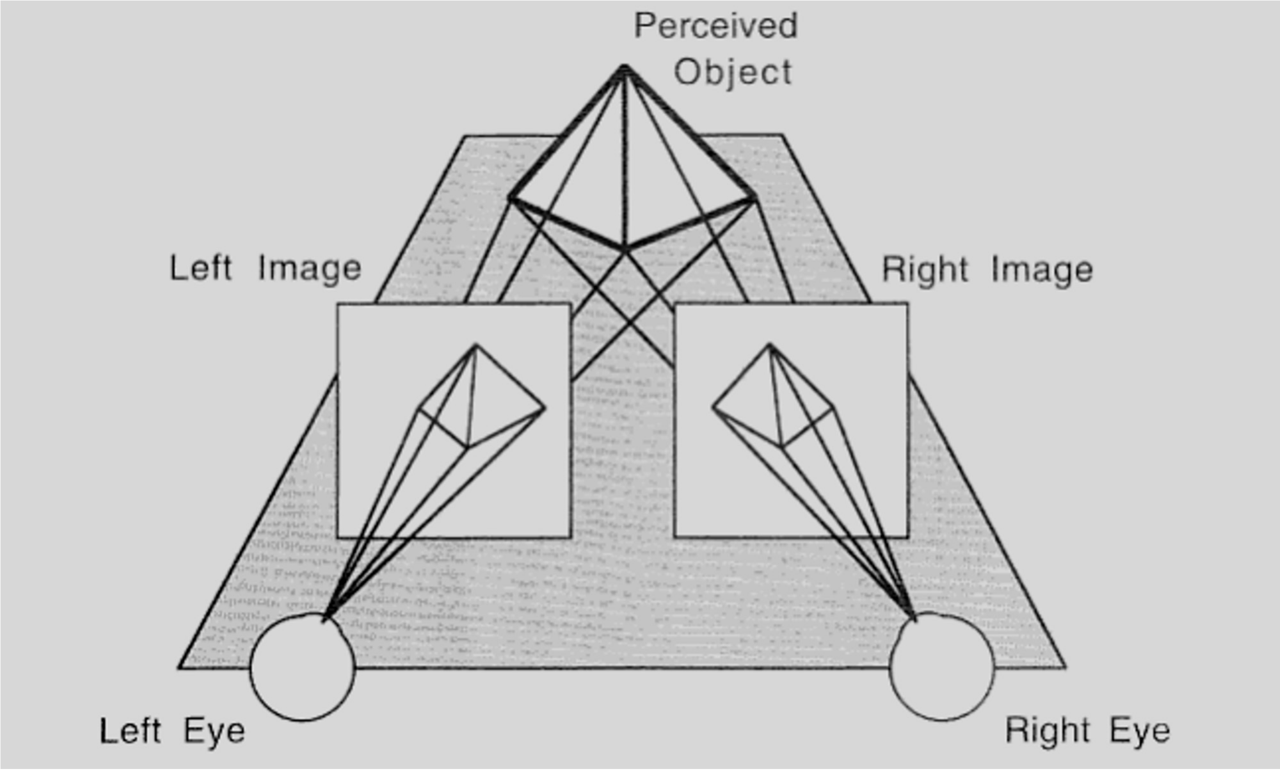
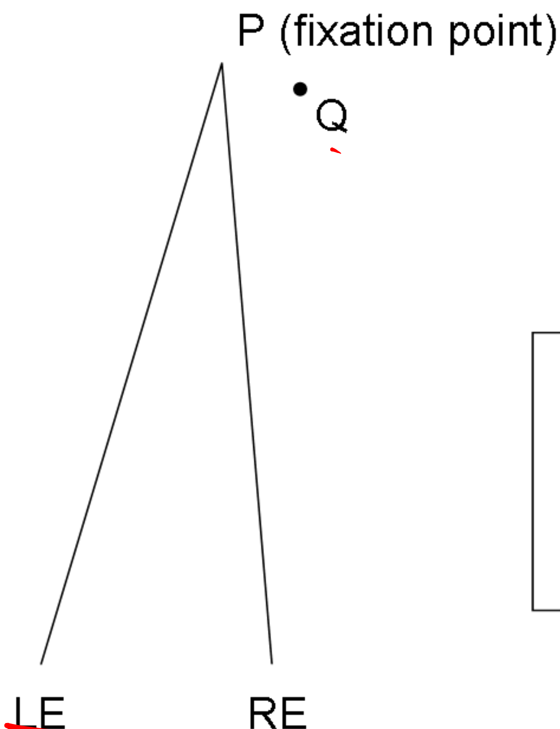
Binocular Stereopsis

