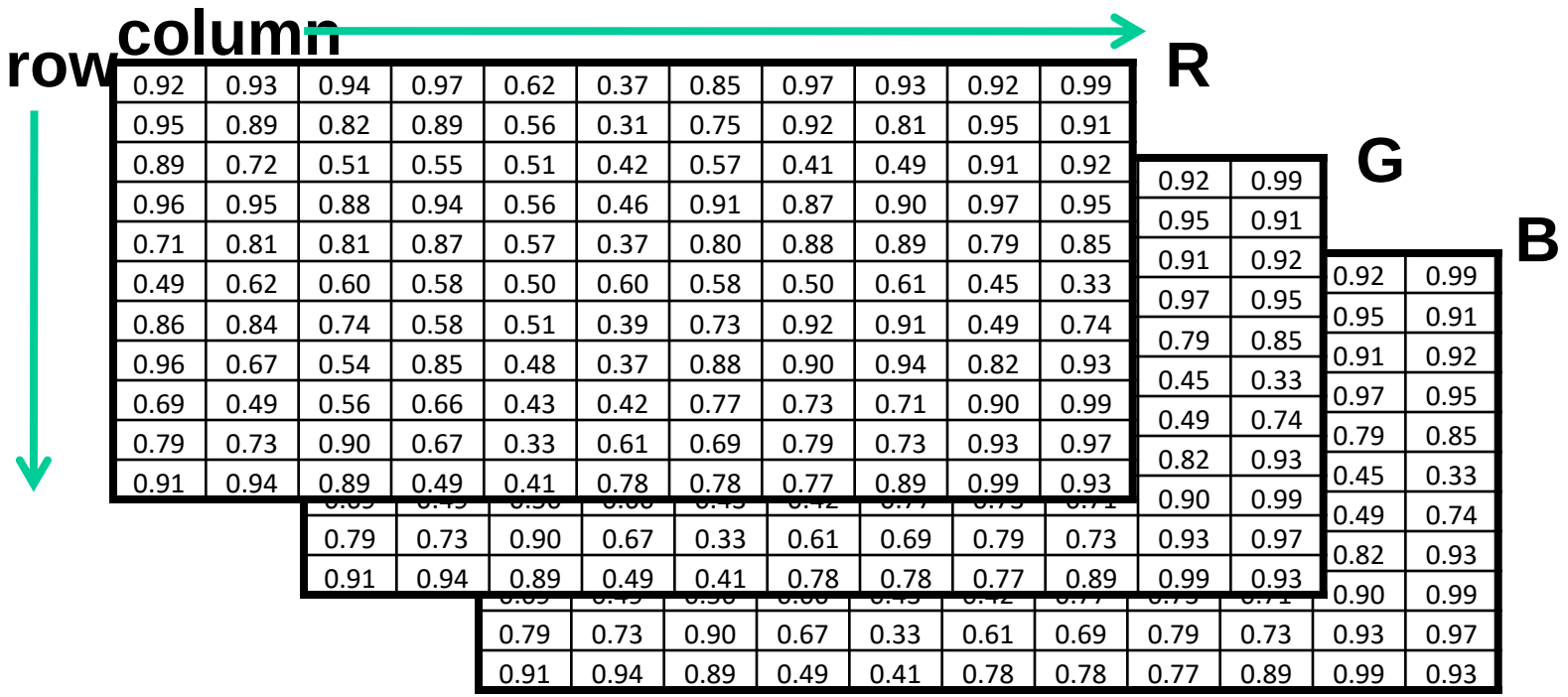
Image Processing

points, lines,
planes, transforms,
irradiance,
projections, …

Pixels!

uint8 [0..255]

Image as a matrix (tensor) or numbers



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

0.92	0.99
0.95	0.91
0.91	0.92
0.97	0.95
0.79	0.85
0.45	0.33
0.49	0.74
0.82	0.93
0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

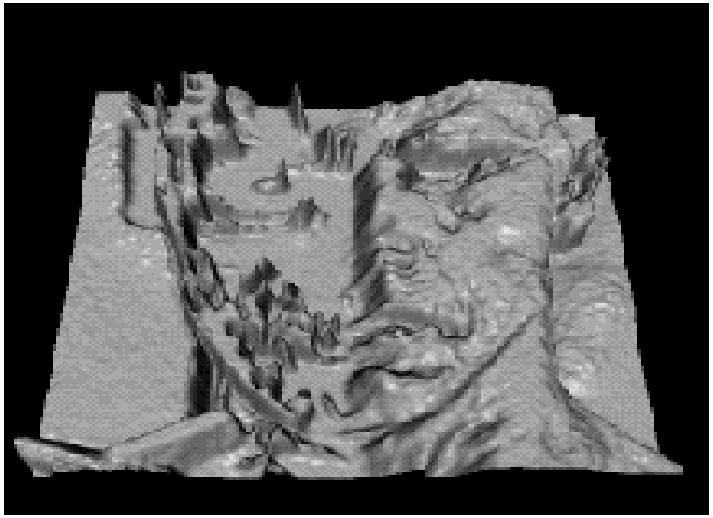
0.92	0.99
0.95	0.91
0.91	0.92
0.97	0.95
0.79	0.85
0.45	0.33
0.49	0.74
0.82	0.93
0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

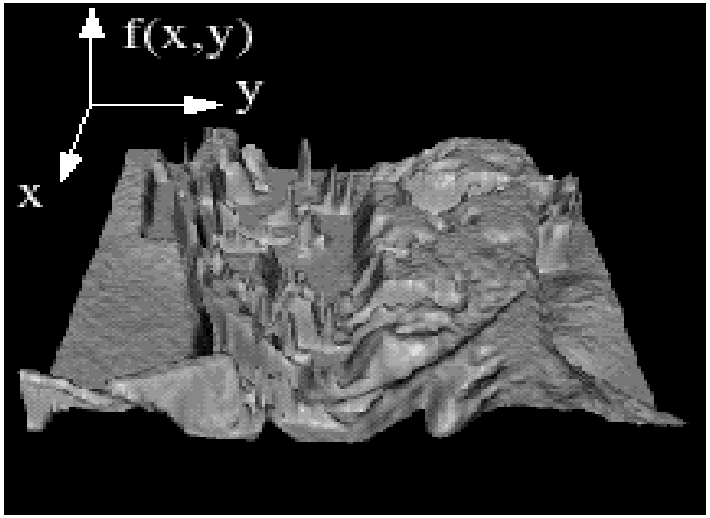
Image as a function

We can think of an **image** as a function, *f*, from R^2 to R:

- $f(x, y)$ gives the **intensity** at position (x, y)
- Realistically, we expect the image only to be defined over a rectangle, with a finite range:
  - $f: [a,b]\mathrm{x}[c,d] \rightarrow [0,1]$

Image as a function





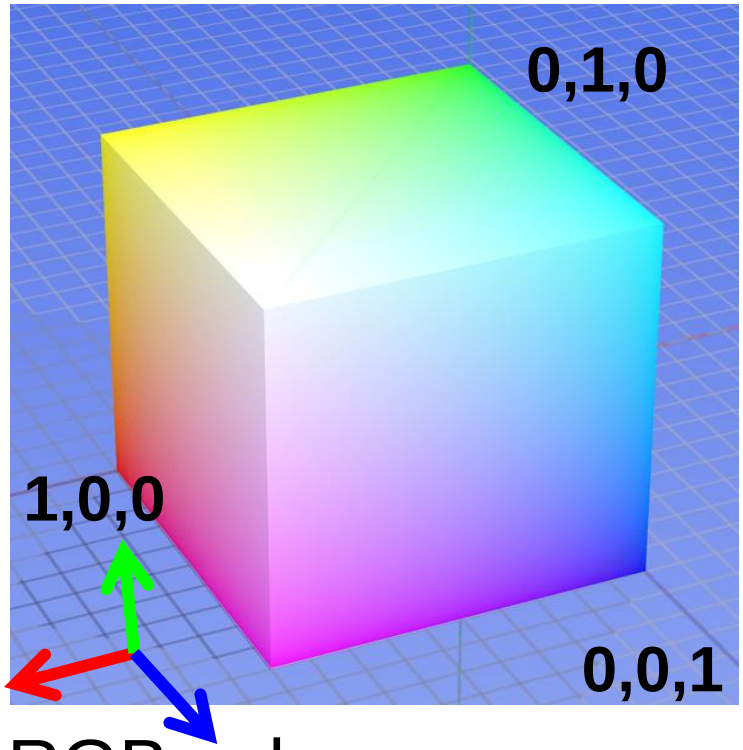
Color image as a function

A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x,y) = \begin{bmatrix} r(x,y) \\ g(x,y) \\ b(x,y) \end{bmatrix}$$

Color spaces: RGB

Default color space



RGB cube

- Easy for devices
- But not perceptual
- Where do the grays live?
- Where is hue and saturation?

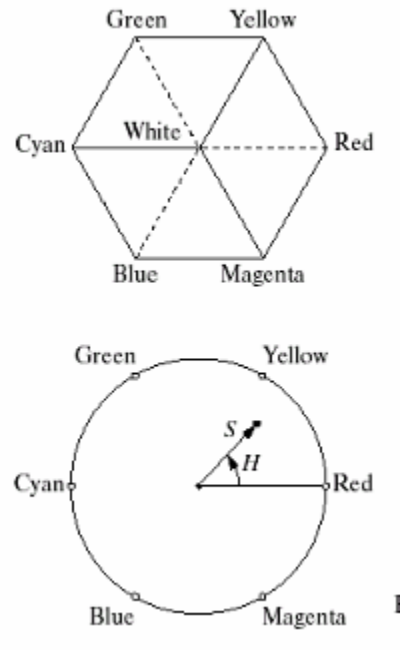
R
(G=0,B=0)

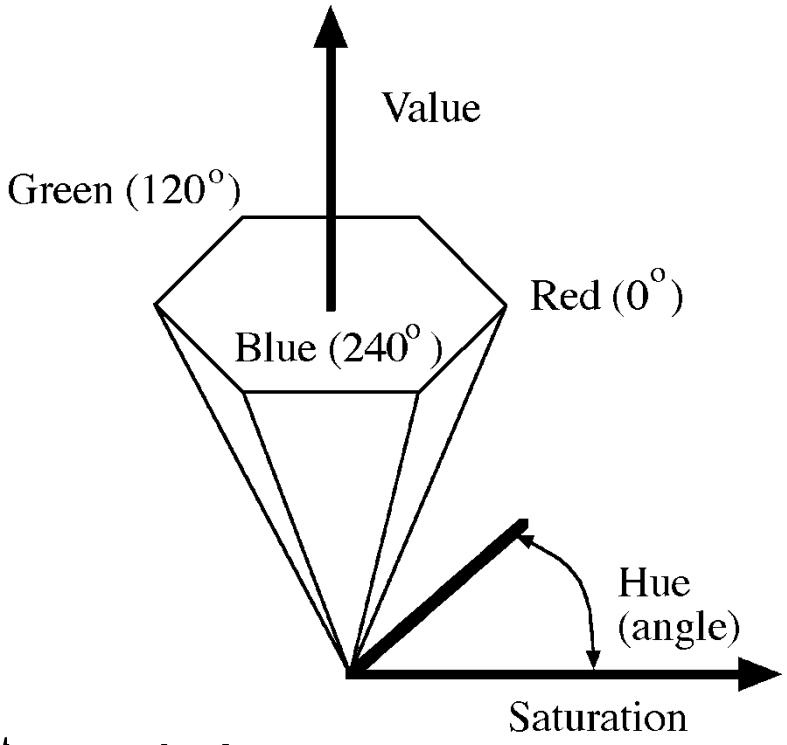
G
(R=0,B=0)

B
(R=0,G=0)

Image from: http://en.wikipedia.org/wiki/File:RGB_color_solid_cube.png

HSV





Hue, Saturation, Value (Intensity)

- RGB cube on its vertex

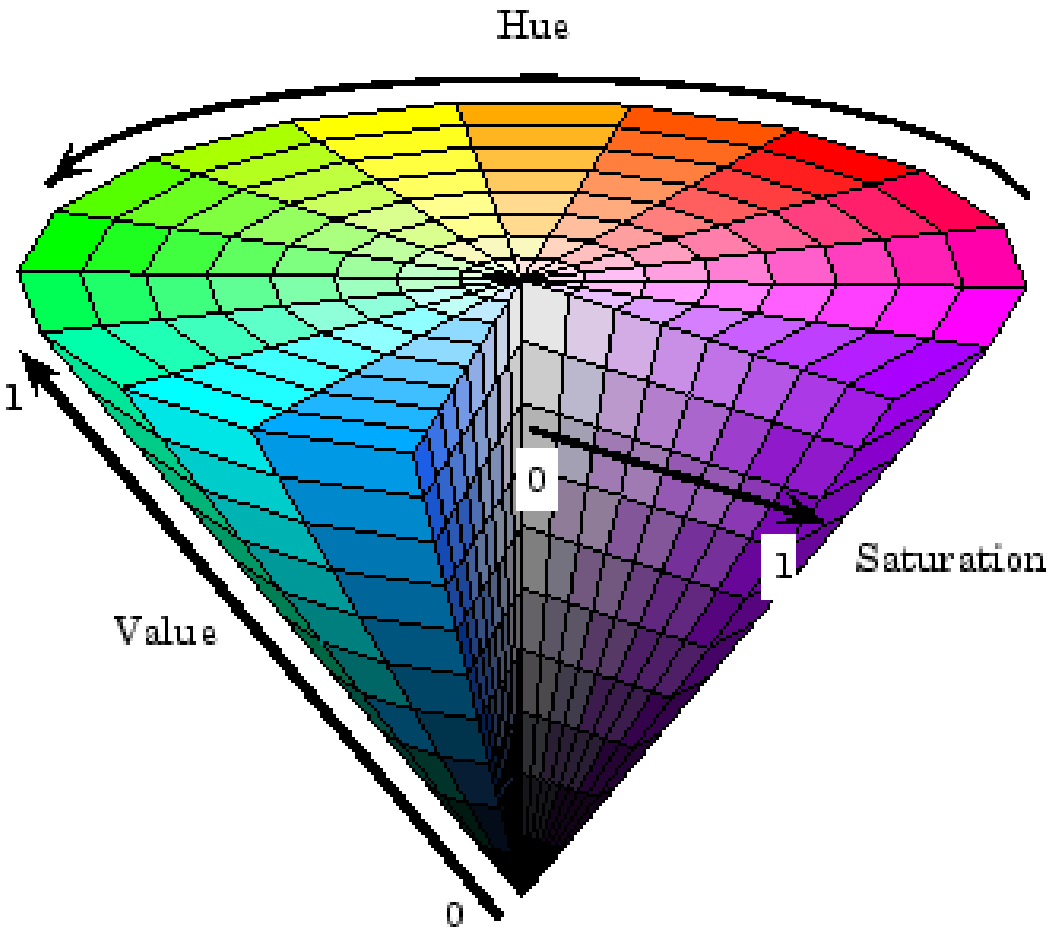
Decouples the three components (a bit)

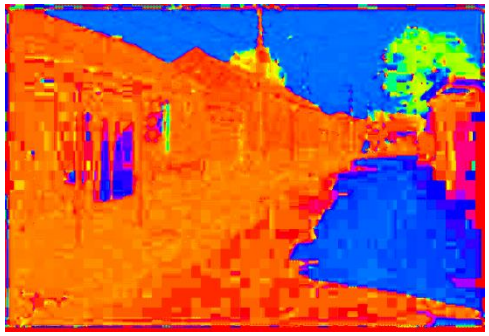
Use rgb2hsv() and hsv2rgb() in Matlab / Python

Slide by Steve Seitz

Color spaces: HSV

Intuitive color space





H

(S=1,V=1)

S

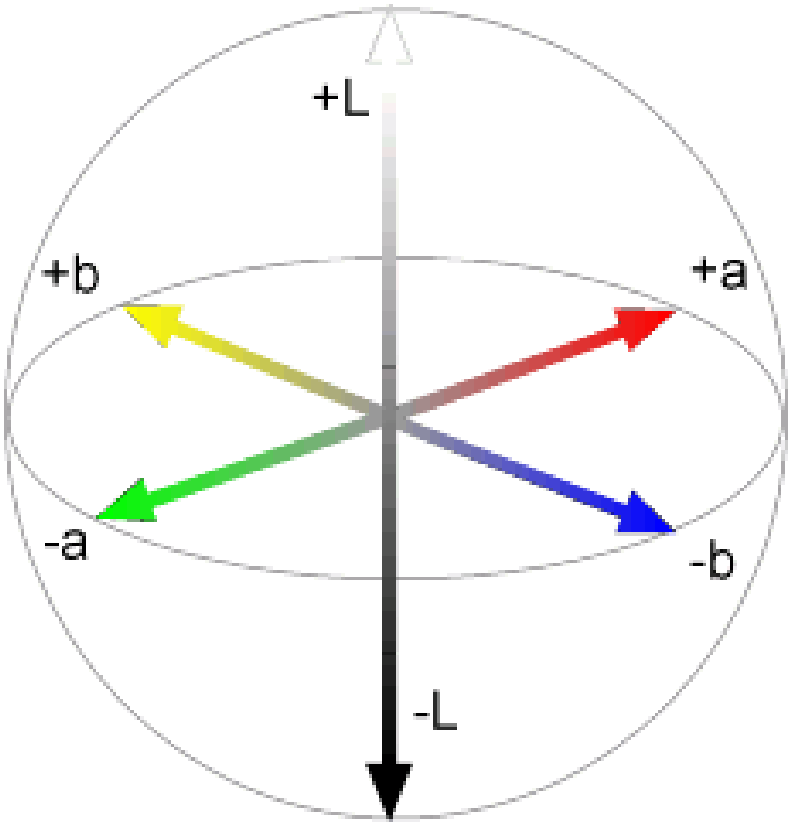
(H=1,V=1)

V

(H=1,S=0)

Color spaces: L*a*b*

"Perceptually uniform"* color space

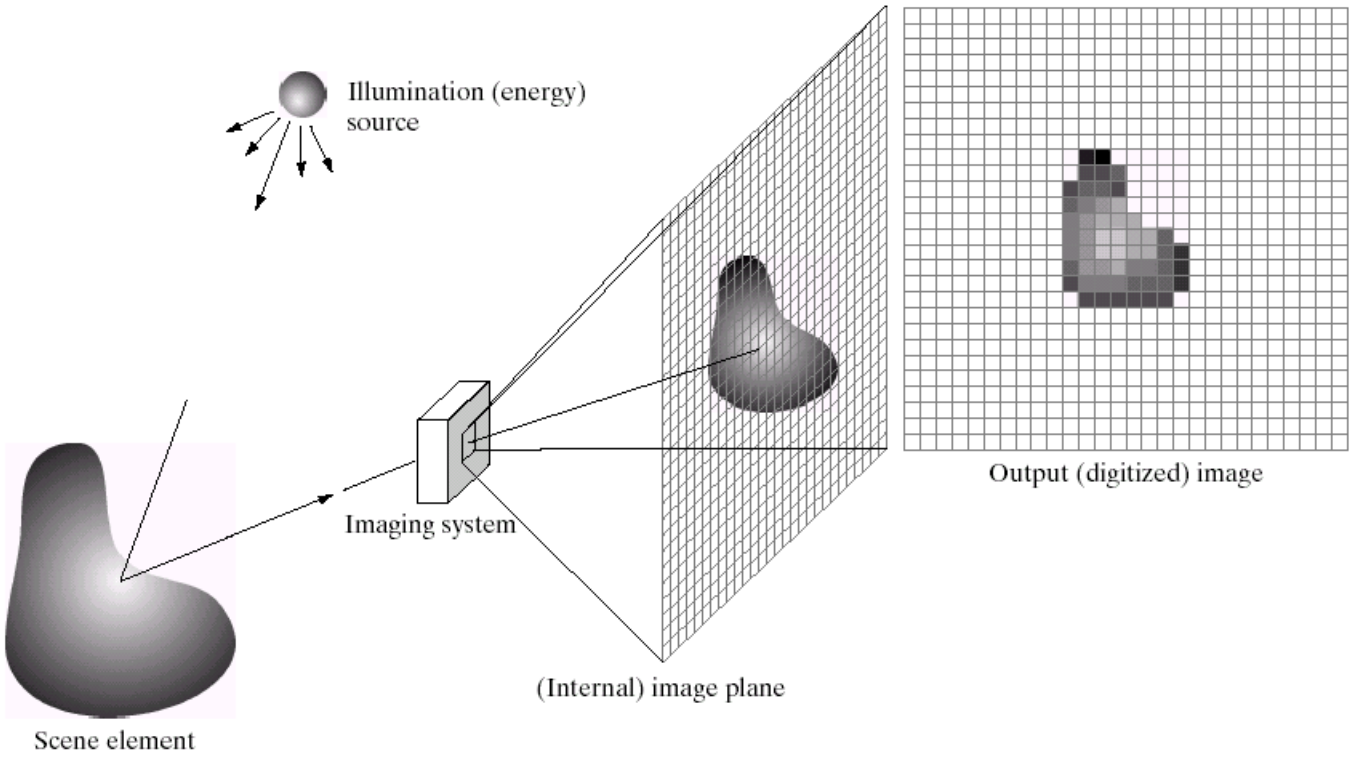


L
(a=0,b=0)

a
(L=65,b=0)

b
(L=65,a=0)

What's in the "pixel intensity"?



a b c d e

FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

*f(x,y) = reflectance(x,y) * illumination(x,y)*

Reflectance in [0,1], illumination in [0,inf]

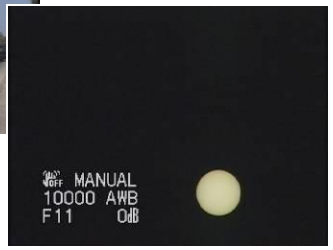
Problem: Dynamic Range

1

1500

25,000

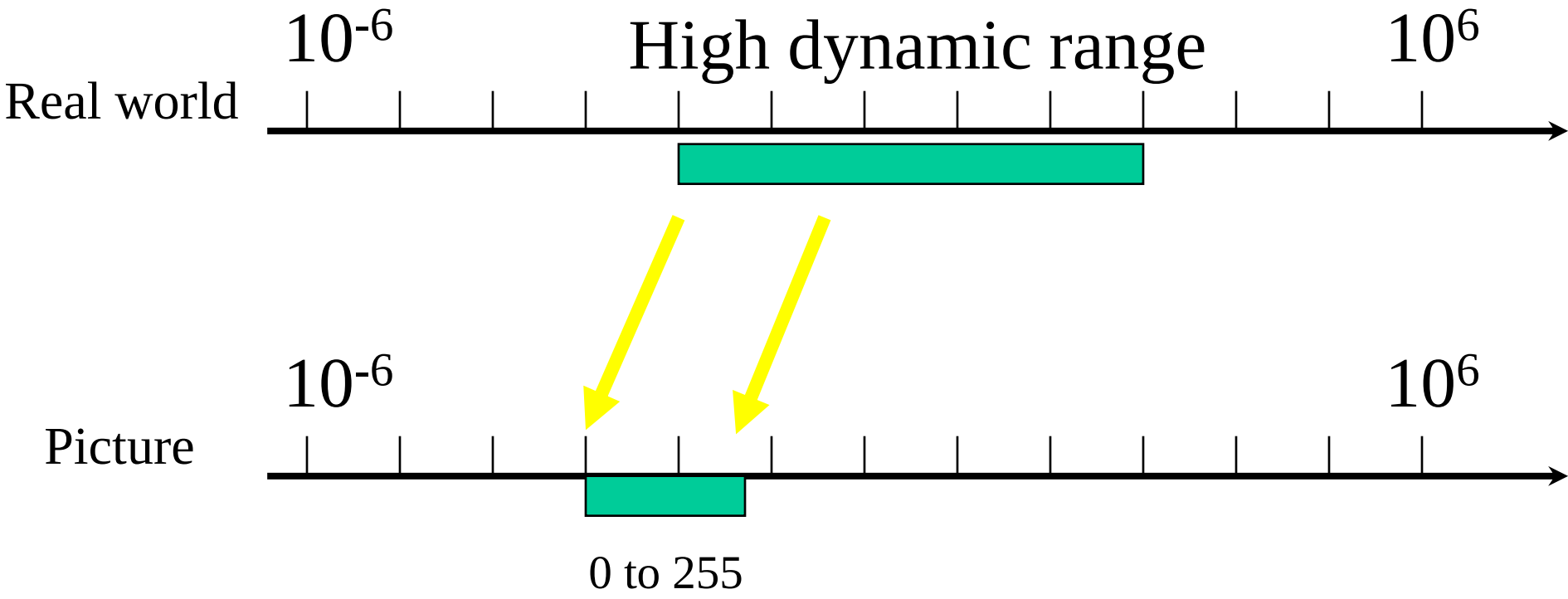
400,000



2,000,000,000

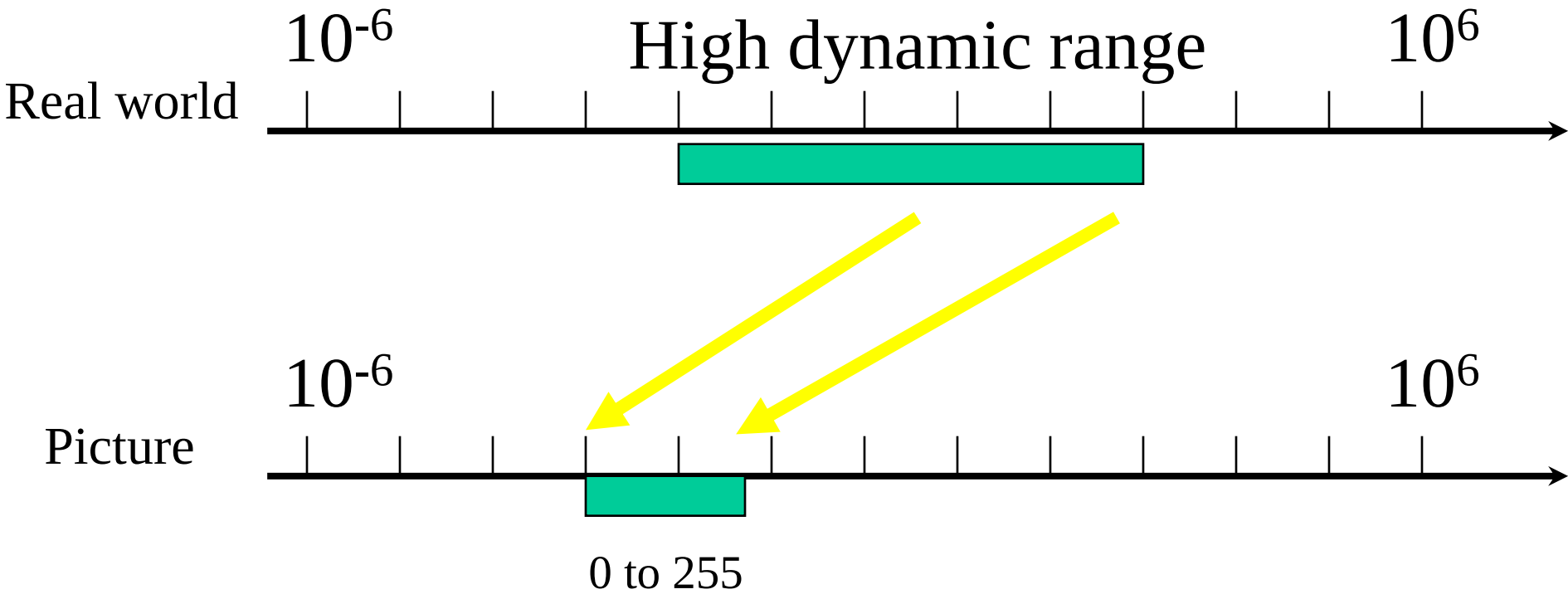
The real world is
High dynamic range

Long Exposure



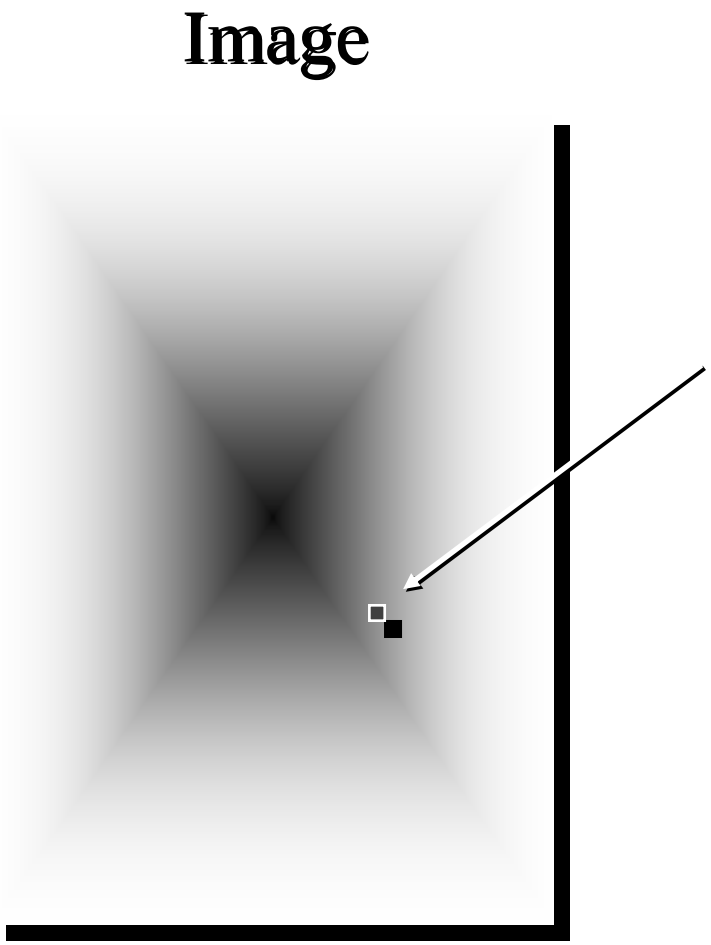
- What does pixel value 255 mean?

Short Exposure



- What does pixel value 0 mean?

Is Camera a photometer?



pixel (312, 284) = 42

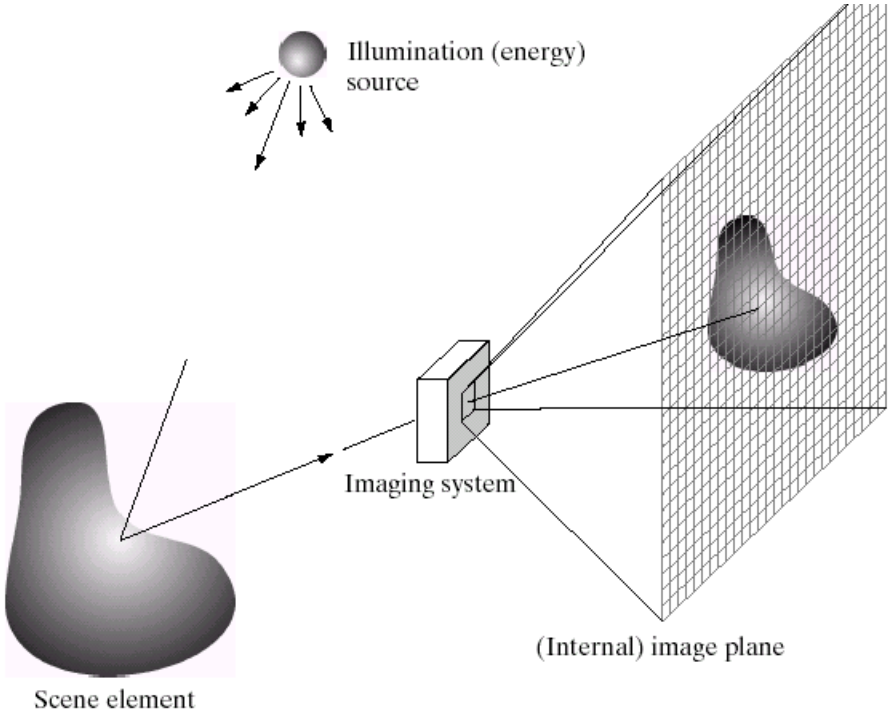
42 photos?

