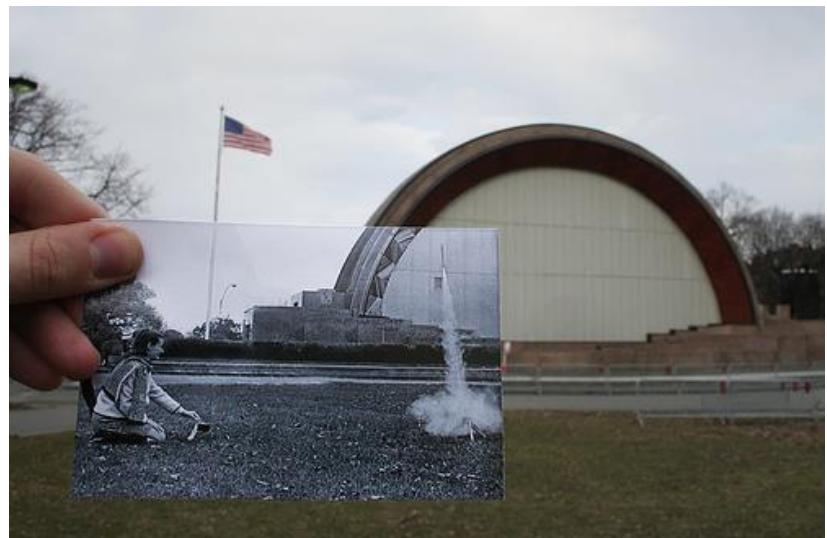
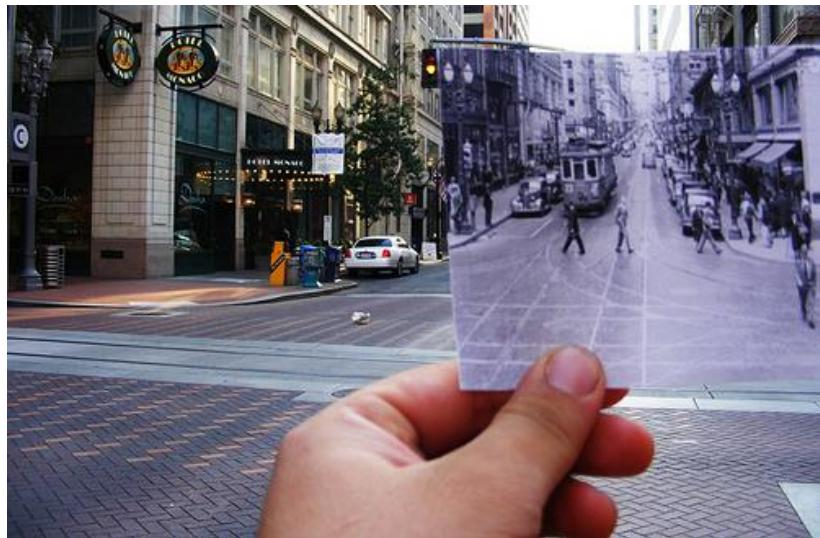
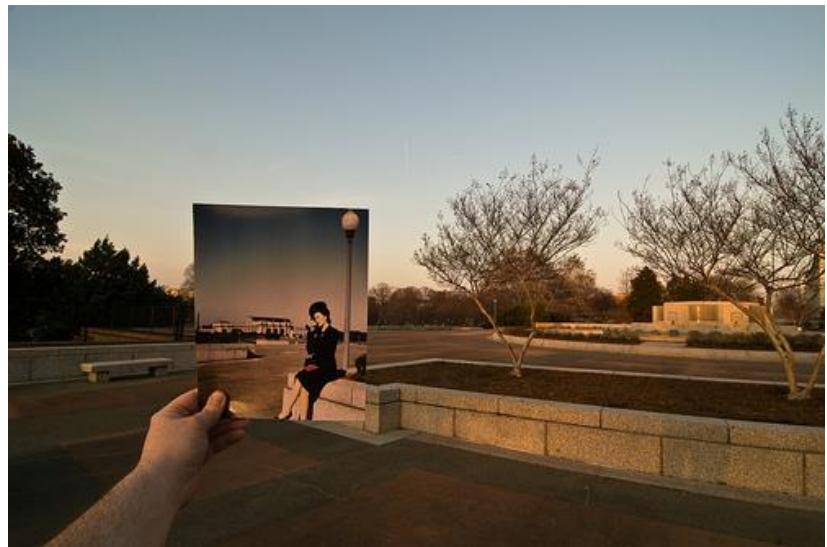
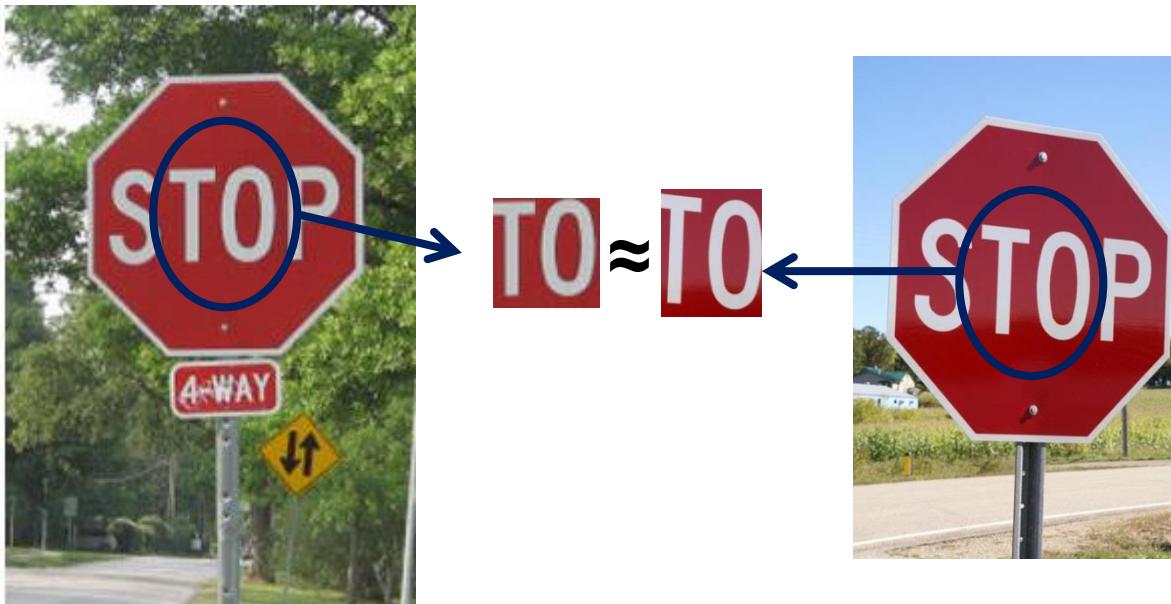


Correspondence and Interest Points

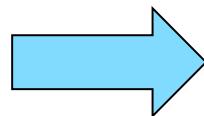


Correspondence across views

Correspondence: matching points, patches, edges, or regions across images

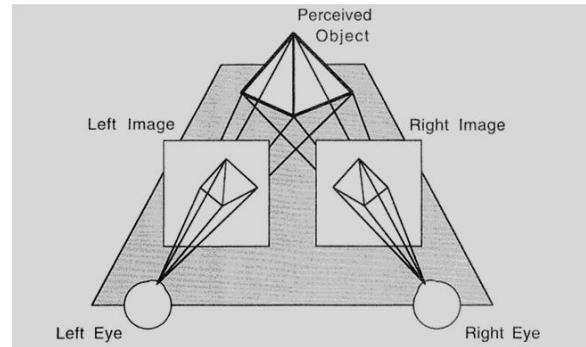


Brightness Constancy may not apply!

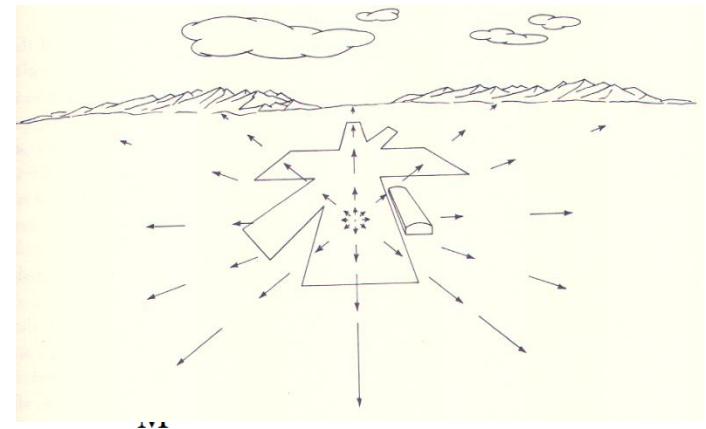


Everything depends on correspondence

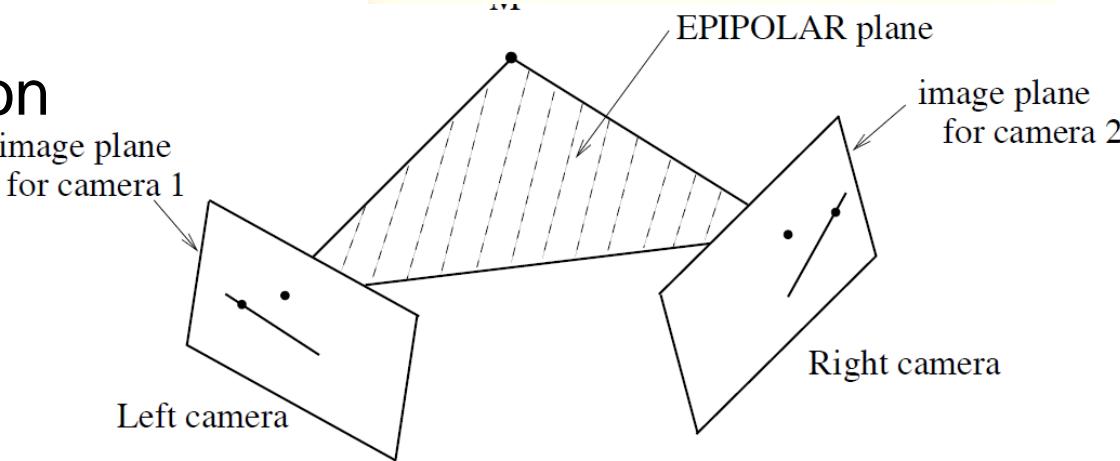
- Stereopsis



- Optical Flow

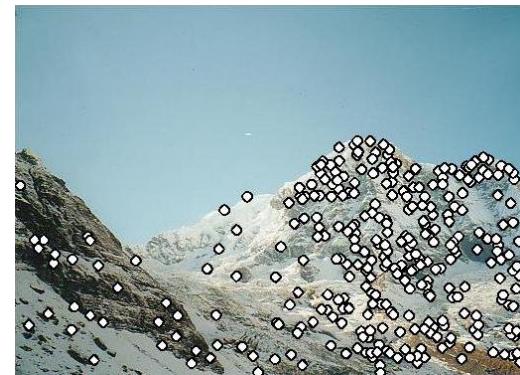


- Structure from Motion



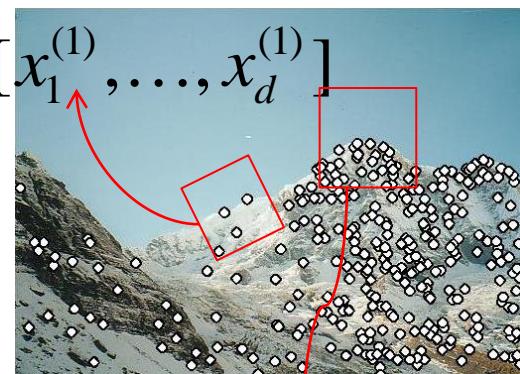
Keypoint Matching: main components

1) Detection: Identify the interest points



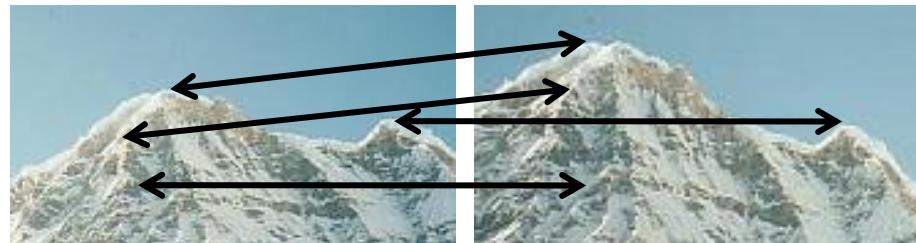
2) Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



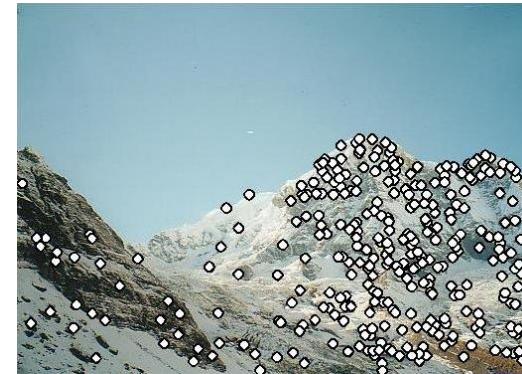
3) Matching: Determine correspondence between descriptors in two views

$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$



Local features: main components

1) Detection: Identify the interest points



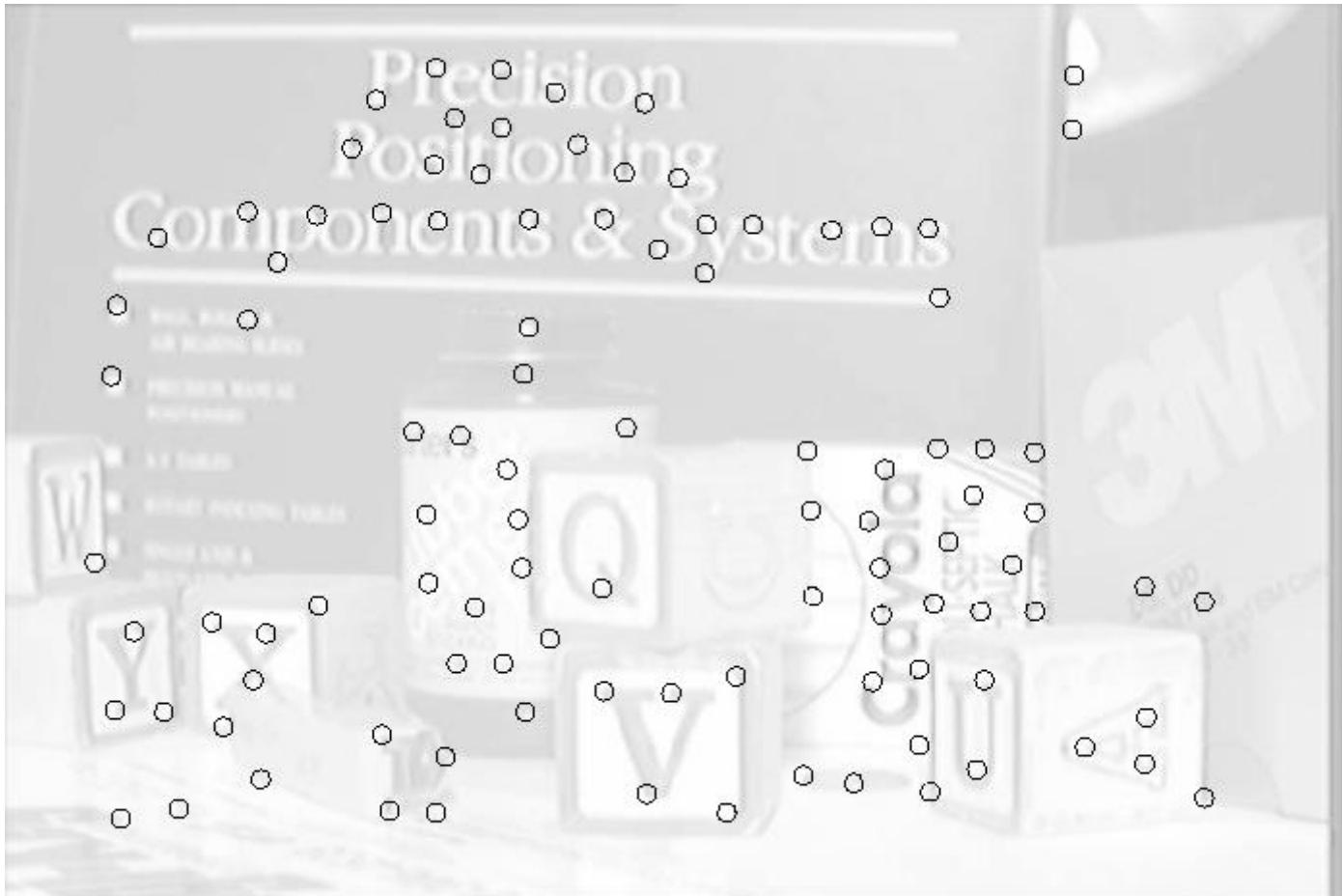
2) Description: Extract vector feature descriptor surrounding each interest point.