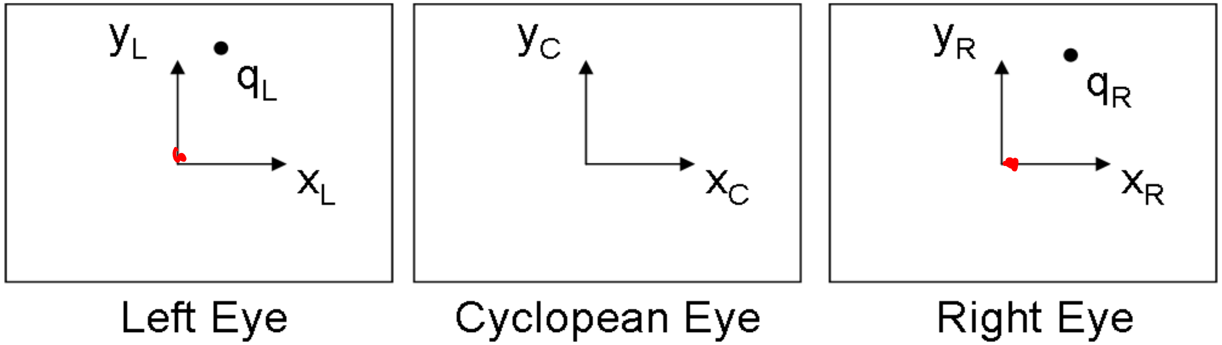


Various camera configurations

- Single point of fixation where optical axes intersect
- Optical axes parallel (fixation at infinity)
- General case

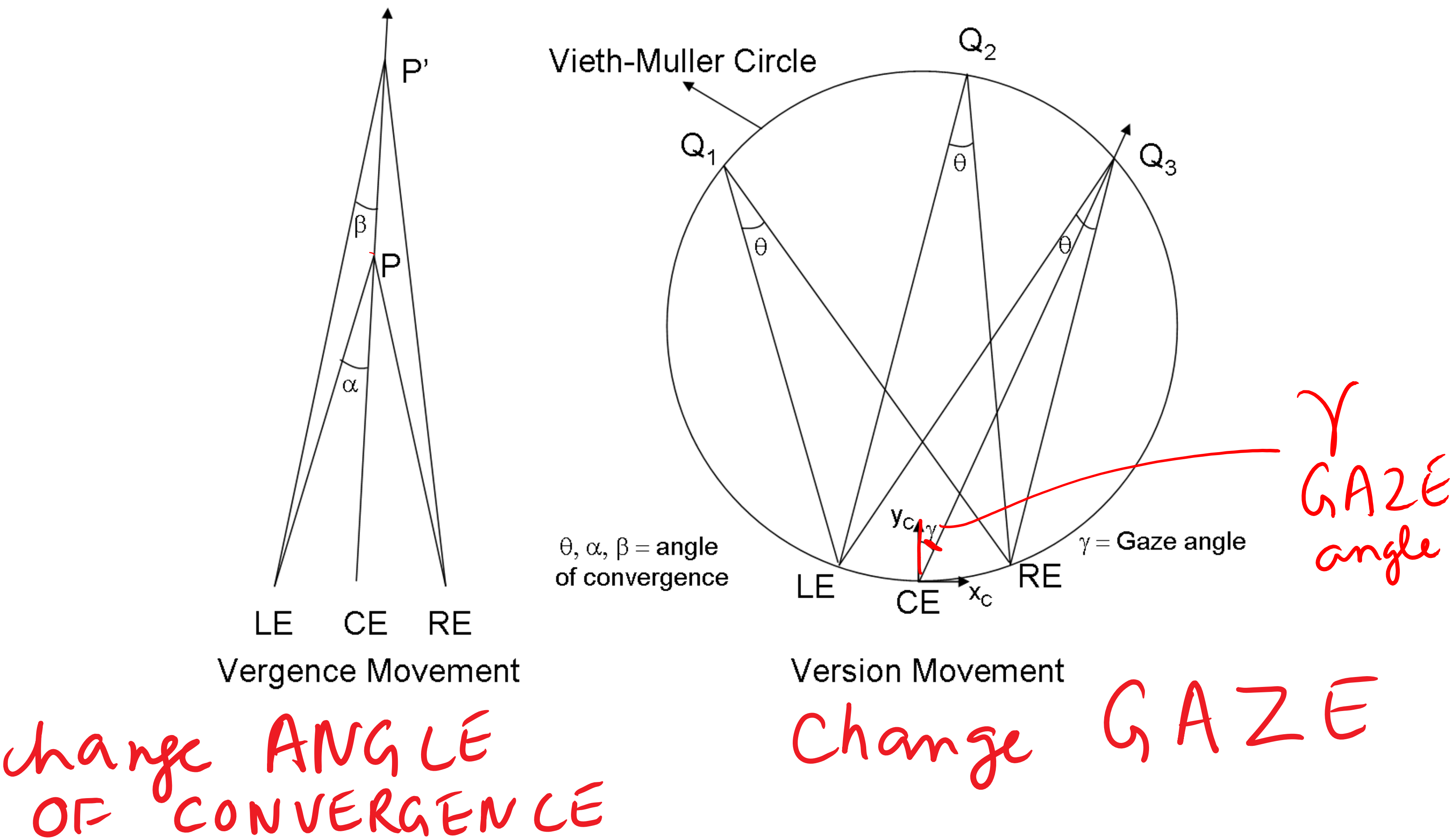
Disparity for a fixating binocular system