Don’t trust the pixel values

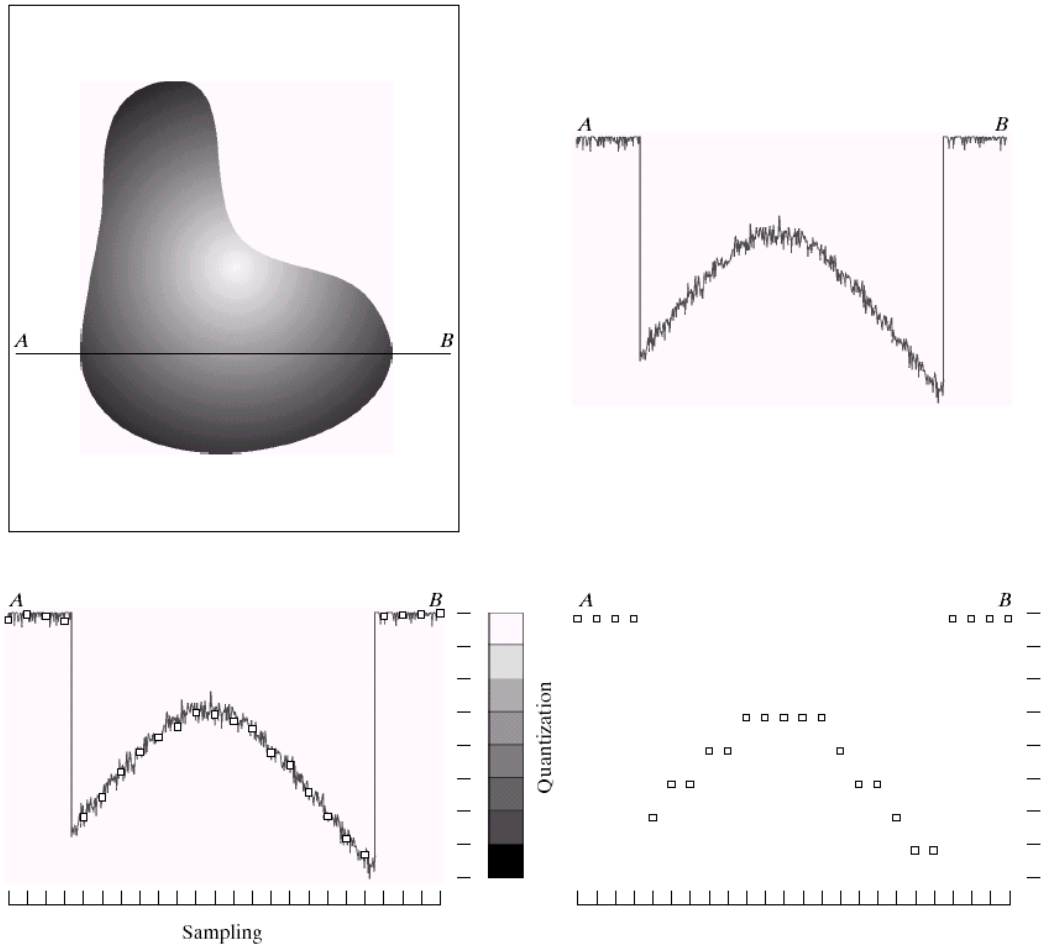
a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0, respectively. (Original image for this example courtesy of NASA.)

Sampling and Quantization



Sampling and Quantization

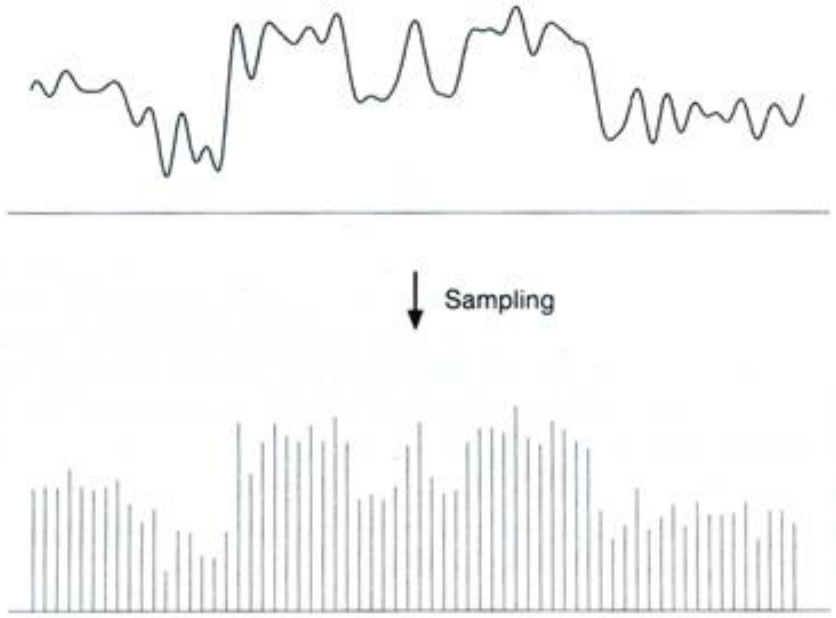


a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Sampled representations

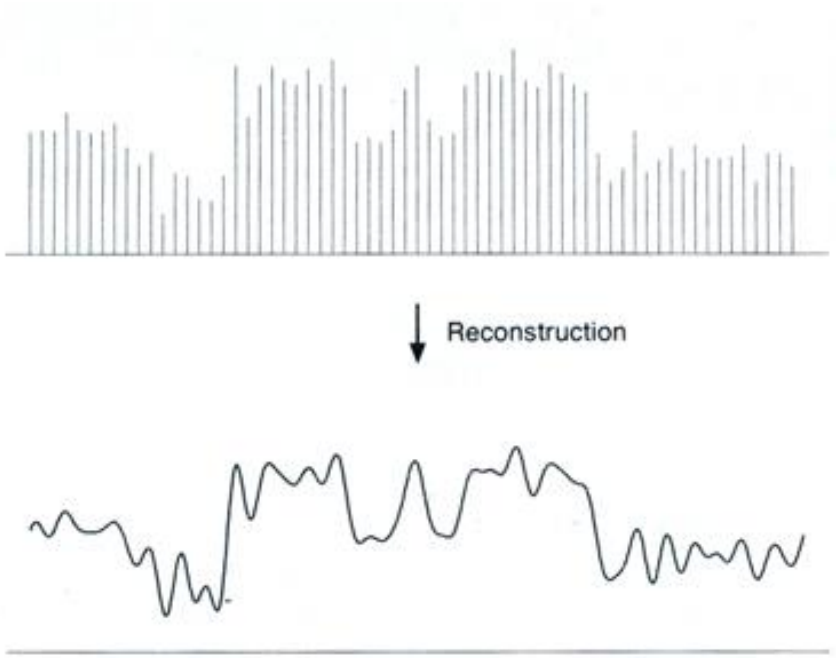
- How to store and compute with continuous functions?
- Common scheme for representation: samples
  - write down the function's values at many points



[FvDFH fig.14.14b / Wolberg]

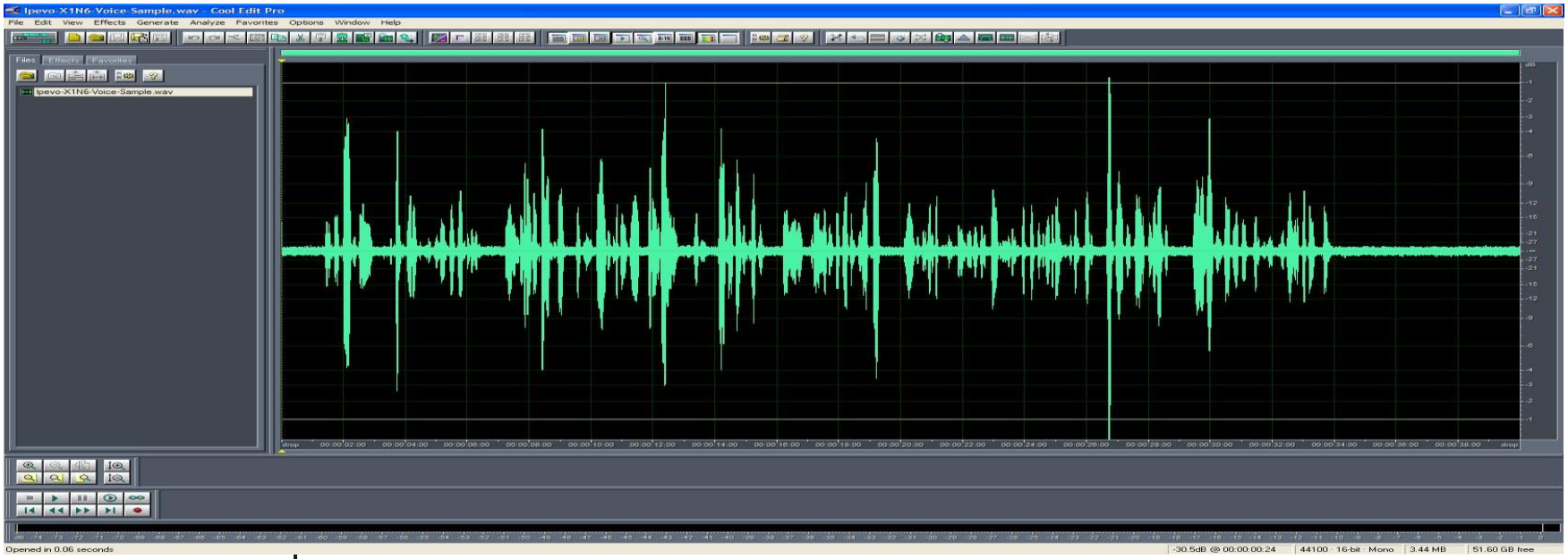
Reconstruction

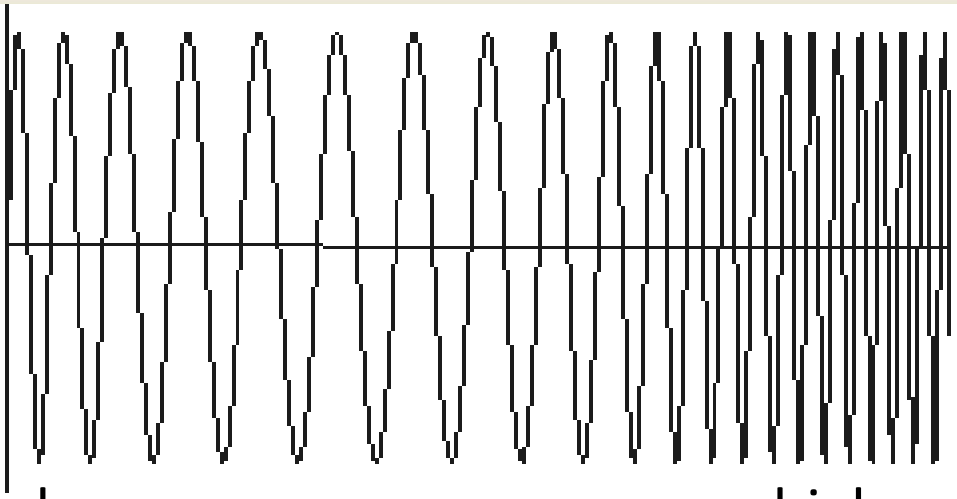
- Making samples back into a continuous function
  - for output (need realizable method)
  - for analysis or processing (need mathematical method)
  - amounts to “guessing” what the function did in between



[FvDFH fig.14.14b / Wolberg]

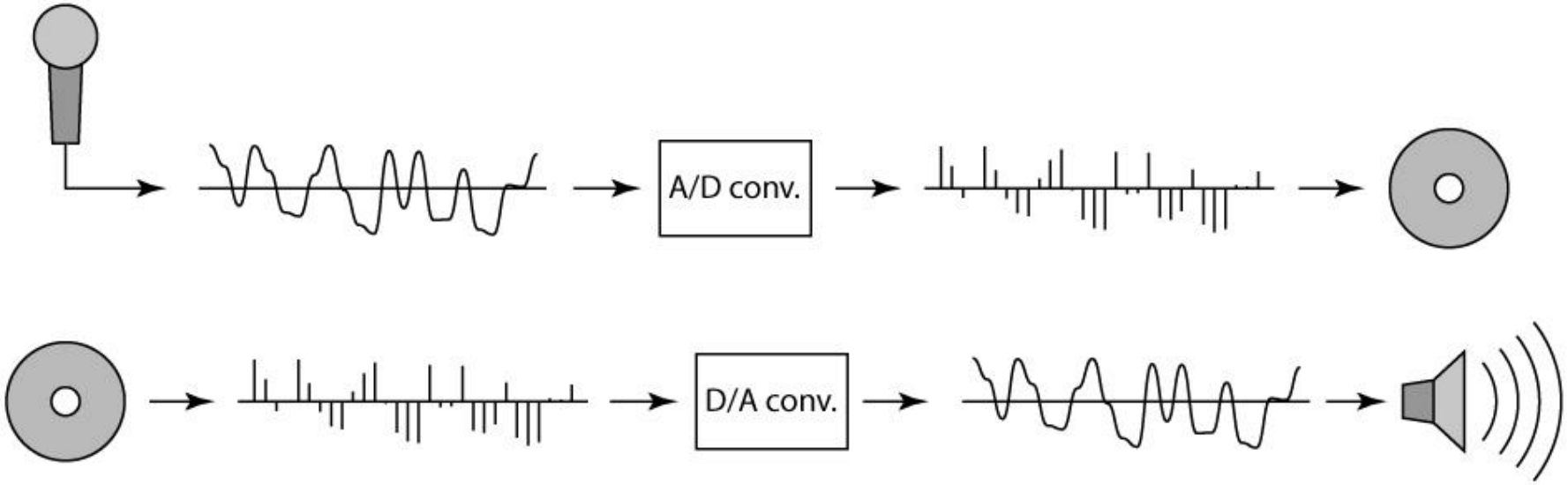
1D Example: Audio





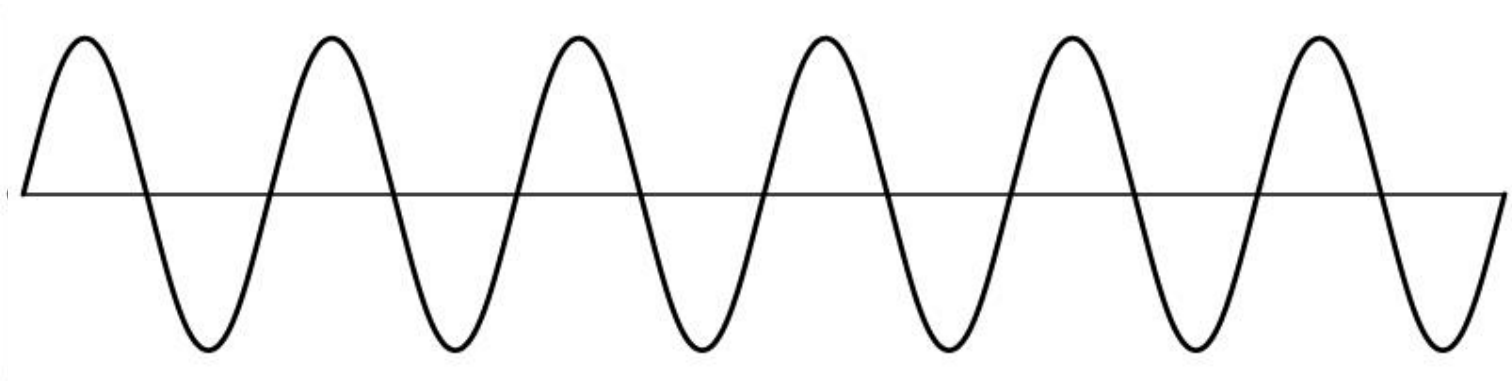
Sampling in digital audio

- Recording: sound to analog to samples to disc
- Playback: disc to samples to analog to sound again
  - how can we be sure we are filling in the gaps correctly?



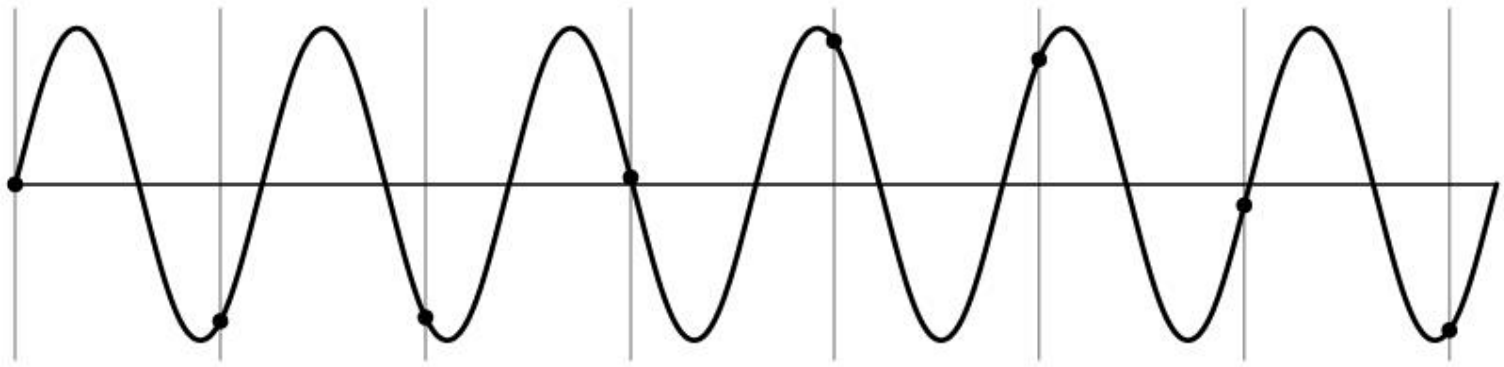
Sampling and Reconstruction

- Simple example: a sign wave



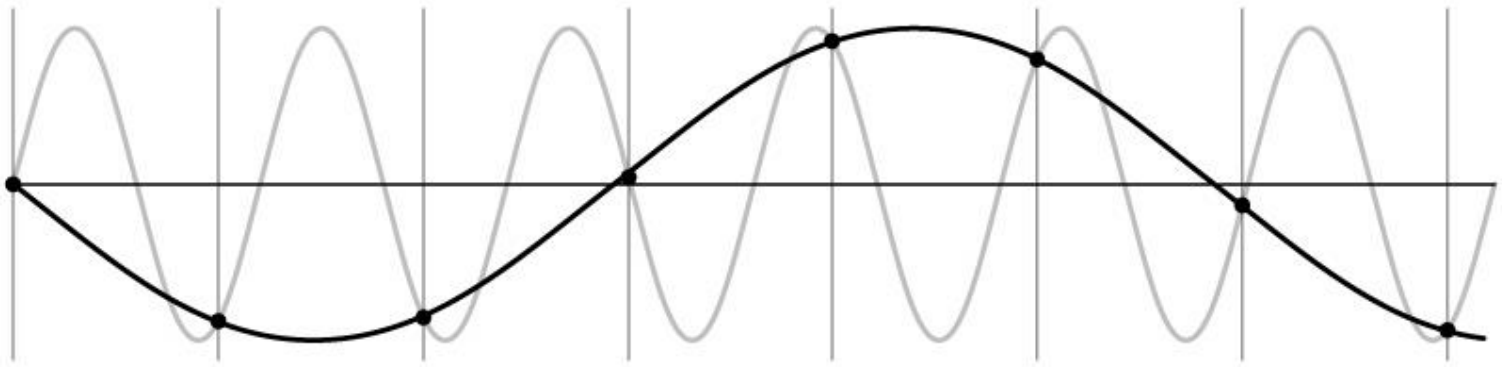
Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost



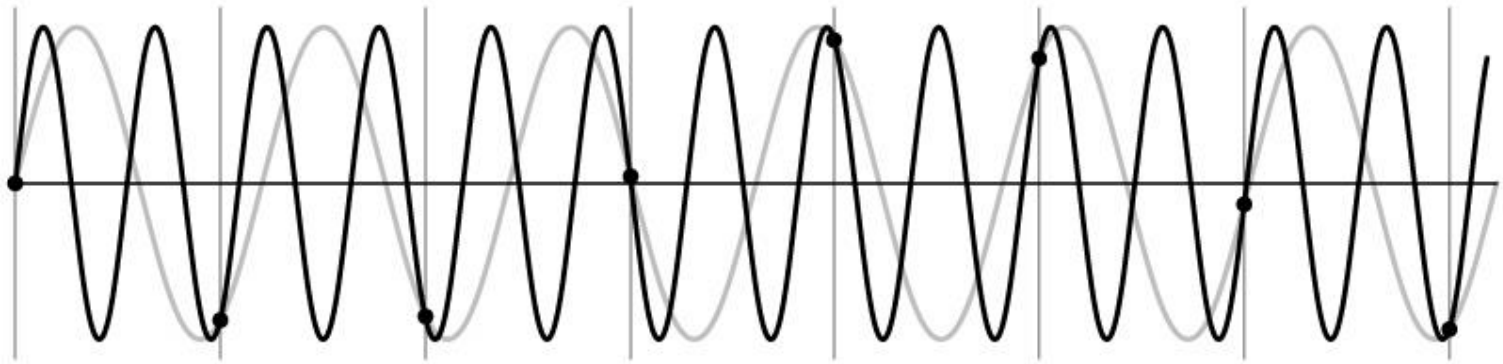
Undersampling

- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency



Undersampling

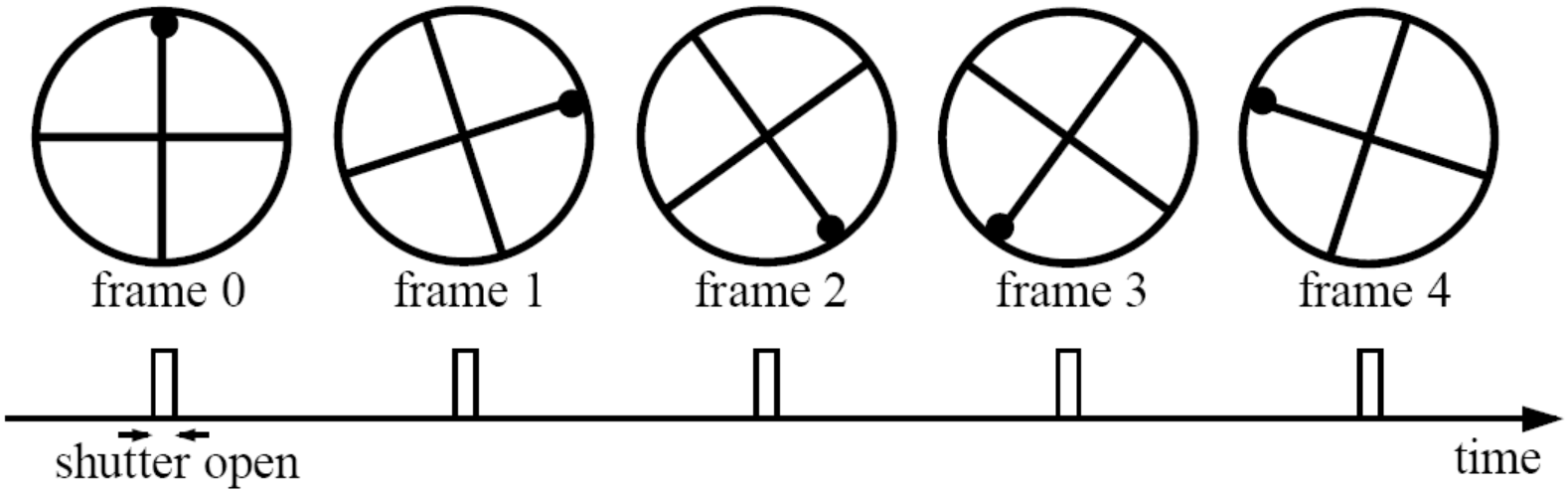
- What if we “missed” things between the samples?
- Simple example: undersampling a sine wave
  - unsurprising result: information is lost
  - surprising result: indistinguishable from lower frequency
  - also, was always indistinguishable from higher frequencies
  - *aliasing*: signals “traveling in disguise” as other frequencies



Aliasing in video

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what's happening.

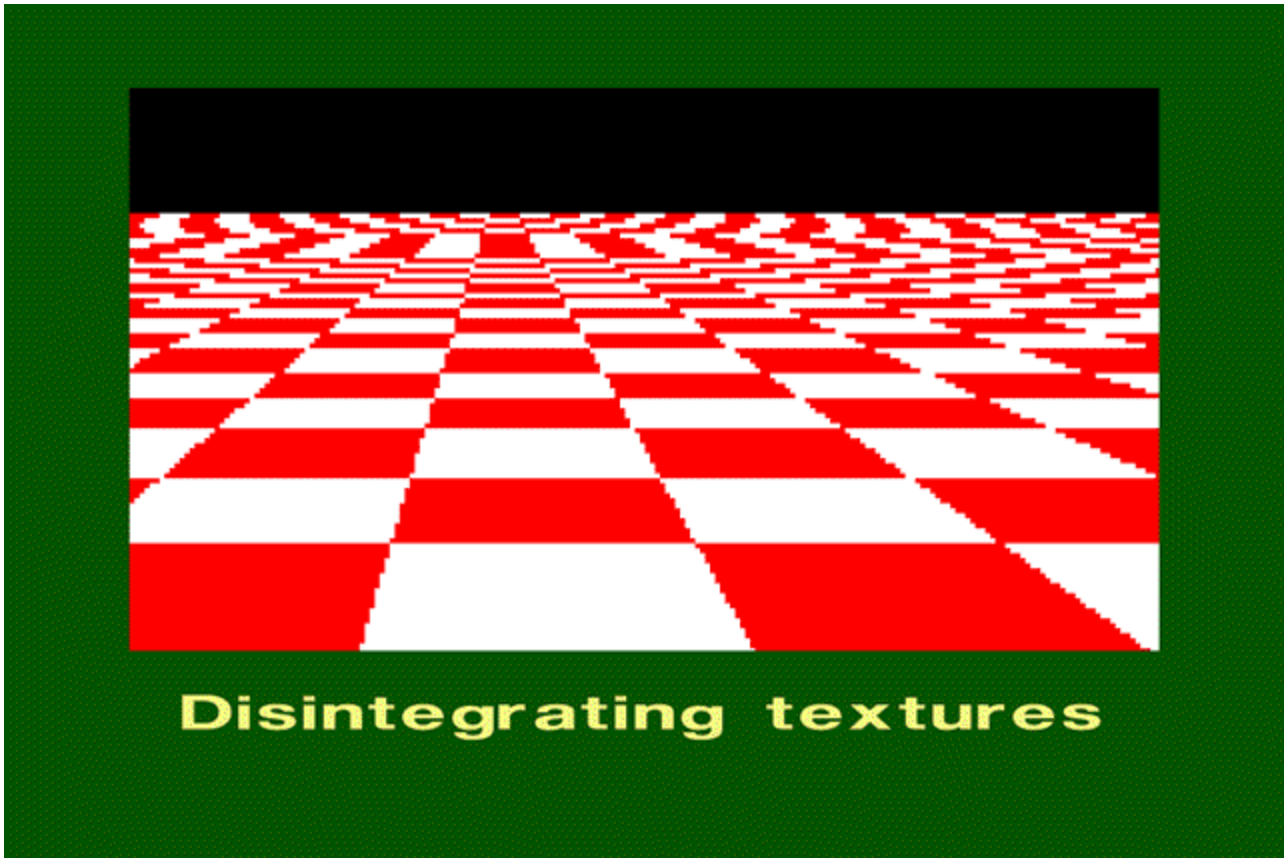
If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):



Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

Slide by Steve Seitz

Aliasing in images



Antialiasing

What can we do about aliasing?

Sample more often

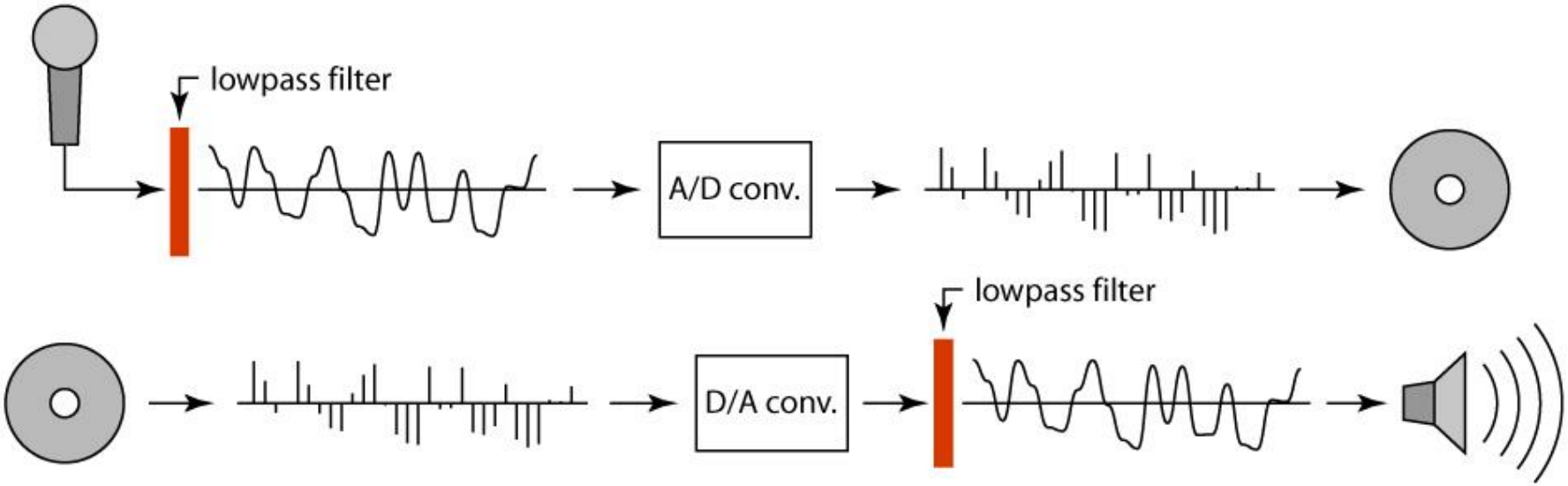
- Join the Mega-Pixel craze of the photo industry
- But this can’t go on forever

Make the signal less “wiggly”

- Get rid of some high frequencies
- Will loose information
- But it’s better than aliasing

Preventing aliasing

- Introduce lowpass filters:
  - remove high frequencies leaving only safe, low frequencies
  - choose lowest frequency in reconstruction (disambiguate)

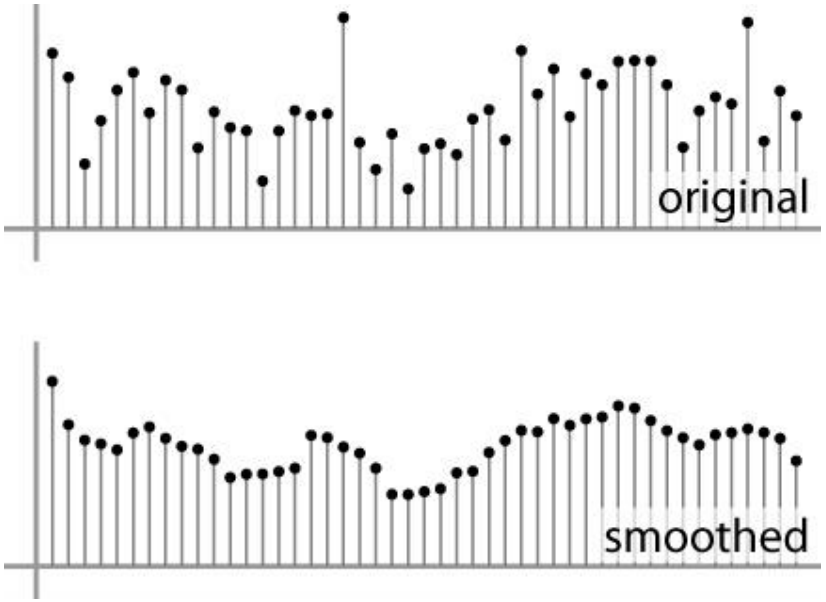


Linear filtering: a key idea

- Transformations on signals; e.g.:
  - bass/treble controls on stereo
  - blurring/sharpening operations in image editing
  - smoothing/noise reduction in tracking
- Key properties
  - linearity: filter($f + g$) = filter(f) + filter(g)
  - shift invariance: behavior invariant to shifting the input
    - delaying an audio signal
    - sliding an image around
- Can be modeled mathematically by *convolution*

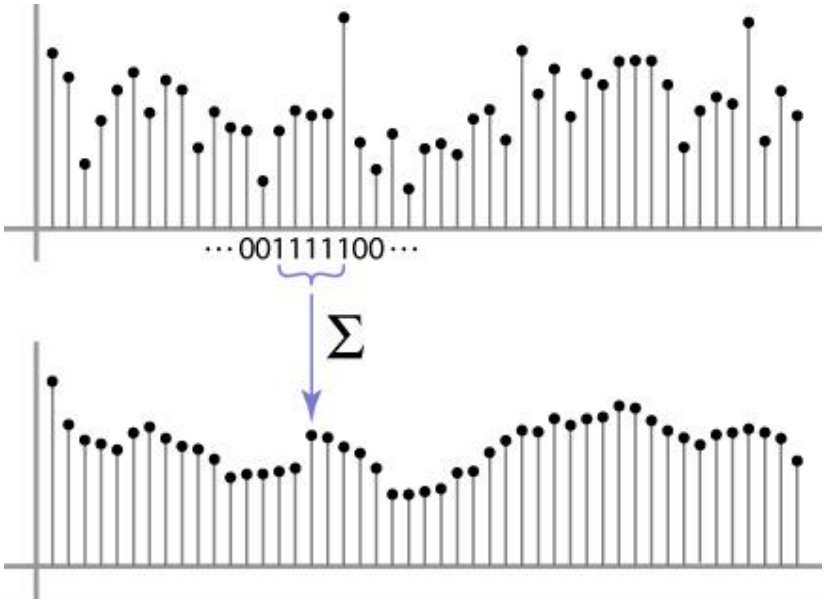
Moving Average

- basic idea: define a new function by averaging over a sliding window
- a simple example to start off: smoothing



Moving Average

- Can add weights to our moving average
- *Weights* […, 0, 1, 1, 1, 1, 1, 0, …] / 5



In 2D: box filter

$$h[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10							

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20						

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
				?					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30				
						?			
				50					

Credit: S. Seitz

Image filtering

$h[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$g[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$g[m,n] = \sum_{k,l} h[k,l]\ f[m+k, n+l]$$

Credit: S. Seitz

Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

- Achieve smoothing effect (remove sharp features)

$$h[\cdot,\cdot]$$

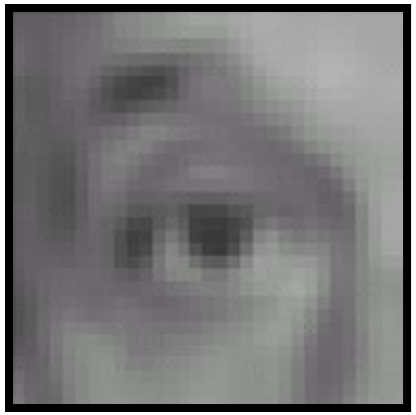
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Linear filters: examples

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

Source: D. Lowe

Cross-correlation

Let F be the image, H be the kernel (of size 2*k*+1 x 2*k*+1), and G be the output image

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u,j+v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

- Can think of as a "dot product" between local neighborhood and kernel for each pixel

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

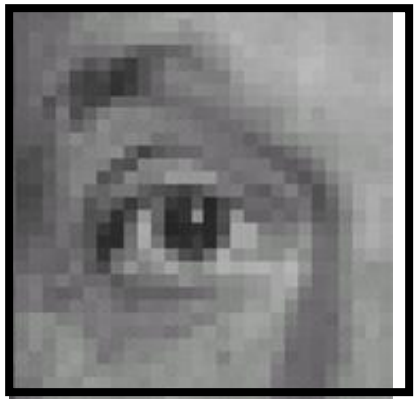
?