3) Matching: Determine correspondence between descriptors in two views

Many Existing Detectors Available

Hessian & Harris	[Beaudet '78], [Harris '88]
Laplacian, DoG	[Lindeberg '98], [Lowe 1999]
Harris-/Hessian-Laplace	[Mikolajczyk & Schmid '01]
Harris-/Hessian-Affine	[Mikolajczyk & Schmid '04]
EBR and IBR	[Tuytelaars & Van Gool '04]
MSER	[Matas '02]
Salient Regions	[Kadir & Brady '01]
Others...	

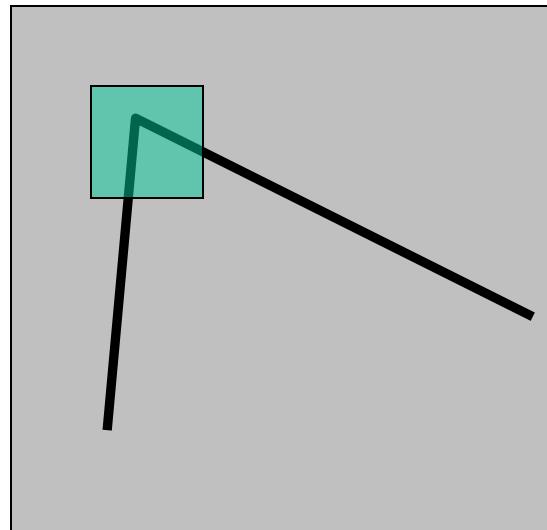
Corner Detection



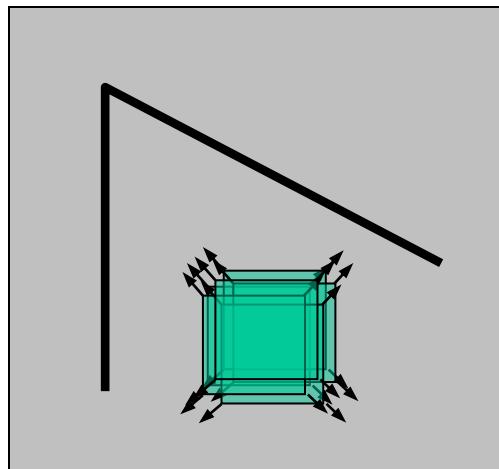
The Basic Idea

We should easily recognize the point by looking through a small window

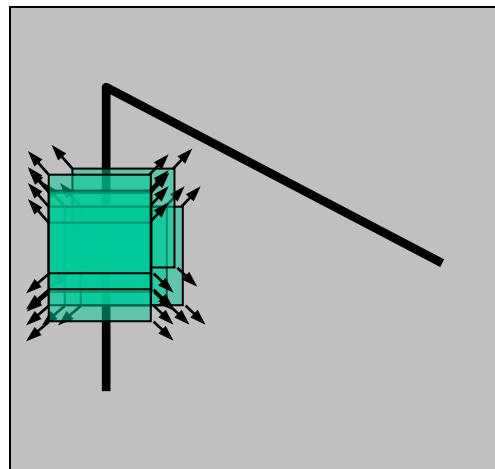
Shifting a window in *any direction* should give a *large change* in intensity



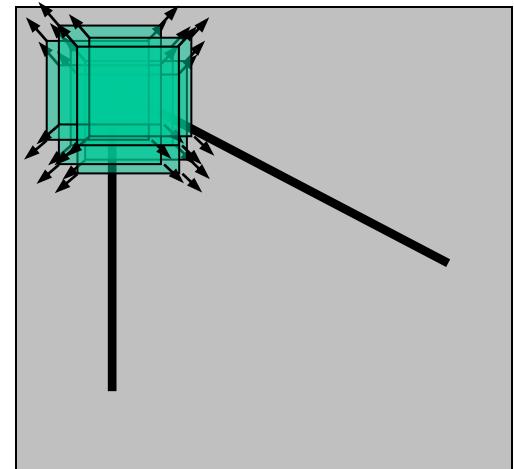
Corner Detector: Basic Idea



“flat” region:
no change in
all directions



“edge”:
no change along
the edge direction

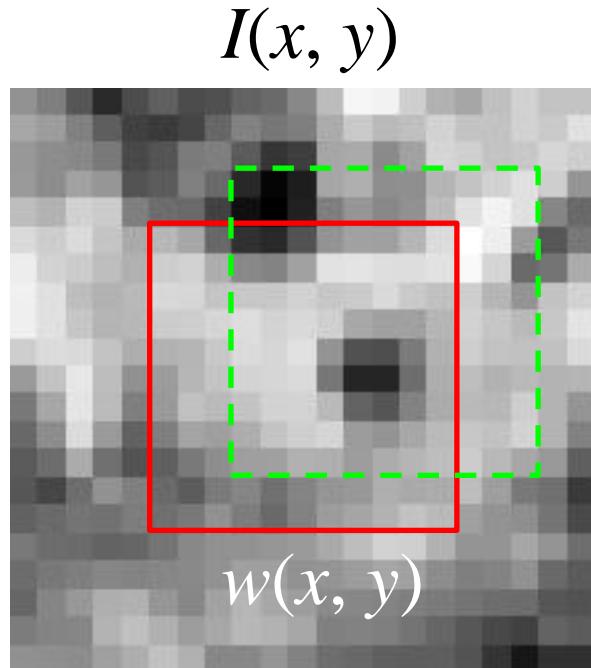


“corner”:
significant change
in all directions

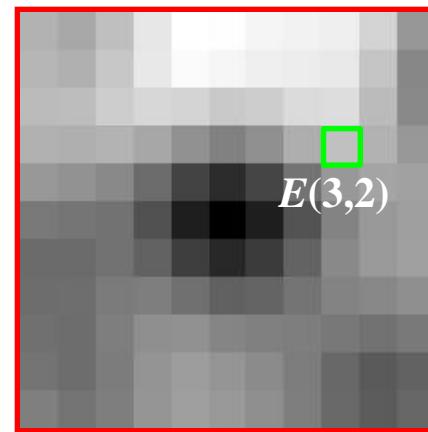
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$



$$E(u, v)$$

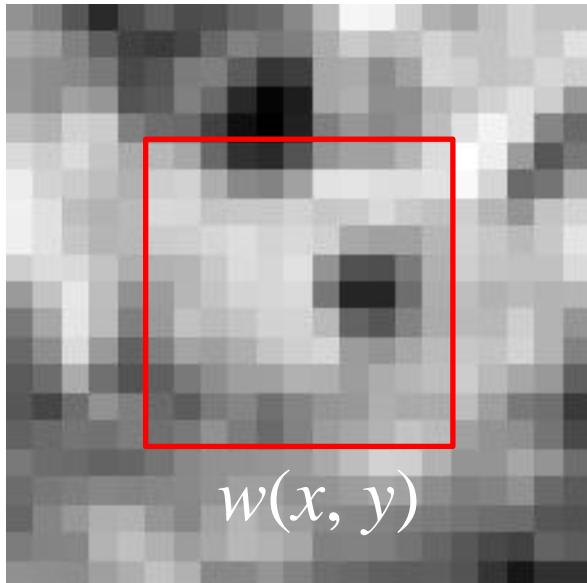


Corner Detection: Mathematics

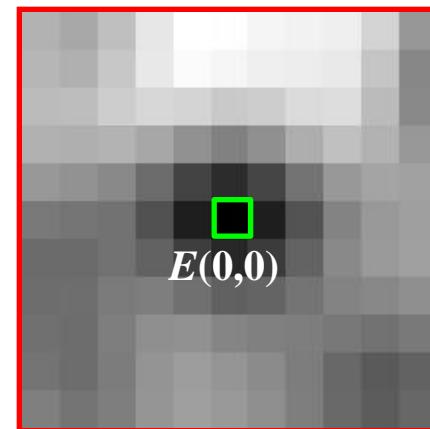
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

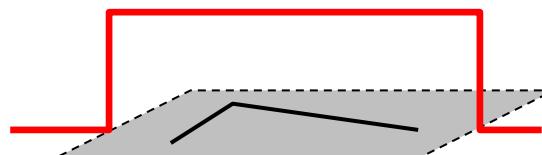
$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u, y+v) - I(x, y)]^2$$

Window
function

Shifted
intensity

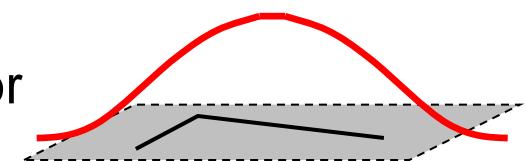
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

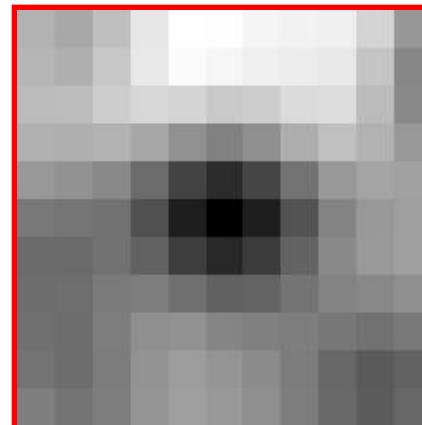
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

We want to find out how this function behaves for small shifts

$$E(u, v)$$



Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u, v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a *second moment matrix* computed from image derivatives:

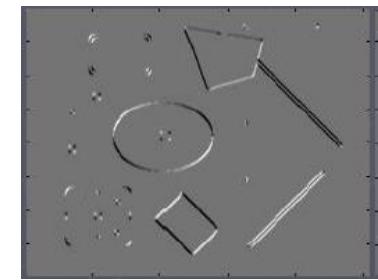
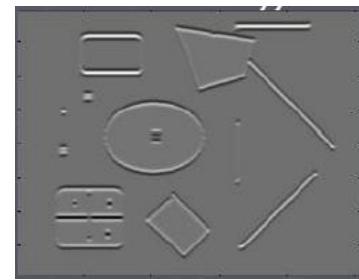
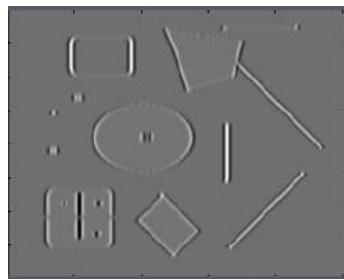
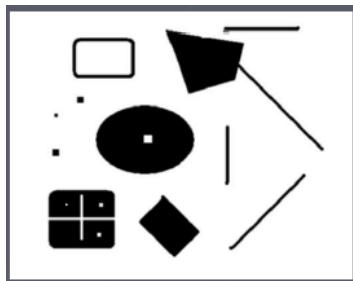
$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

Corners as distinctive interest points

$$M = \sum w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



Notation:

$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$

$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$

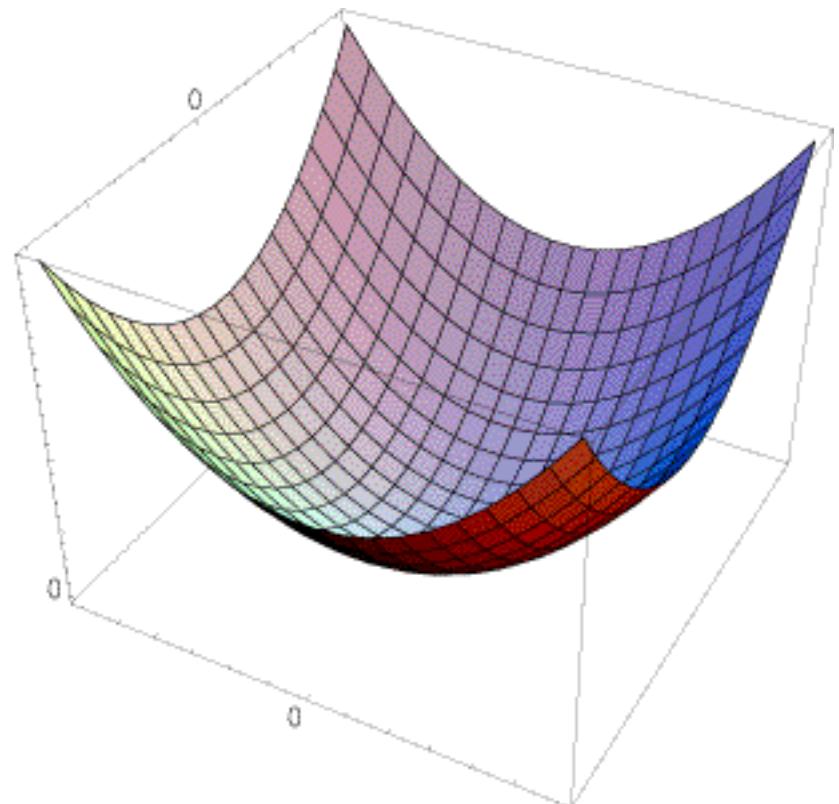
$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$

Interpreting the second moment matrix

The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

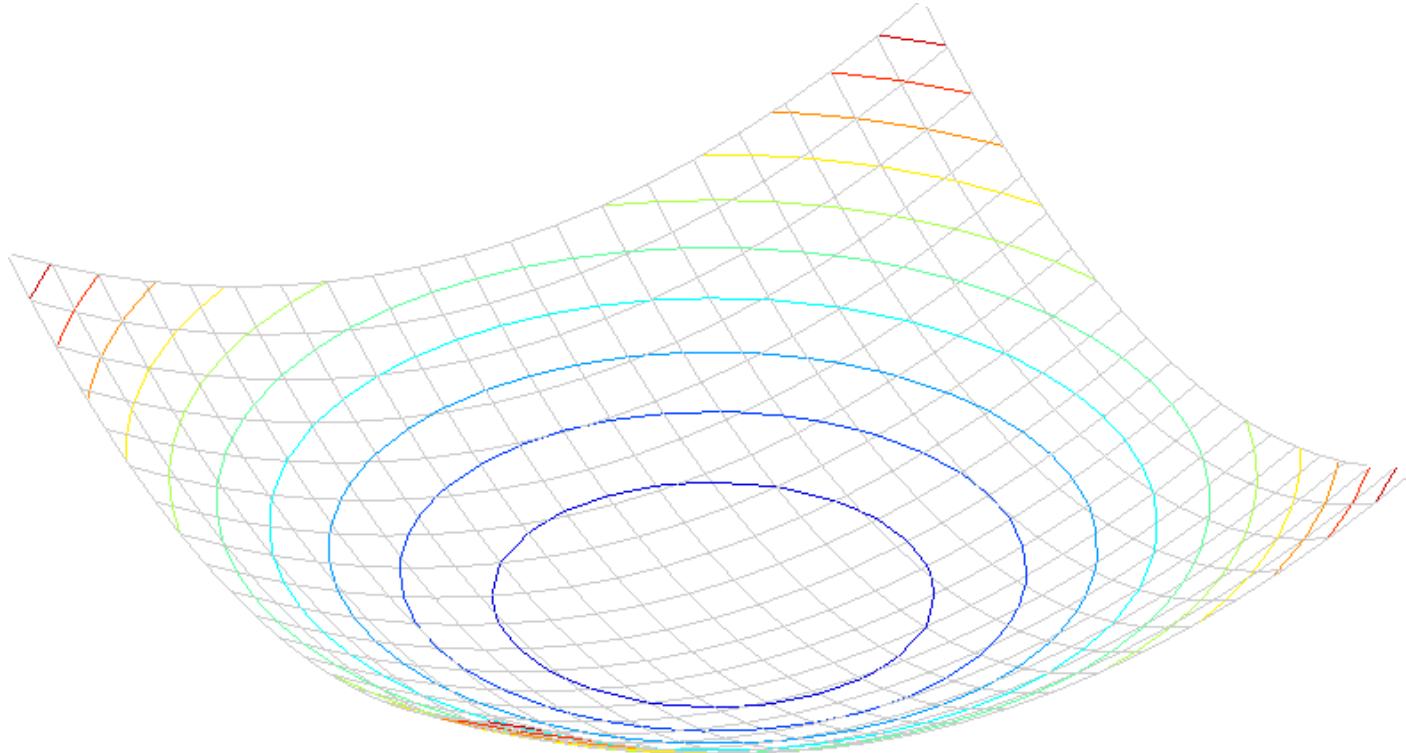
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