The Cyclopean eye is an imaginary eye at the midpoint of LE & RE

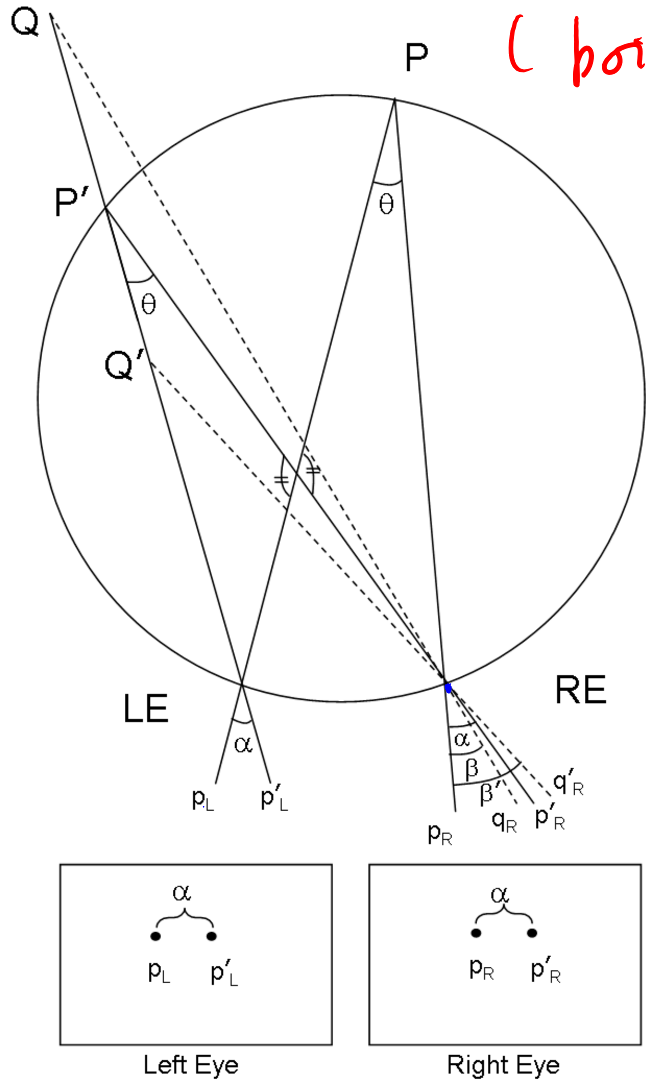
The two basic binocular eye movements



Disparity is zero on Vieth-Muller circle

(defined by P, LE, RE)

(point of fixation)



P has zero disparity

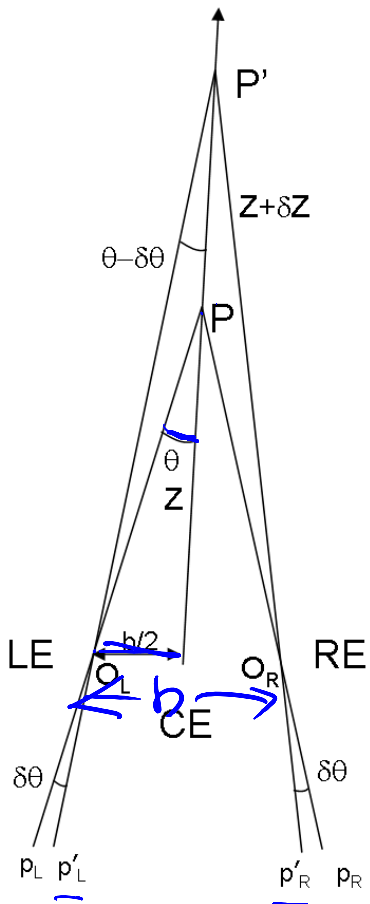
P′ ??

$2\delta\theta$ $\qquad$ δZ

Relation between disparity and depth

$b =$ baseline

$p'_R - p'_L$ (disparity)



$\tan\theta = \frac{b/2}{Z}$ $\qquad$ $\tan(\theta - \delta\theta) = \frac{b/2}{Z+\delta Z}$