Source: D. Lowe

Practice with linear filters

Original

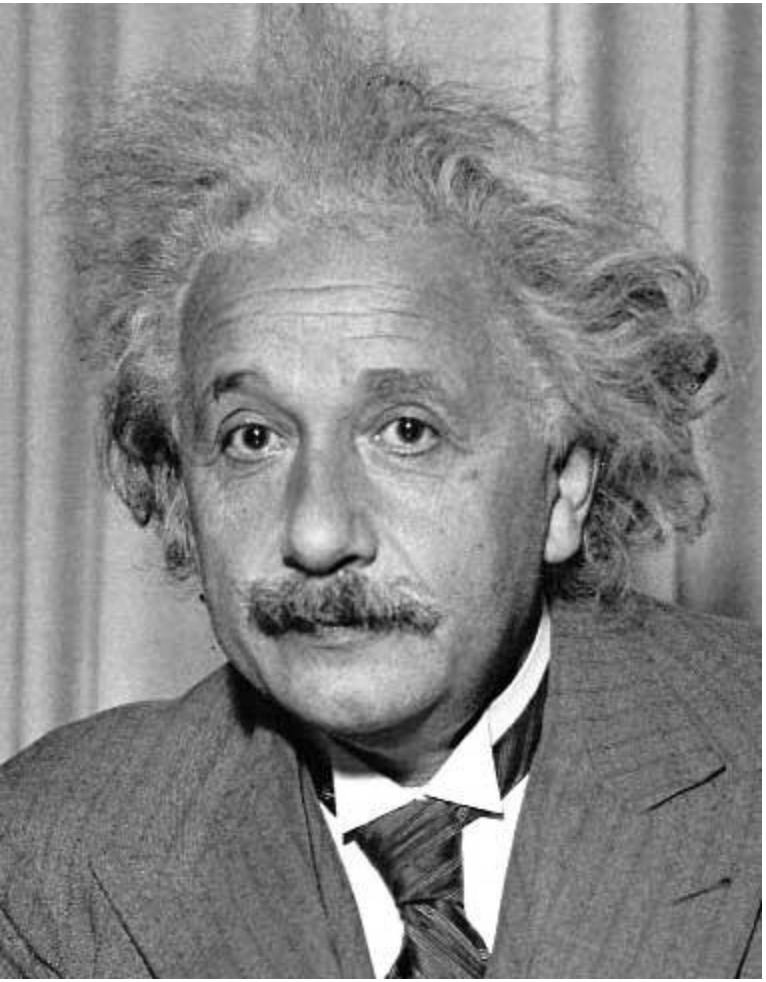
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

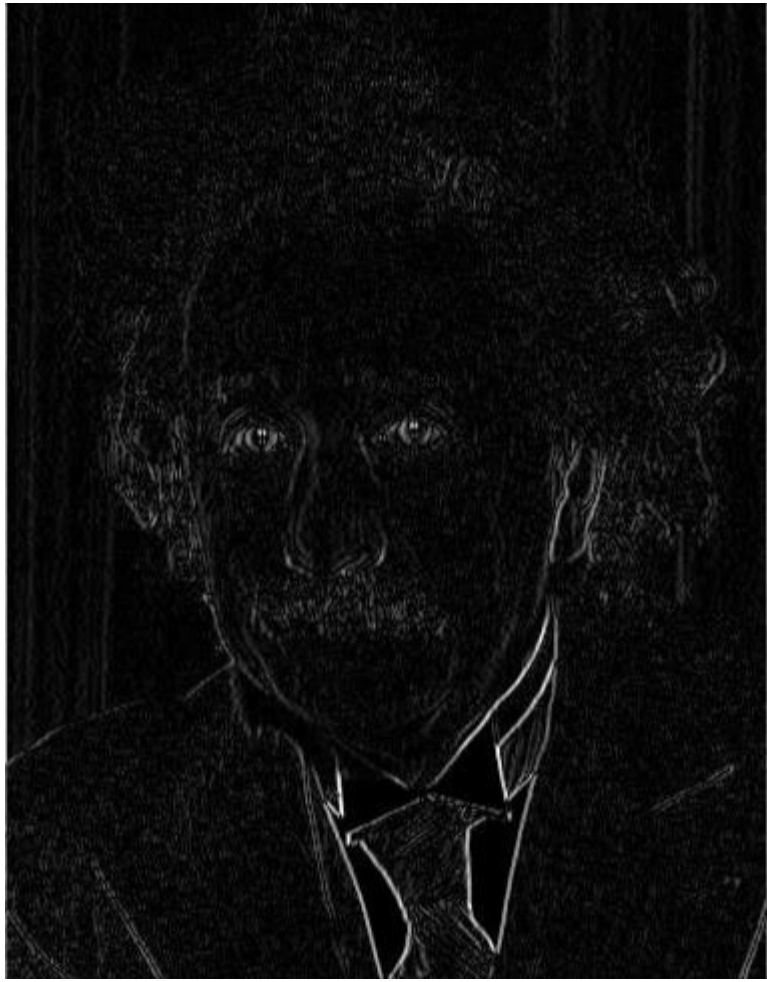
Source: D. Lowe

Other filters



1	0	-1
2	0	-2
1	0	-1

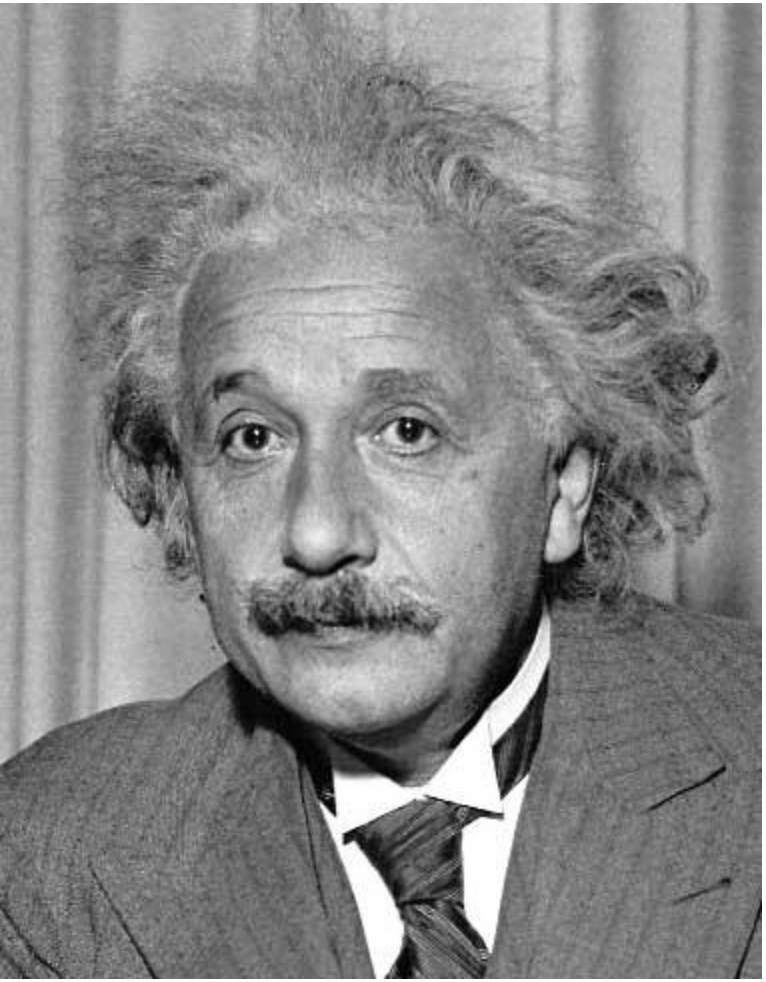
Sobel



Vertical Edge
(absolute value)

Q?

Other filters

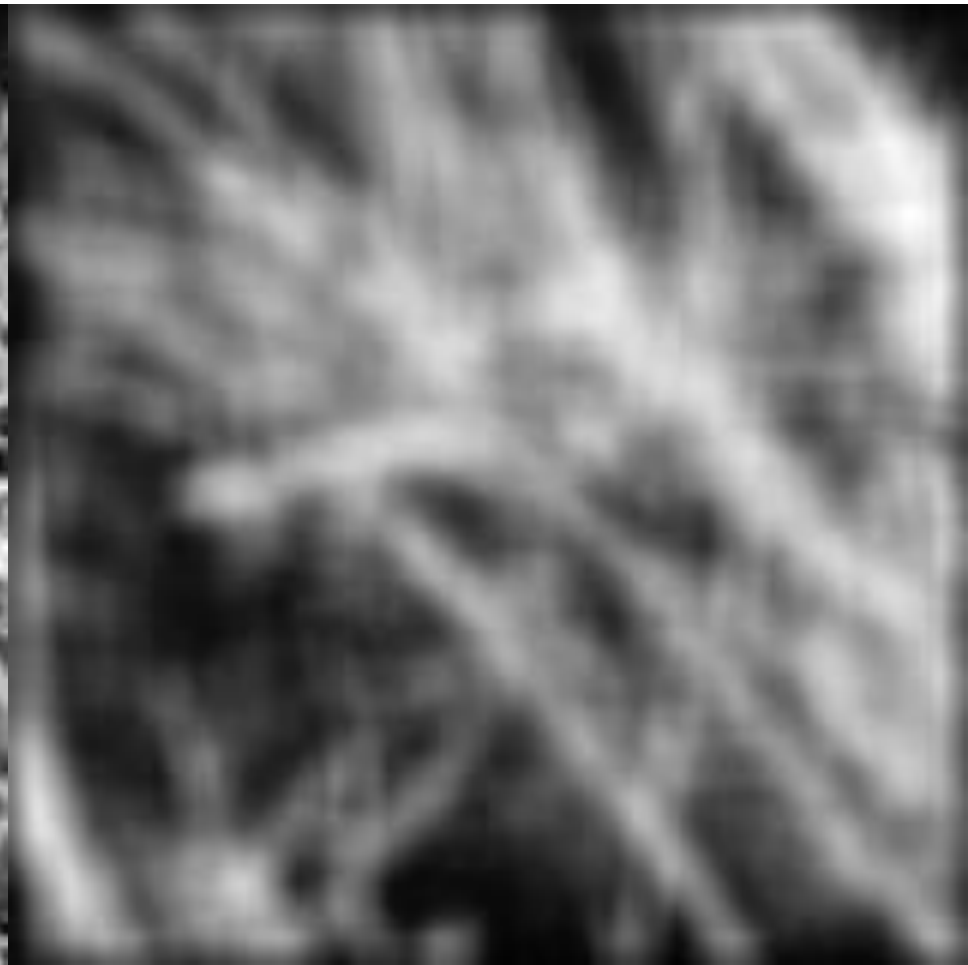


1	2	1
0	0	0
-1	-2	-1

Sobel

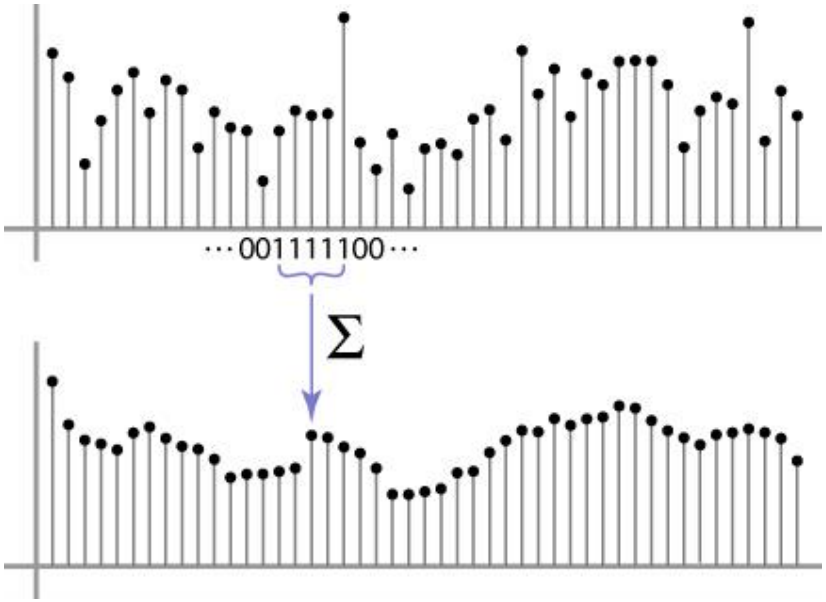
Horizontal Edge
(absolute value)

Back to the box filter



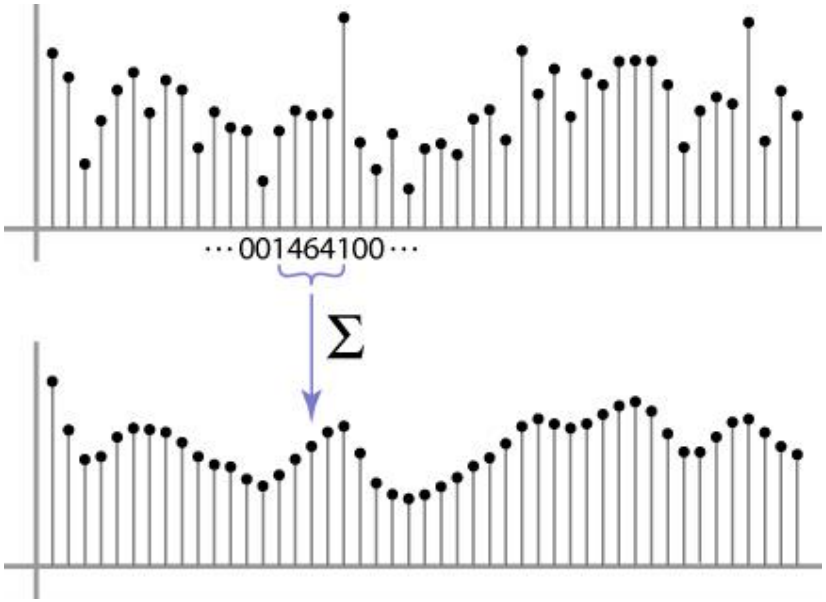
Moving Average

- Can add weights to our moving average
- *Weights* […, 0, 1, 1, 1, 1, 1, 0, …] / 5



Weighted Moving Average

- bell curve (gaussian-like) weights […, 1, 4, 6, 4, 1, …]

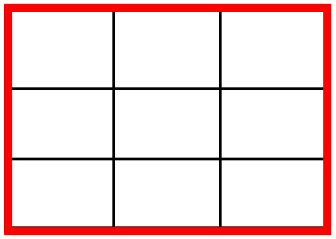


Moving Average In 2D

What are the weights H?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$



$H[u, v]$

 Slide by Steve Seitz

Gaussian filtering

A Gaussian kernel gives less weight to pixels further from the center of the window

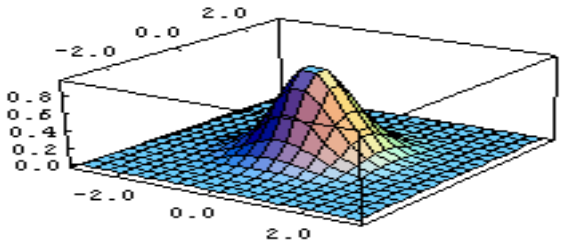
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

$$\frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}$$

$H[u, v]$

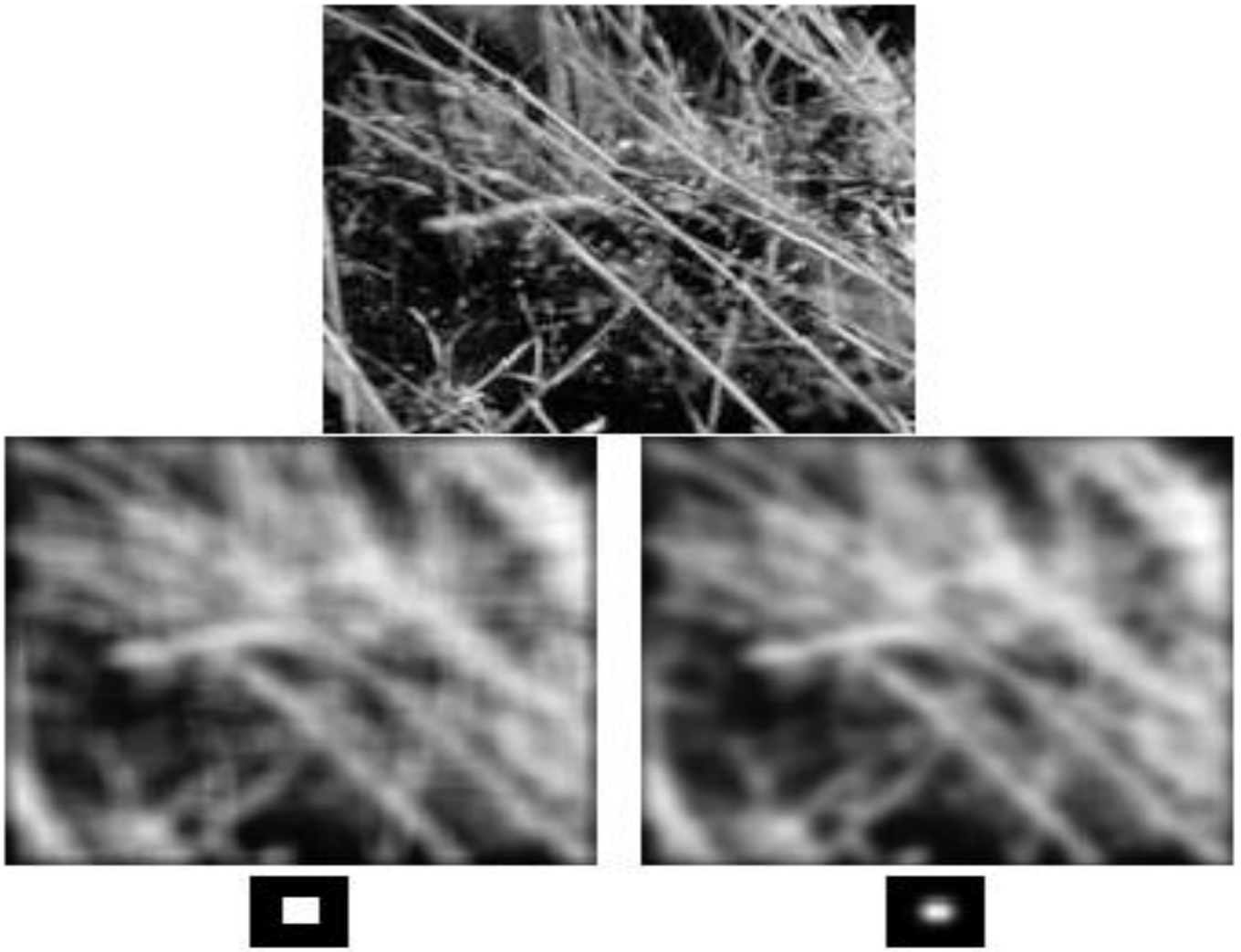
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



This kernel is an approximation of a Gaussian function:

Slide by Steve Seitz

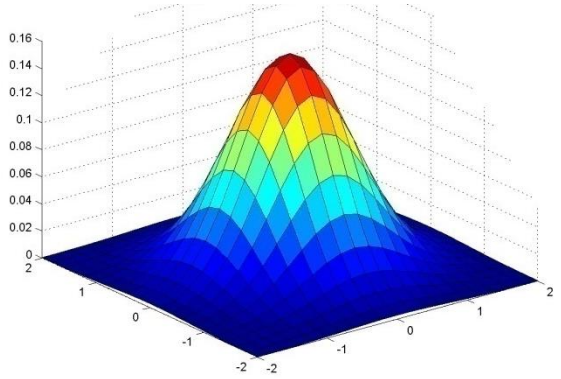
Mean vs. Gaussian filtering

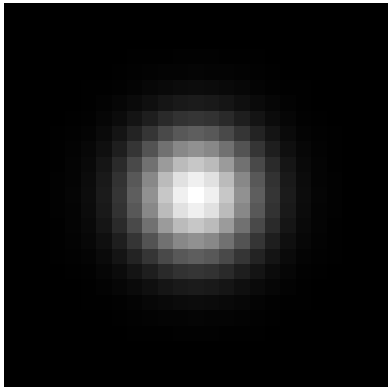


Slide by Steve Seitz

Important filter: Gaussian

Weight contributions of neighboring pixels by nearness





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

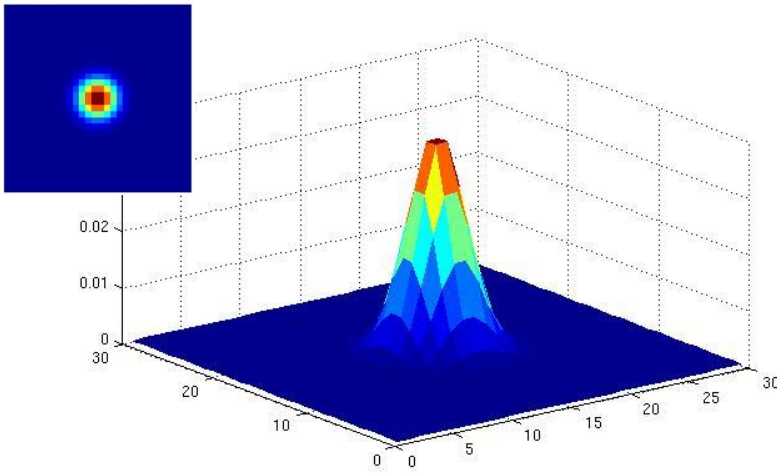
5 x 5, $\sigma = 1$

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

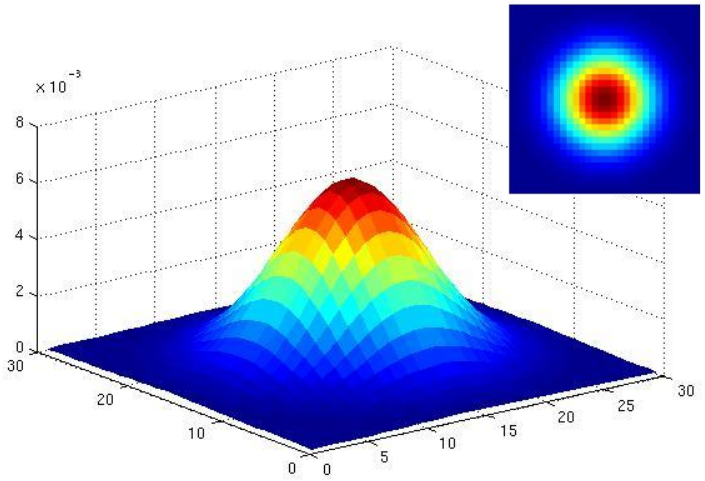
Slide credit: Christopher Rasmussen

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



σ = 2 with 30 x 30 kernel

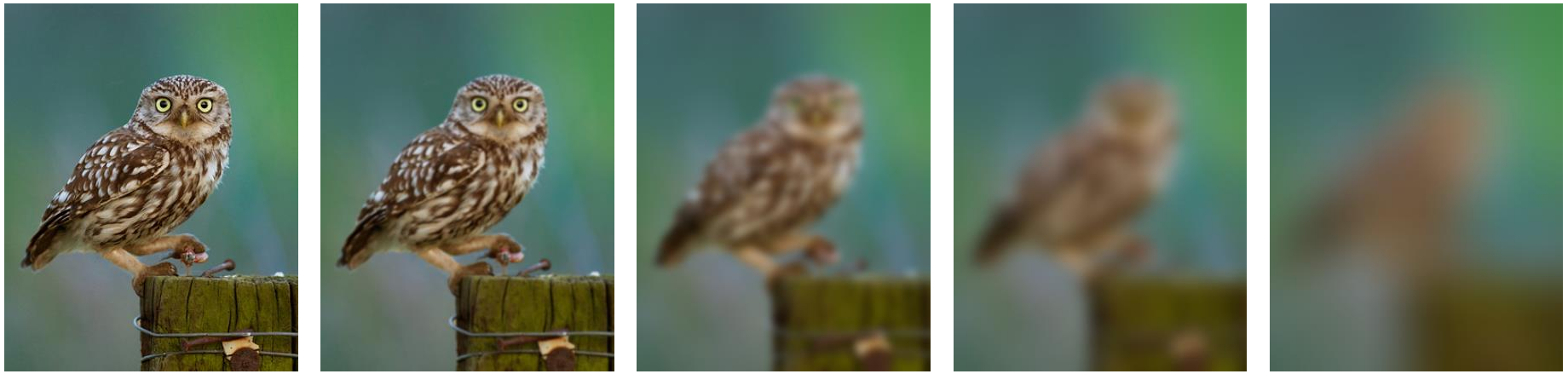


σ = 5 with 30 x 30 kernel

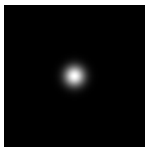
- Standard deviation σ: determines extent of smoothing

Source: K. Grauman

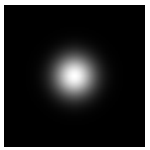
Gaussian filters



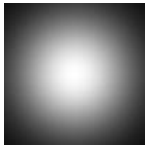
σ = 1 pixel



σ = 5 pixels



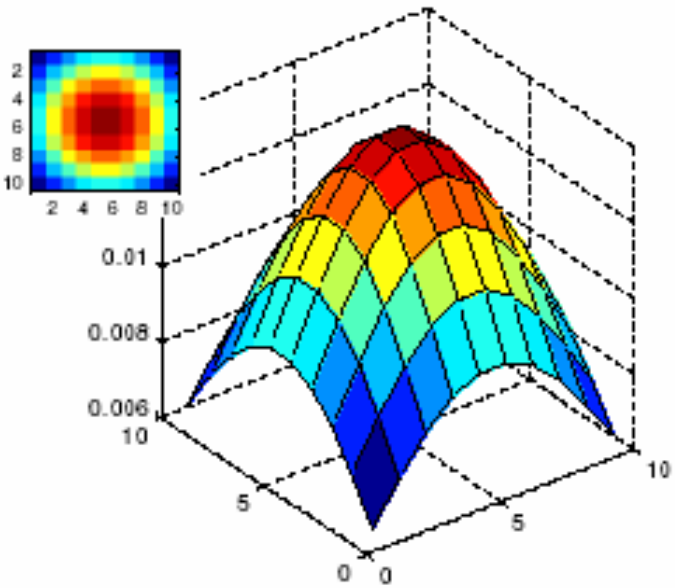
σ = 10 pixels



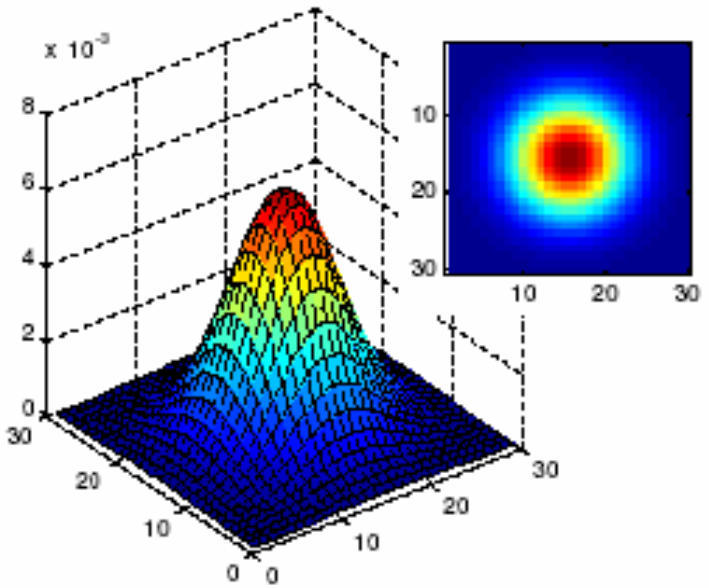
σ = 30 pixels

Choosing kernel width

- The Gaussian function has infinite support, but discrete filters use finite kernels



σ = 5 with 10x10 kernel



σ = 5 with 30x30 kernel

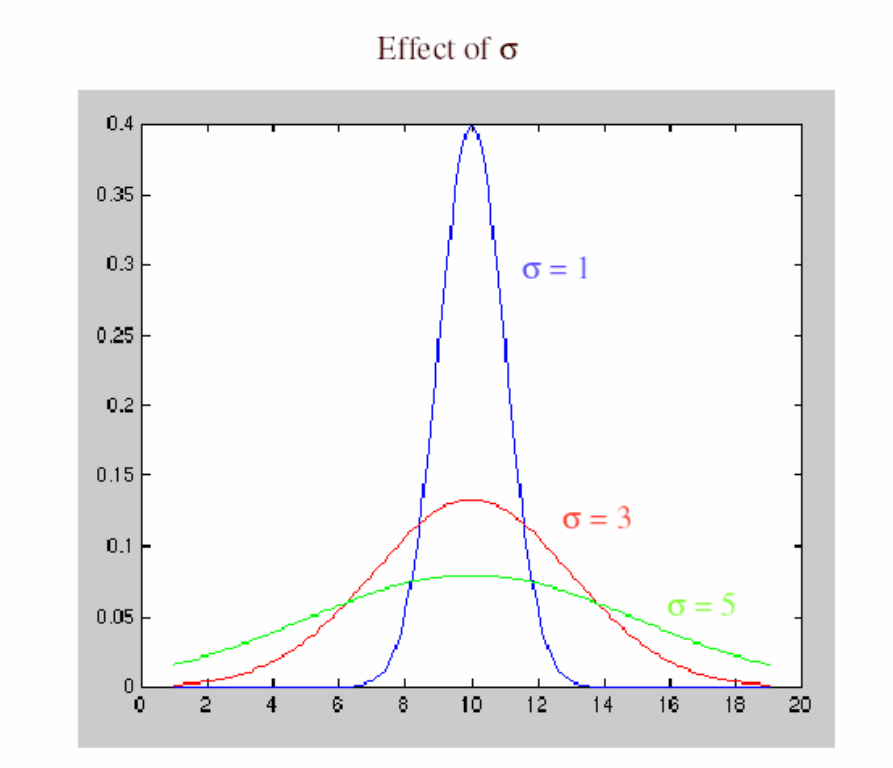
Source: K. Grauman

Practical matters

How big should the filter be?