Consider a horizontal “slice” of $E(u, v)$: $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



Interpreting the second moment matrix

First, consider the axis-aligned case
(gradients are either horizontal or vertical)

$$M = \sum_{x,y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

Interpreting the second moment matrix

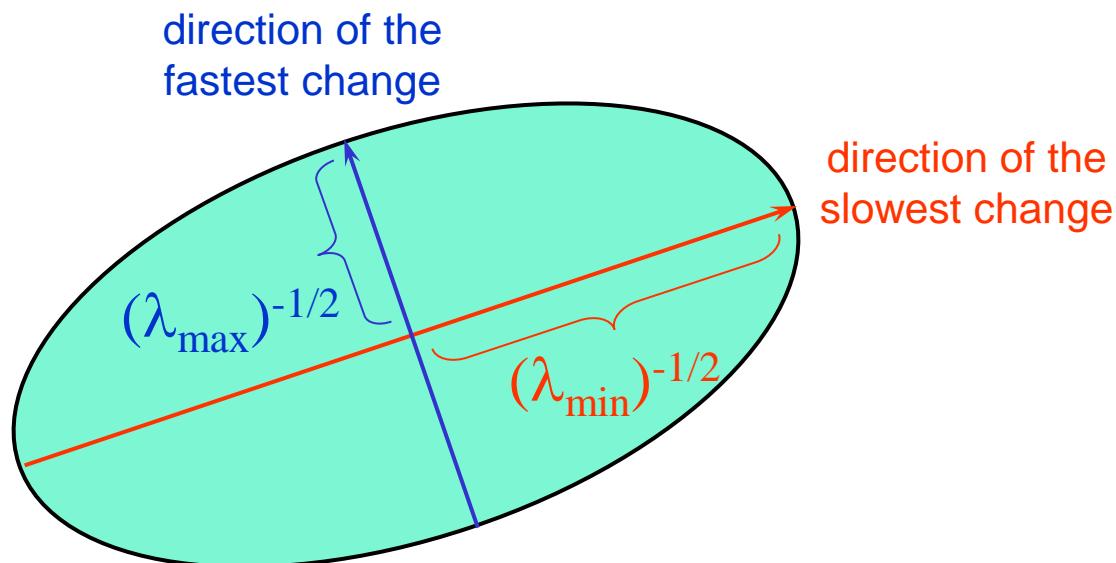
Consider a horizontal “slice” of $E(u, v)$: $[u \ v] M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

Diagonalization of M :

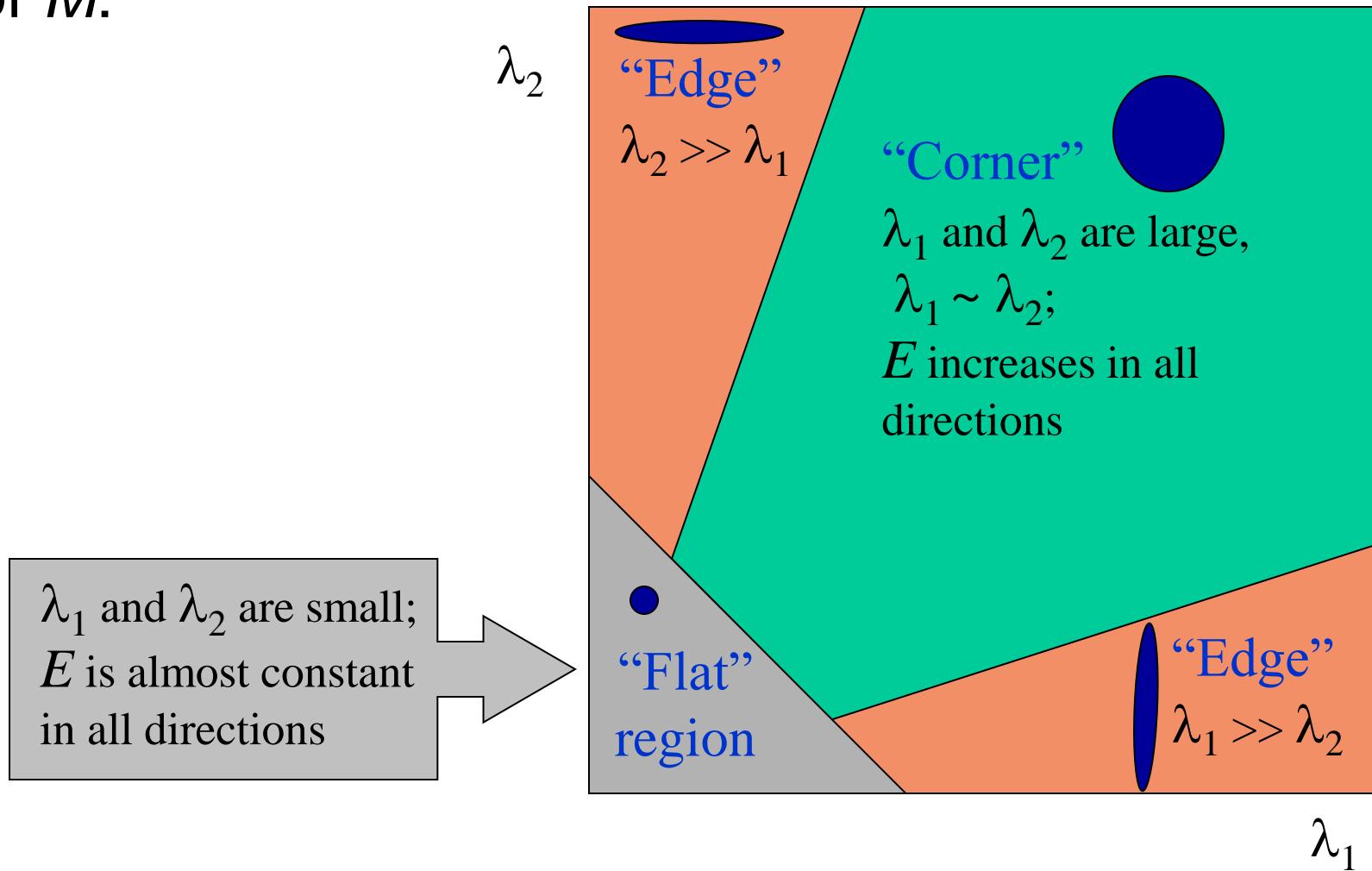
$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Interpreting the eigenvalues

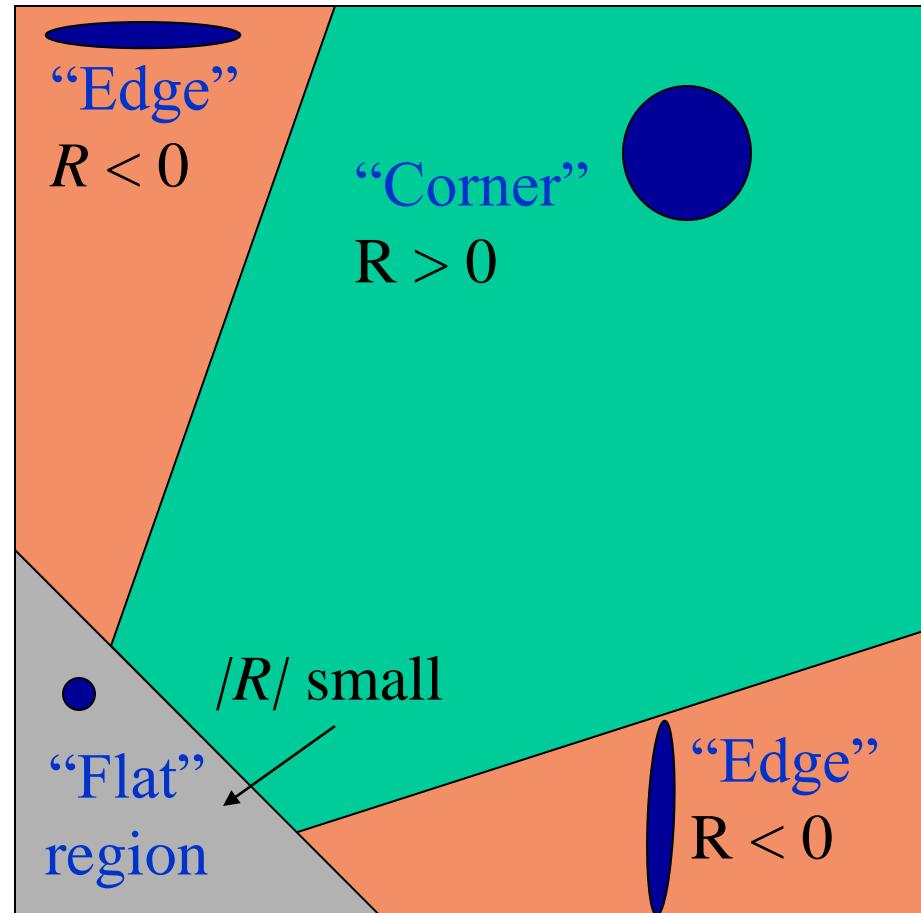
Classification of image points using eigenvalues of M :



Corner response function

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

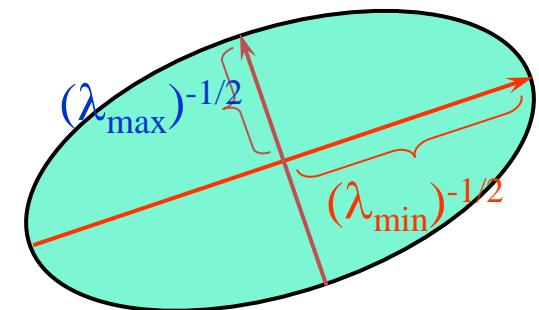
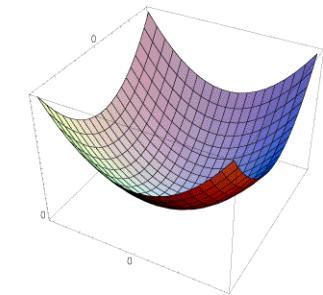
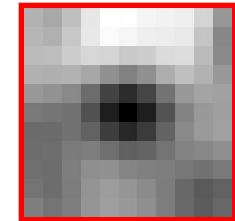
α : constant (0.04 to 0.06)



Review: Harris corner detector

$E(u, v)$

- Approximate distinctiveness by local auto-correlation.
- Approximate local auto-correlation by second moment matrix
- Quantify distinctiveness (or cornerness) as function of the eigenvalues of the second moment matrix.
- But we don't actually need to compute the eigenvalues by using the determinant and trace of the second moment matrix.

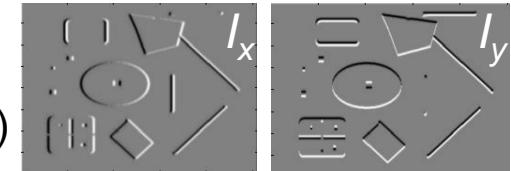
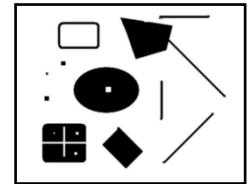


Harris Detector [Harris88]

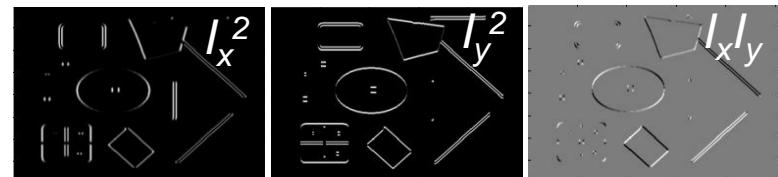
- Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives
(optionally, blur first)



2. Square of derivatives



3. Gaussian filter $g(\sigma_\nu)$



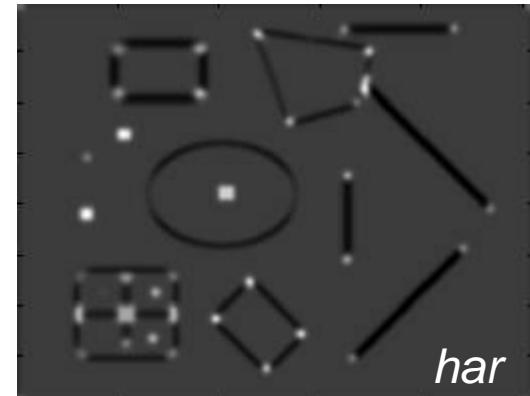
$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

4. Cornerness function – both eigenvalues are strong

$$\begin{aligned} har &= \det[\mu(\sigma_I, \sigma_D)] - \alpha[\text{trace}(\mu(\sigma_I, \sigma_D))^2] = \\ &= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2 \end{aligned}$$