For small angles $\tan\theta \approx \theta$

$$\theta = \frac{b/2}{Z}$$

$$(\theta - \delta\theta) = \frac{b/2}{Z + \delta Z}$$

$$\delta\theta = \frac{b/2}{Z} - \frac{b/2}{Z + \delta Z}$$

$$\approx \frac{b\delta Z}{2Z^2}$$

DISPARITY $= \frac{b\delta Z}{Z^2}$

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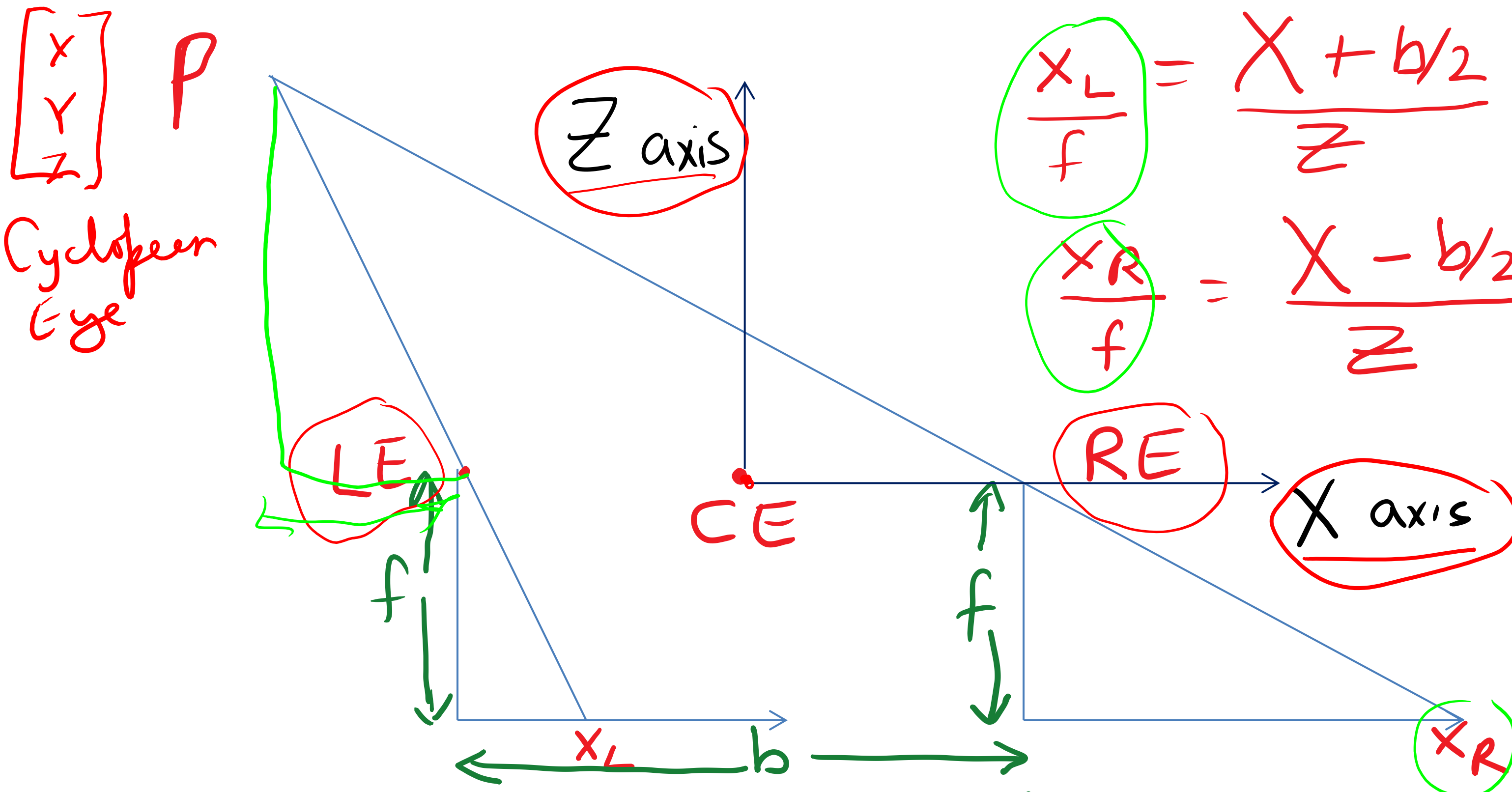
Parallel Optical Axes
(fixation at infinity)