Values at edges should be near zero

Rule of thumb for Gaussian: set filter half-width to about 3 σ



Side by Derek Hoiem

Cross-correlation vs. Convolution

cross-correlation: $G = H \otimes F$

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

A **convolution** operation is a cross-correlation where the filter is flipped both horizontally and vertically before being applied to the image:

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u, j-v]$$

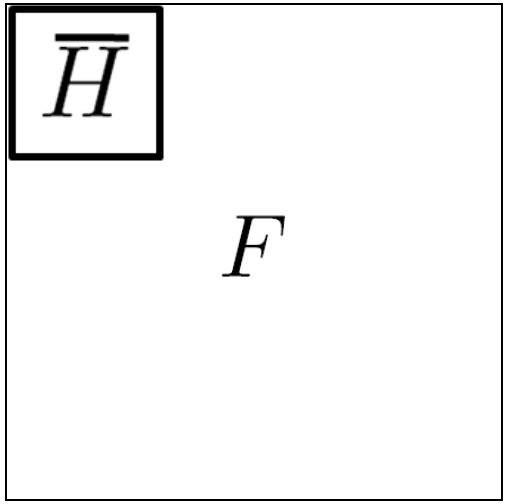
It is written:

$$G = H \star F$$

Convolution is **commutative** and **associative**

Slide by Steve Seitz

Convolution



Adapted from F. Durand

Convolution is nice!

- Notation: $b = c \star a$
- Convolution is a multiplication-like operation
  - commutative $a \star b = b \star a$
  - associative $a \star (b \star c) = (a \star b) \star c$
  - distributes over addition $a \star (b + c) = a \star b + a \star c$
  - scalars factor out $\alpha a \star b = a \star \alpha b = \alpha(a \star b)$
  - identity: unit impulse *e* = […, 0, 0, 1, 0, 0, …]

    $$a \star e = a$$
- Conceptually no distinction between filter and signal
- Usefulness of associativity
  - often apply several filters one after another: (((*a* * b_1) * b_2) * b_3)
  - this is equivalent to applying one filter: a * (b_1 * b_2 * b_3)

The Convolution Theorem

The greatest thing since sliced (banana) bread!

- The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$\mathrm{F}[g * h] = \mathrm{F}[g]\mathrm{F}[h]$$

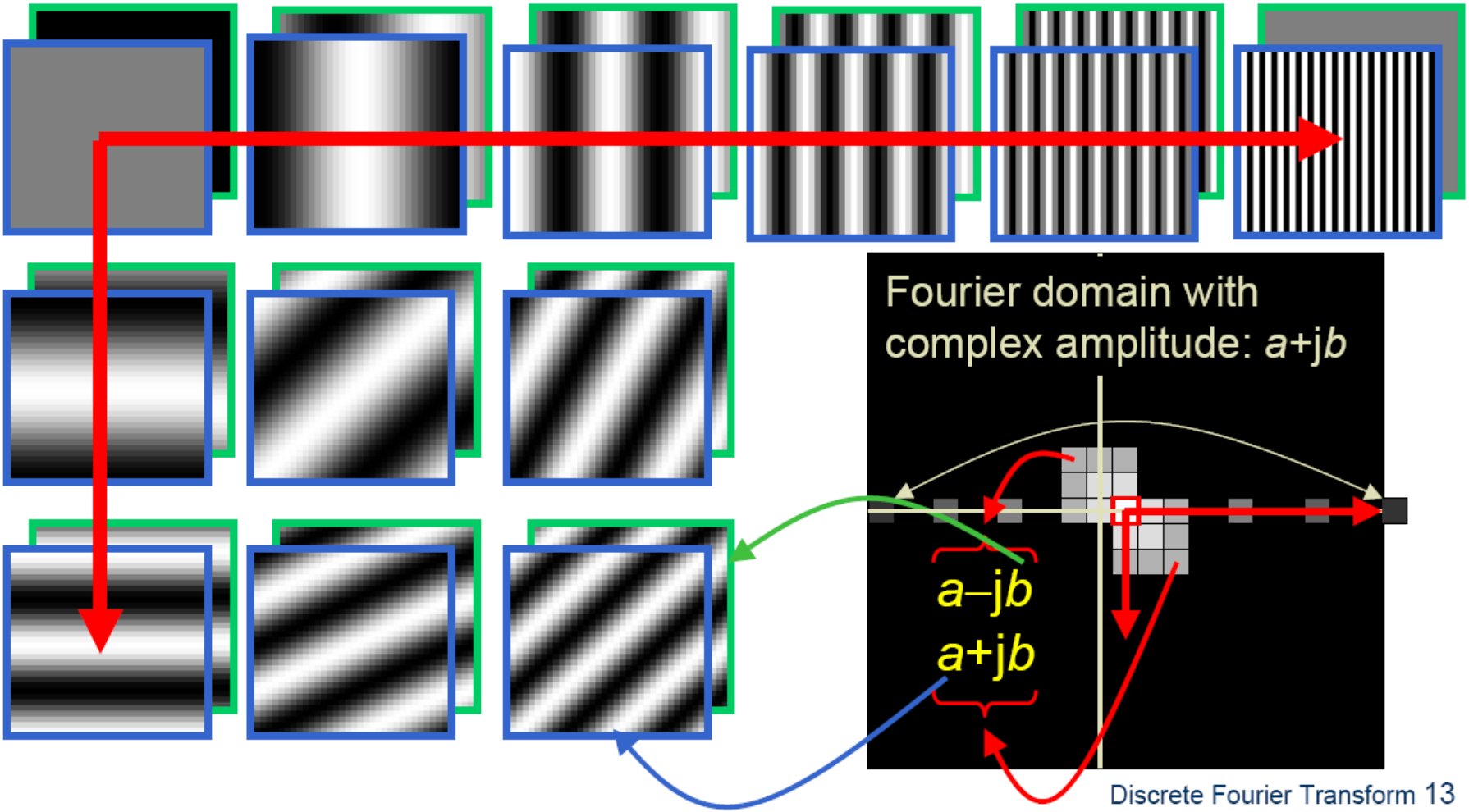
- The inverse Fourier transform of the product of two Fourier transforms is the convolution of the two inverse Fourier transforms

$$\mathrm{F}^{-1}[gh] = \mathrm{F}^{-1}[g] * \mathrm{F}^{-1}[h]$$

- **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

2D Fourier transform

Teases away fast vs. slow changes in the image.

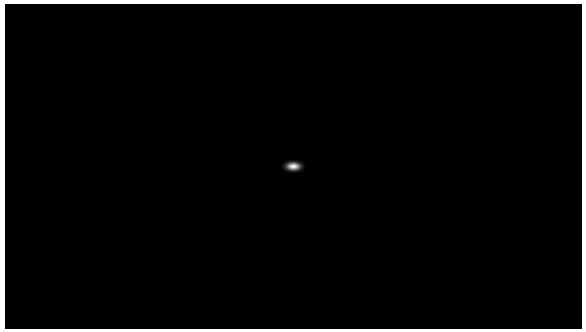


2D convolution theorem example

$f(x,y)$

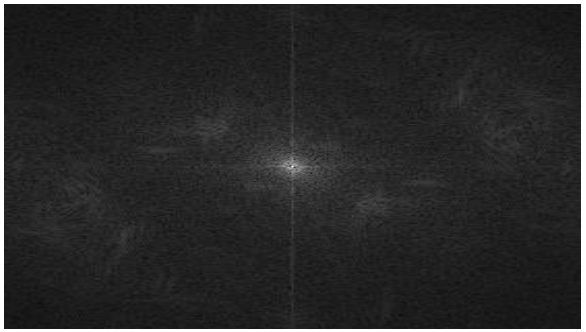
$*$

$h(x,y)$

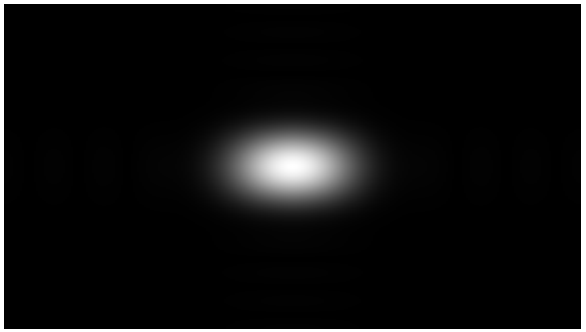


$\Downarrow$

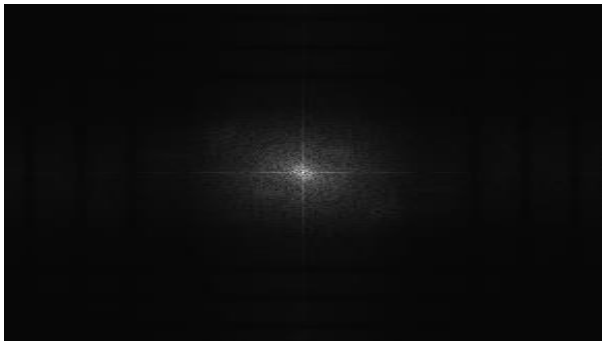
$g(x,y)$



$\times$



$\Downarrow$



$|F(s_x,s_y)|$

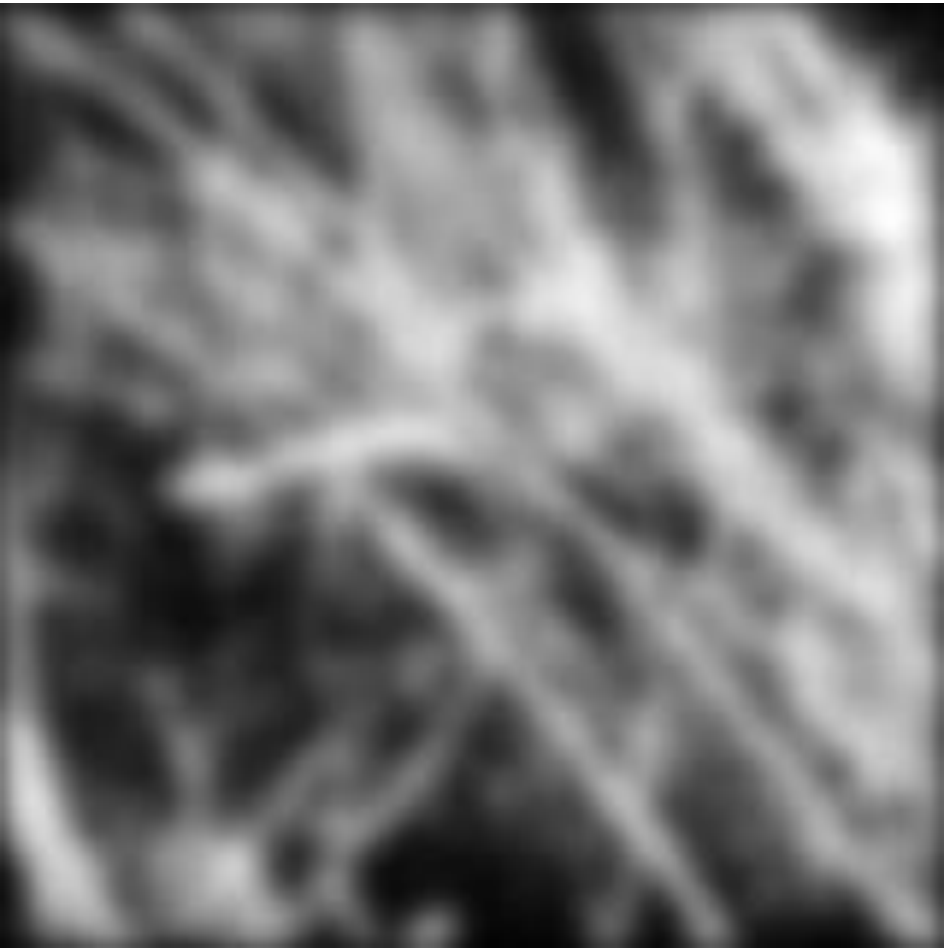
$|H(s_x,s_y)|$

$|G(s_x,s_y)|$

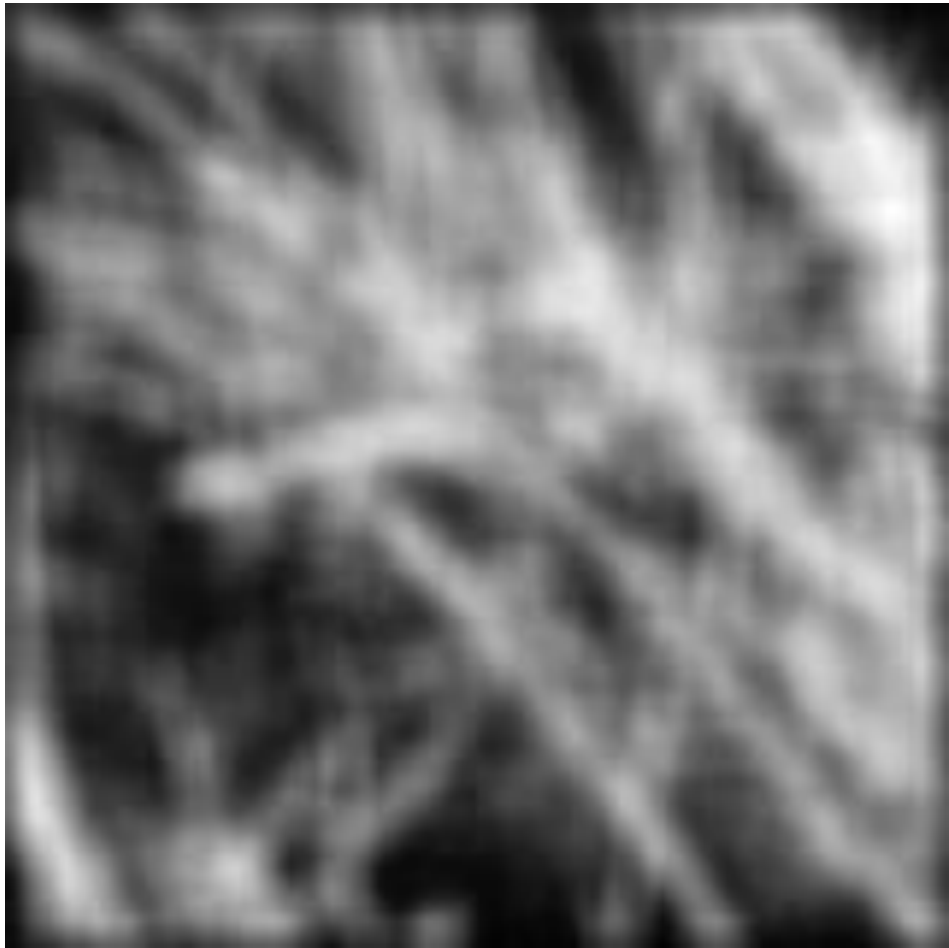
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

Gaussian 



Box filter 



Fourier Transform pairs

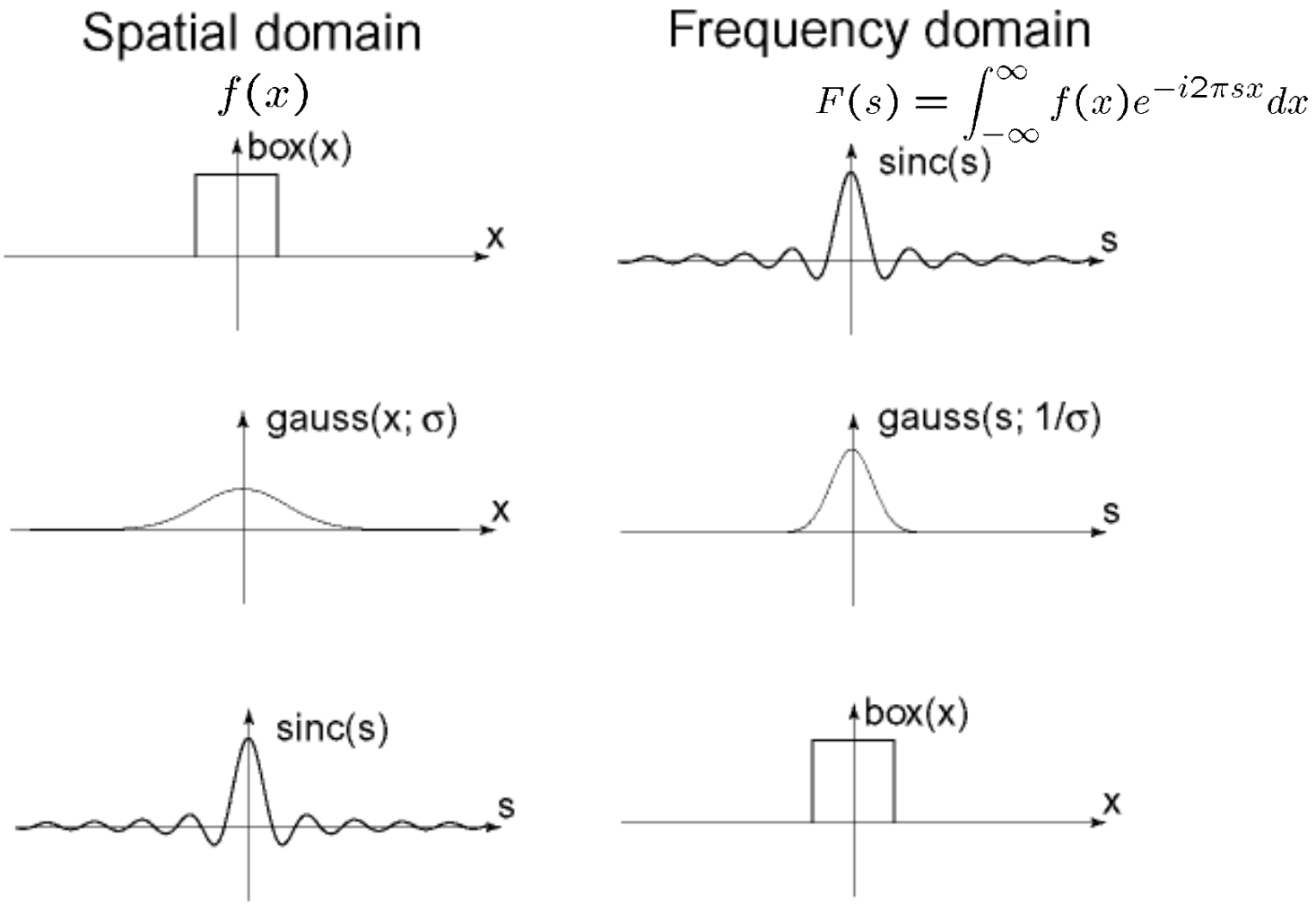


Image half-sizing

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

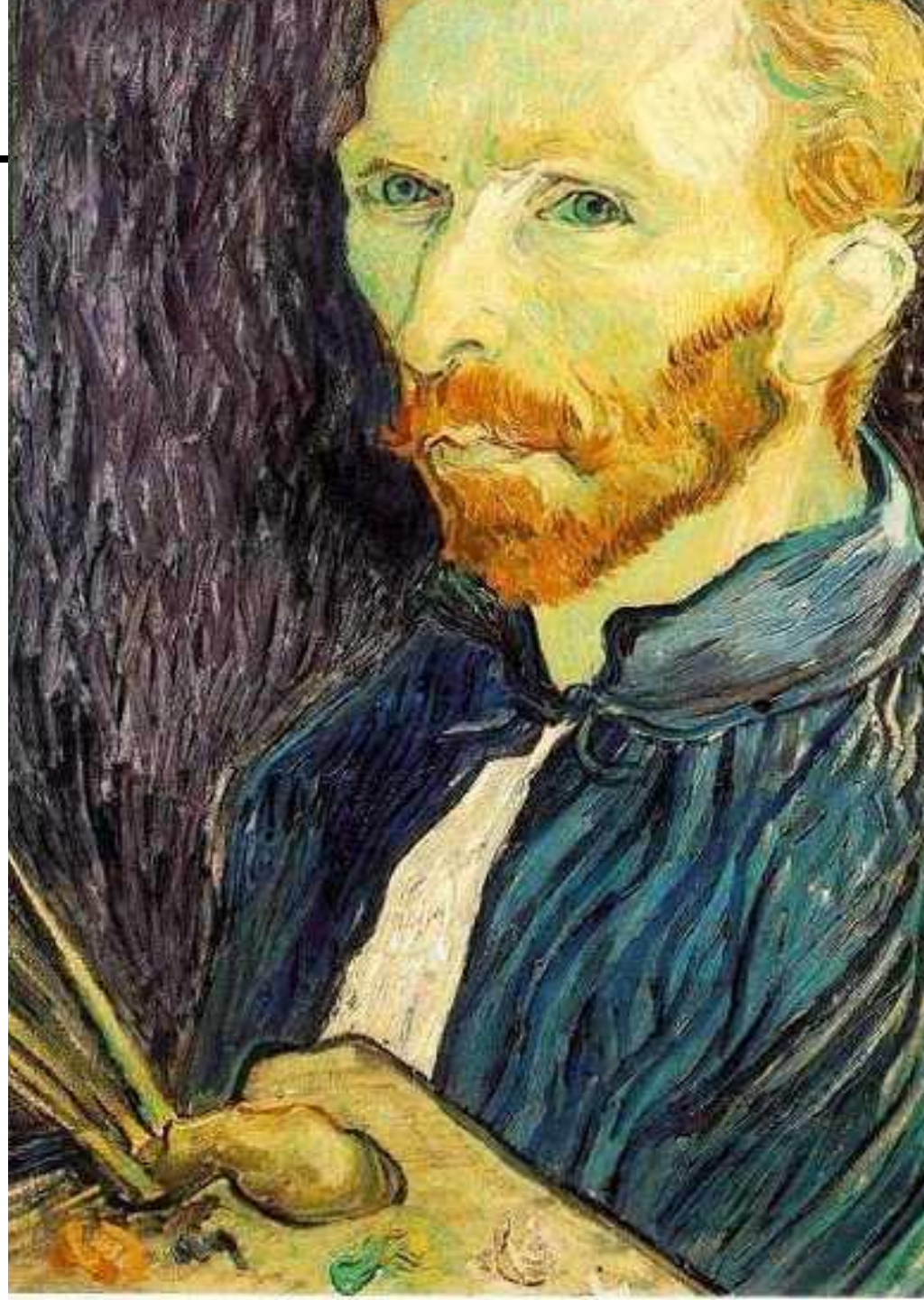
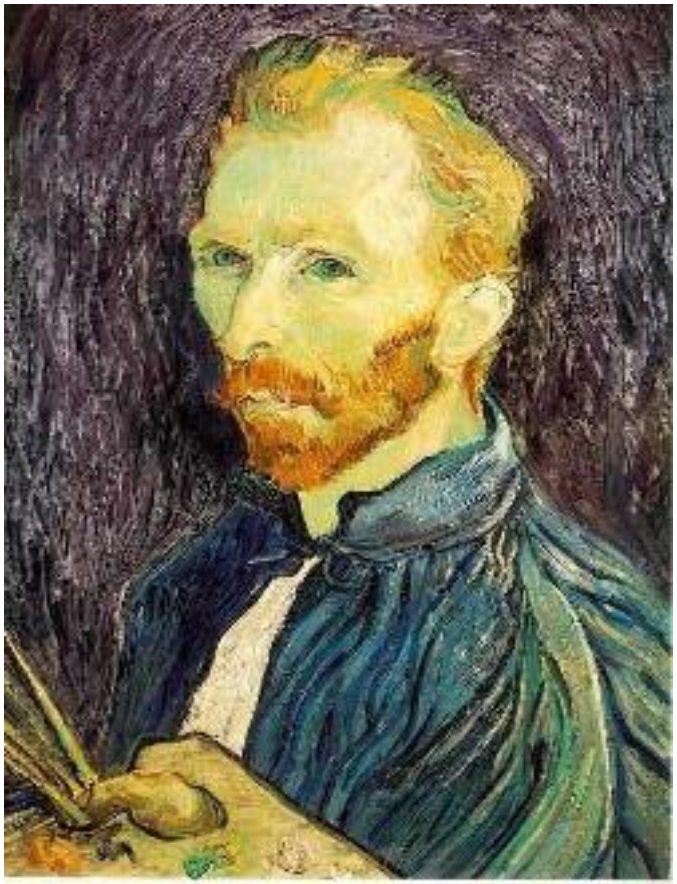


Image sub-sampling



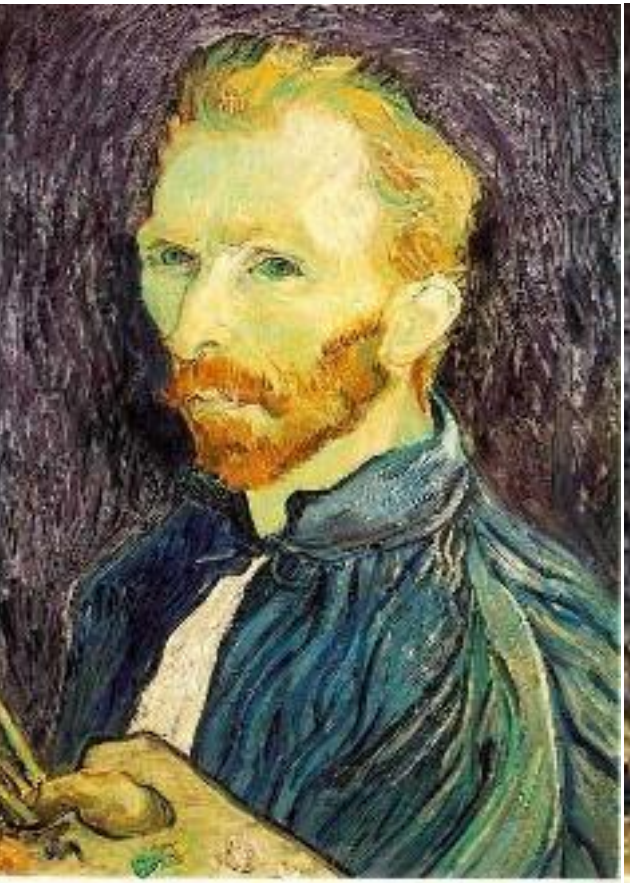
1/4

1/8

Throw away every other row and column to create a 1/2 size image
- called *image sub-sampling*

Slide by Steve Seitz

Image sub-sampling



1/2

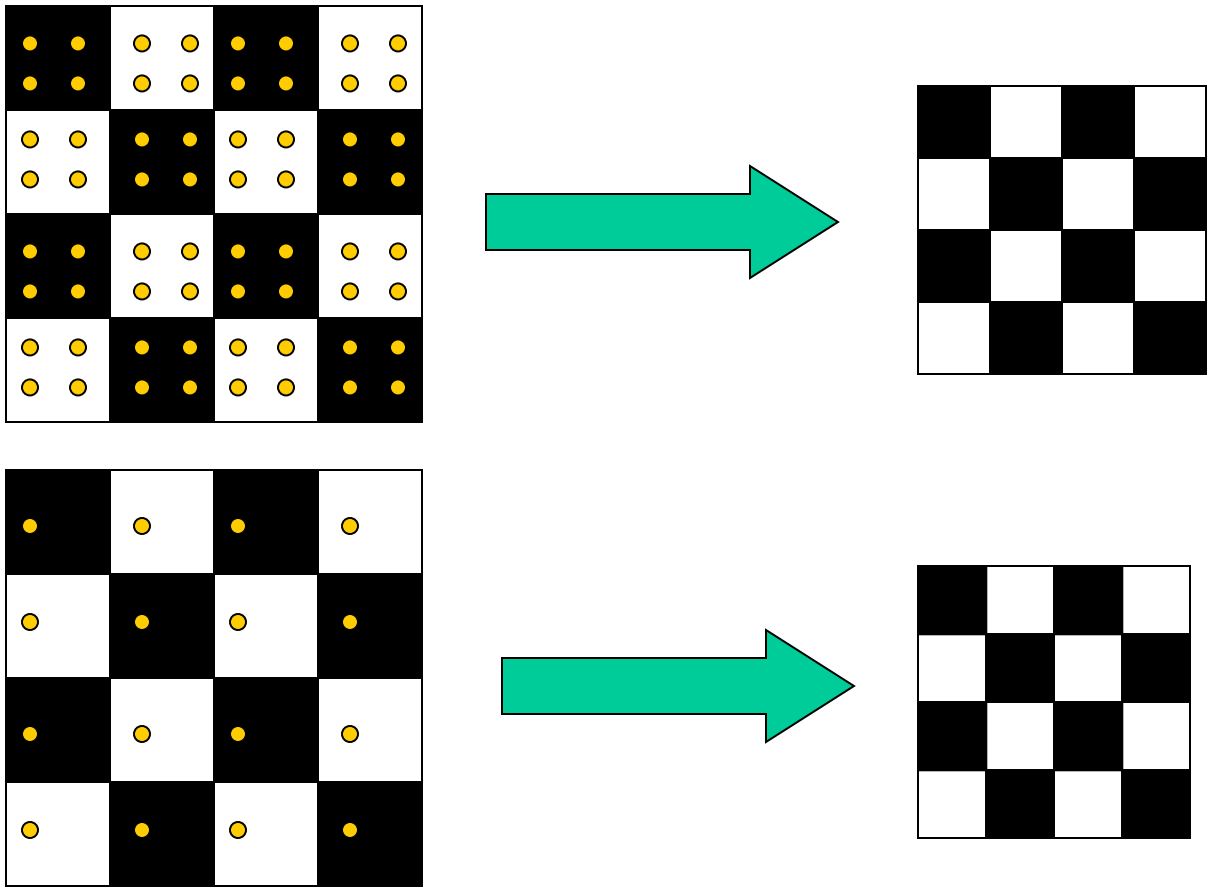
1/4 (2x zoom)

1/8 (4x zoom)

Aliasing! What do we do?