5. Non-maxima suppression

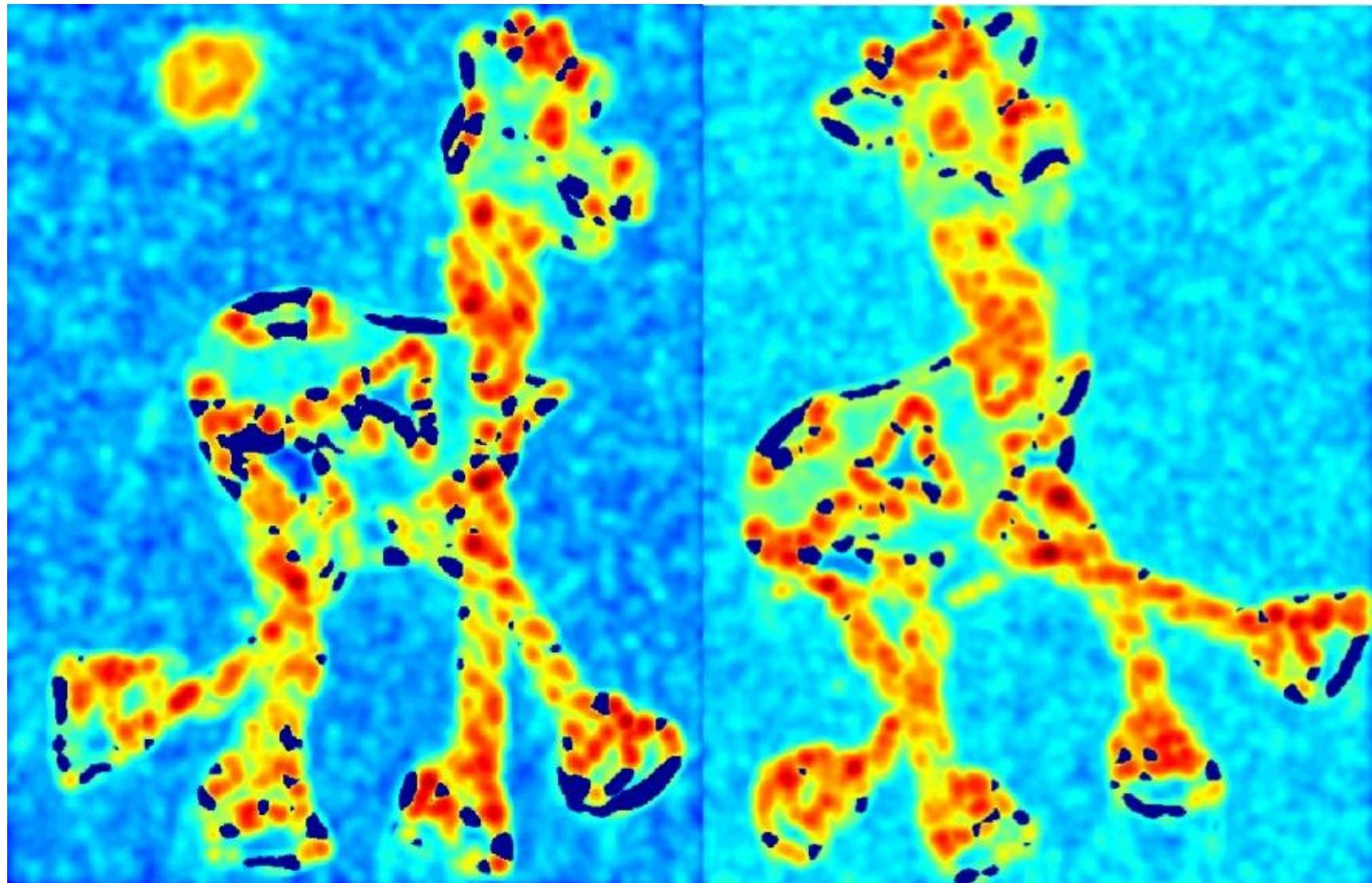


Harris Detector: Steps



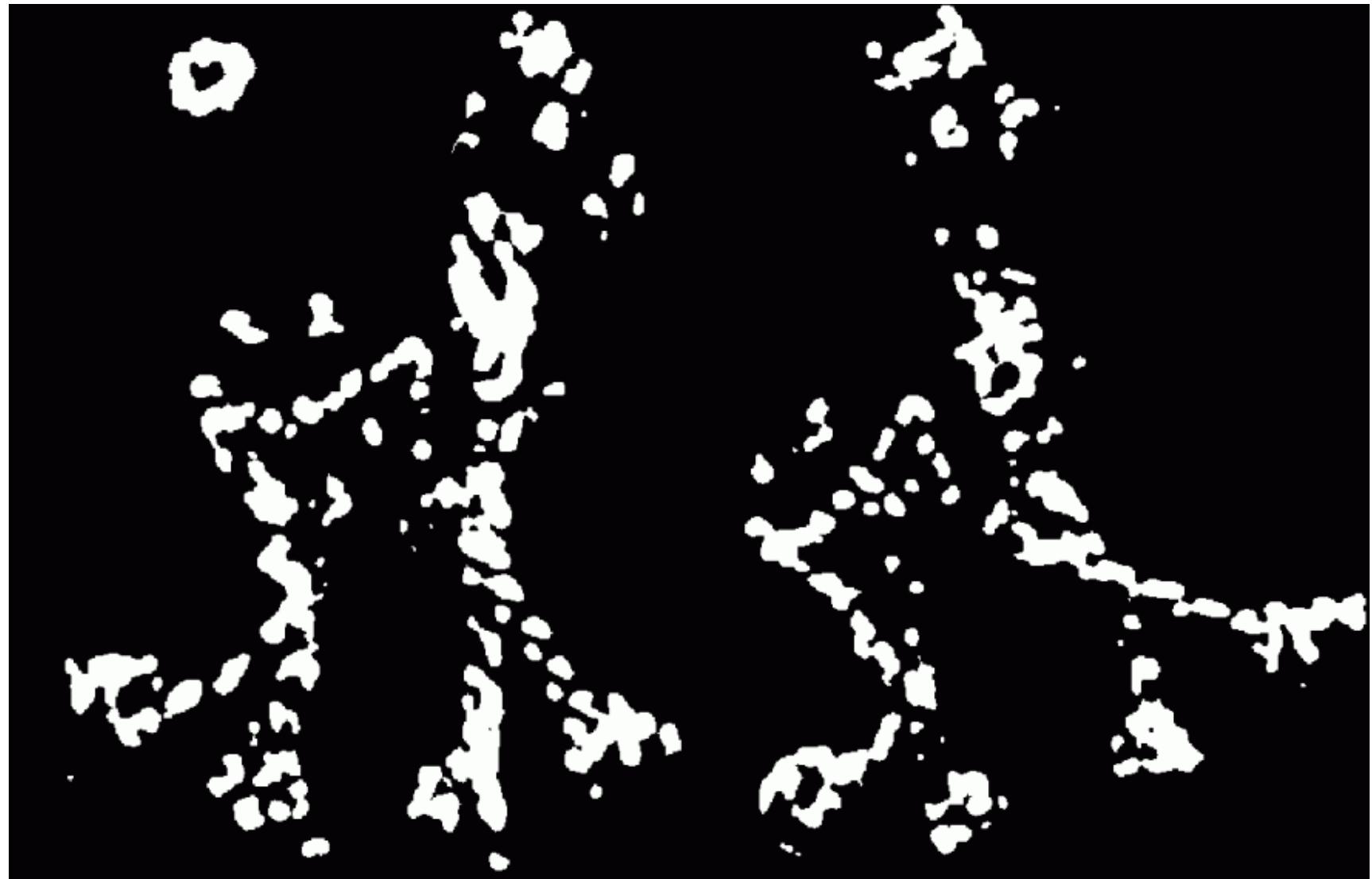
Harris Detector: Steps

Compute corner response R



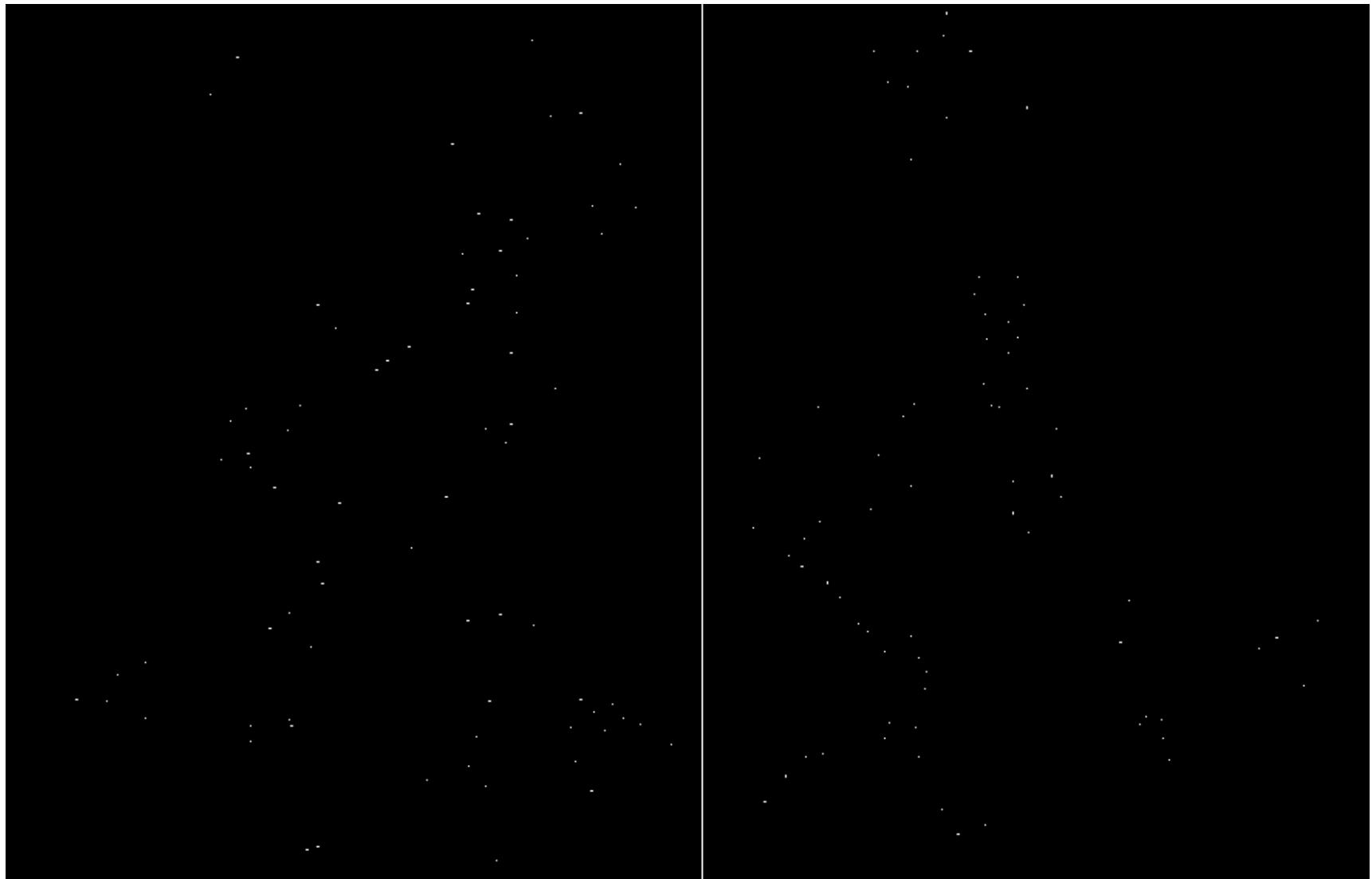
Harris Detector: Steps

Find points with large corner response: $R>\text{threshold}$



Harris Detector: Steps

Take only the points of local maxima of R



Harris Detector: Steps



Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
 - **Invariance:** image is transformed and corner locations do not change
 - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

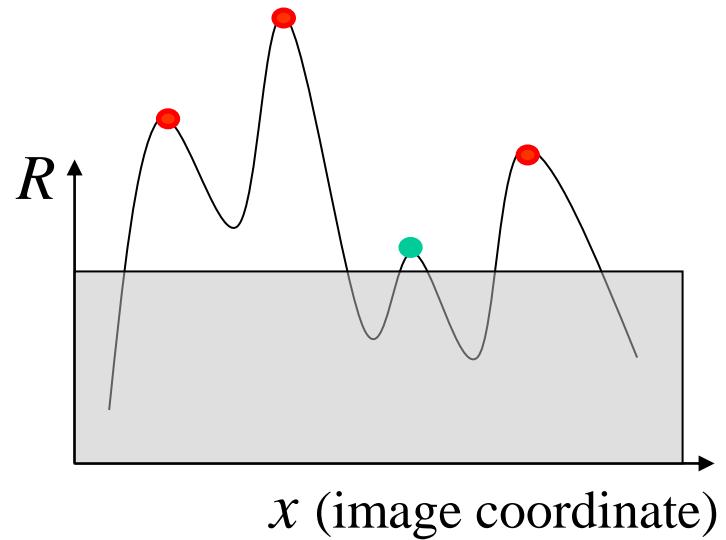
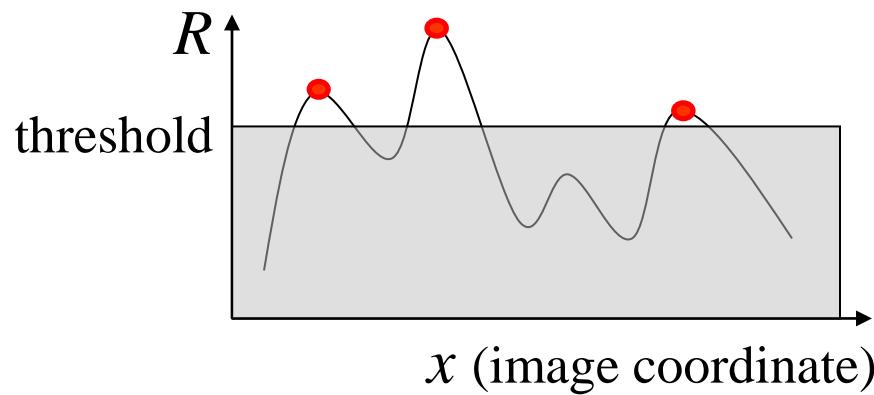


Affine intensity change



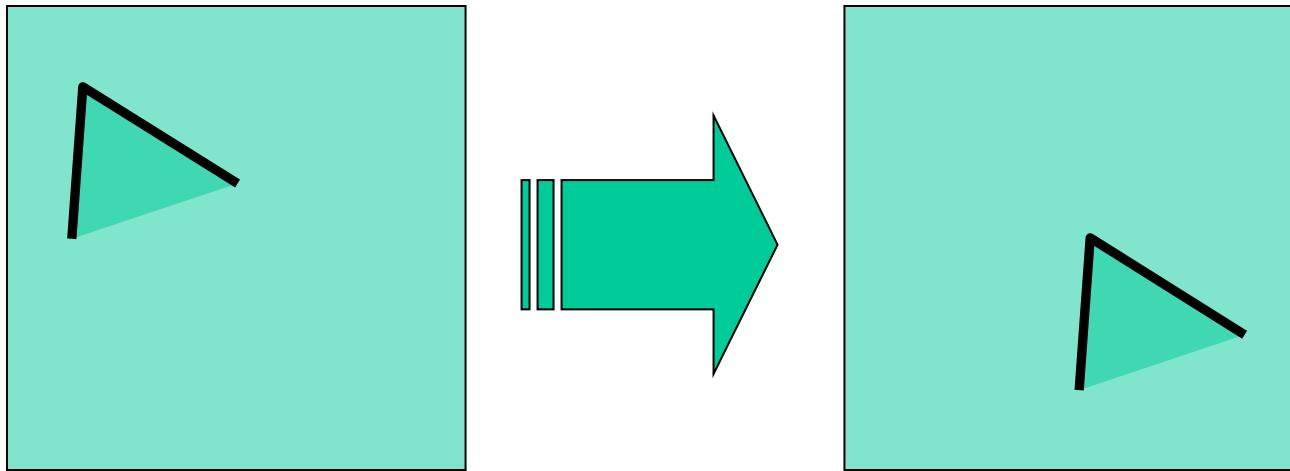
$$I \rightarrow a I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a I$



Partially invariant to affine intensity change

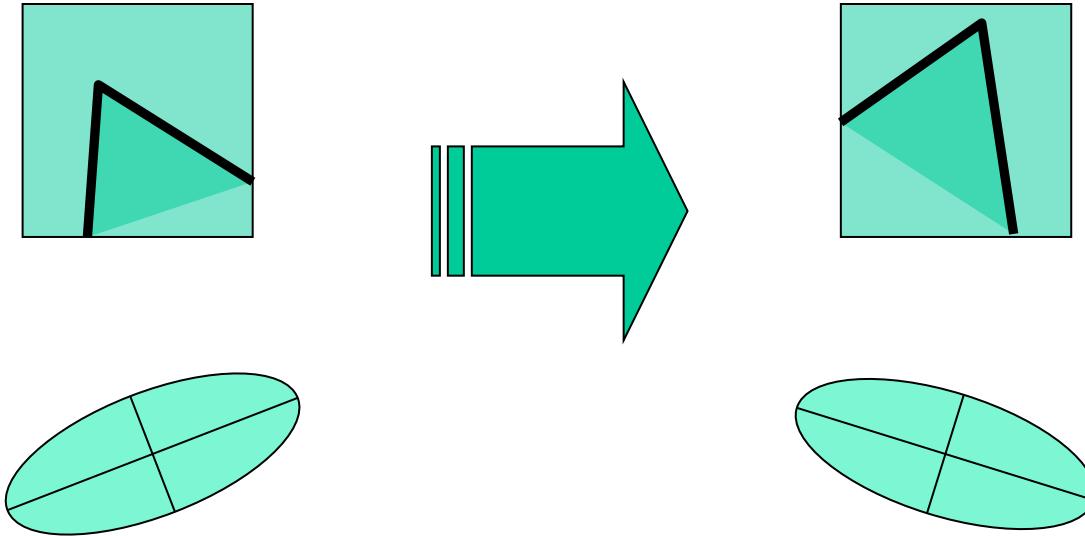
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