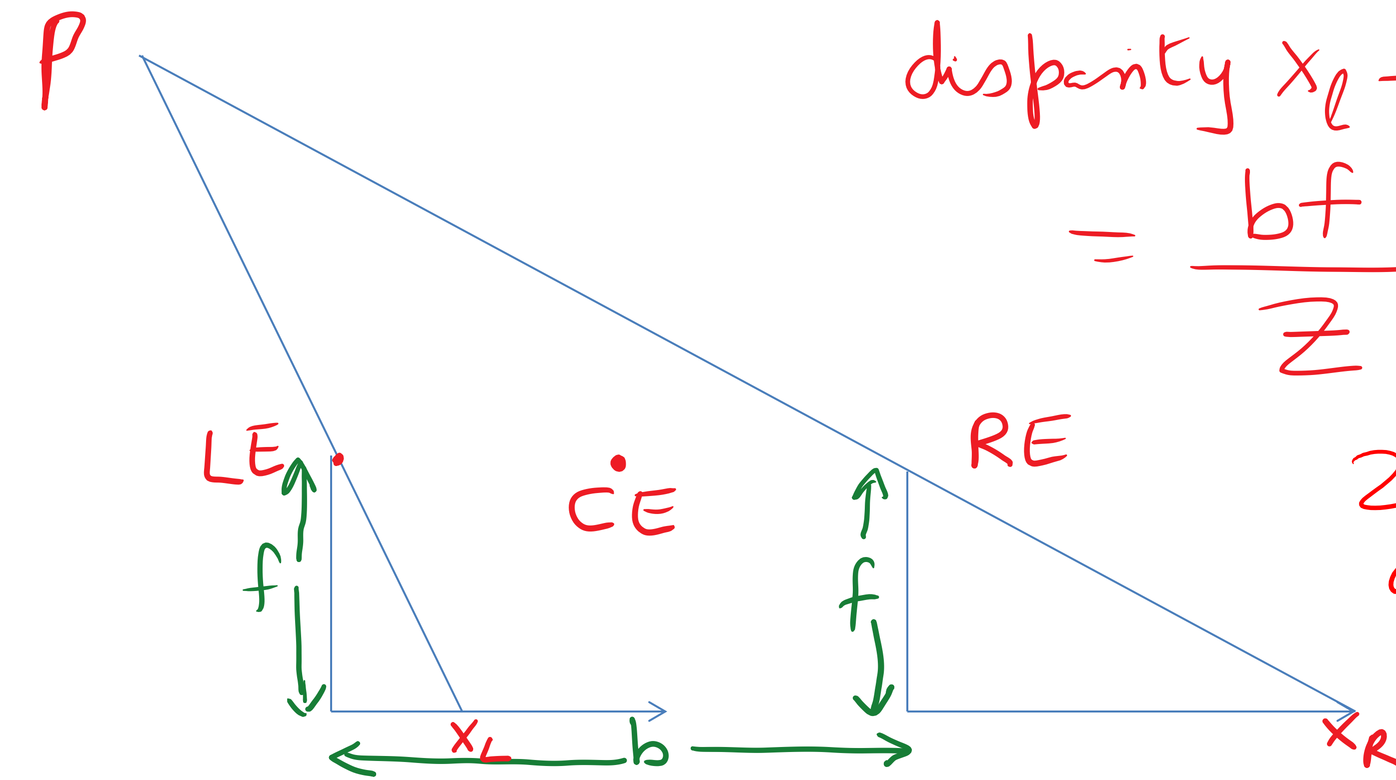


$$\frac{x_L}{f} = \frac{X + b/2}{Z}$$

$$\frac{x_R}{f} = \frac{X - b/2}{Z}$$

b baseline f focal length

Parallel Optical Axes
(fixation at infinity)



disparity $x_l - x_r$

$$= \frac{bf}{Z}$$

$Z \to \infty$

disparity $\to 0$

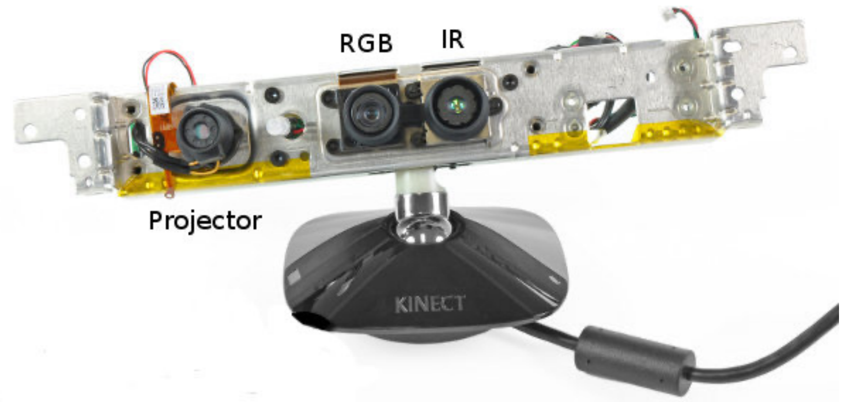
b baseline

f focal length

P in world

$P_L \leftrightarrow P_R$ CORRESPONDENCE

Range Sensors

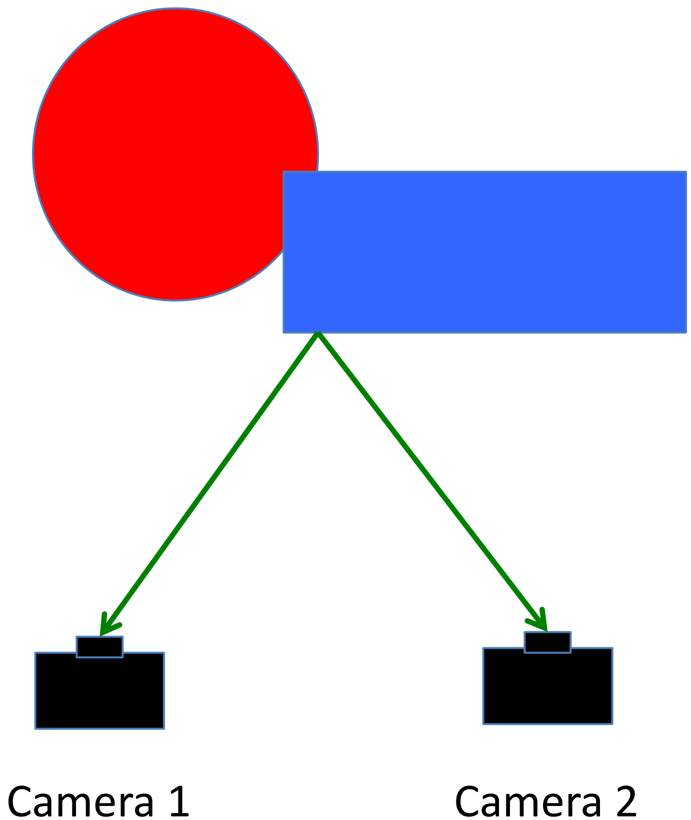


primesense sensor (used in Kinect)