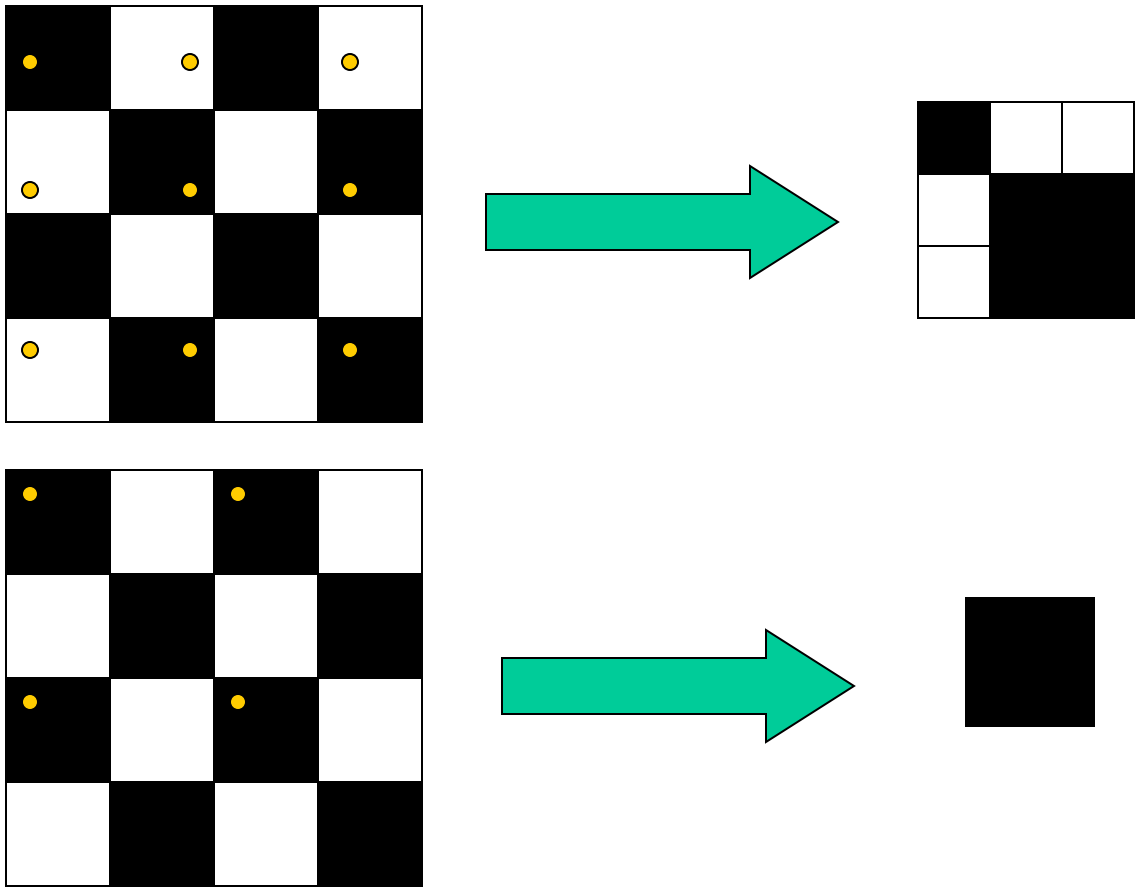
Slide by Steve Seitz

Sampling an image



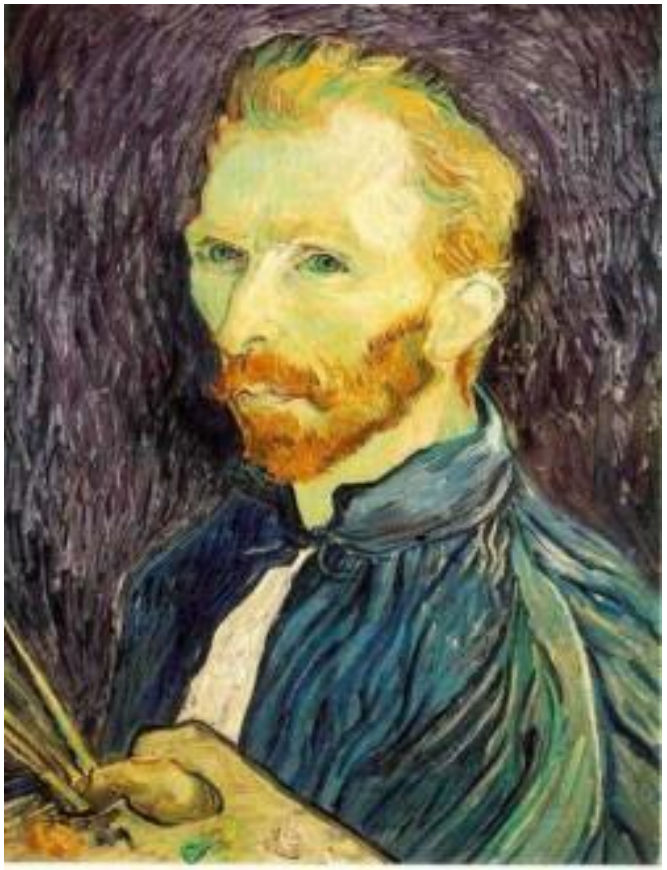
Examples of GOOD sampling

Undersampling



Examples of BAD sampling -> Aliasing

Gaussian (lowpass) pre-filtering



Gaussian 1/2

G 1/4

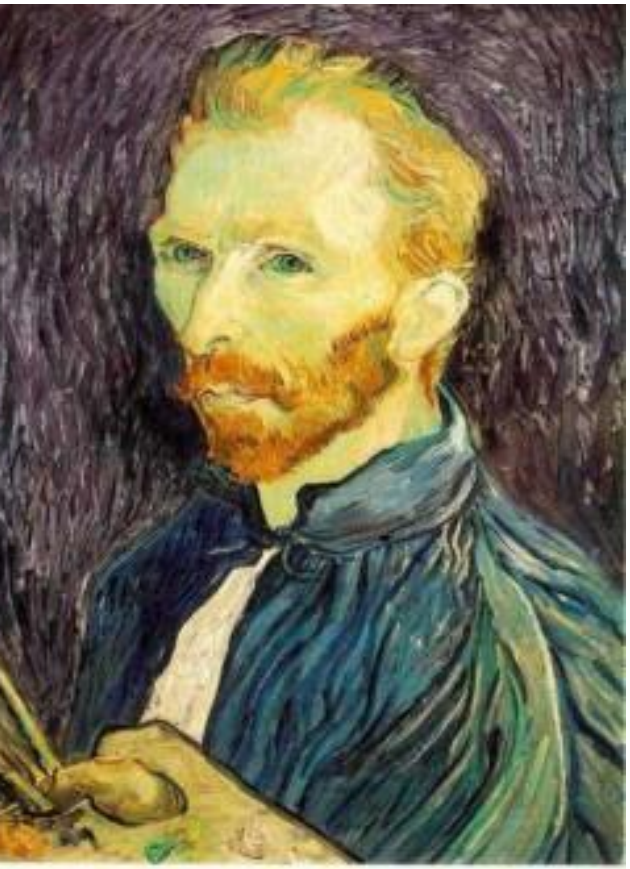
G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?

Slide by Steve Seitz

Subsampling with Gaussian pre-filtering



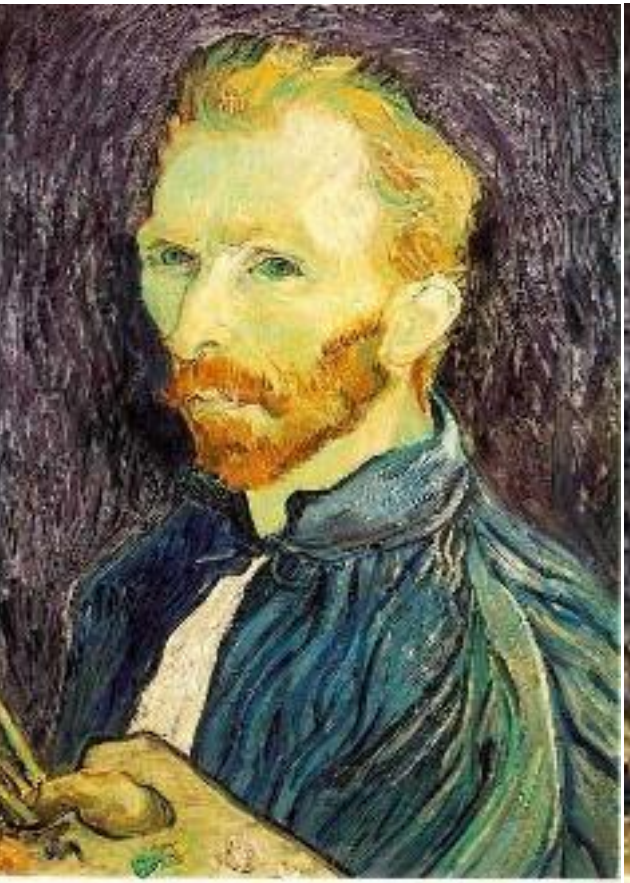
Gaussian 1/2

G 1/4

G 1/8

Slide by Steve Seitz

Compare with...



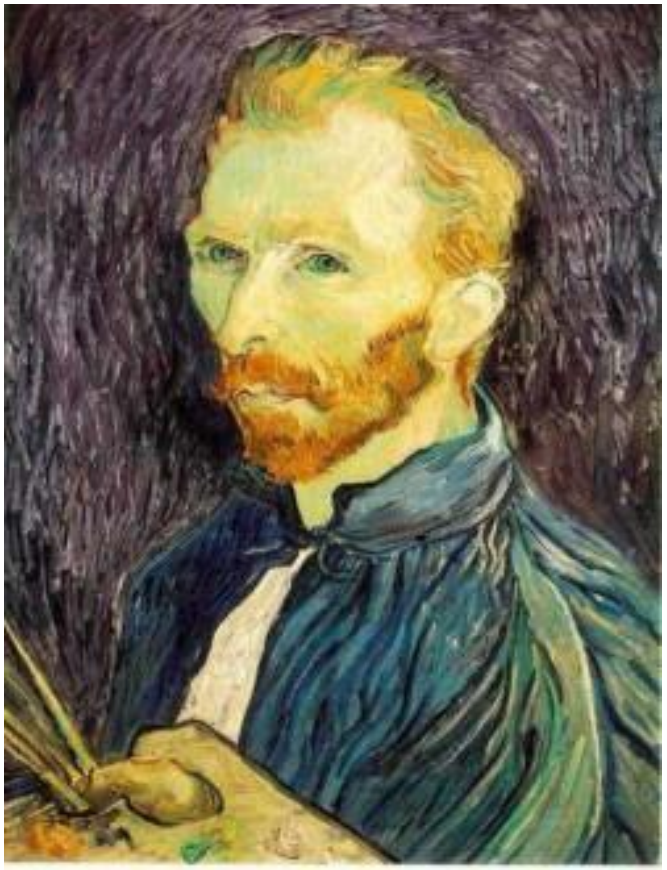
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Slide by Steve Seitz

Gaussian (lowpass) pre-filtering



Gaussian 1/2

G 1/4

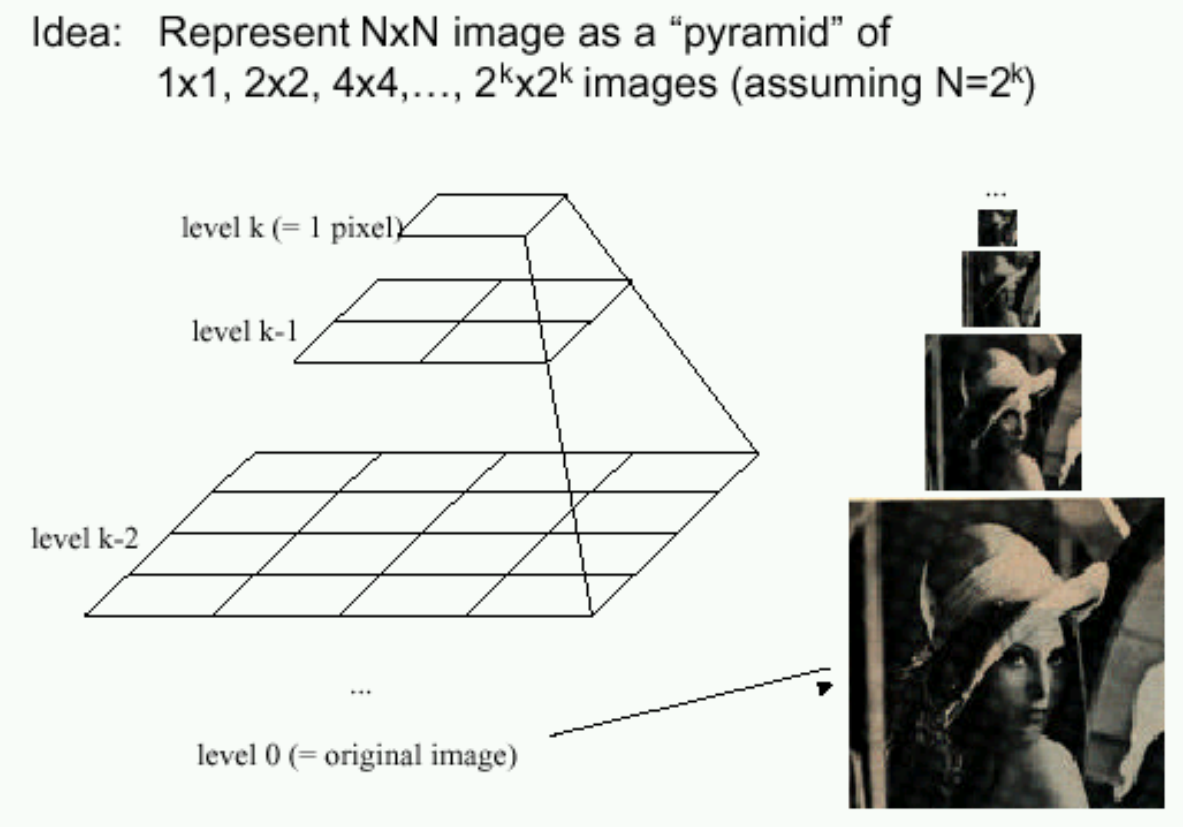
G 1/8

Solution: filter the image, *then* subsample

- Filter size should double for each ½ size reduction. Why?
- How can we speed this up?

Slide by Steve Seitz

Image Pyramids



Known as a **Gaussian Pyramid** [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

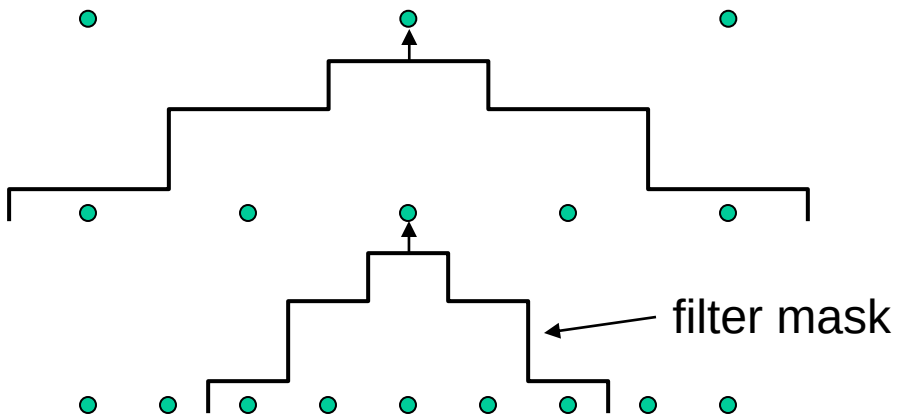
Slide by Steve Seitz

512 256 128 64 32 16 8

A bar in the big images is a hair on the zebra's nose; in smaller images, a stripe; in the smallest, the animal's nose

Figure from David Forsyth

Gaussian pyramid construction



Repeat

- Filter
- Subsample

Until minimum resolution reached

- can specify desired number of levels (e.g., 3-level pyramid)

The whole pyramid is only 4/3 the size of the original image!

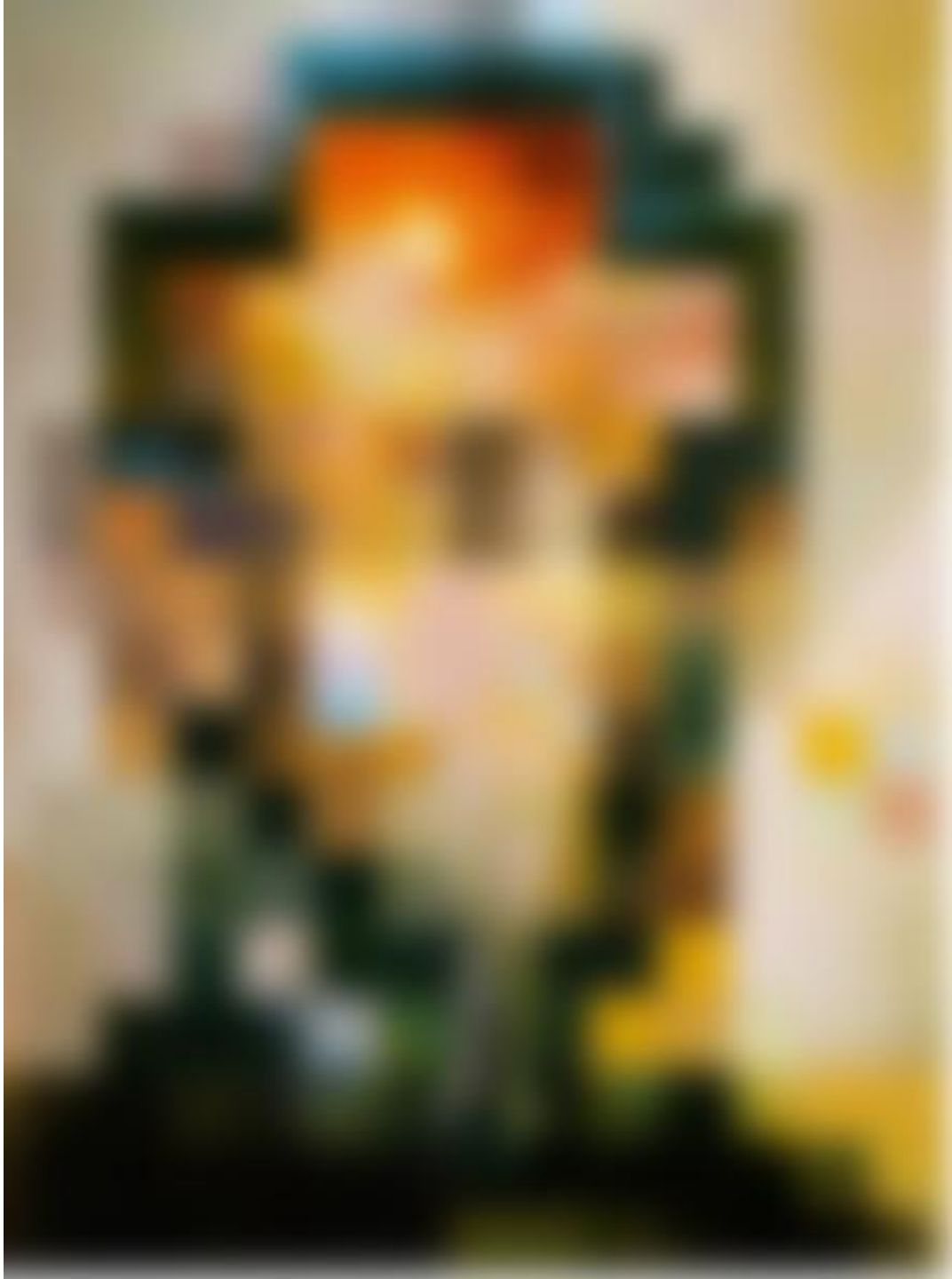
Slide by Steve Seitz

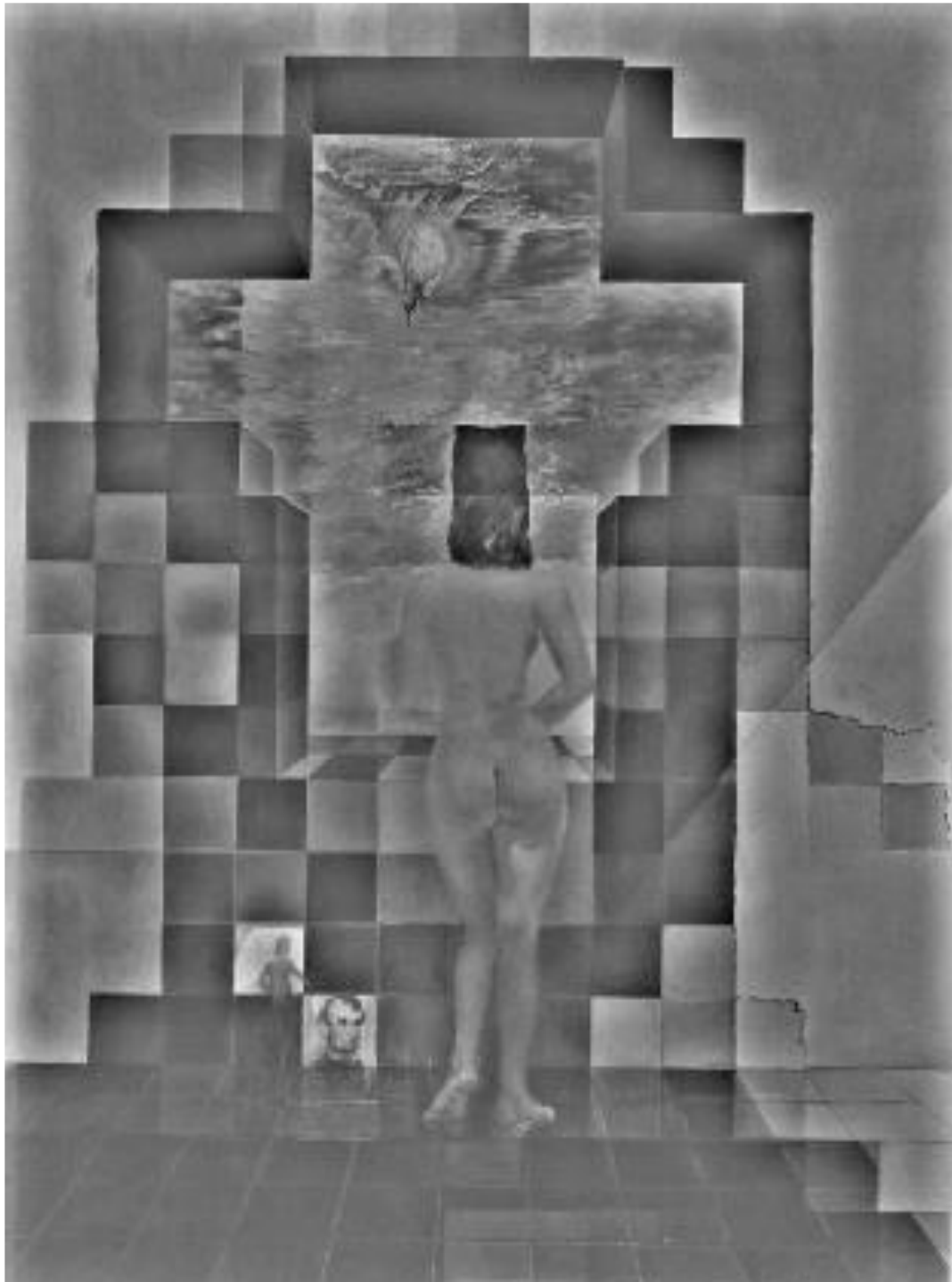
What are they good for?

Improve Search

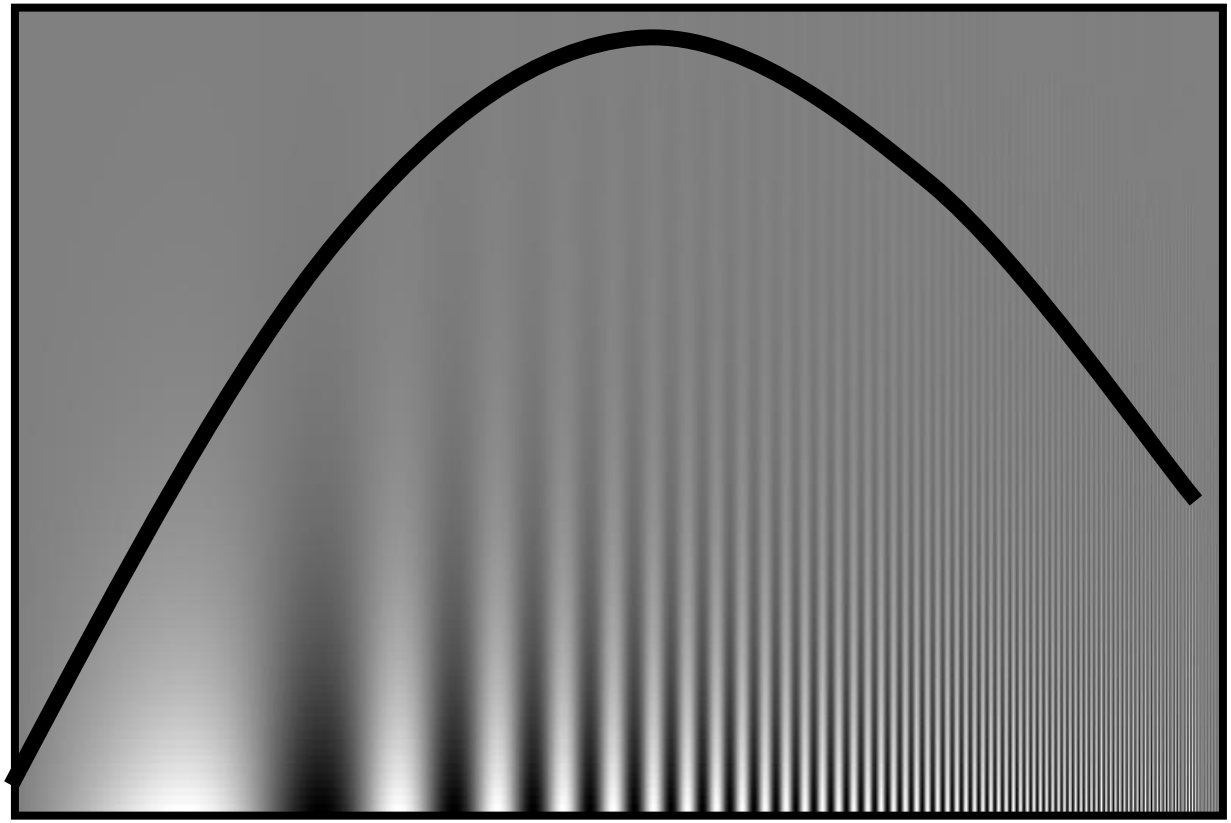
- Search over translations
  - Classic coarse-to-fine strategy
- Search over scale
  - Template matching
  - E.g. find a face at different scales