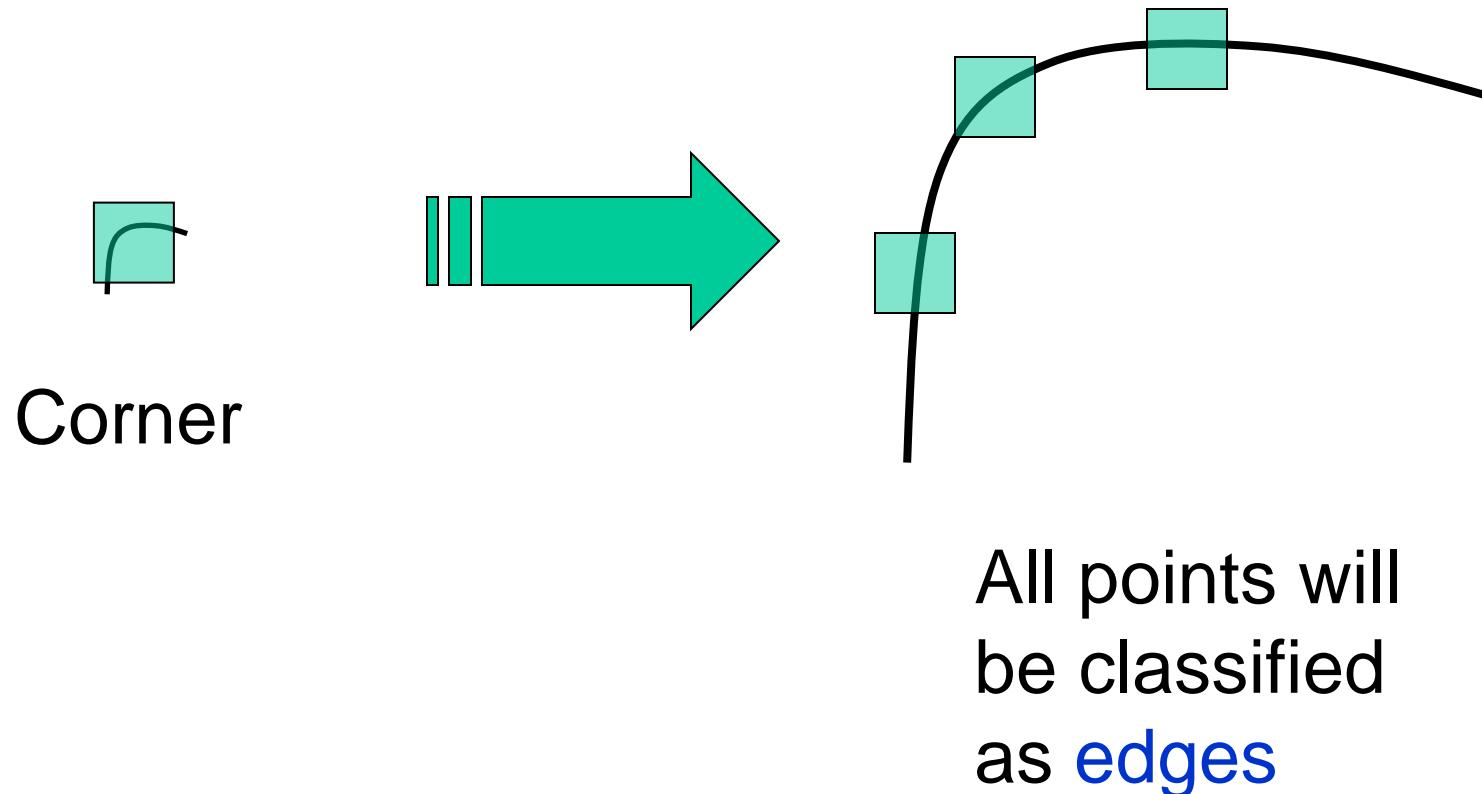
Image rotation



Second moment ellipse rotates but its shape
(i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

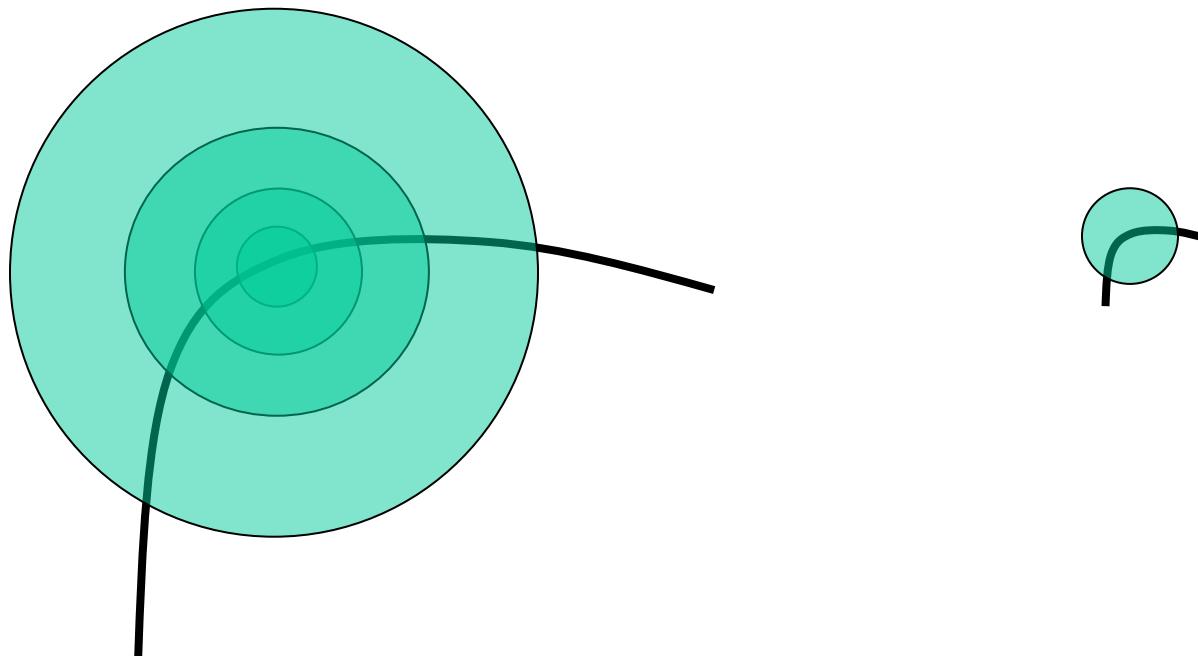
Scaling



Corner location is not covariant to scaling!

Scale Invariant Detection

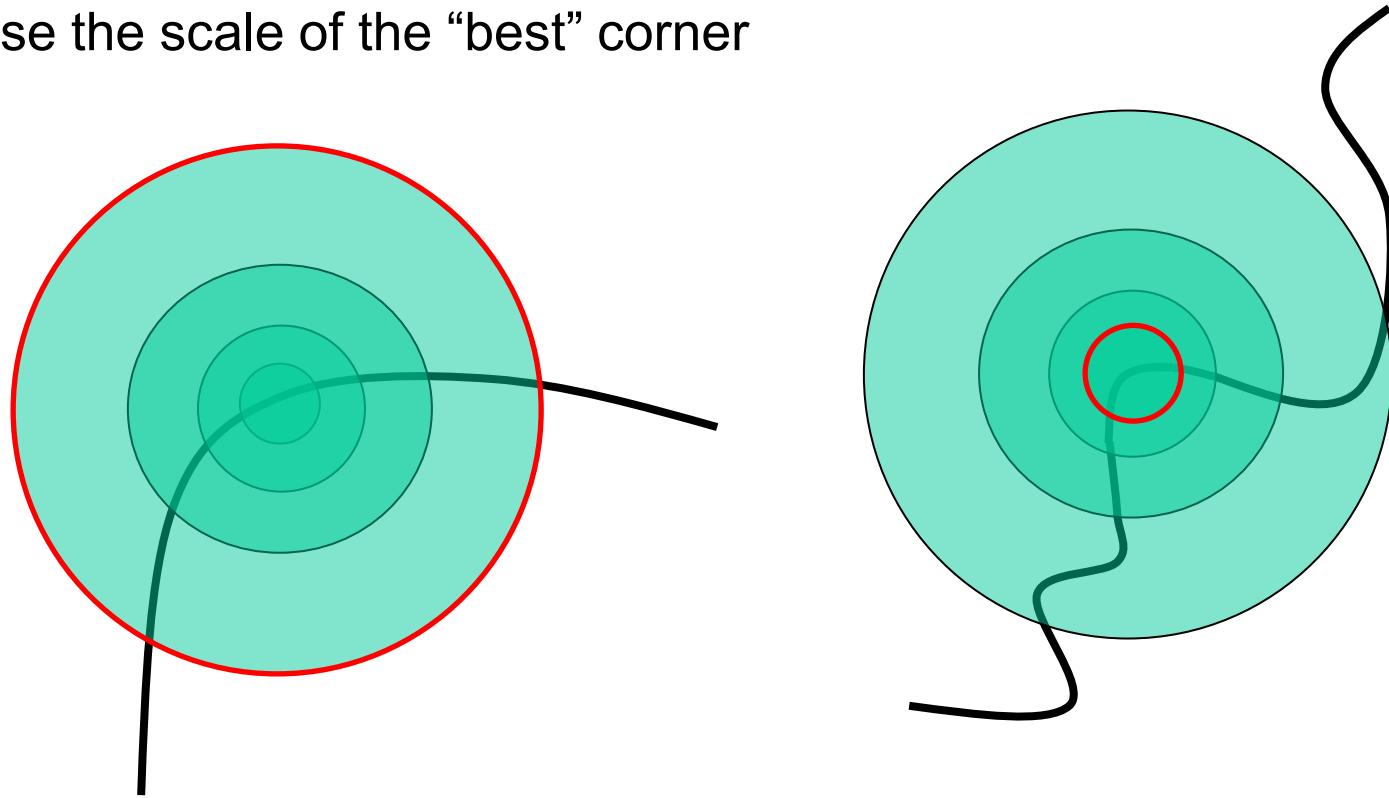
Consider regions (e.g. circles) of different sizes around a point
Regions of corresponding sizes will look the same in both images



Scale Invariant Detection

The problem: how do we choose corresponding circles
independently in each image?

Choose the scale of the “best” corner

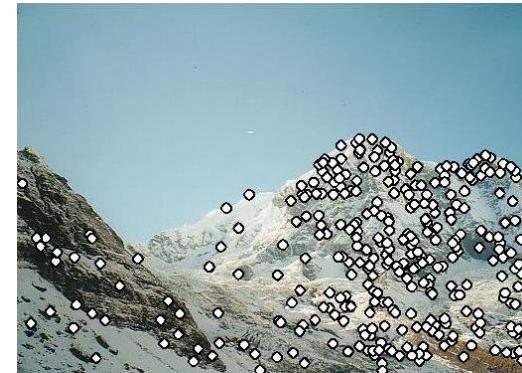


Pick how many corners you want



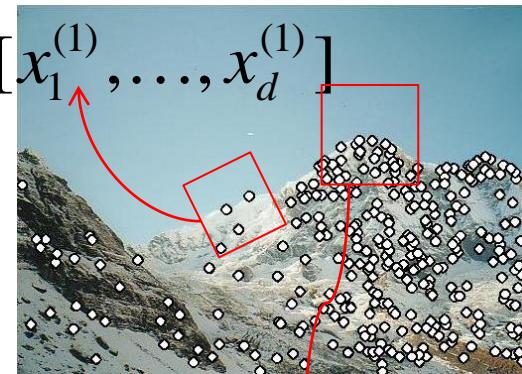
Local features: main components

- 1) Detection: Identify the interest points



- 2) Description: Extract vector feature descriptor surrounding each interest point.

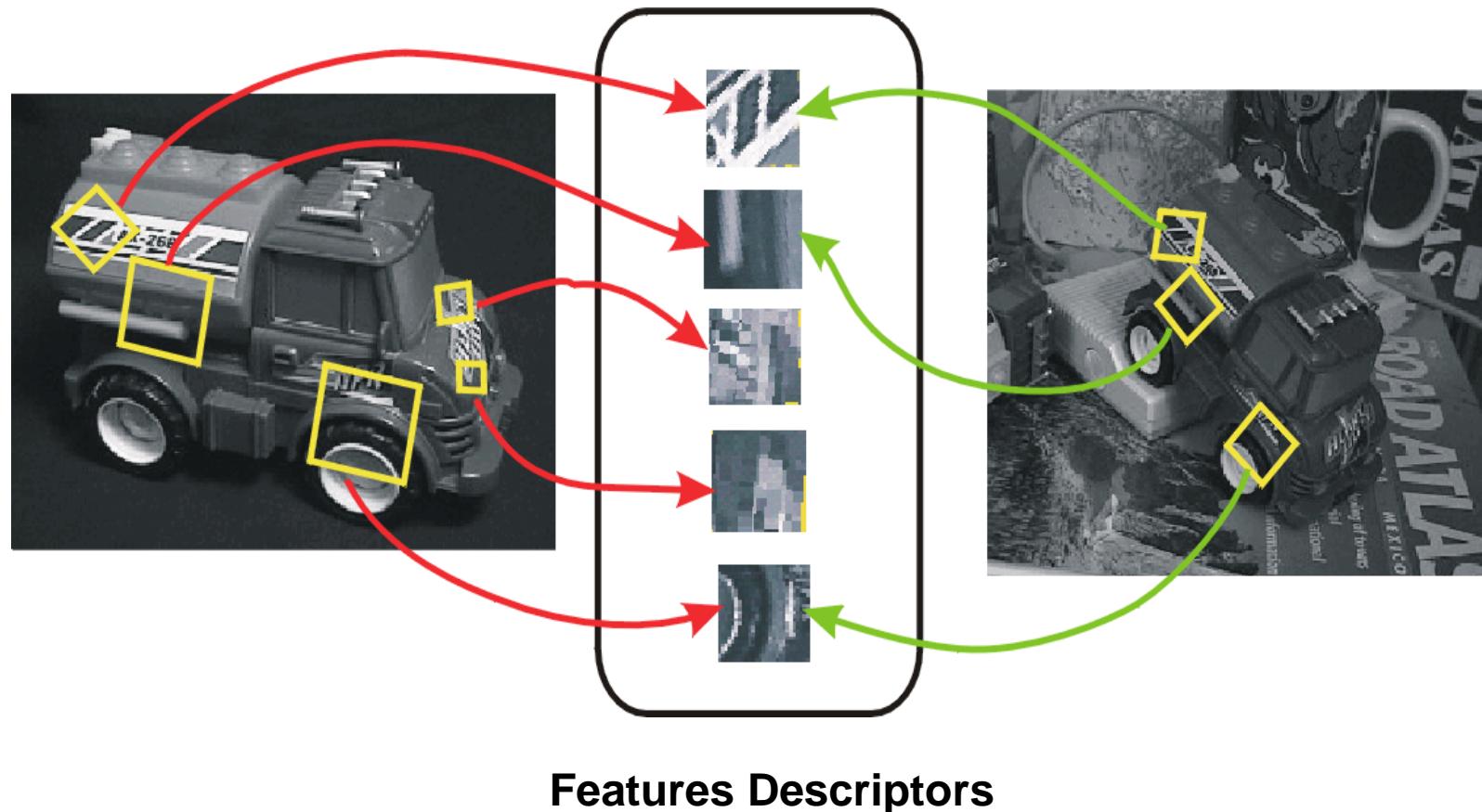
$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

Invariant Local Features

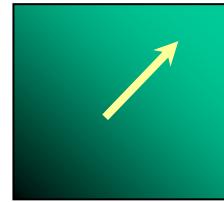
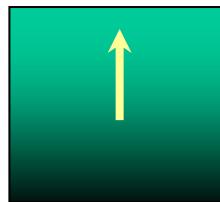
- Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters



Descriptors Invariant to Rotation

Find local orientation

Dominant direction of gradient



- Extract image patches relative to this orientation

Descriptor Vector

Rotation Invariant Frame

- Scale-space position (x, y, s) + orientation (θ)