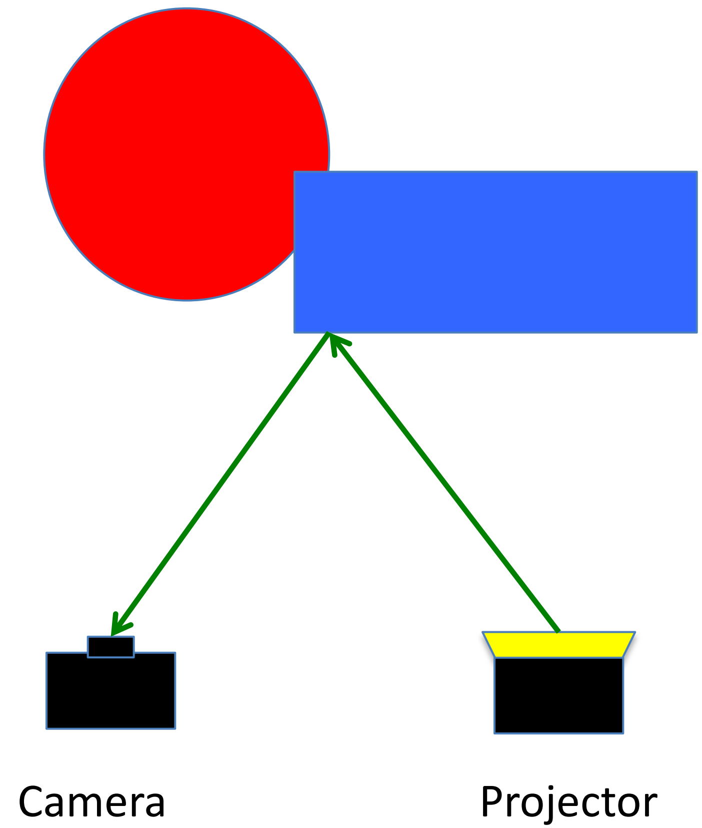
Velodyne LIDAR Sensor

http://www.primesense.com/, http://www.ifixit.com/,
http://mirror.umd.edu/roswiki/kinect_calibration(2f)technical.html
http://velodynelidar.com/lidar/lidar.aspx