Salvador Dali
“Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976





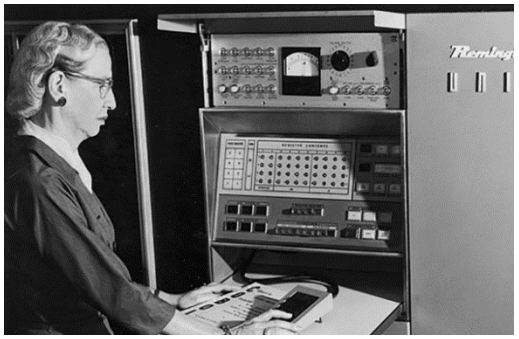
Spatial Frequencies and Perception



Campbell-Robson contrast sensitivity curve

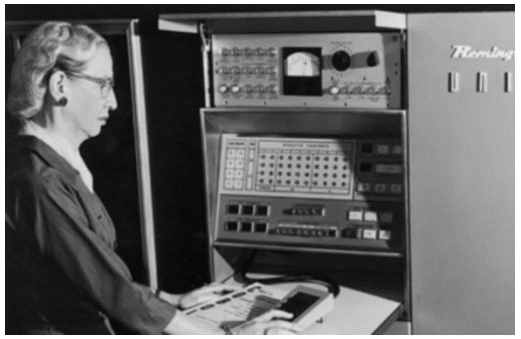
Filtering – Sharpening

Image



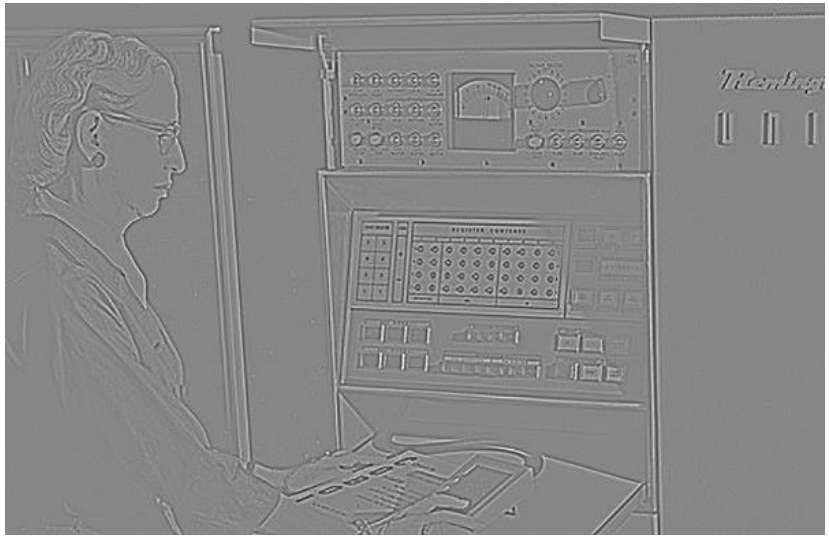
-

Smoothed



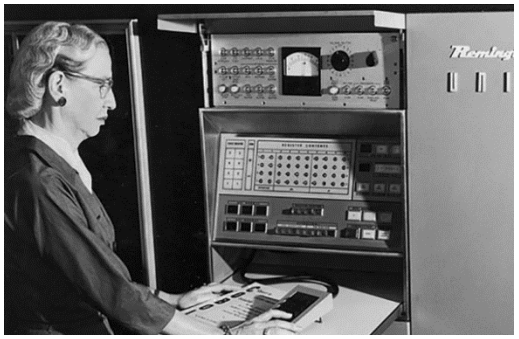
Details

=



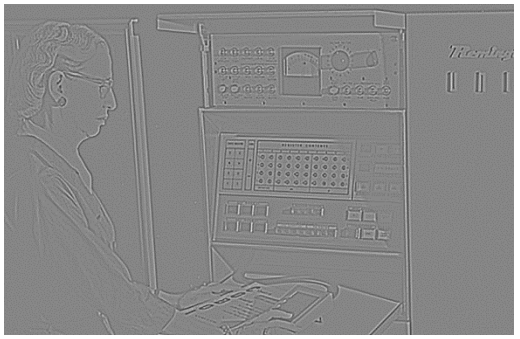
Filtering – Sharpening

Image



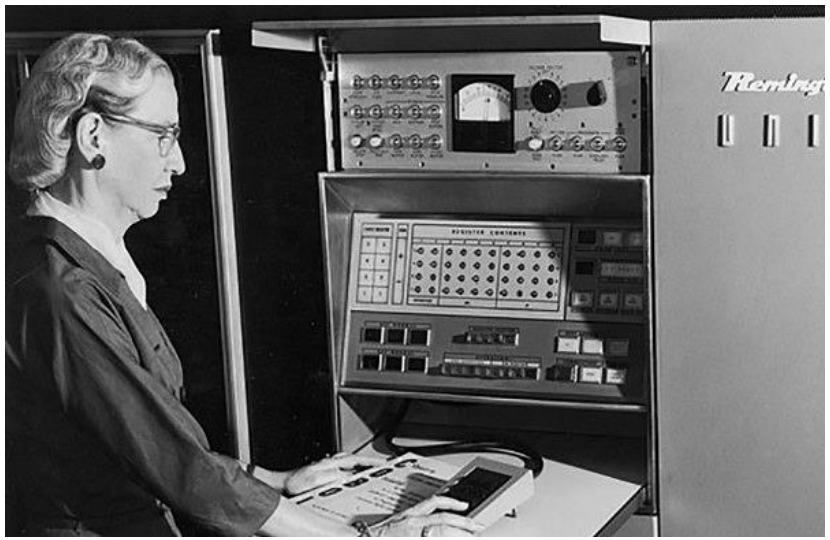
+α

Details



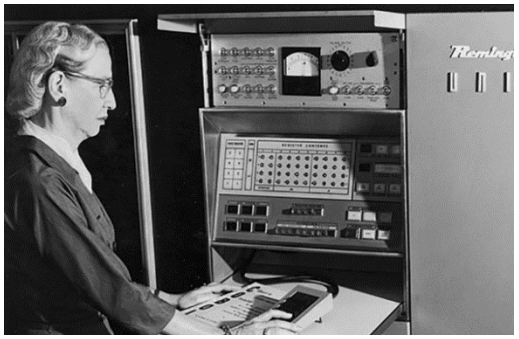
“Sharpened” α=1

=



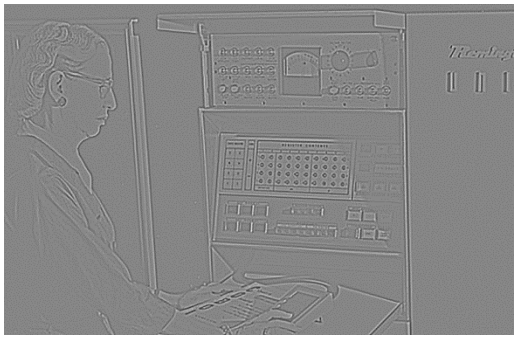
Filtering – Sharpening

Image



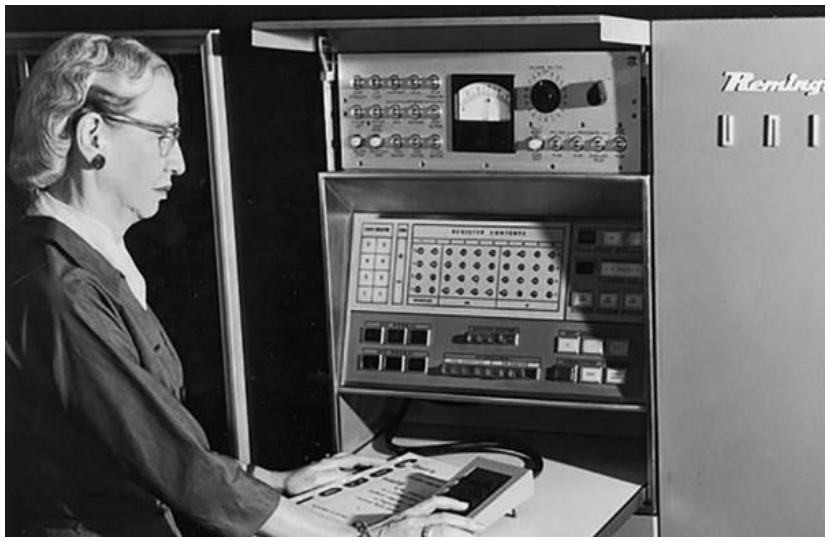
+α

Details



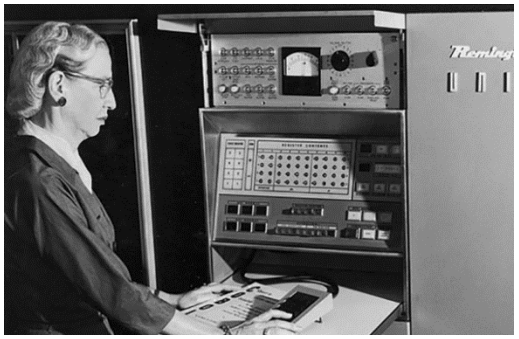
“Sharpened” α=0

=



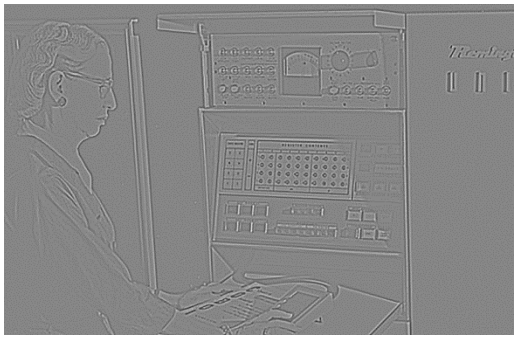
Filtering – Sharpening

Image



+α

Details

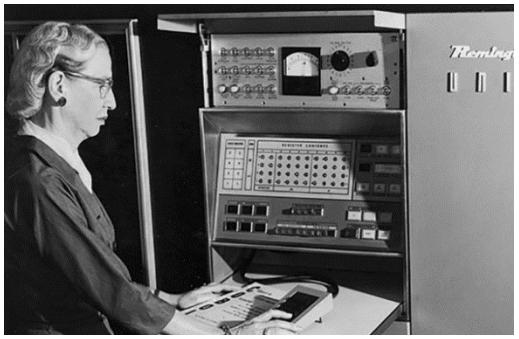


“Sharpened” α=2

=

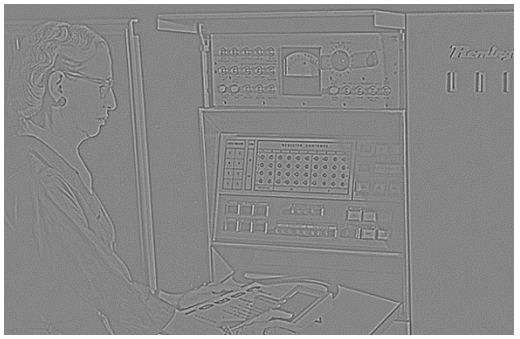
Filtering – Sharpening

Image



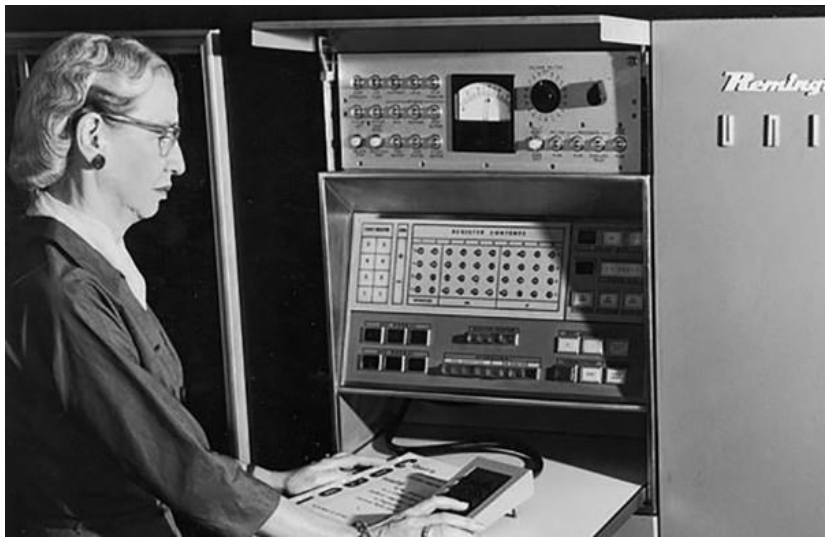
+α

Details



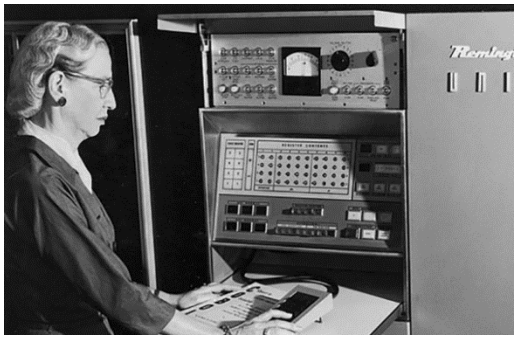
“Sharpened” α=0

=



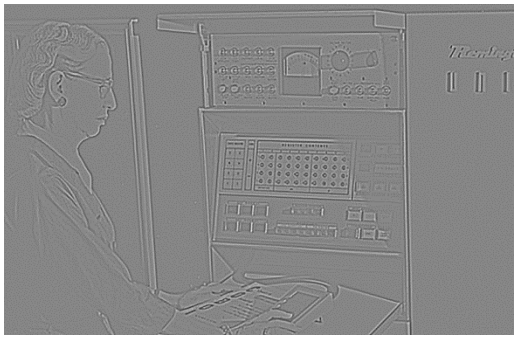
Filtering – Extreme Sharpening

Image



+α

Details



“Sharpened” α=10

=

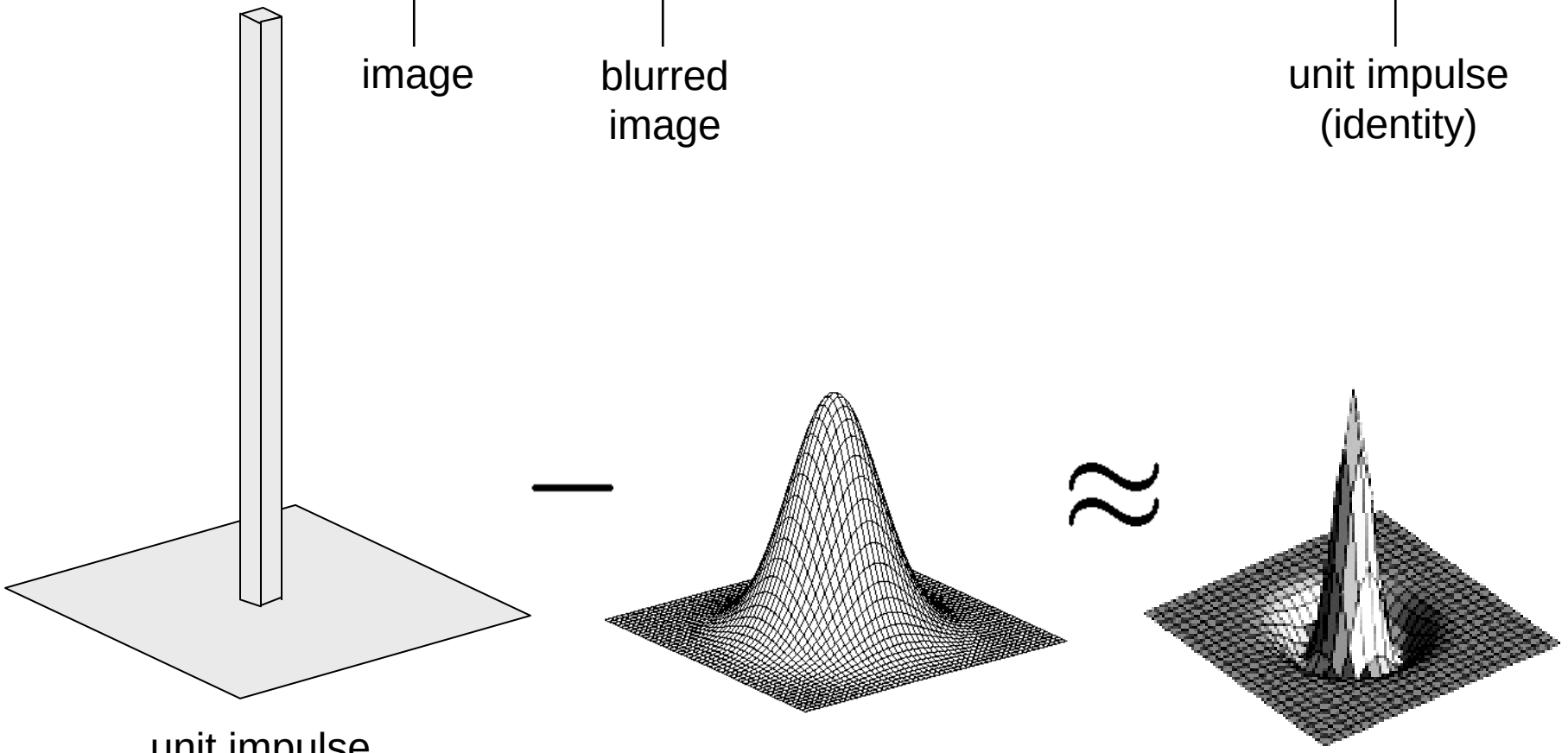
Unsharp mask filter

$$f + \alpha(f - f * g) = (1+\alpha) f - \alpha\, f * g = f * ((1+\alpha)e - \alpha g)$$

image

blurred image

unit impulse (identity)



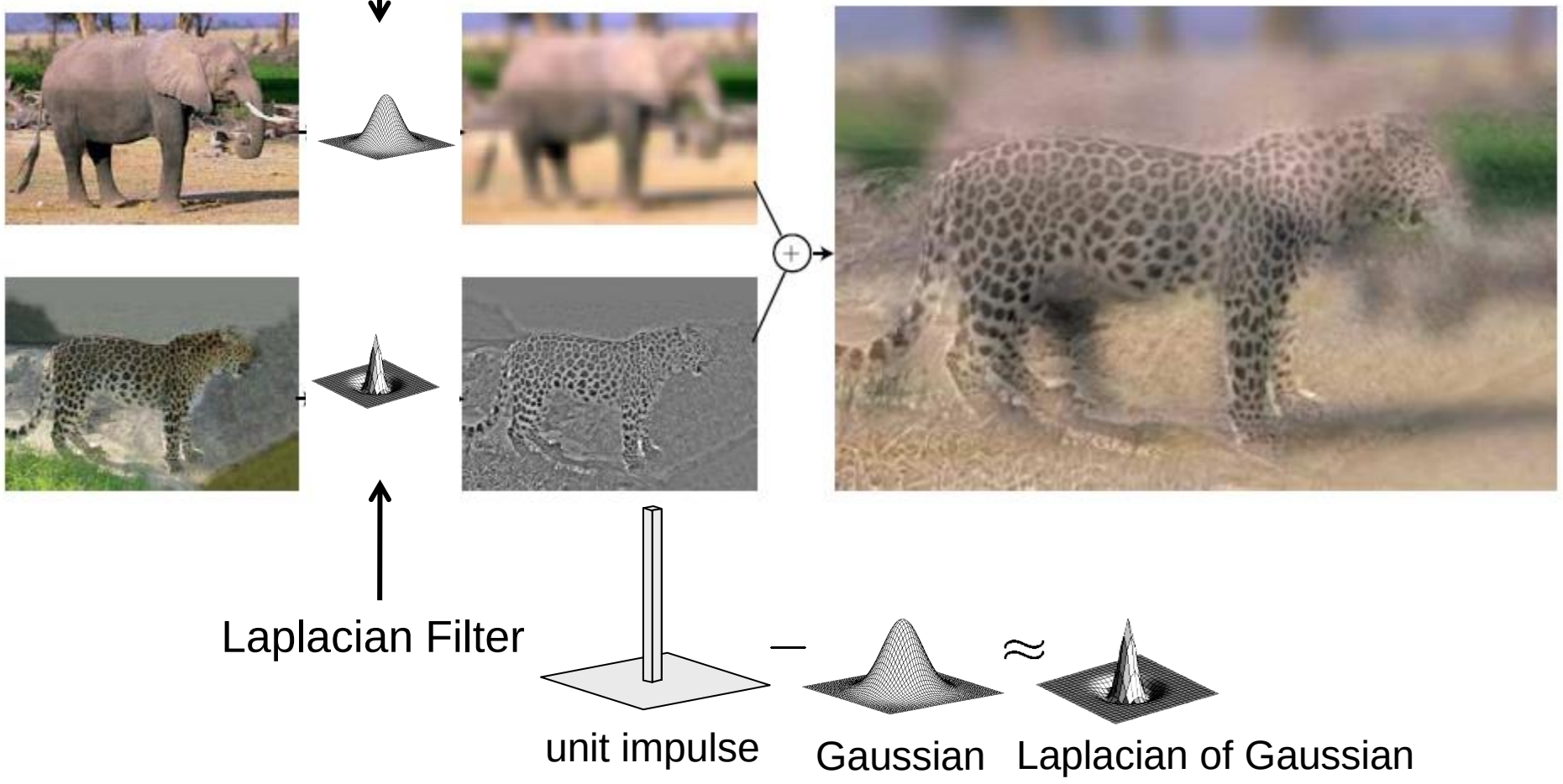
unit impulse

Gaussian

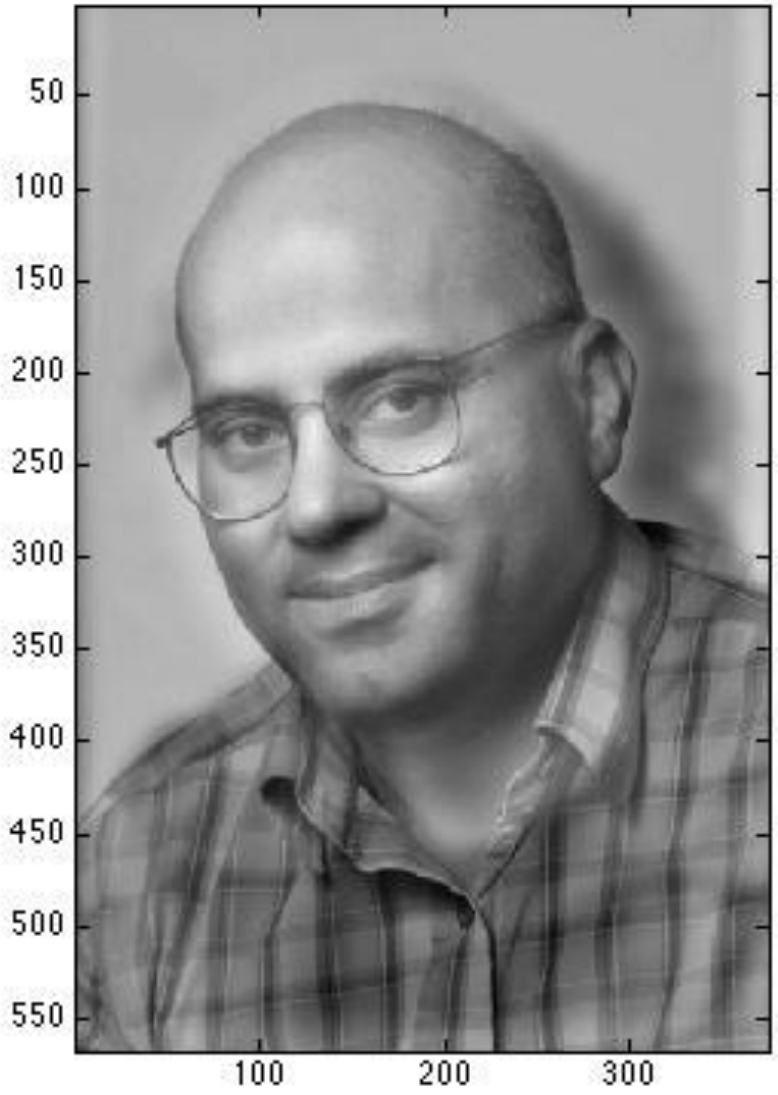
Laplacian of Gaussian

Application: Hybrid Images

A. Oliva, A. Torralba, P.G. Schyns, "Hybrid Images," SIGGRAPH 2006

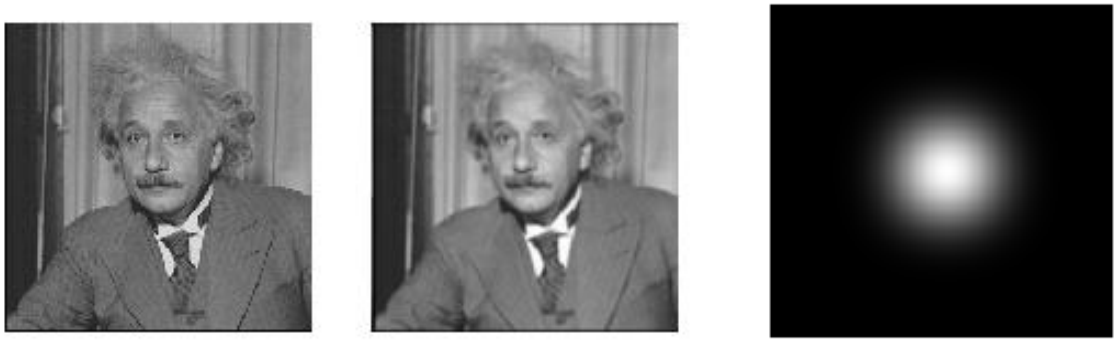


Prof. Jitendros Papadimalik



Low-pass, Band-pass, High-pass filters

low-pass:



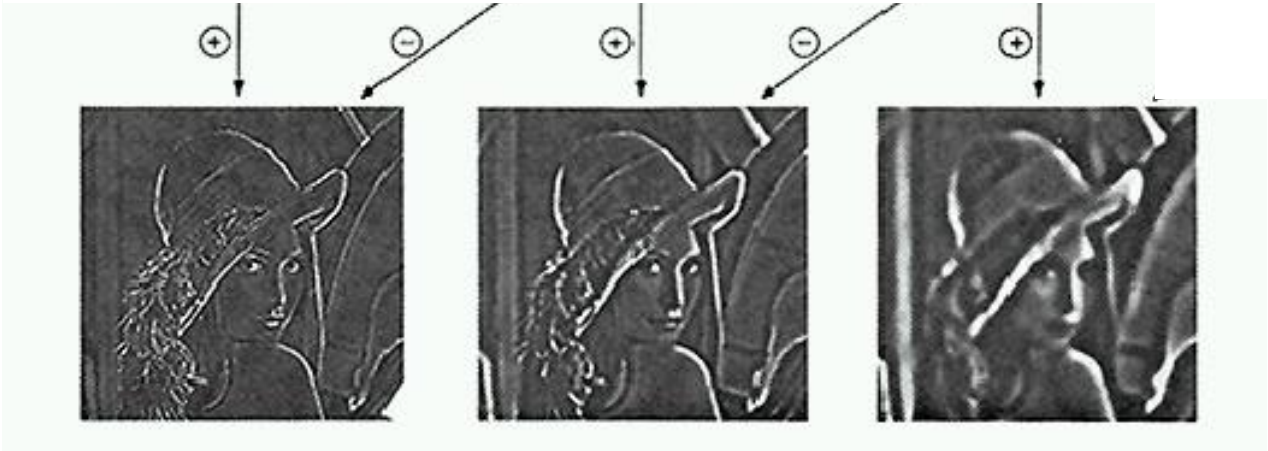
High-pass / band-pass:

Band-pass filtering

Gaussian Pyramid (low-pass images)

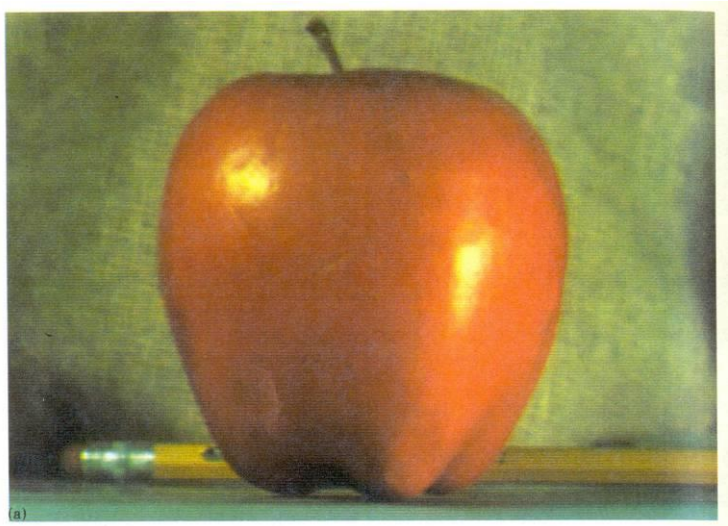
Laplacian Pyramid (Burt and Adelson, 83)

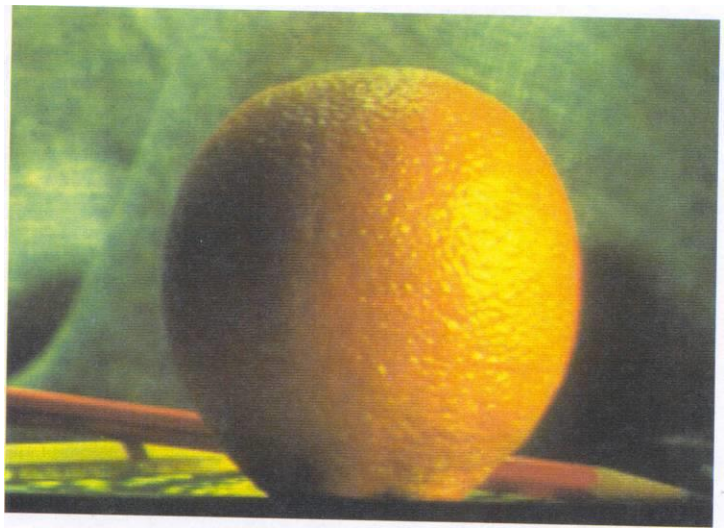
Original image

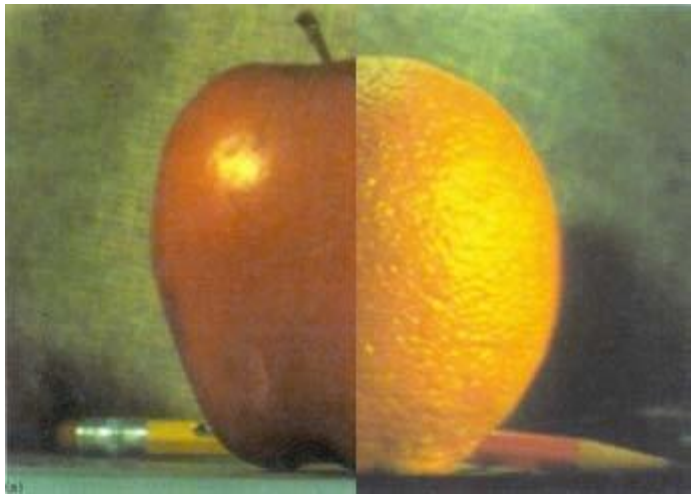


How can we reconstruct (collapse) this pyramid into the original image?

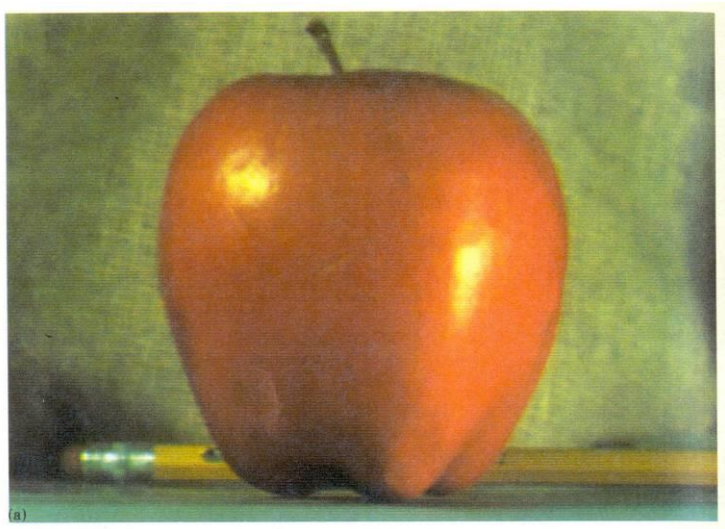
Cut and Paste Blending

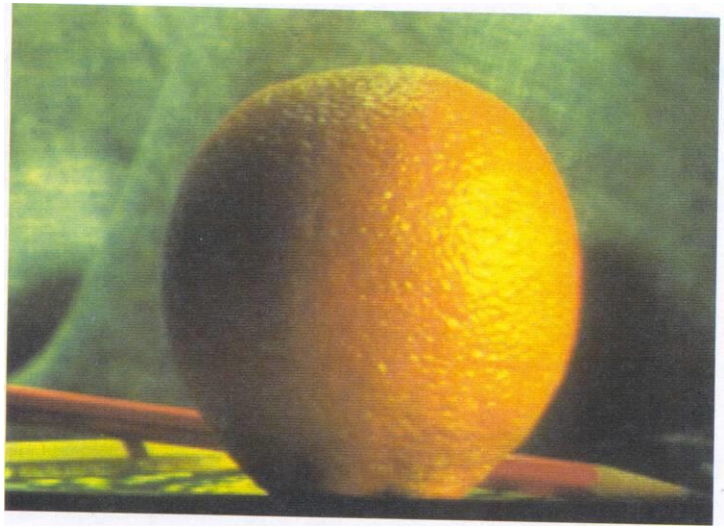


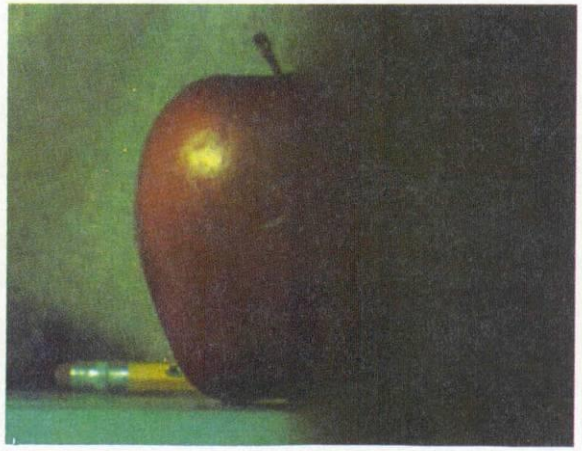




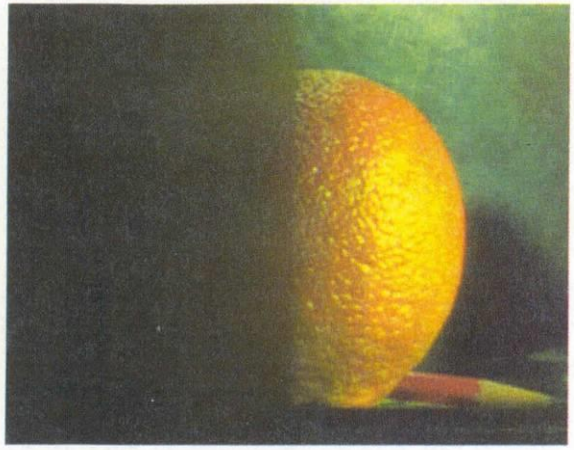
Pyramid Blending



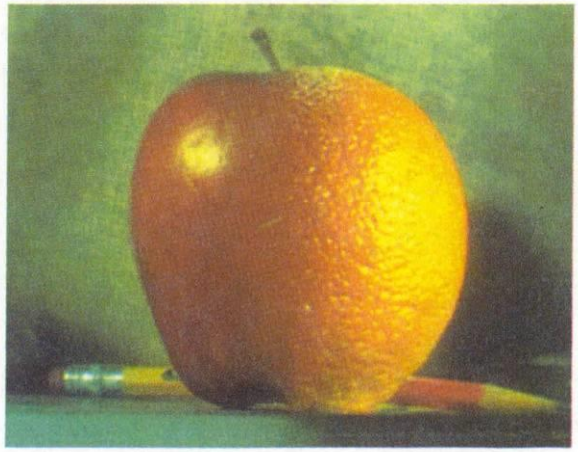




(d)

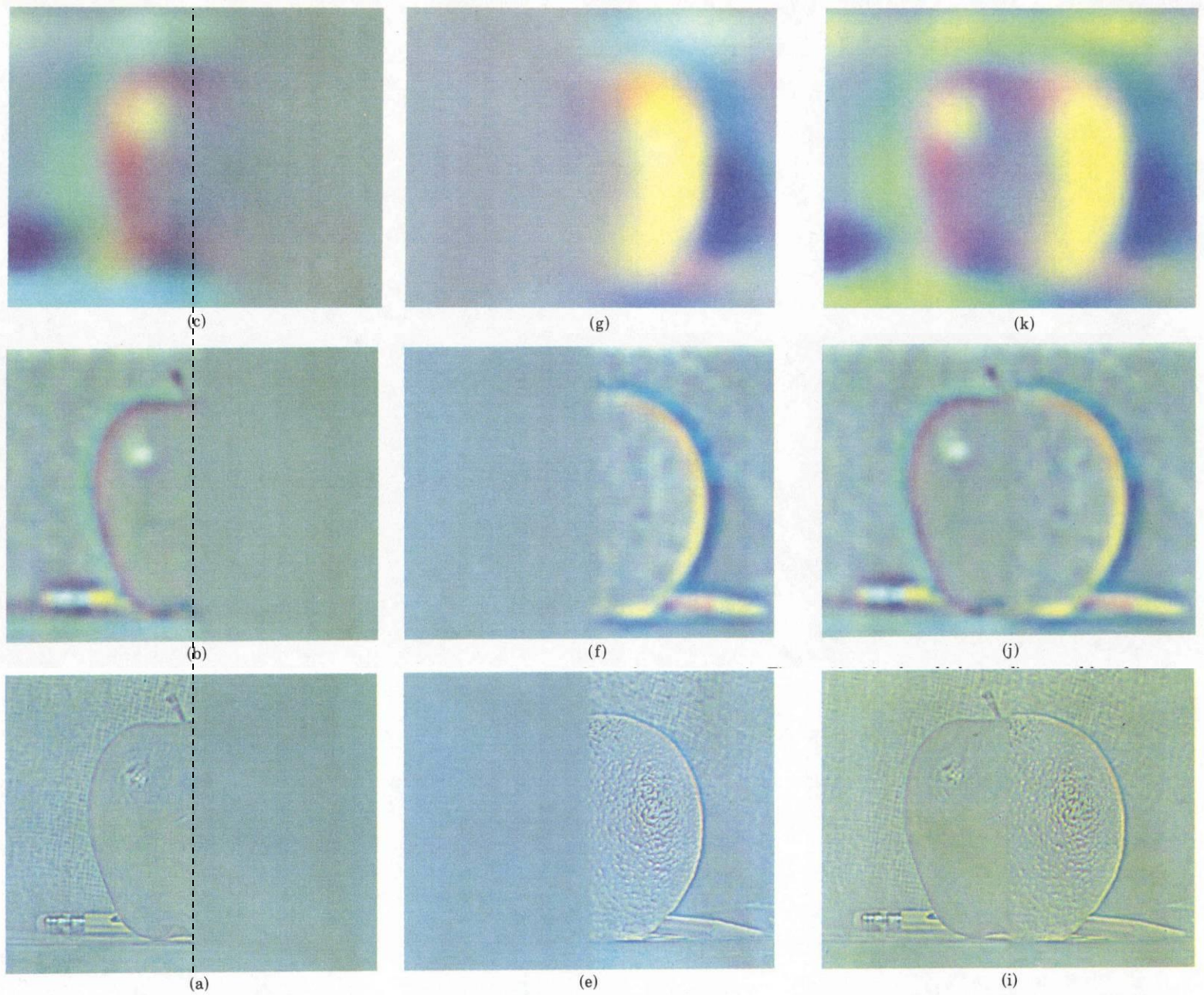


(h)