Detections at multiple scales

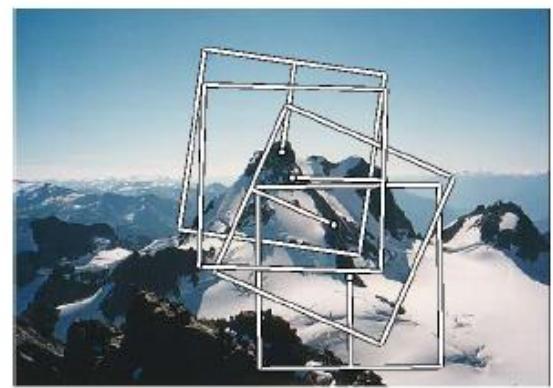
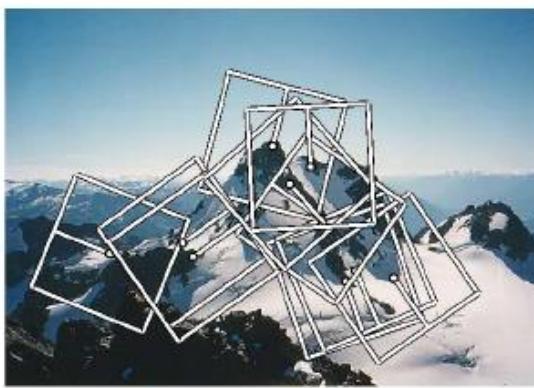
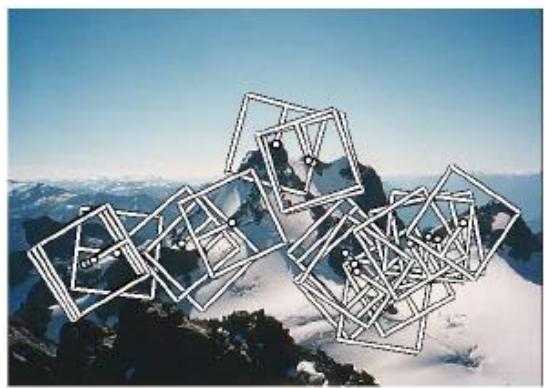
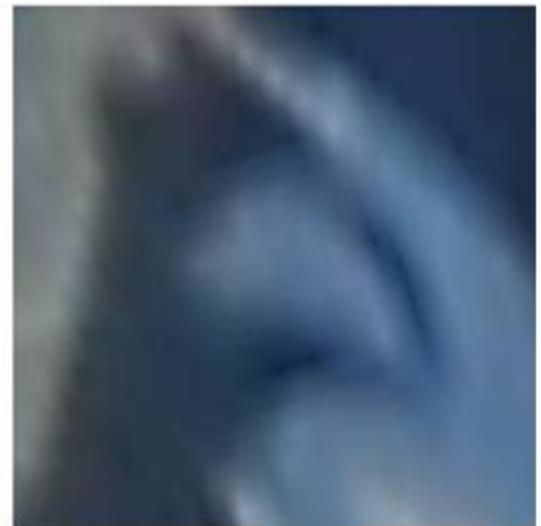
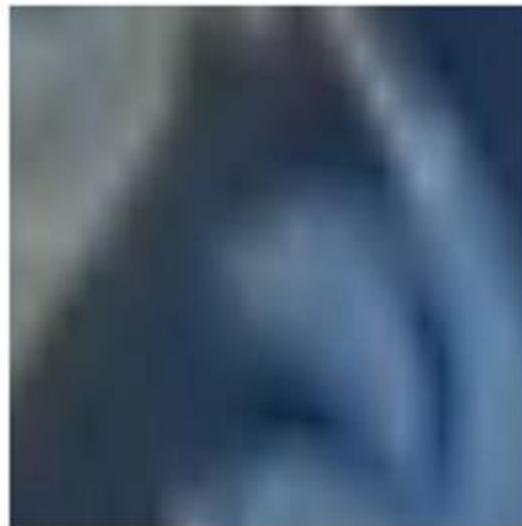
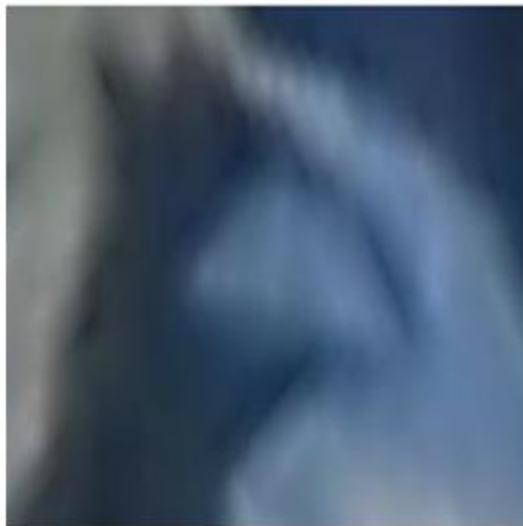


Figure 1. Multi-scale Oriented Patches (MOPS) extracted at five pyramid levels from one of the Matier images. The boxes show the feature orientation and the region from which the descriptor vector is sampled.

Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot

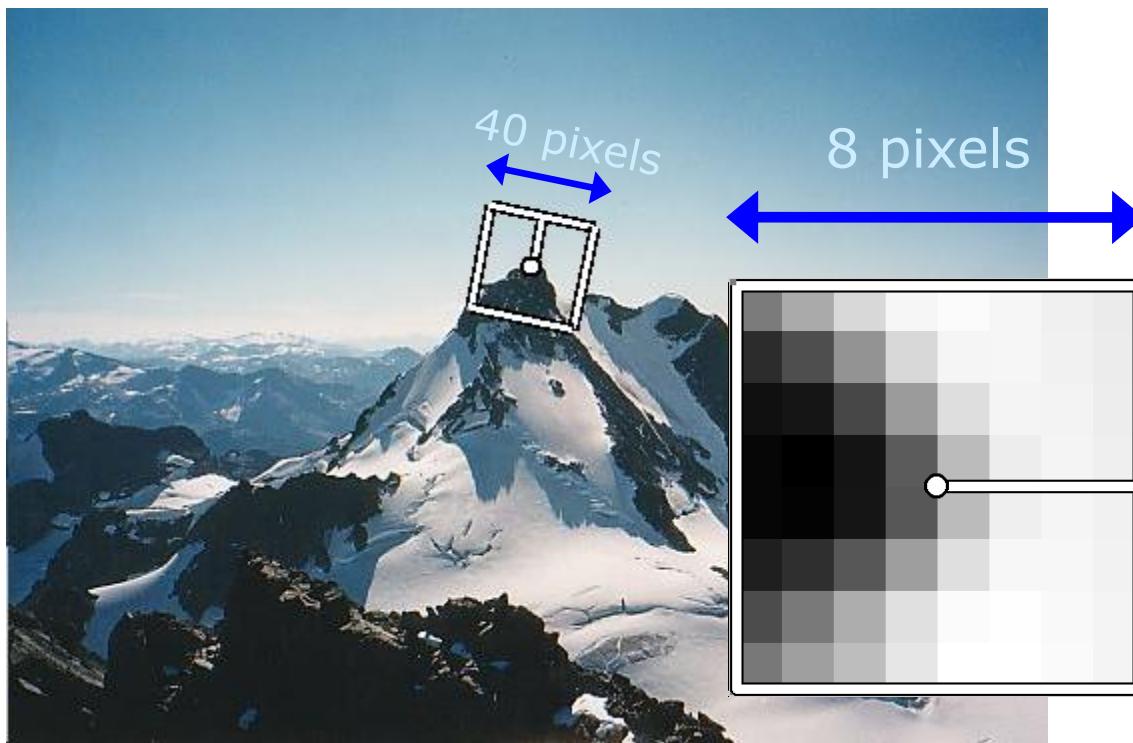


MOPS descriptor vector

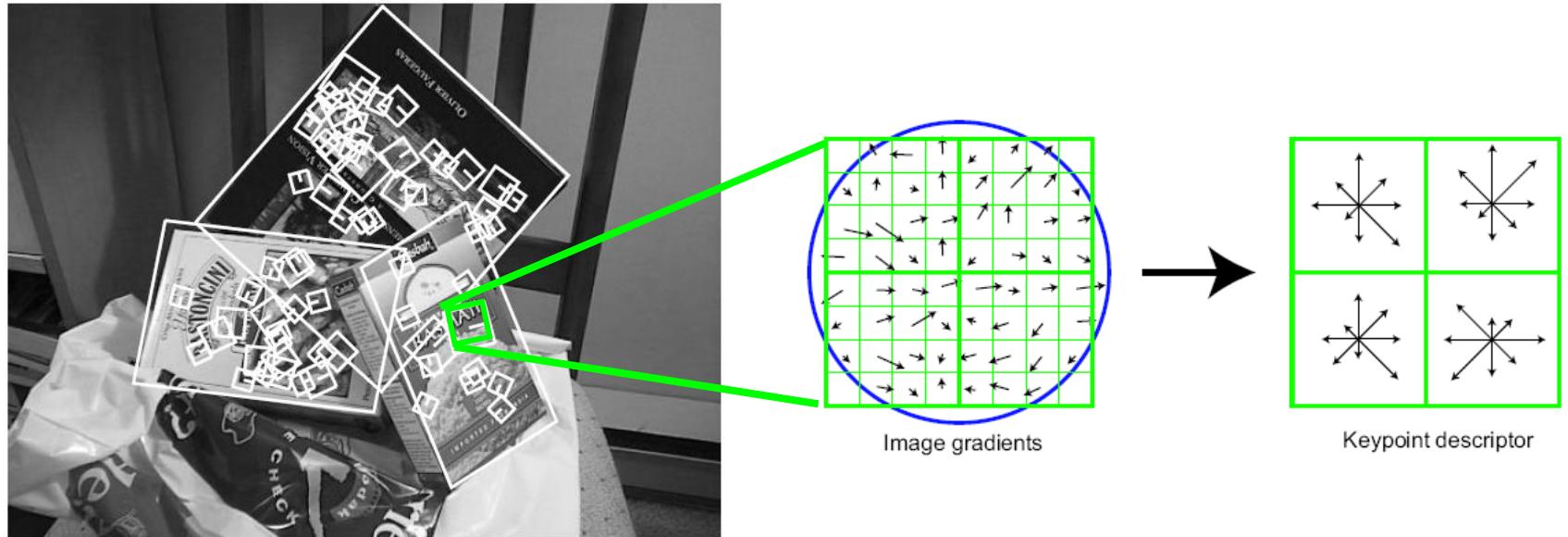
8x8 oriented patch

- Sampled at 5 x scale

Bias/gain normalisation: $I' = (I - \mu)/\sigma$



Local Descriptors: SIFT Descriptor



Histogram of oriented gradients

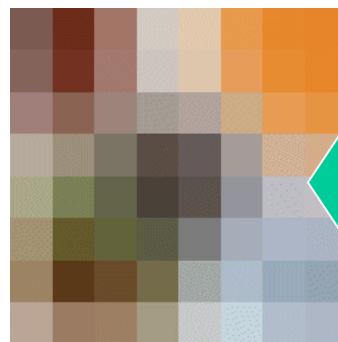
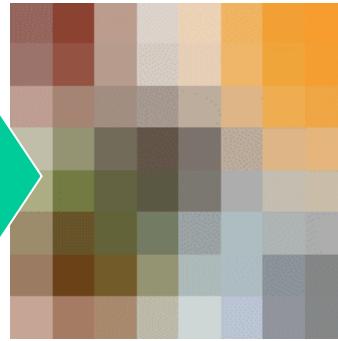
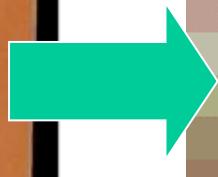
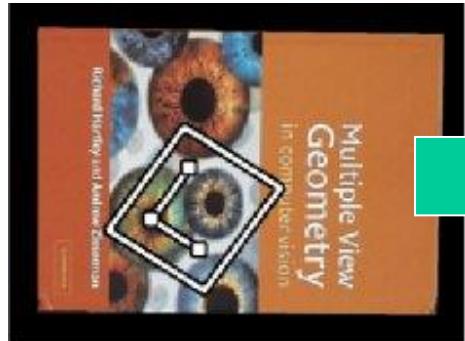
- Captures important texture information
- Robust to small translations / affine deformations

[Lowe, ICCV 1999]

K. Grauman, B. Leibe

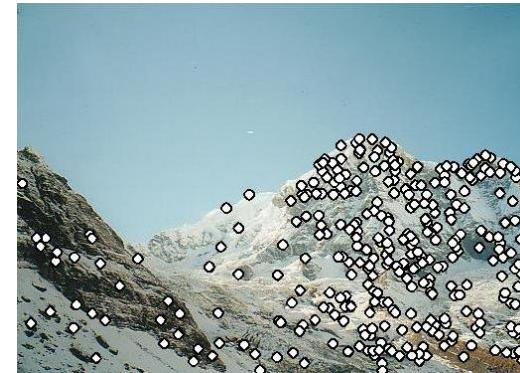
Invariant Feature Descriptors

Schmid & Mohr 1997, Lowe 1999, Baumberg 2000, Tuytelaars & Van Gool 2000, Mikolajczyk & Schmid 2001, Brown & Lowe 2002, Matas et. al. 2002, Schaffalitzky & Zisserman 2002



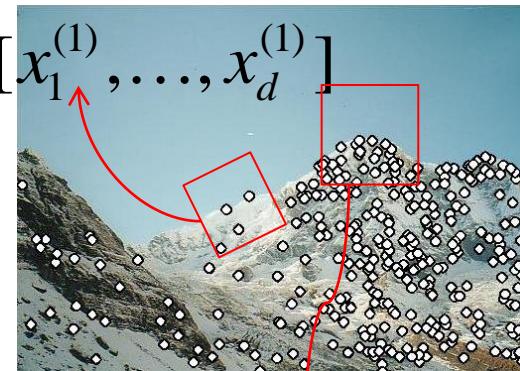
Local features: main components

1) Detection: Identify the interest points



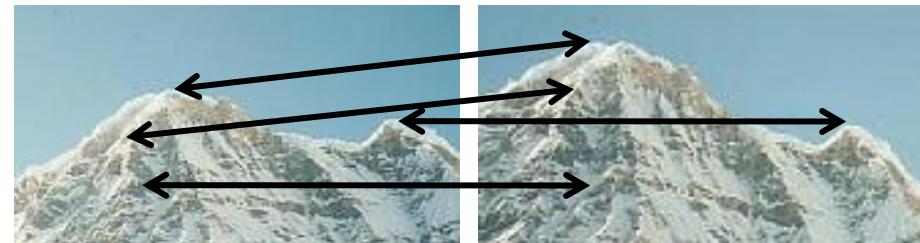
2) Description: Extract vector feature descriptor surrounding each interest point.

$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

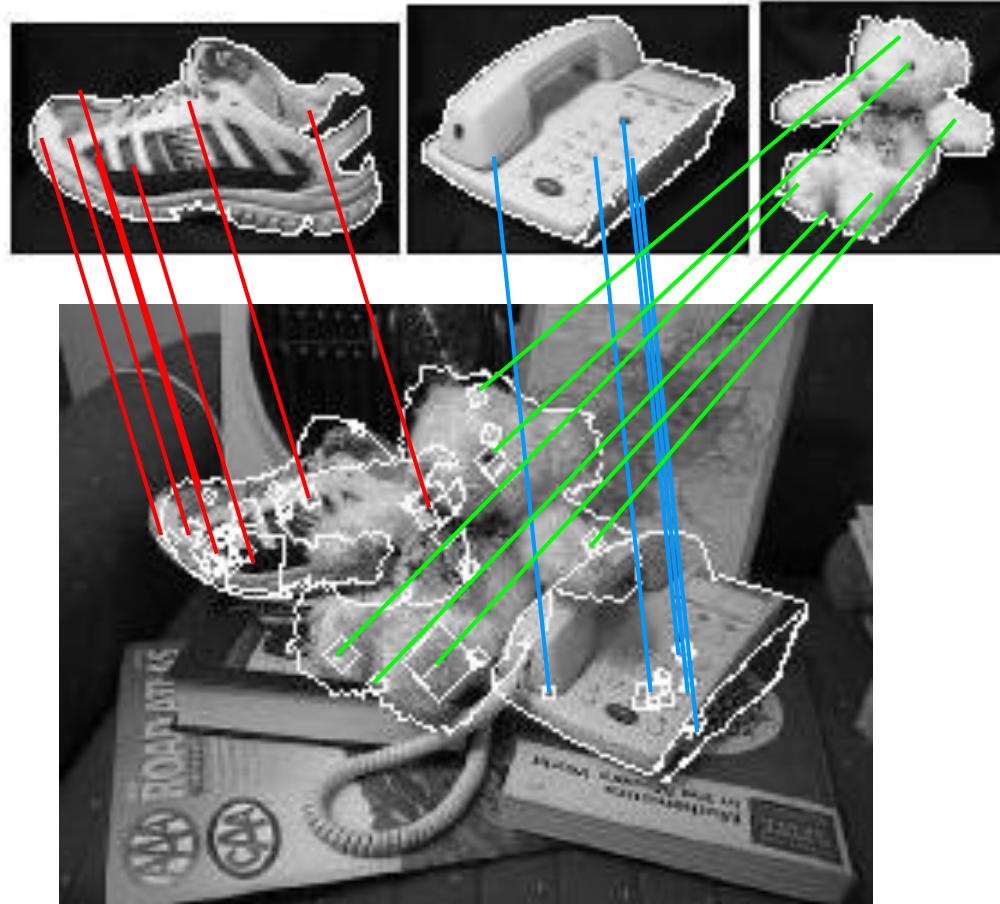


3) Matching: Determine correspondence between descriptors in two views

$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

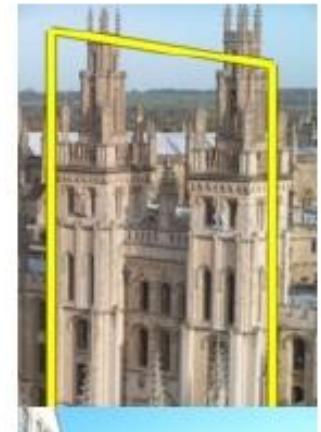
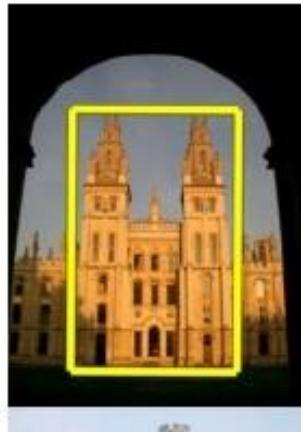
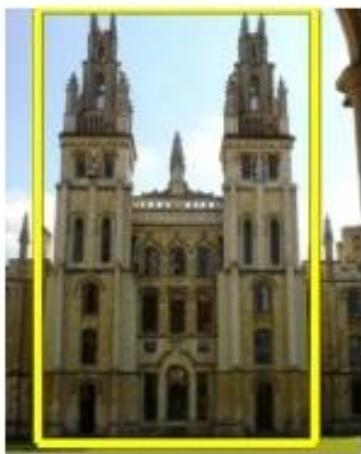


Large-scale Instance Retrieval



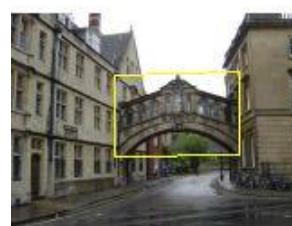
Slide Credits: Kristen Grauman, Josef Sivic, James Hays

How to quickly find images in a large database that match a given image region?