Depth from Triangulation



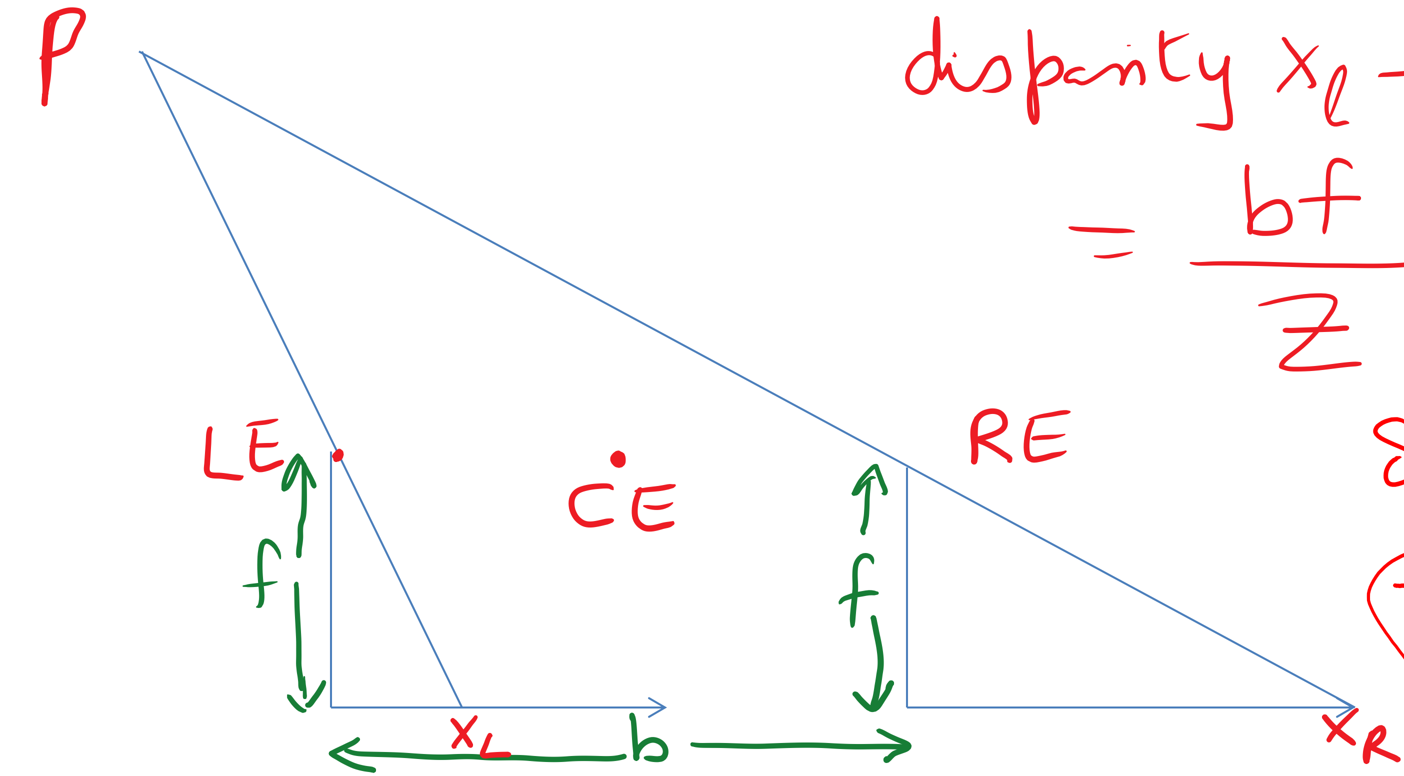
Passive Stereopsis



Active Stereopsis

Active sensing simplifies the problem of estimating point correspondences

Recall the formula for disparity with parallel optical axes…



disparity $x_l - x_r$

$$= \frac{bf}{Z}$$

$$\delta = \frac{c_1}{Z}$$

$$Z = \frac{c_2}{\delta}$$

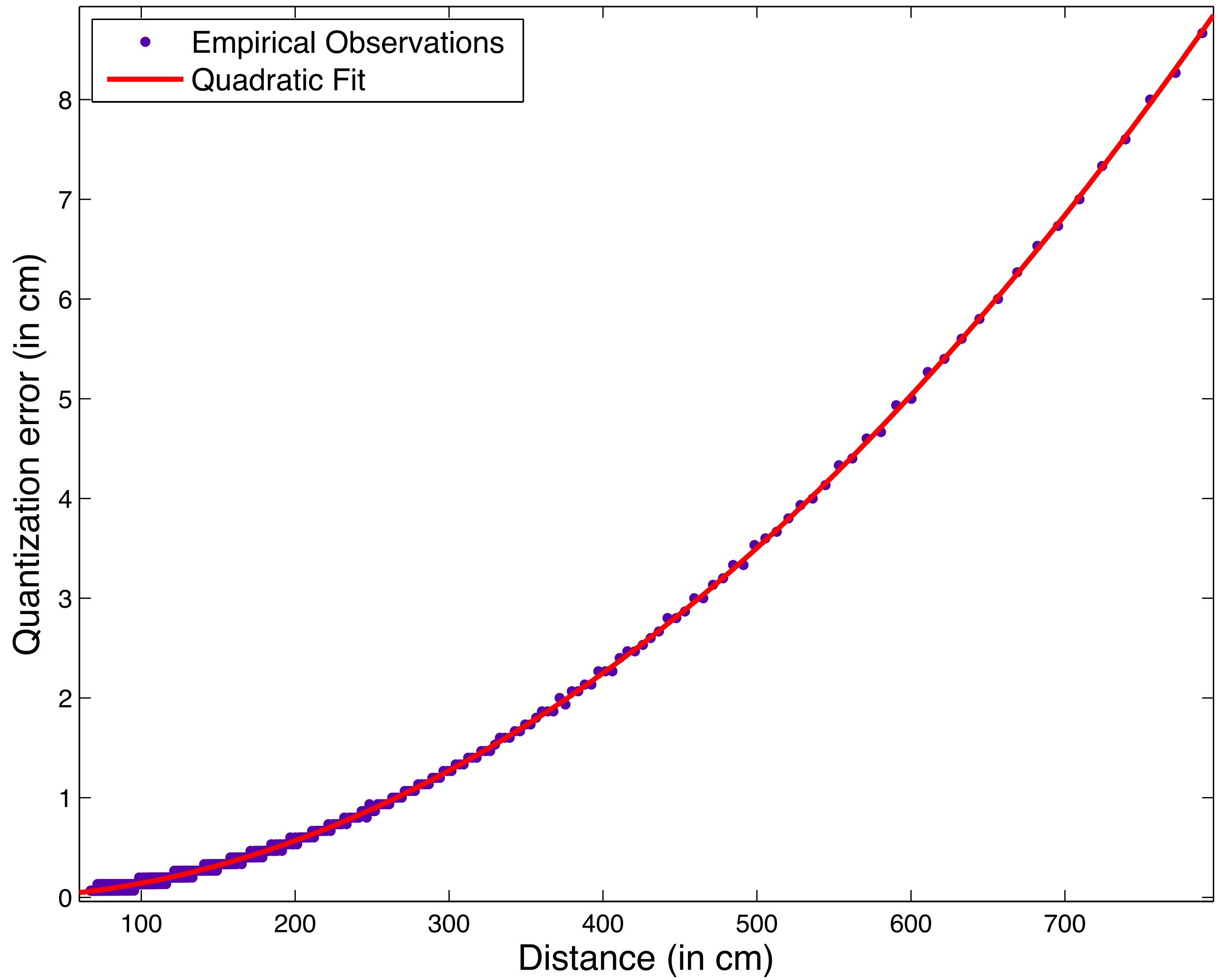
b baseline

f focal length

error(distance) – Kinect type sensor

Error in distance estimate increases quadratically with the distance

$$\begin{aligned} Z &= \text{distance} \\ d &= \text{disparity} \\ Z &= \frac{C}{d} \\ \delta Z &= \frac{-Z^2}{C}\delta d \\ |\delta Z| &= \frac{Z^2}{C}|\delta d| \\ \text{error} &\propto \text{distance}^2 \end{aligned}$$



Bessel chose the star 61 Cygni as a likely star to be near the Sun, and therefore to have appreciable parallax. 61 Cygni is not nearly so bright as α Lyræ, but has a very great angular movement or proper-motion among the stars. Bessel used an instrument called a heliometer. Like Struve's telescope, it was mounted so that it could be driven by clock-work to point always at the same star. The object-glass of Bessel's telescope was made by the great optician Fraunhofer, with the intention of cutting it in halves. Fraunhofer died before the time came to carry out this delicate operation, but it was successfully accomplished after his death.

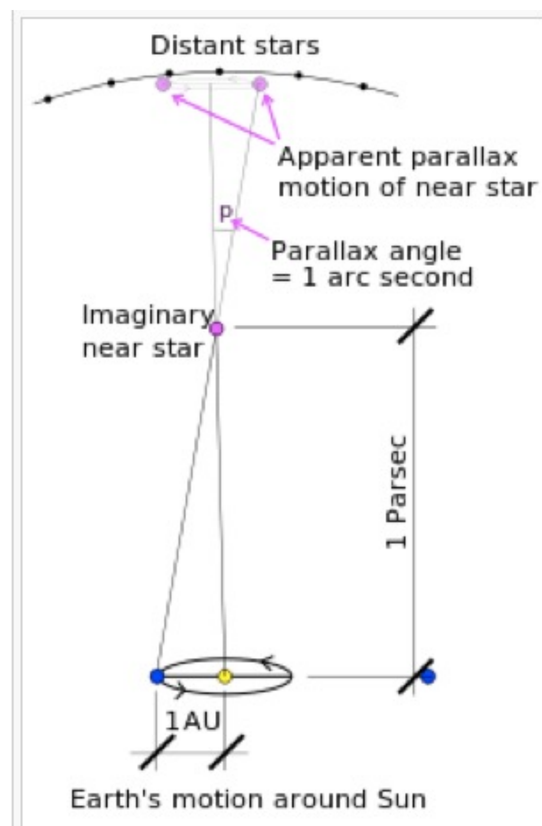