(l)

http://persci.mit.edu/pub_pdfs/spline83.pdf

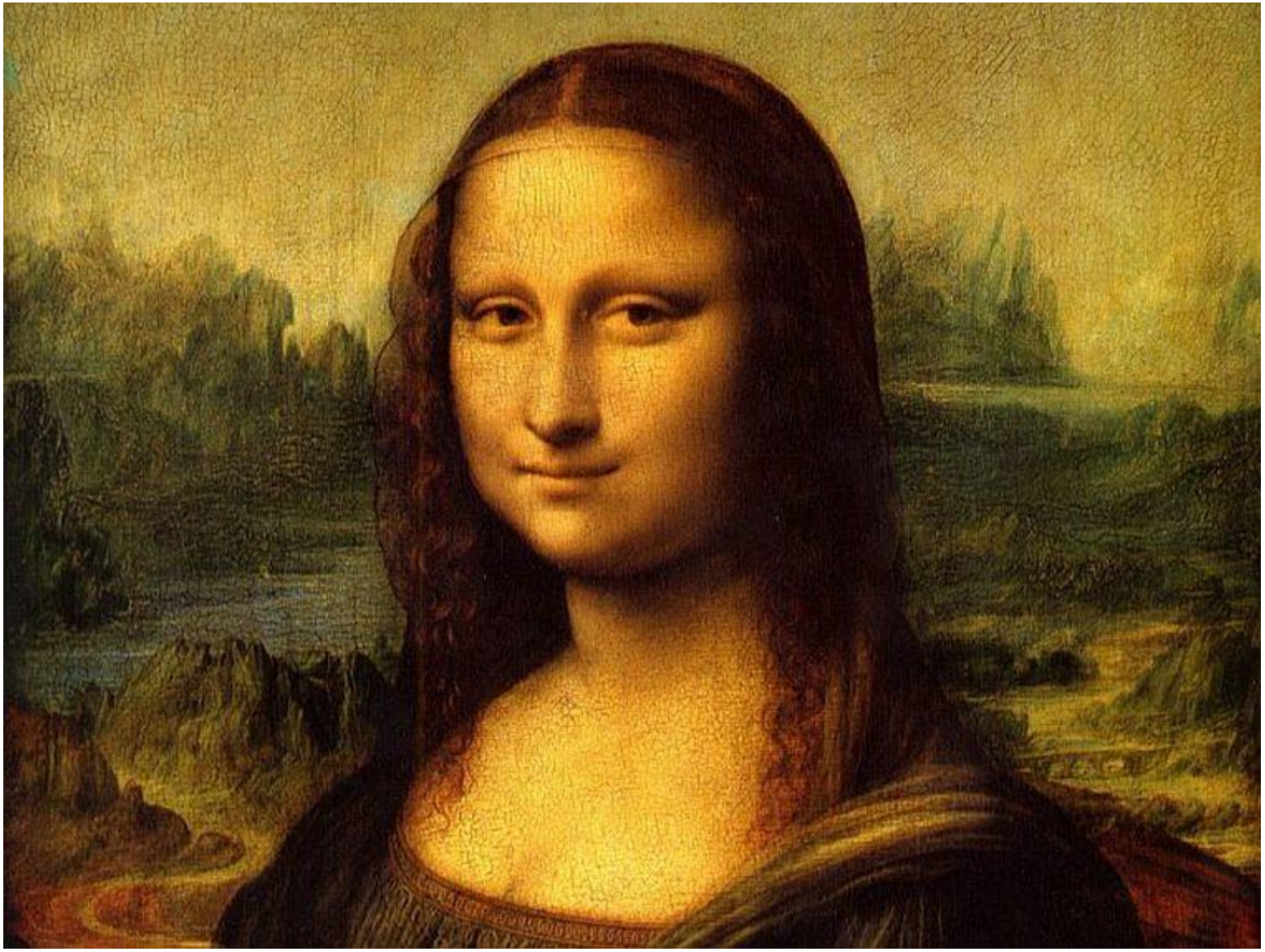

laplacian level 4
laplacian level 2
laplacian level 0
(c)
(g)
(k)
(b)
(f)
(j)
(a)
(e)
(i)
left pyramid
right pyramid
blended pyramid

Blending Regions

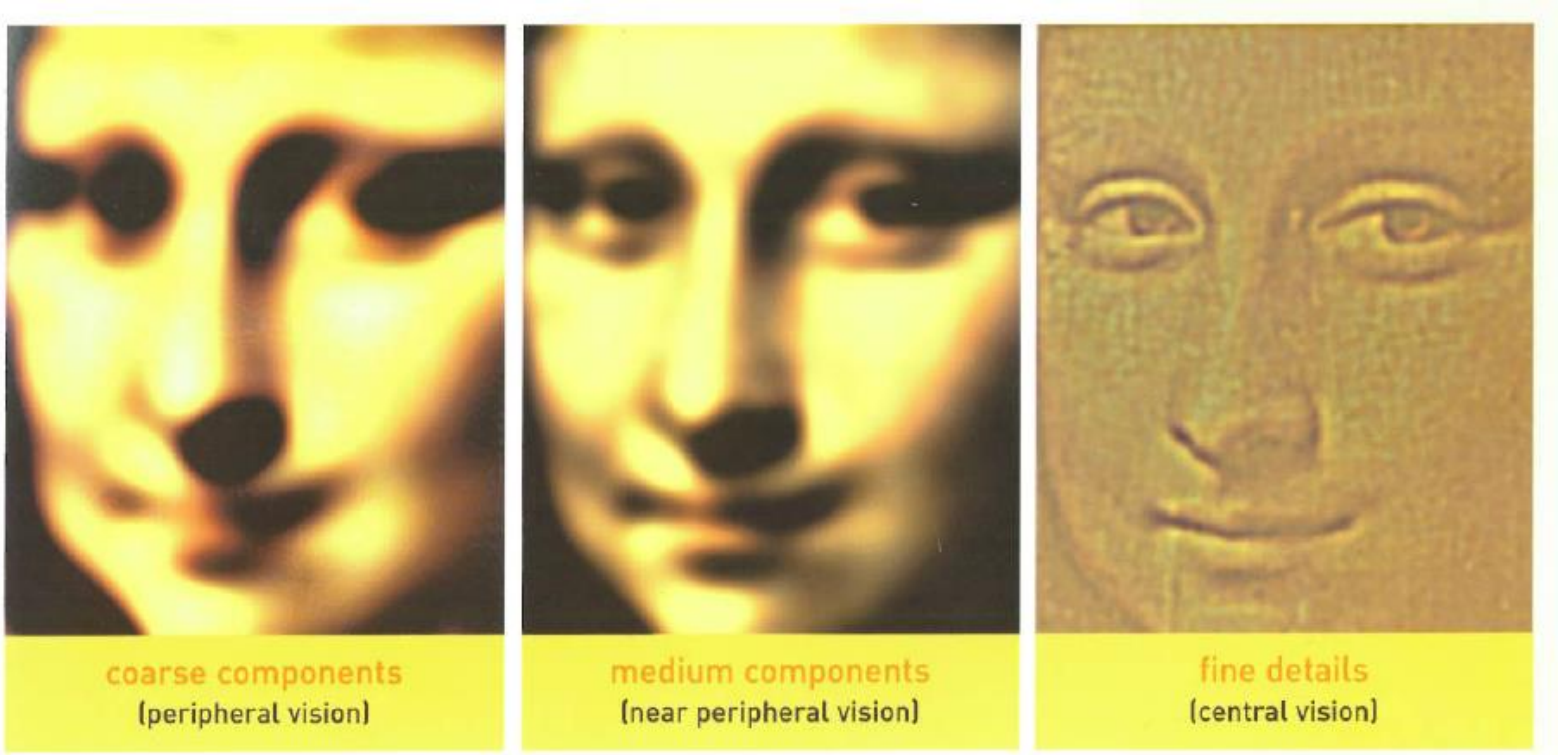
Results from previous class

© Chris Cameron

Da Vinci and The Laplacian Pyramid



Da Vinci and The Laplacian Pyramid



Leonardo playing with peripheral vision

Livingstone, Vision and Art: The Biology of Seeing

Taking derivative by convolution

Partial derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

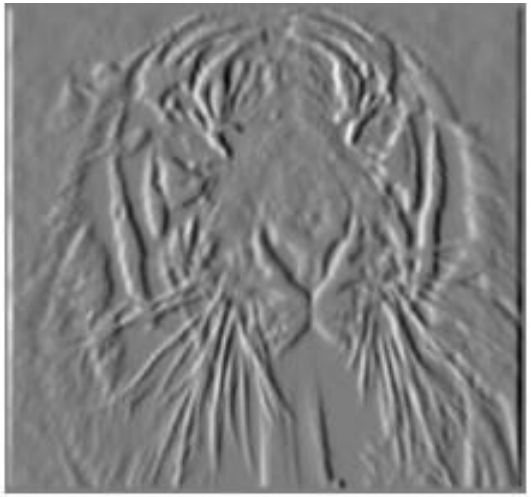
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement above as convolution, what would be the associated filter?

Source: K. Grauman

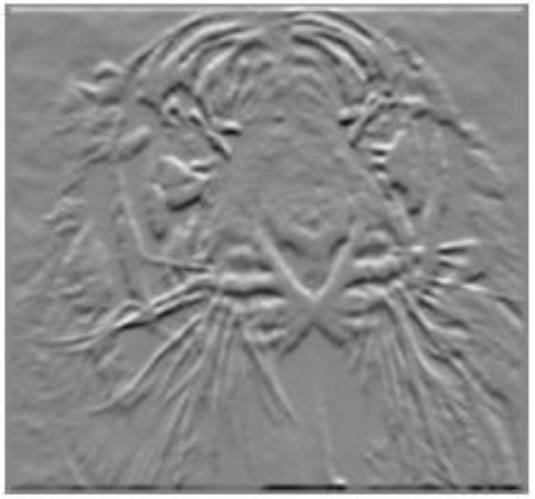
Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$



-1	1

$$\frac{\partial f(x,y)}{\partial y}$$



-1
1

or

1
-1

Which shows changes with respect to x?

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

; $M_y =$

1	2	1
0	0	0
-1	-2	-1

Roberts: $M_x =$

0	1
-1	0

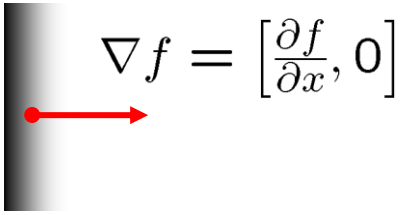
; $M_y =$

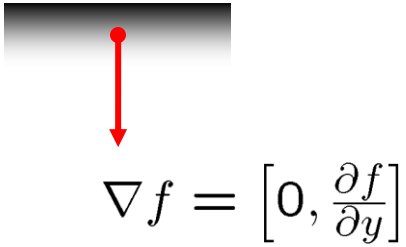
1	0
0	-1

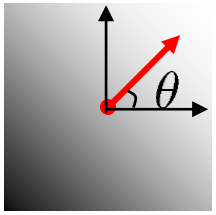
Source: K. Grauman

Image gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$







The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

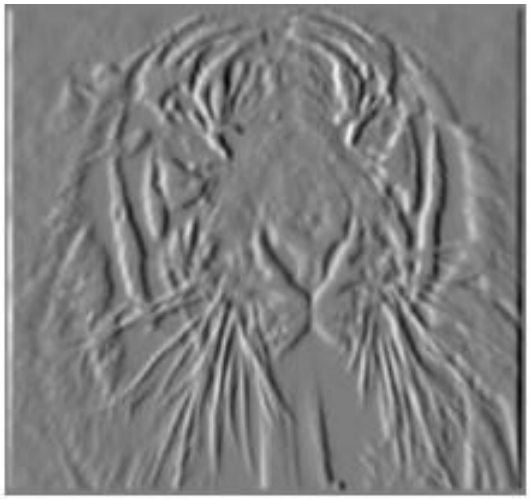
The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

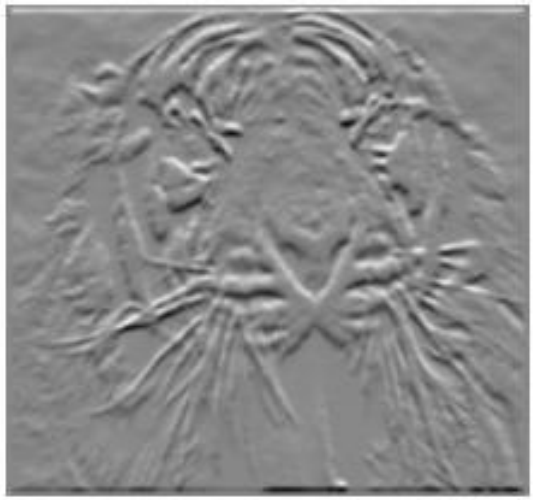
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Image Gradient



$$\frac{\partial f(x,y)}{\partial x}$$



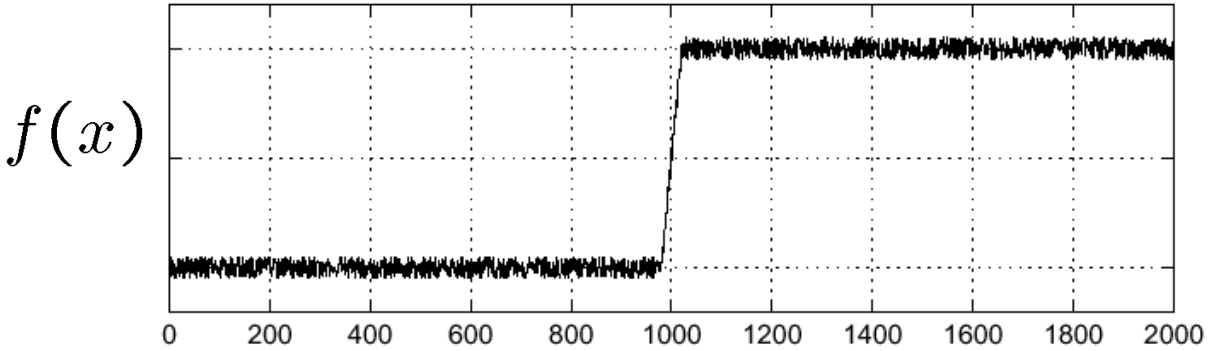
$$\frac{\partial f(x,y)}{\partial y}$$

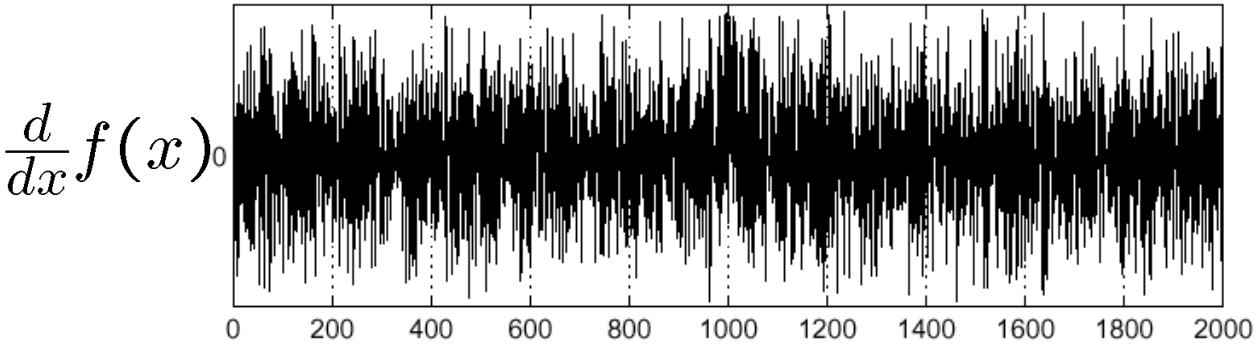
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

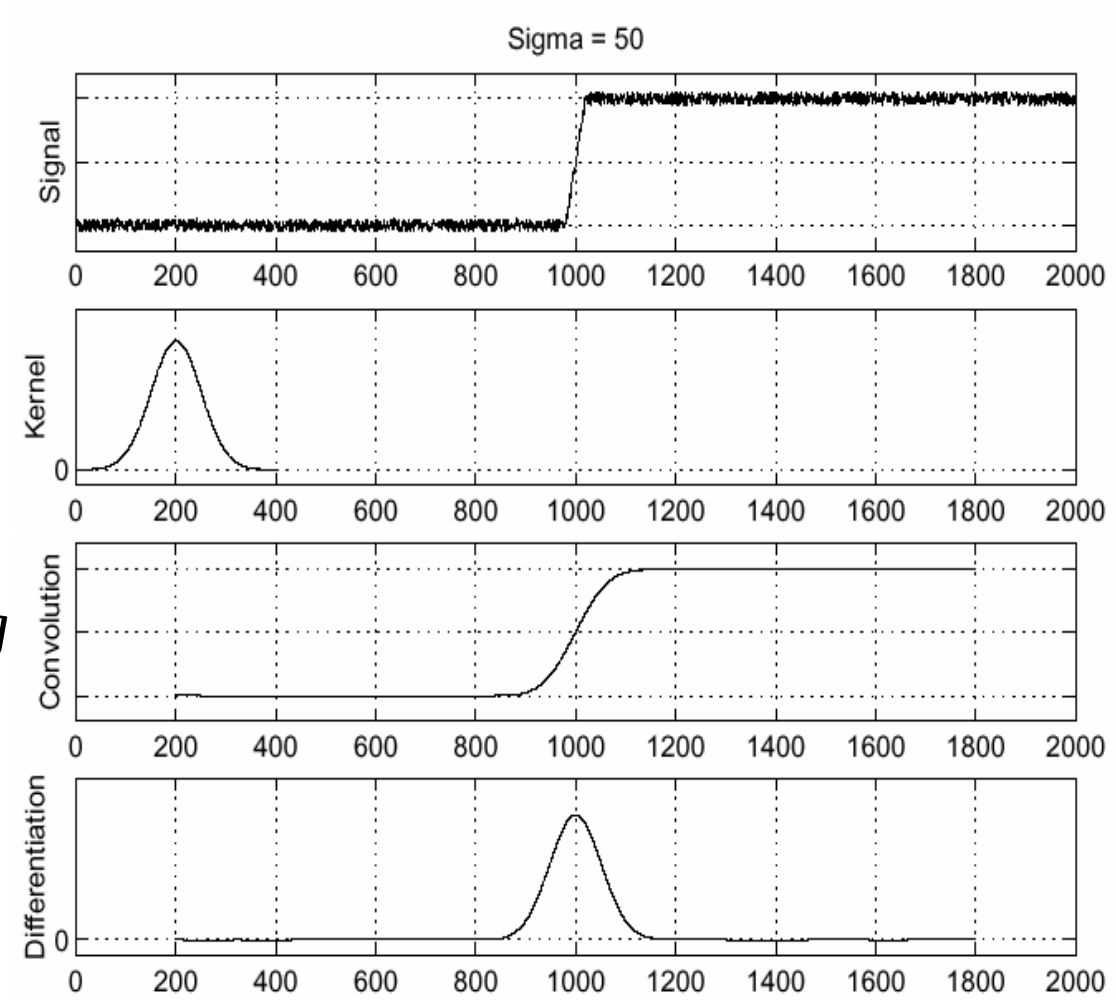




Where is the edge?

Source: S. Seitz

Solution: smooth first



f

g

$f * g$

$\frac{d}{dx}(f * g)$

- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

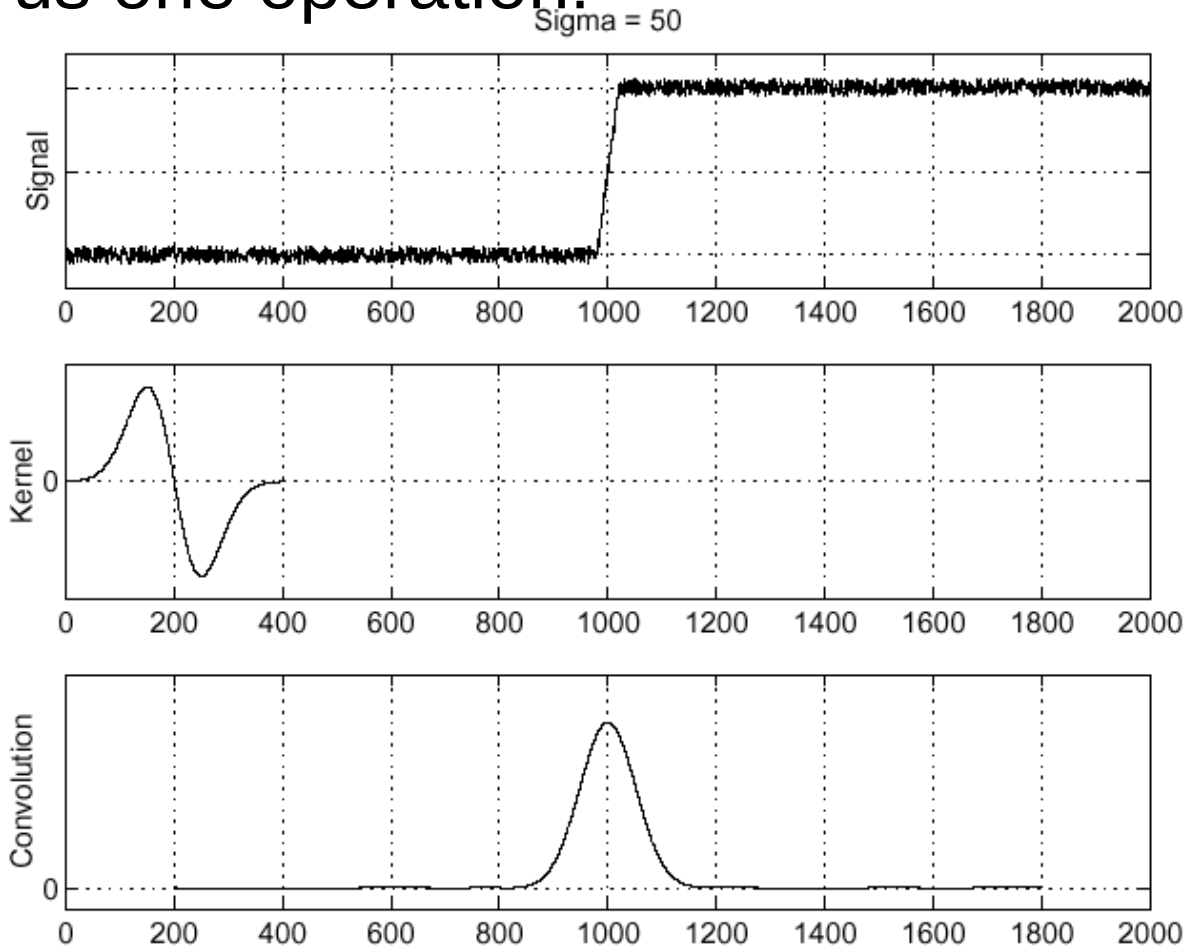
Derivative theorem of convolution

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

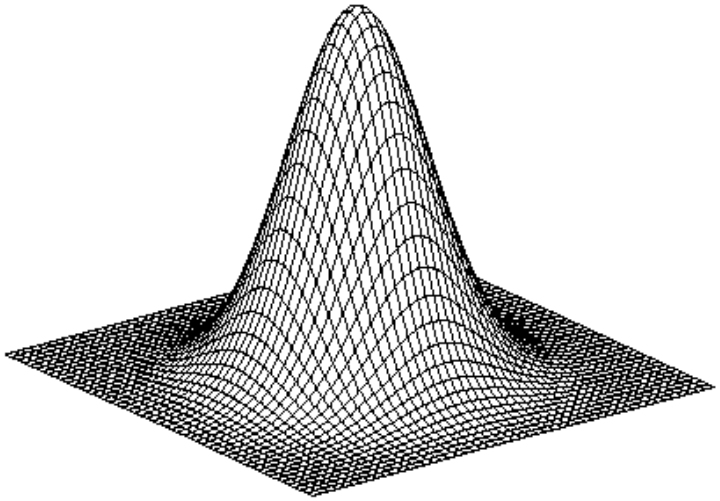
This saves us one operation:

$\frac{\partial}{\partial x}h$

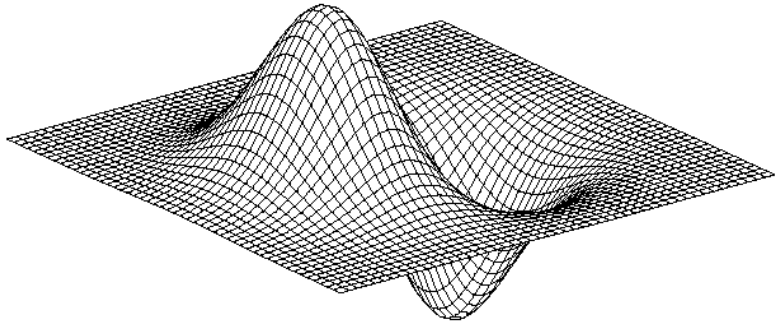
$(\frac{\partial}{\partial x}h) \star f$



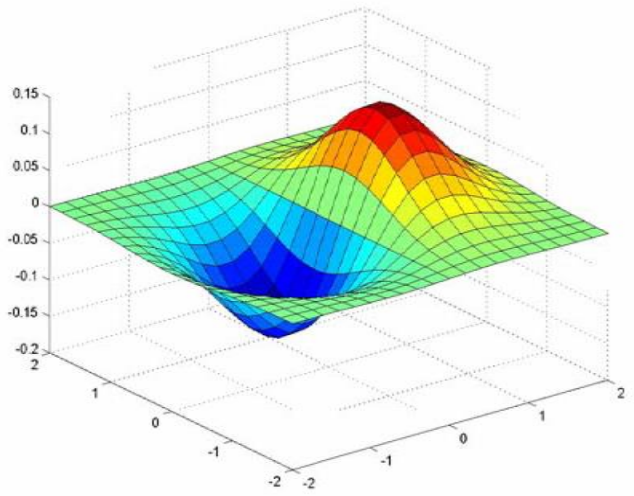
Derivative of Gaussian filter



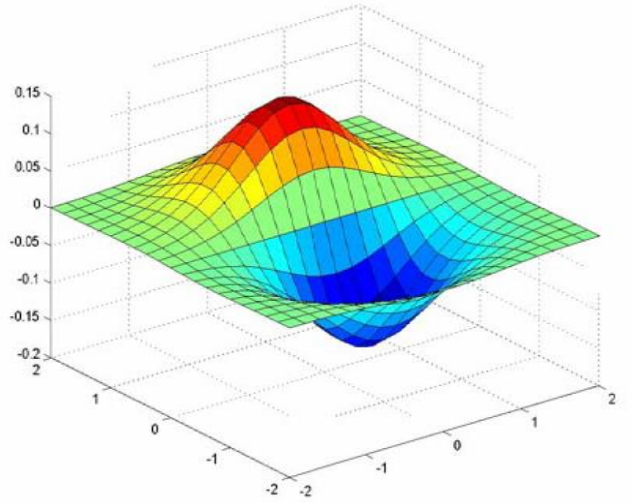
* [1 -1] =



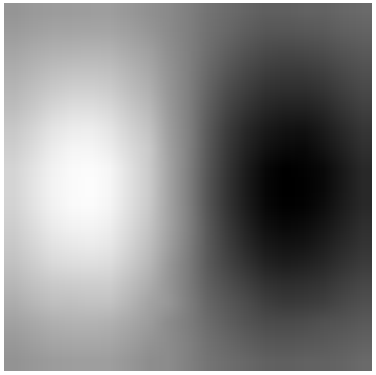
Derivative of Gaussian filter

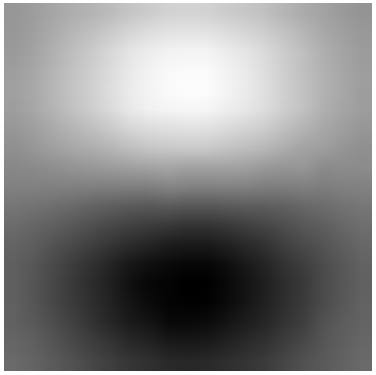


x-direction



y-direction

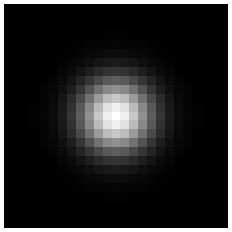




Which one finds horizontal/vertical edges?

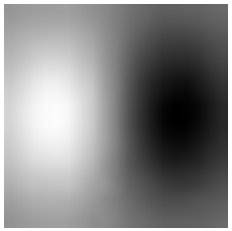
Review: Smoothing vs. derivative filters

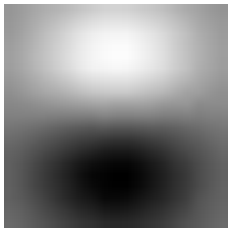
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters





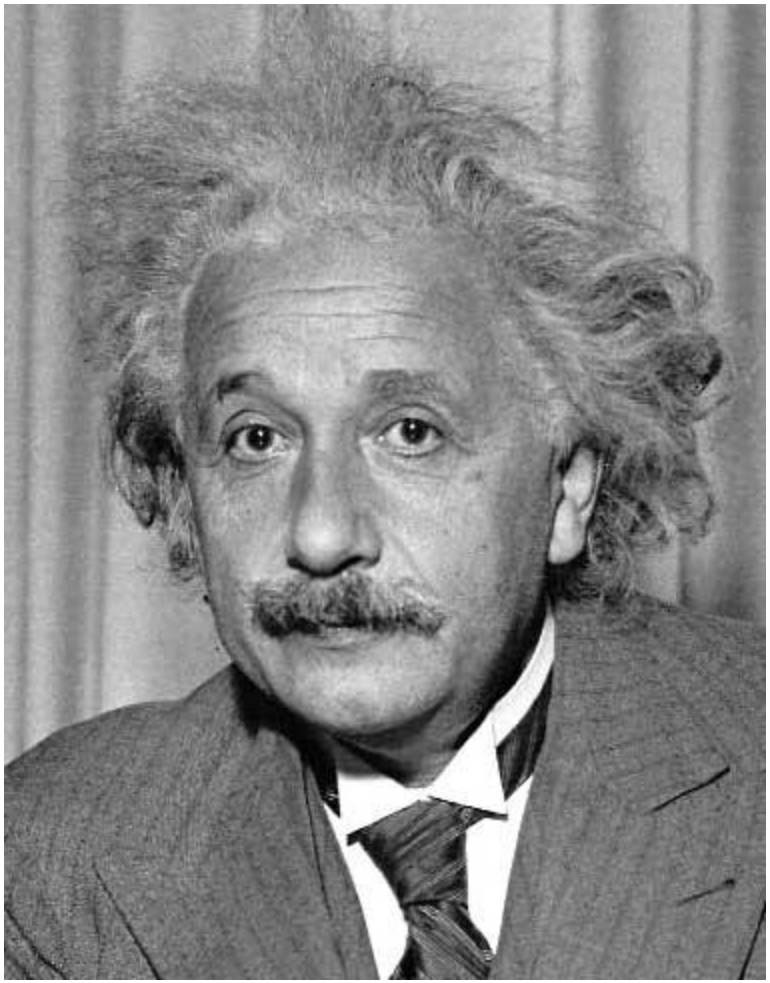
- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions
- High absolute value at points of high contrast

Template matching

Goal: find  in image

Main challenge: What is a good similarity or distance measure between two patches?

- Correlation
- Zero-mean correlation
- Sum Square Difference
- Normalized Cross Correlation



Side by Derek Hoiem

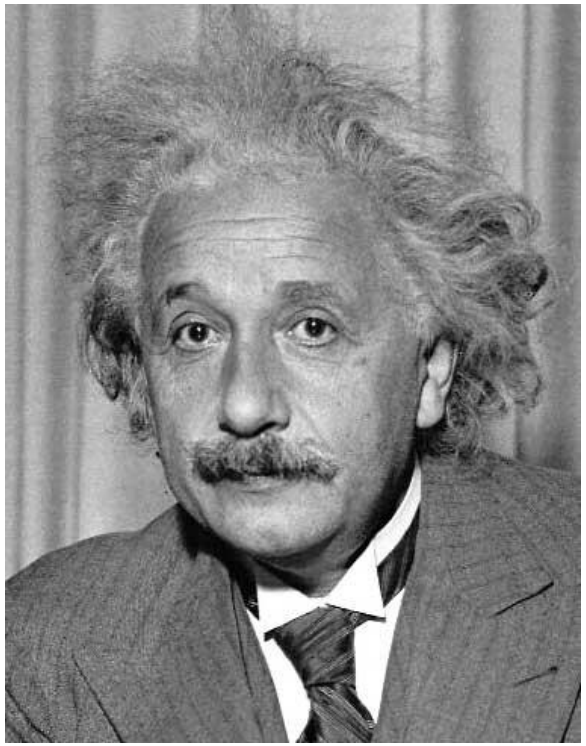
Matching with filters

Goal: find  in image

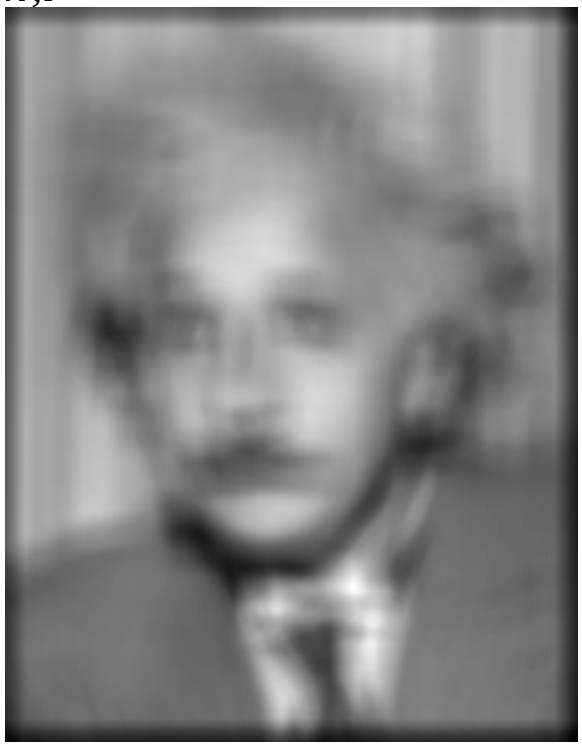
Method 0: filter the image with eye patch

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k, n+l]$$

f = image
g = filter



Input



Filtered Image

What went wrong?