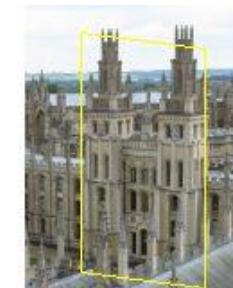
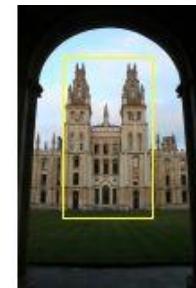


Why is it difficult?

Want to establish correspondence despite possibly large changes in scale, viewpoint, lighting and partial occlusion



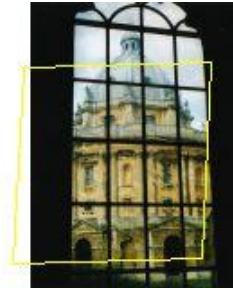
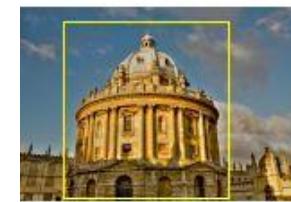
Scale



Viewpoint



Lighting



Occlusion

... and the image collection can be very large (e.g. 1M images)

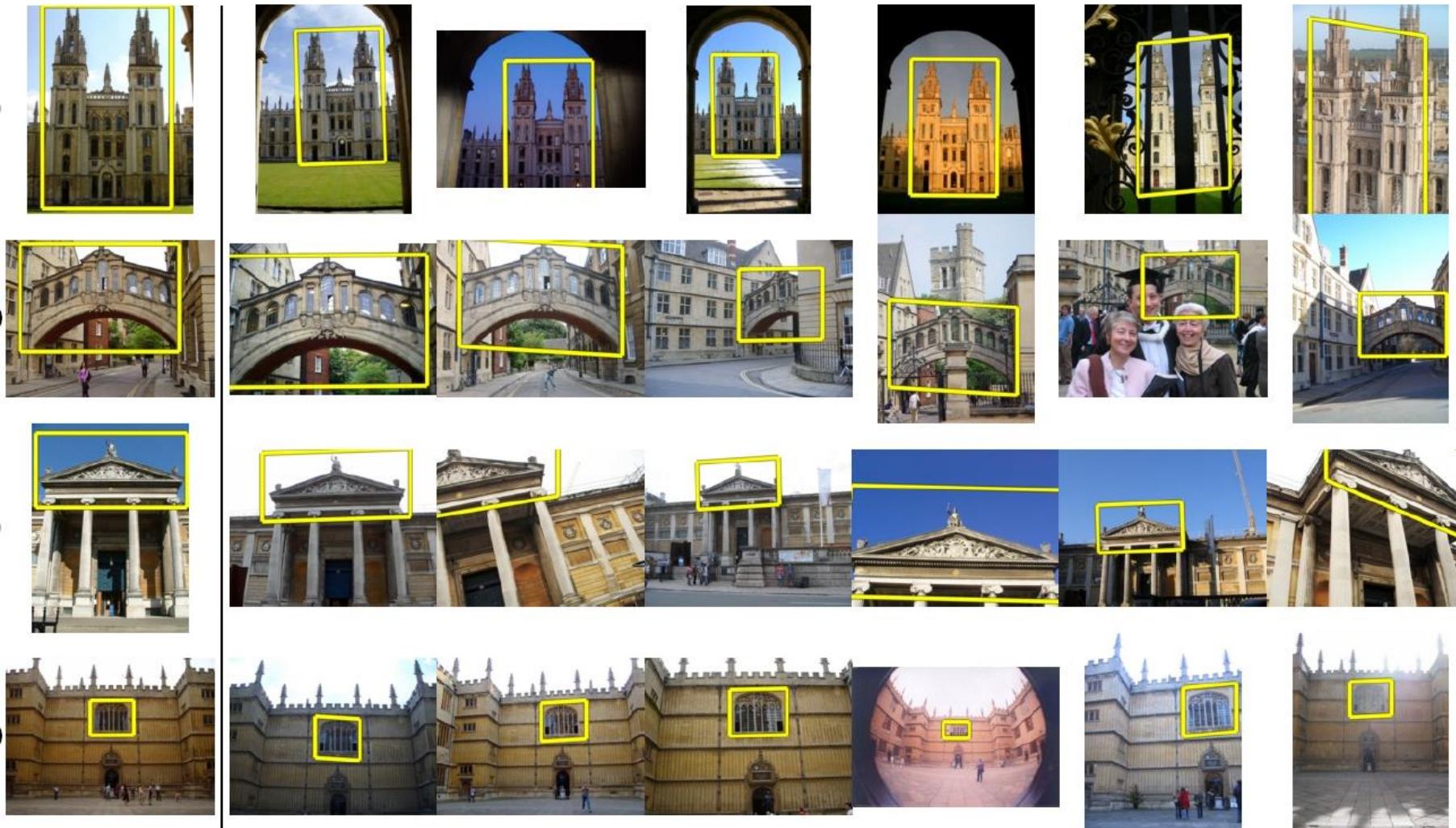
Example Applications



Mobile tourist guide

- Self-localization
- Object/building recognition
- Photo/video augmentation

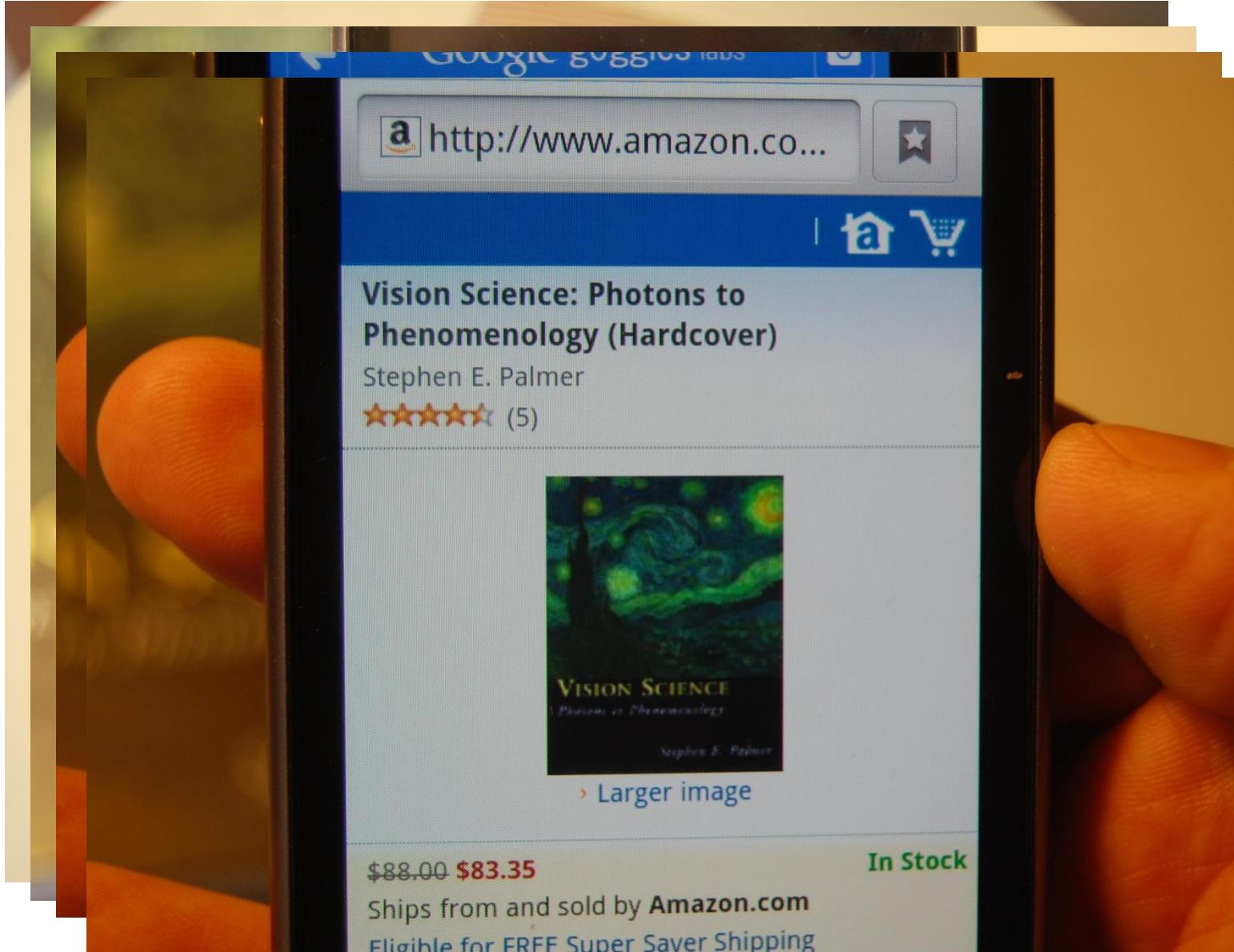
Application: Large-Scale Retrieval



Query

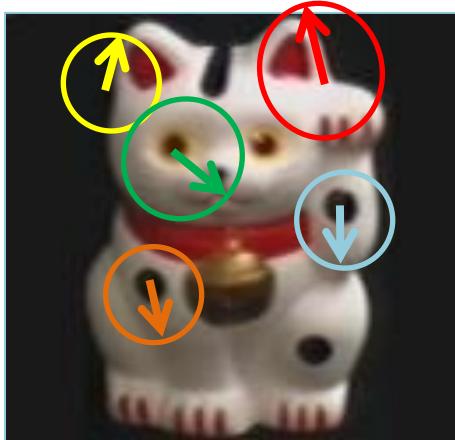
Results from 5k Flickr images (demo available for 100k set)

Example

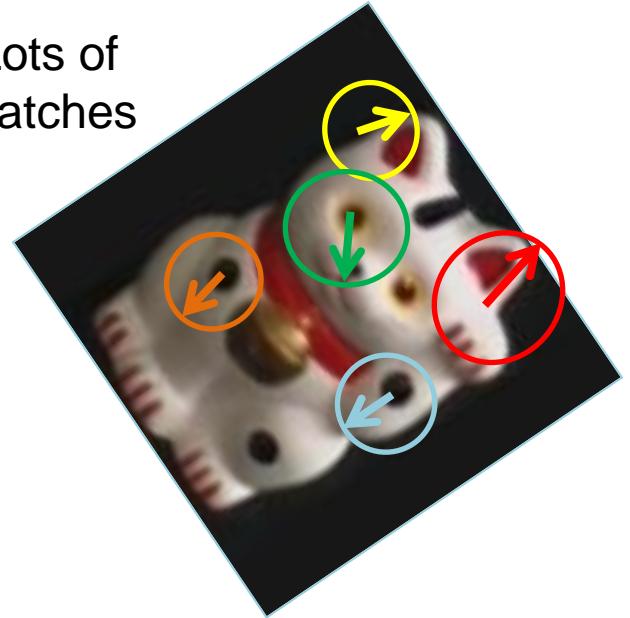


Simple idea

See how many keypoints
are close to keypoints in
each other image



Lots of
Matches



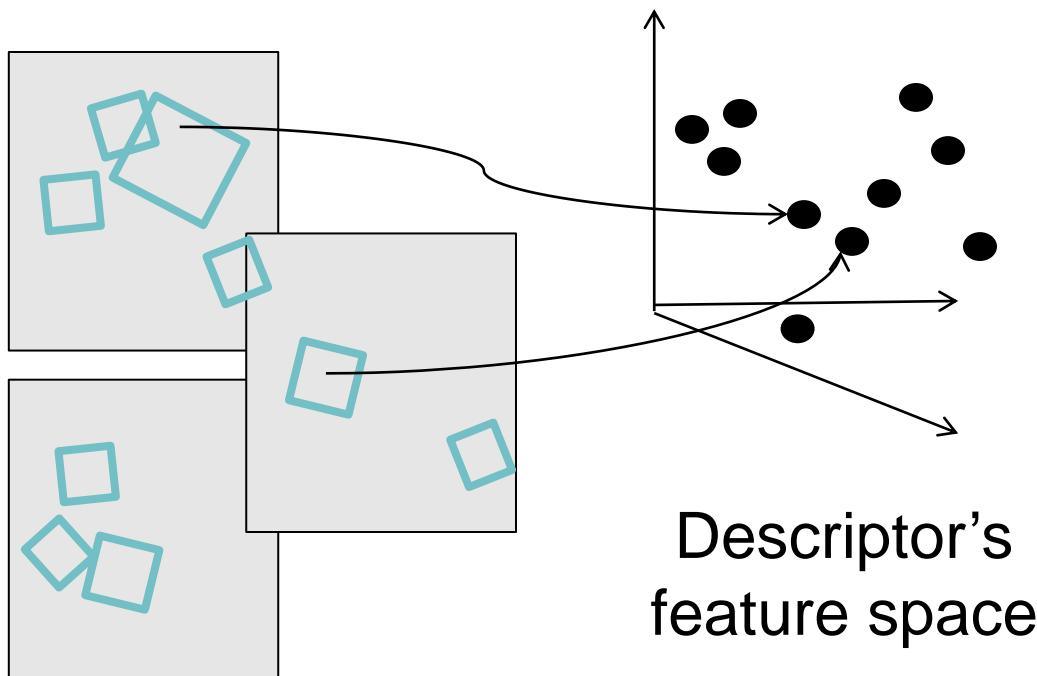
Few or No
Matches



But this will be really, really slow!