Delicate mechanism was provided for turning the glass, and also for moving the two halves relatively to each other; the amount of movement being very accurately measured by screws. Each half gives a perfect image of any object which is examined, but the two images are shifted by an amount equal to the distance one half of the lens is moved along the other. Thus when a bright star and faint star are looked at, one half of the object-glass can be made to give images S and *s*, and the other half S′ and *s*′. By moving the screw exactly the right amount *s*′ can be made to coincide with S, and the reading of the screw gives a measure of the angular distance between the two stars. Bessel made observations on 98 nights extending from August 1837 to September 1838. The following table, taken from a report by Main (*Mem. R. A. S.* vol. xii. p. 29), shows how closely the mean of the observations for each month accords with the supposition that the star has the parallax 0″·369:—

Mean date.	Observed Displacement.	Effect of parallax 0″·369.	Mean date.	Observed Displacement.	Effect of parallax 0″·369.
1837. Aug. 23	+0″·197	+0″·212	1838. Feb. 5	−0″·223	−0″·266
Sept. 14	+0·100	+0·100	May 14	+0·245	+0·238
Oct. 12	+0·040	−0·057	June 19	+0·360	+0·332
Nov. 22	−0·214	−0·258	July 13	+0·216	+0·332
Dec. 21	−0·322	−0·317	Aug. 19	+0·151	+0·227
1838. Jan. 14	−0·376	−0·318	Sept. 19	+0·040	+0·073

The great and difficult problem which had occupied astronomers for many generations was thus solved for three separate stars in 1838:—

	Parallax.	Distance.	Modern observations.	
			Parallax.	Distance.
α Centauri (Henderson)......	1″·0	200,000	0″·750	270,000
61 Cygni (Bessel)	0·314	640,000	·285	700,000
α Lyræ (Struve)	0·262	760,000	·10	2,000,000

(The unit of distance is that from the Earth to the Sun.)



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images from internet

Recover
R, t
Z

C_1 • • C_2

R, t