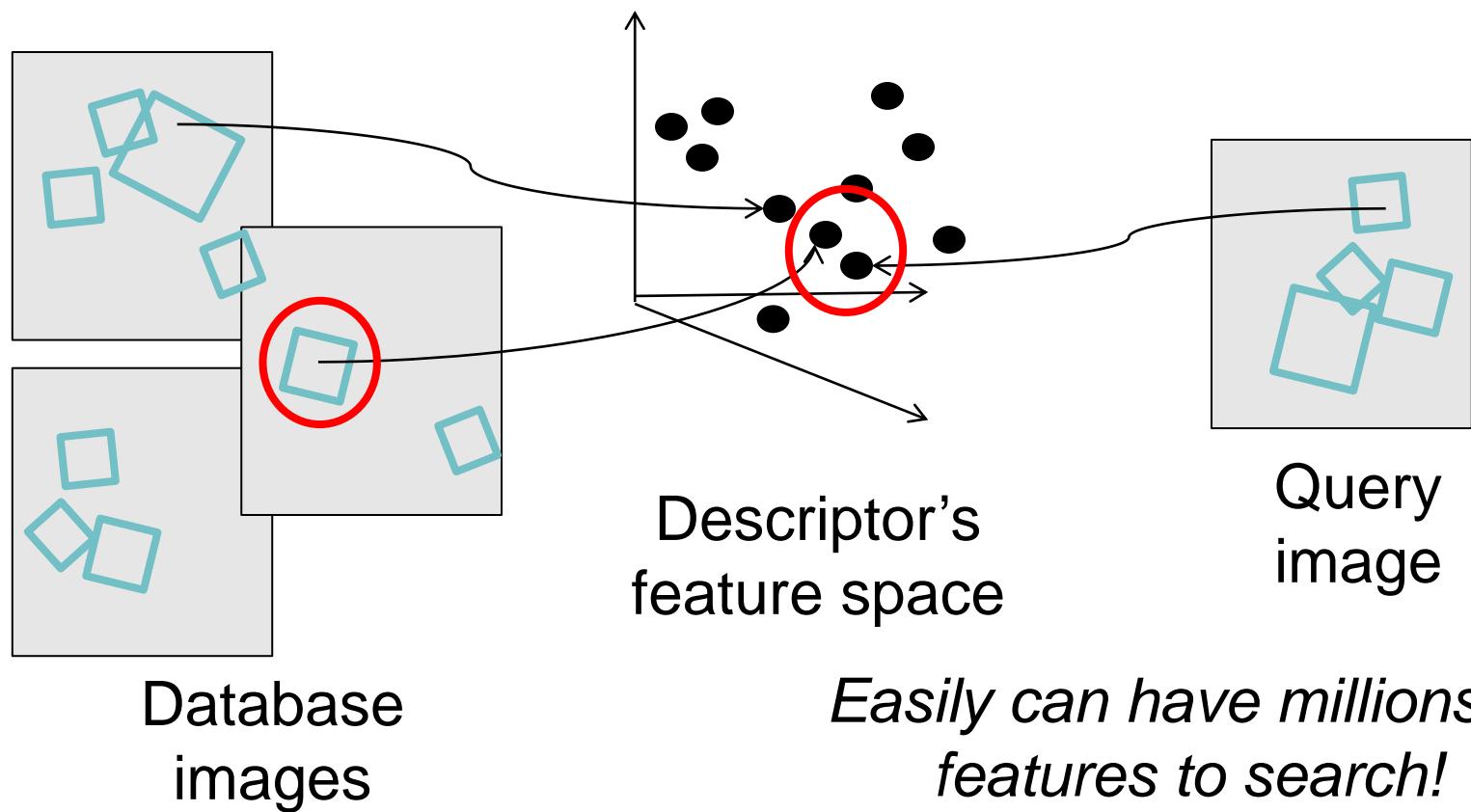
Indexing local features

- Each patch / region has a descriptor, which is a point in some high-dimensional feature space (e.g., SIFT)

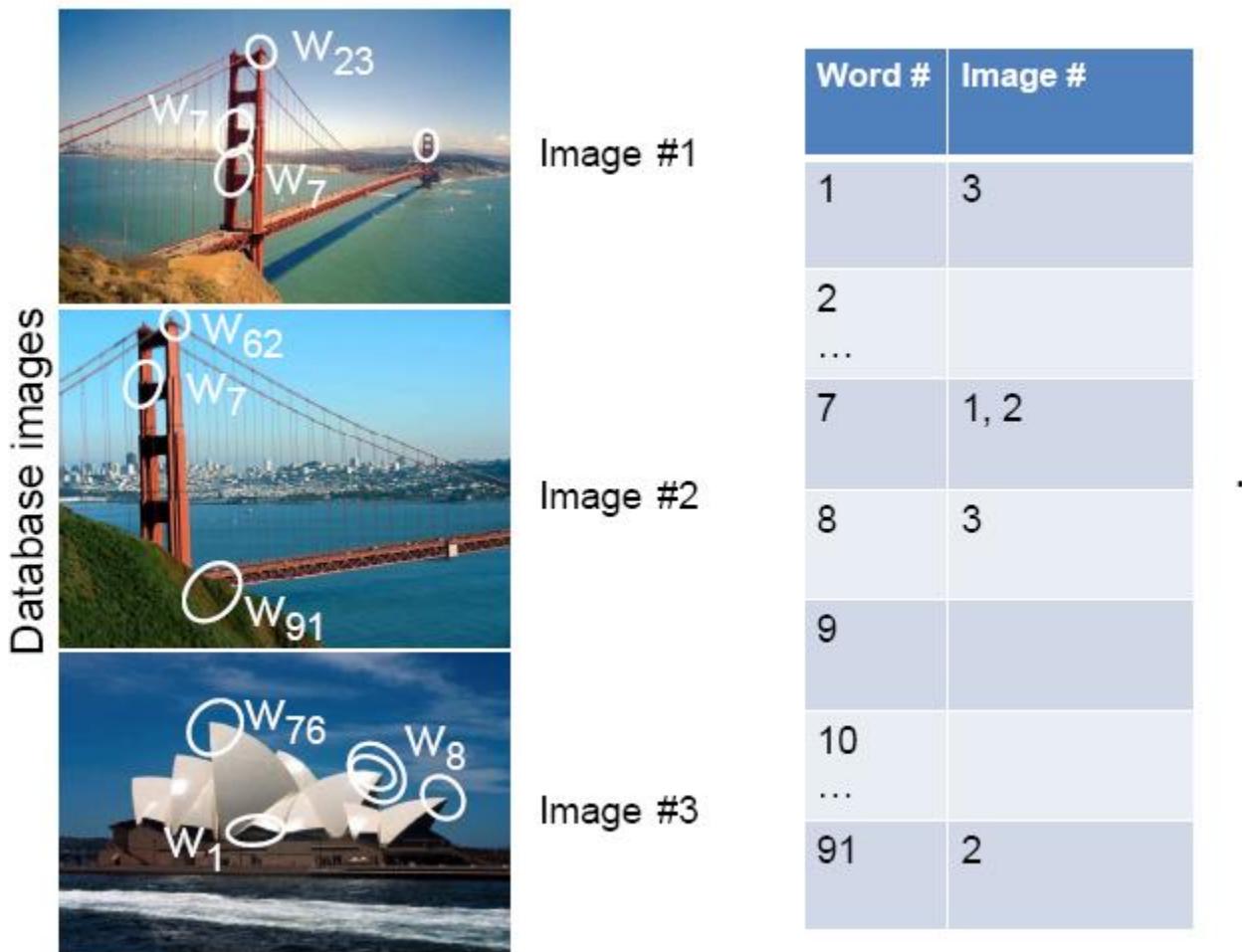


Indexing local features

- When we see close points in feature space, we have similar descriptors, which indicates similar local content.



Inverted file index



- Database images are loaded into the index mapping words to image numbers

Inverted file index



New query image

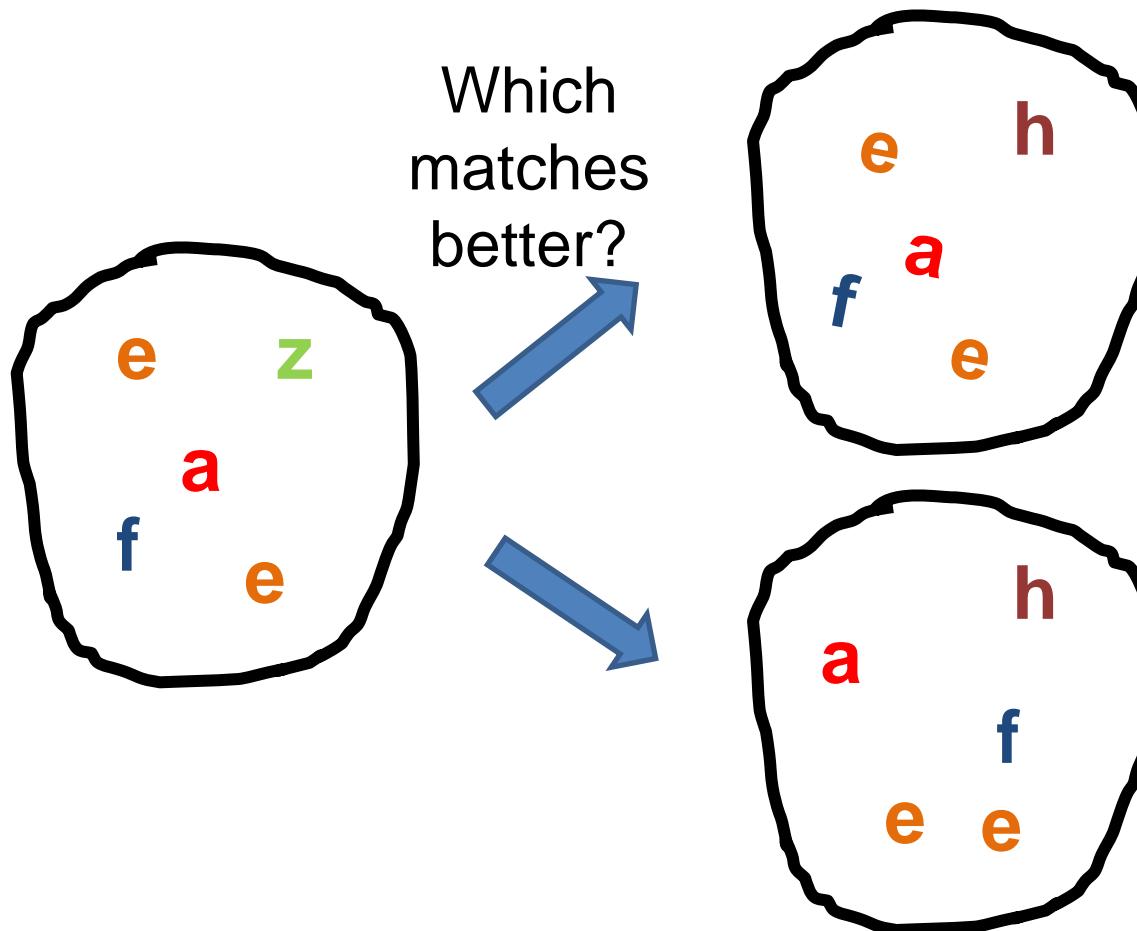
Word #	Image #
1	3
2	
7	1, 2
8	3
9	
10	
...	
91	2



- New query image is mapped to indices of database images that share a word.

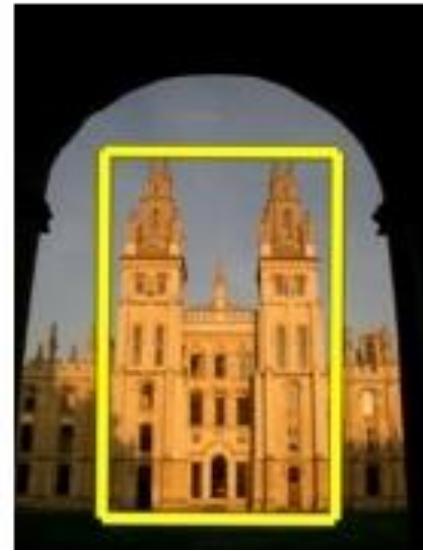
Can we be more accurate?

So far, we treat each image as containing a “bag of words”, with no spatial information



Can we be more accurate?

So far, we treat each image as containing a “bag of words”, with no spatial information



Real objects have consistent geometry

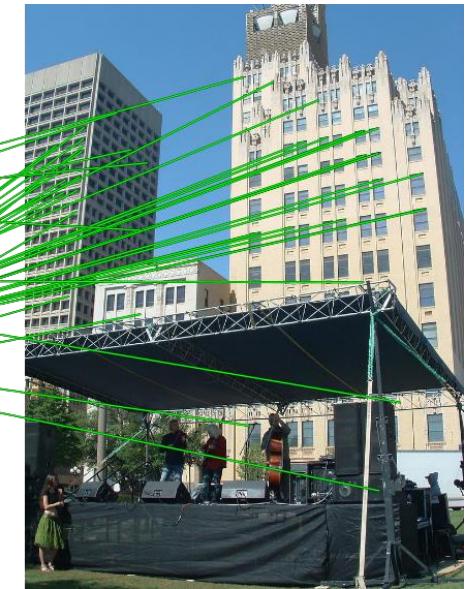
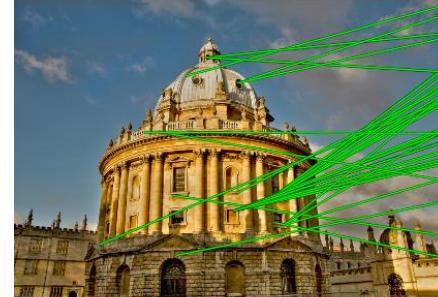
Spatial Verification

Query



DB image with high BoW
similarity

Query

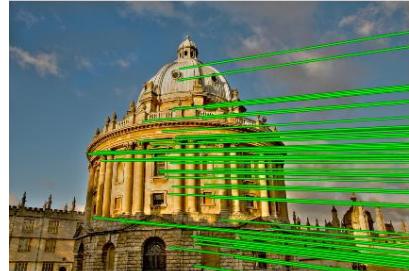


DB image with high BoW
similarity

Both image pairs have many visual words in common.

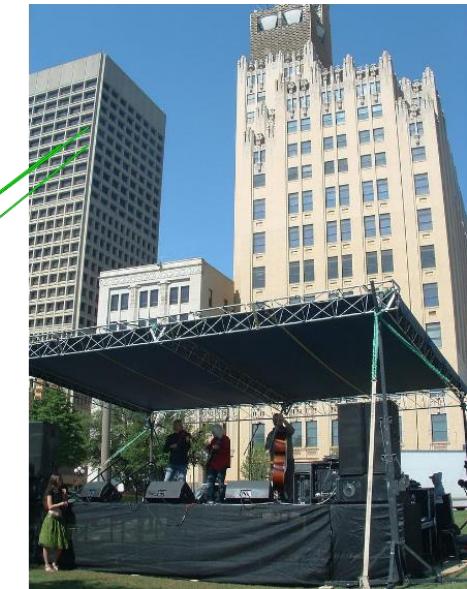
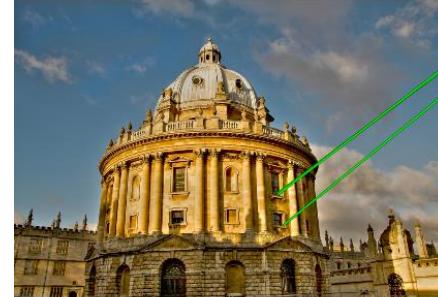
Spatial Verification

Query



DB image with high BoW
similarity

Query



DB image with high BoW
similarity

Only some of the matches are mutually consistent

Geometric verification with global constraints

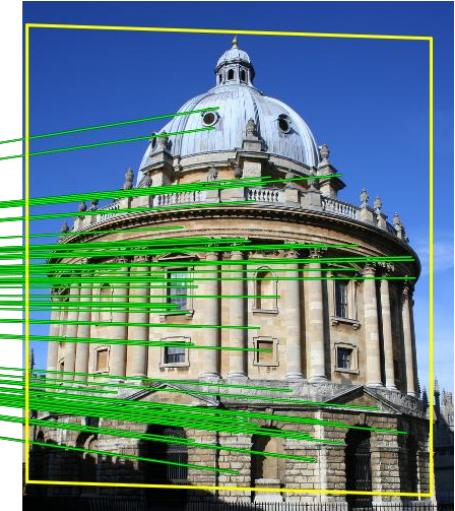
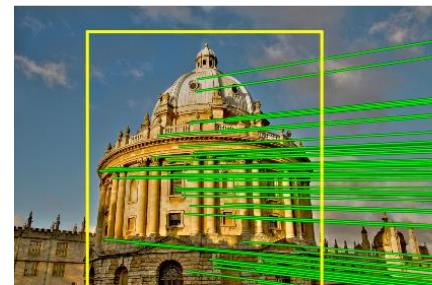
- All matches must be consistent with a global geometric relation / transformation.
- Need to simultaneously (i) estimate the geometric relation / transformation and (ii) the set of consistent matches



Tentative matches



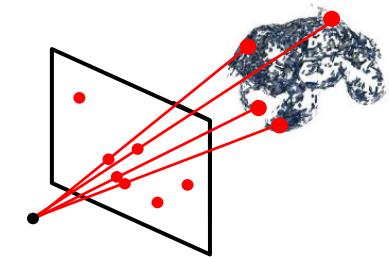
Matches consistent with an affine transformation



Examples of global constraints

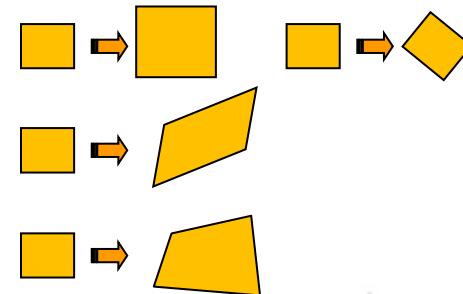
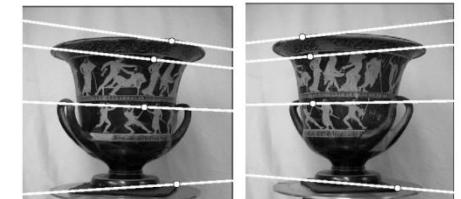
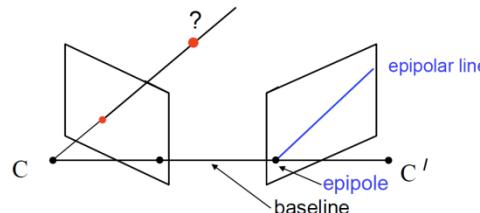
1 view and known 3D model.

- Consistency with a (known) 3D model.



2 views

- Epipolar constraint
- **2D transformations**
 - Similarity transformation
 - Affine transformation
 - Projective transformation



N-views

Are images consistent with a 3D model?