Side by Derek Hoiem

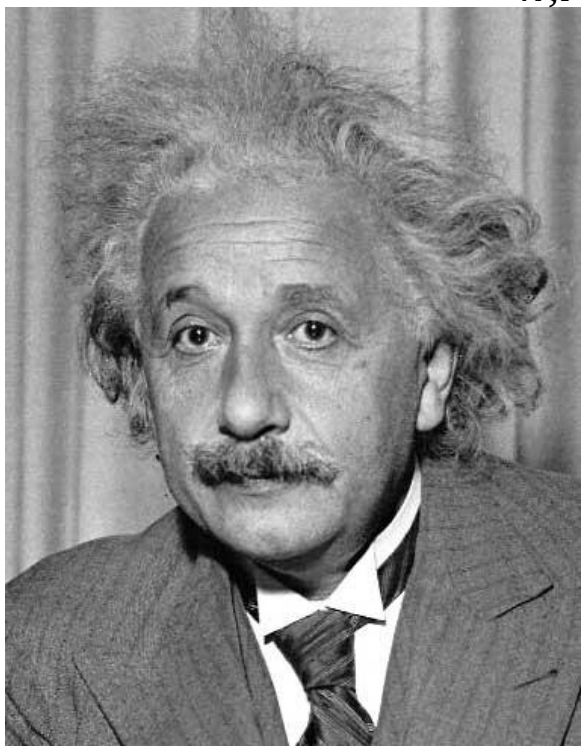
Matching with filters

Goal: find  in image

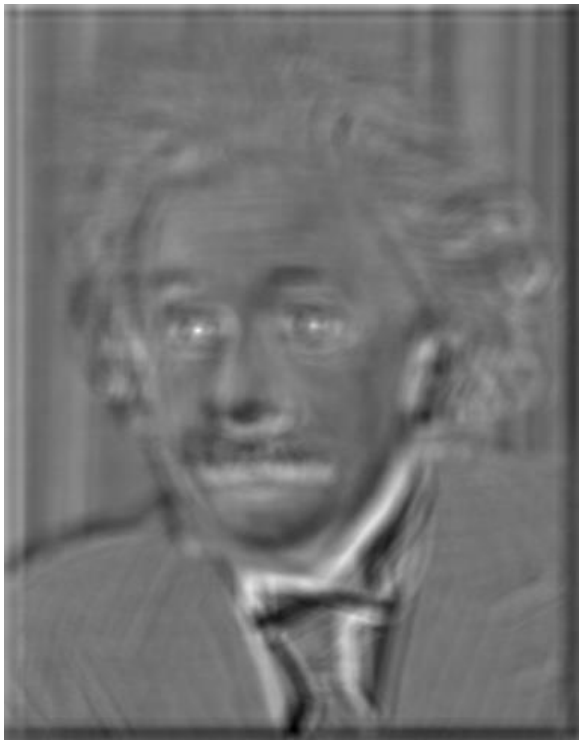
Method 1: filter the image with zero-mean eye

$$h[m,n] = \sum_{k,l} (f[k,l] - \bar{f})(g[m+k,n+l])$$

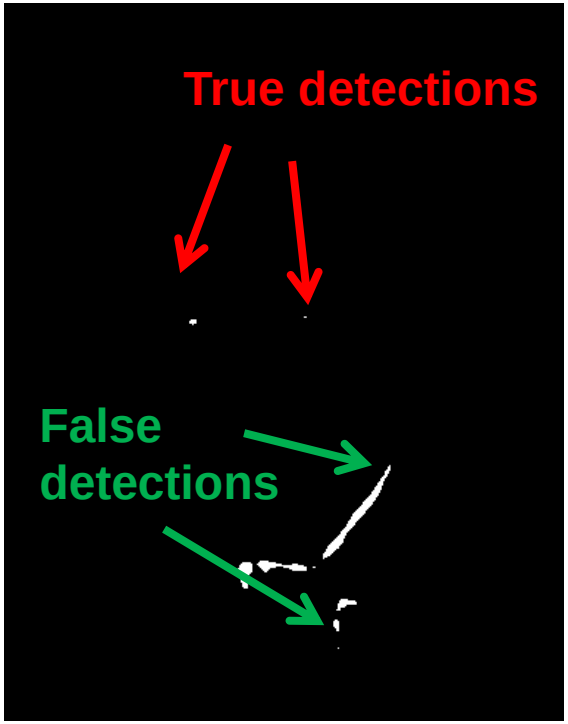
mean of f



Input



Filtered Image (scaled)



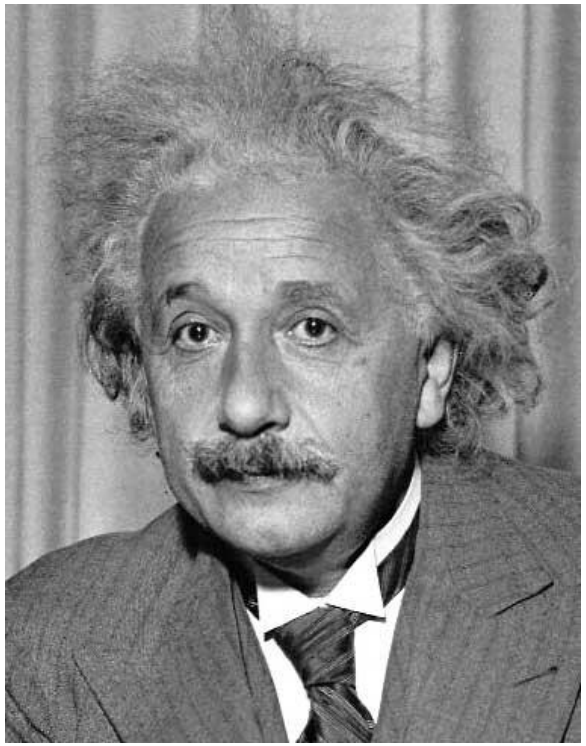
Thresholded Image

Matching with filters

Goal: find  in image

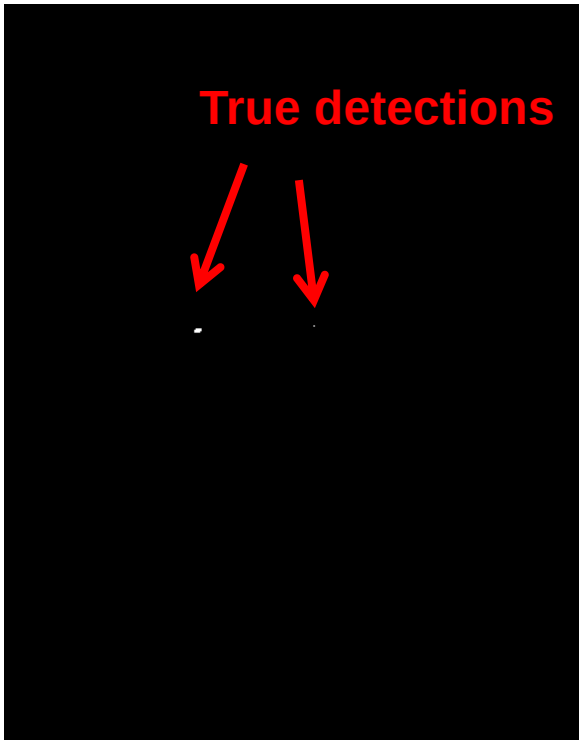
Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)



Thresholded Image

Matching with filters

Can SSD be implemented with linear filters?

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$

Side by Derek Hoiem

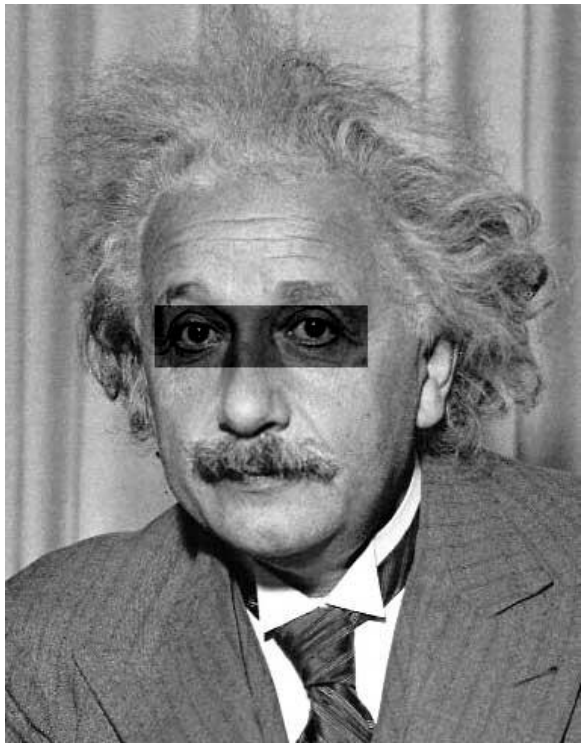
Matching with filters

Goal: find  in image

What's the potential downside of SSD?

Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

Side by Derek Hoiem

Matching with filters

Goal: find  in image

Method 3: Normalized cross-correlation

mean template

mean image patch

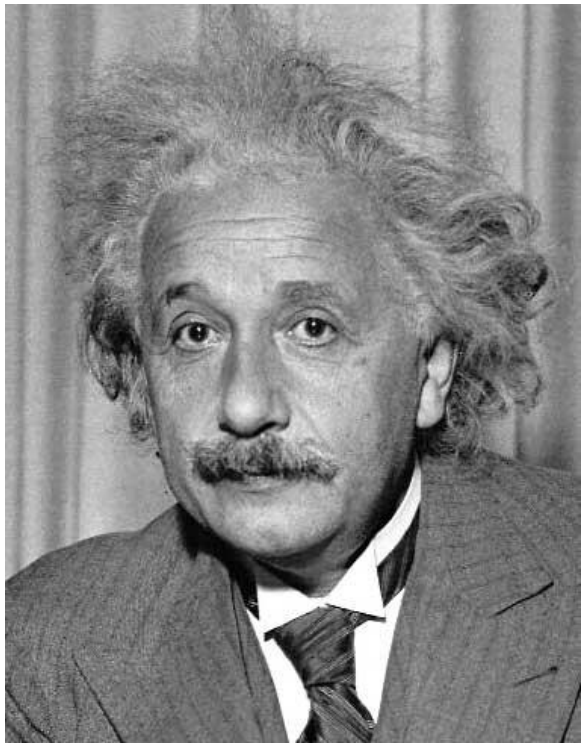
$$h[m,n] = \frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m+k,n+l]-\bar{f}_{m,n})}{\left(\sum_{k,l}(g[k,l]-\bar{g})^2 \sum_{k,l}(f[m+k,n+l]-\bar{f}_{m,n})^2\right)^{0.5}}$$

Side by Derek Hoiem

Matching with filters

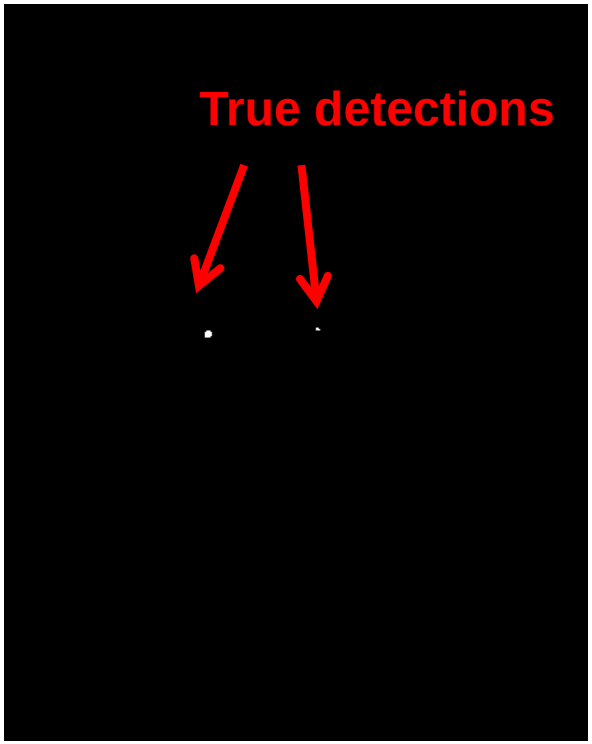
Goal: find  in image

Method 3: Normalized cross-correlation



Input

Normalized X-Correlation

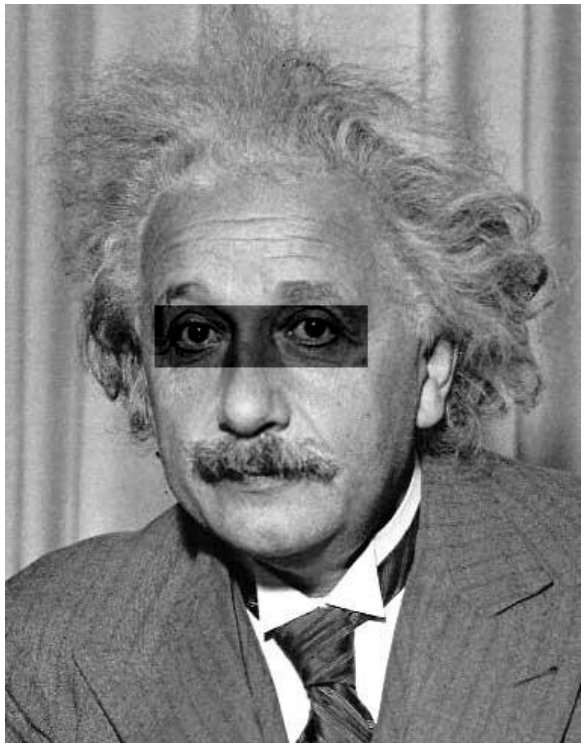


Thresholded Image

Matching with filters

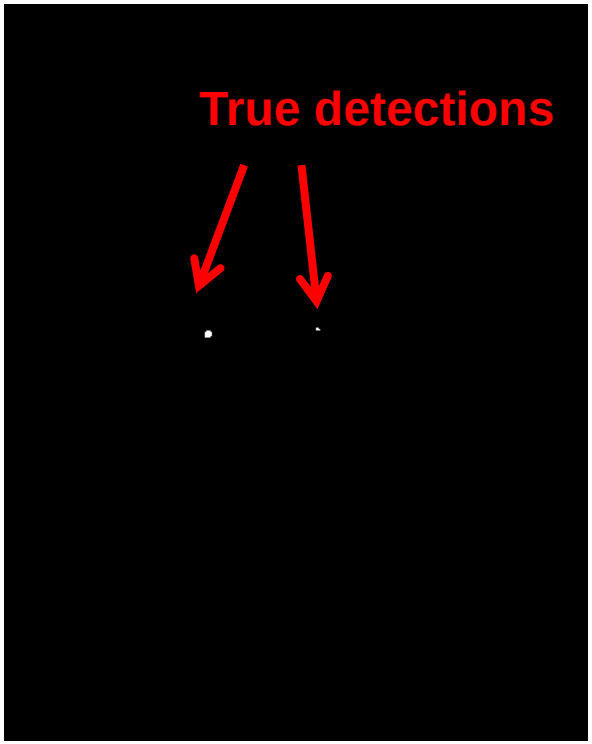
Goal: find  in image

Method 3: Normalized cross-correlation



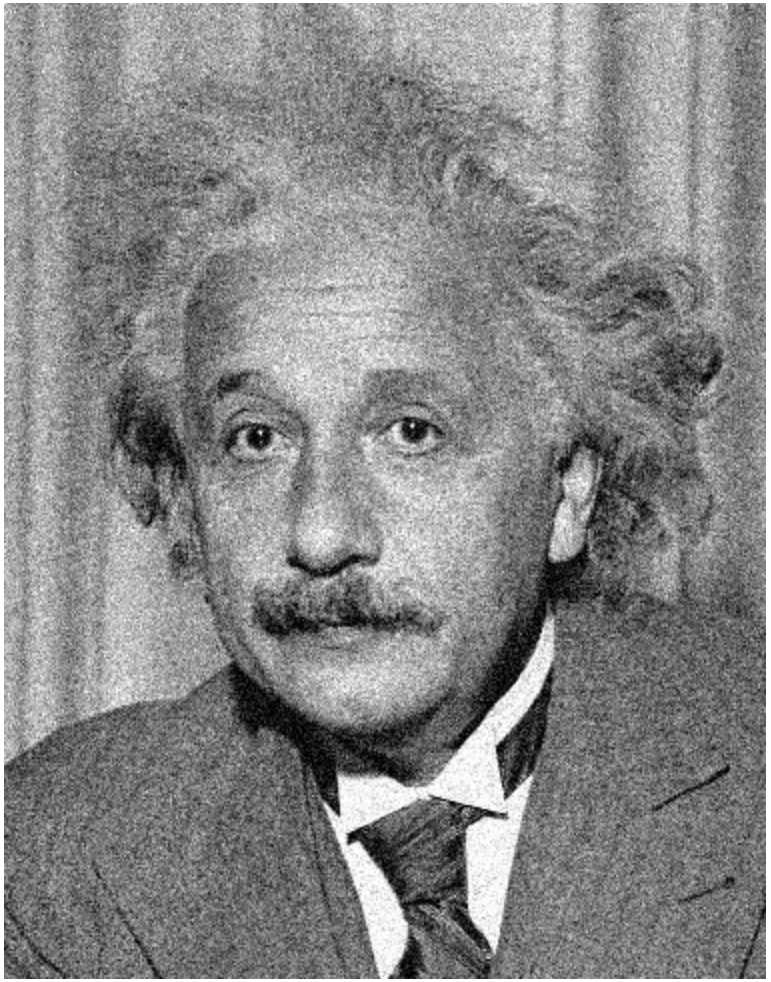
Input

Normalized X-Correlation



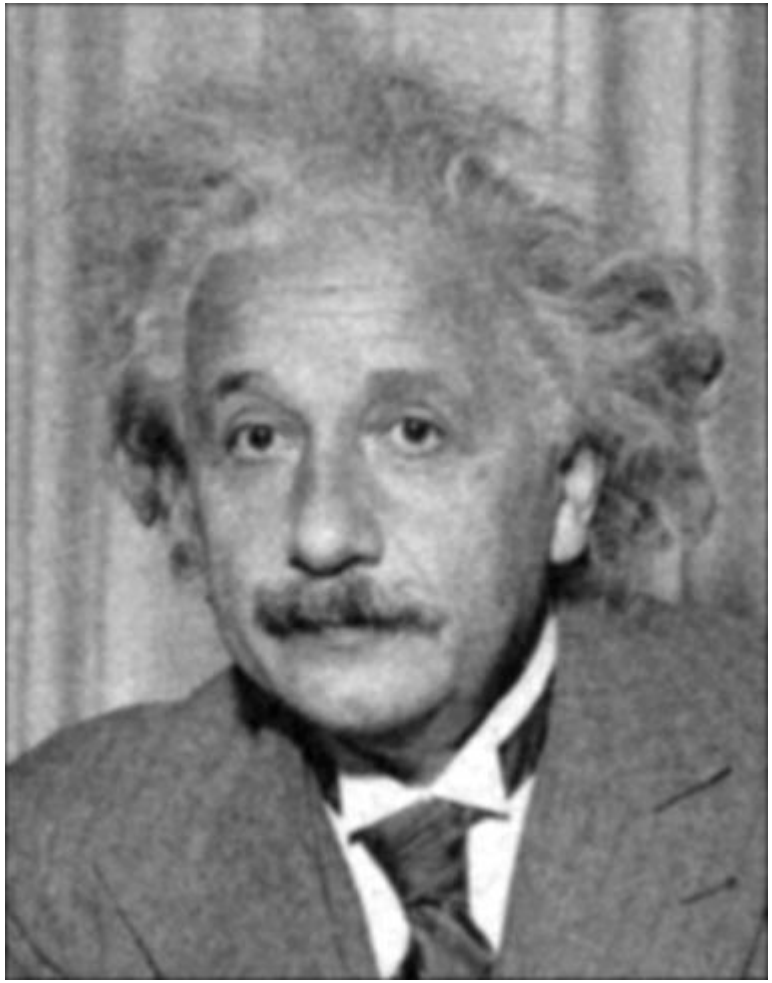
Thresholded Image

Non-linear Filters

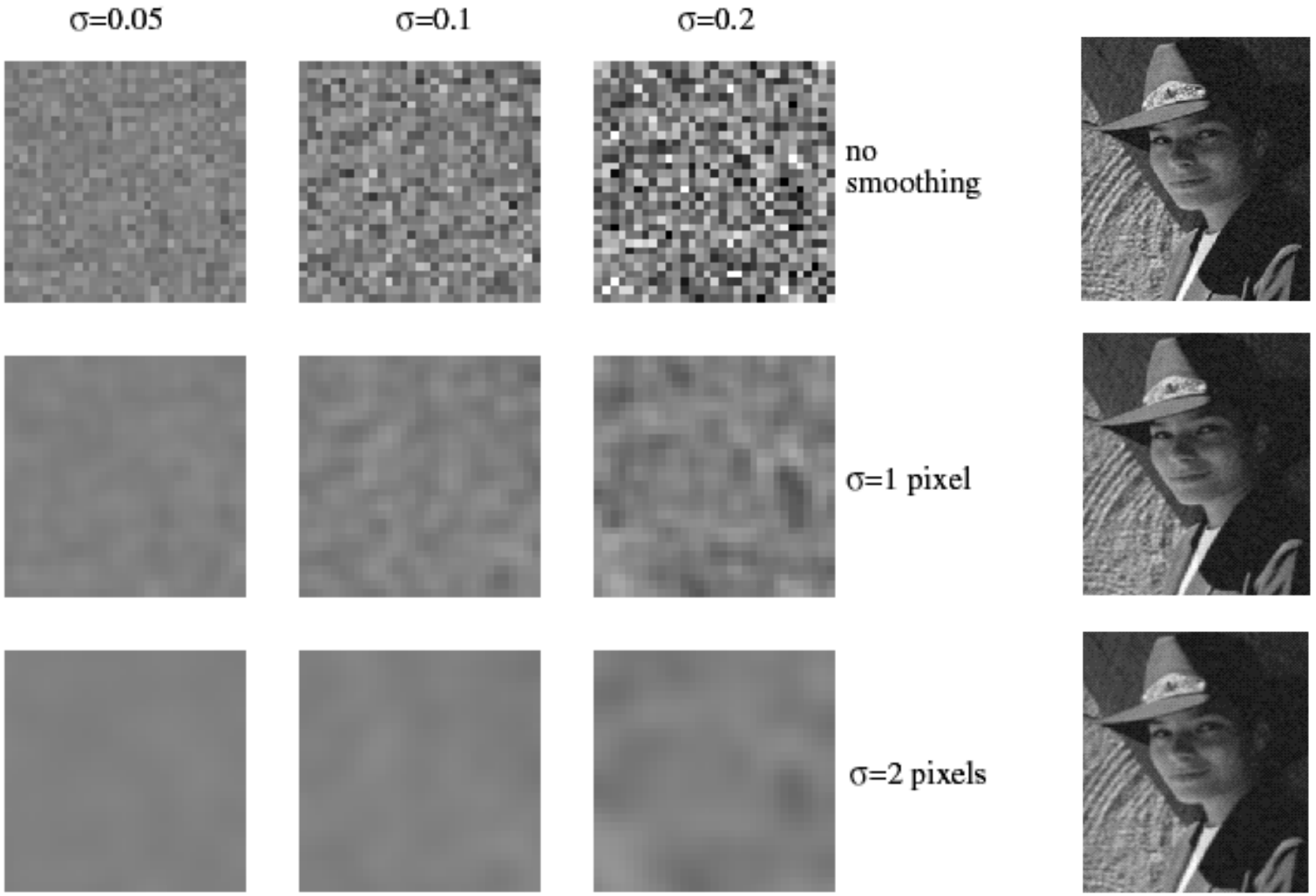


Additive Gaussian Noise

Gaussian
Filter



Reducing Gaussian noise



Smoothing with larger standard deviations suppresses noise, but also blurs the image

Source: S. Lazebnik

Reducing salt-and-pepper noise by Gaussian smoothing

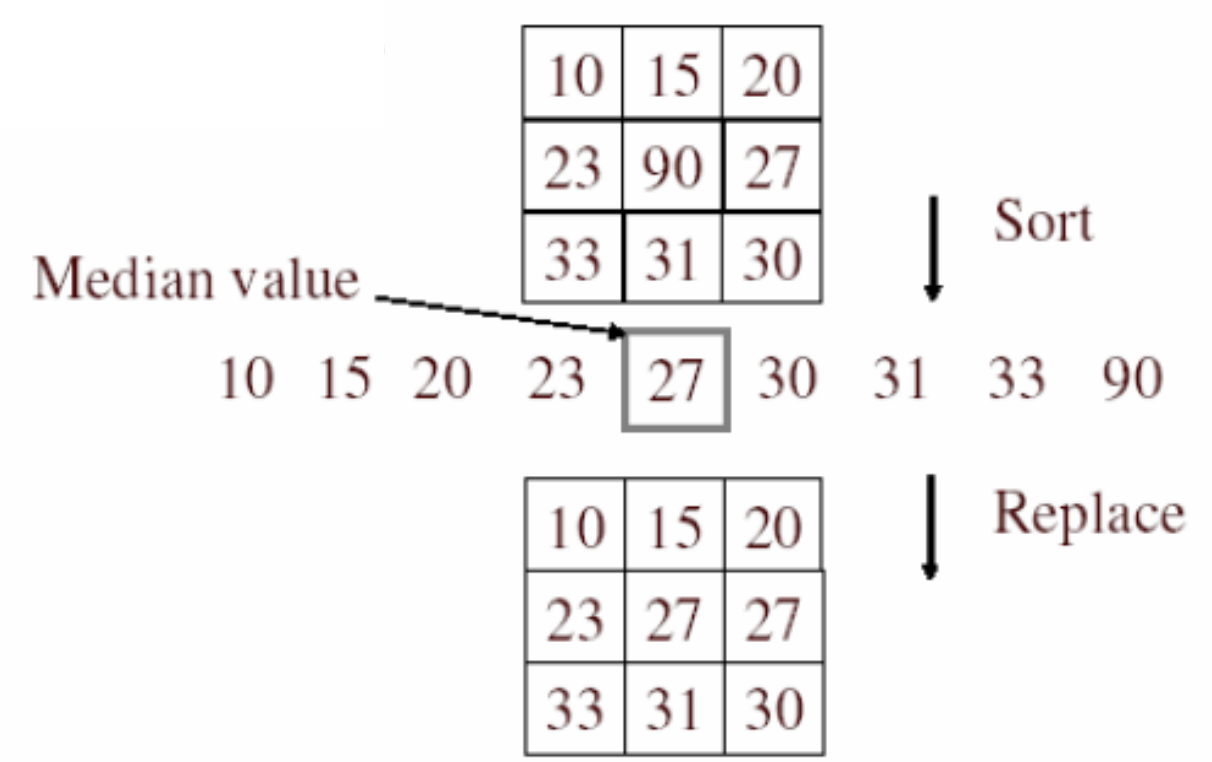
3x3

5x5

7x7

Alternative idea: Median filtering

A **median filter** operates over a window by selecting the median intensity in the window



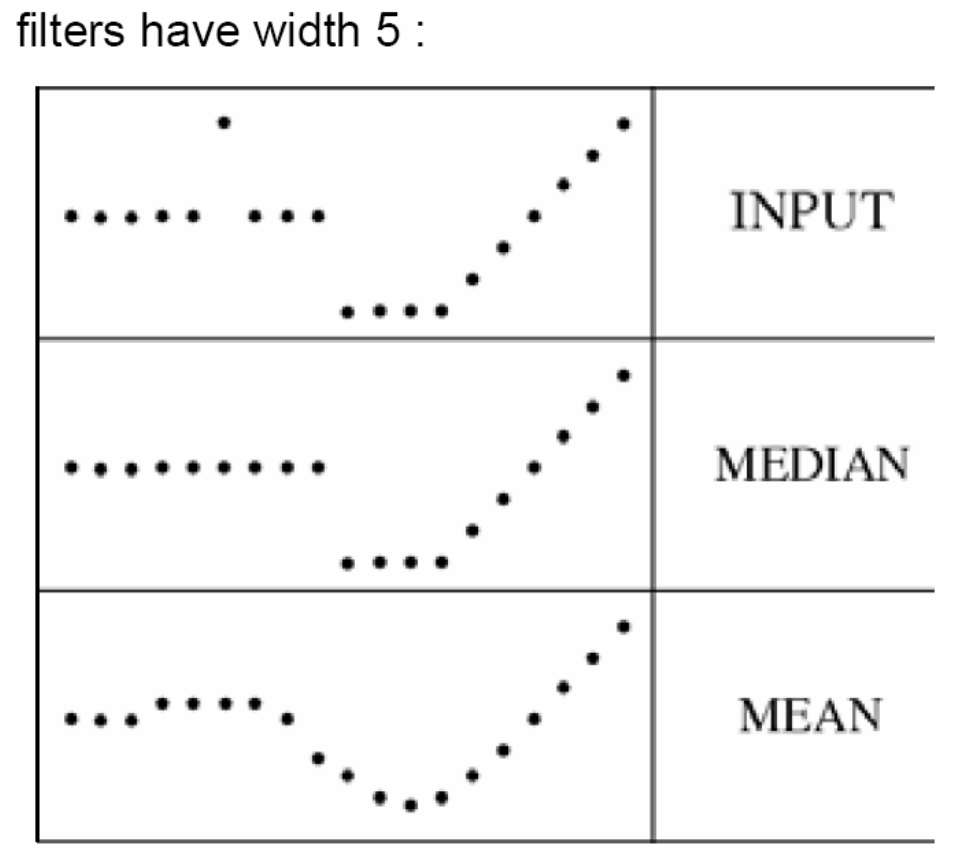
- Is median filtering linear?

Source: K. Grauman

Median filter

What advantage does median filtering have over Gaussian filtering?

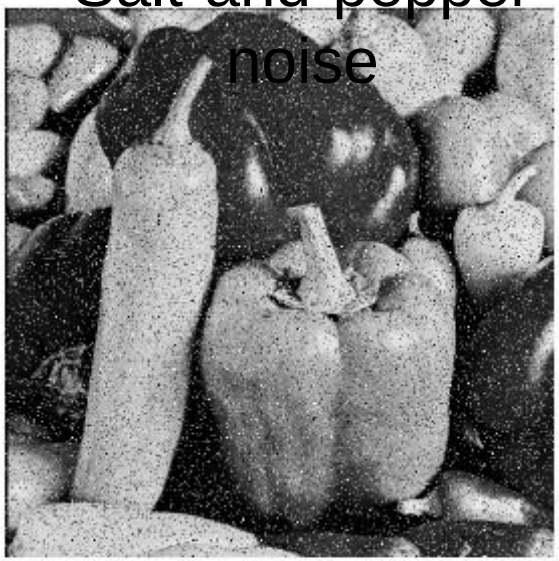
- Robustness to outliers



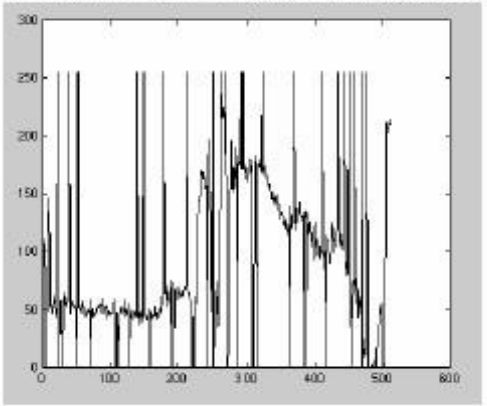
Source: K. Grauman

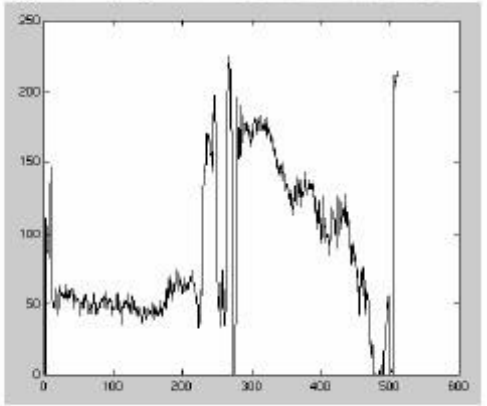
Median filter

Salt-and-pepper noise



Median filtered

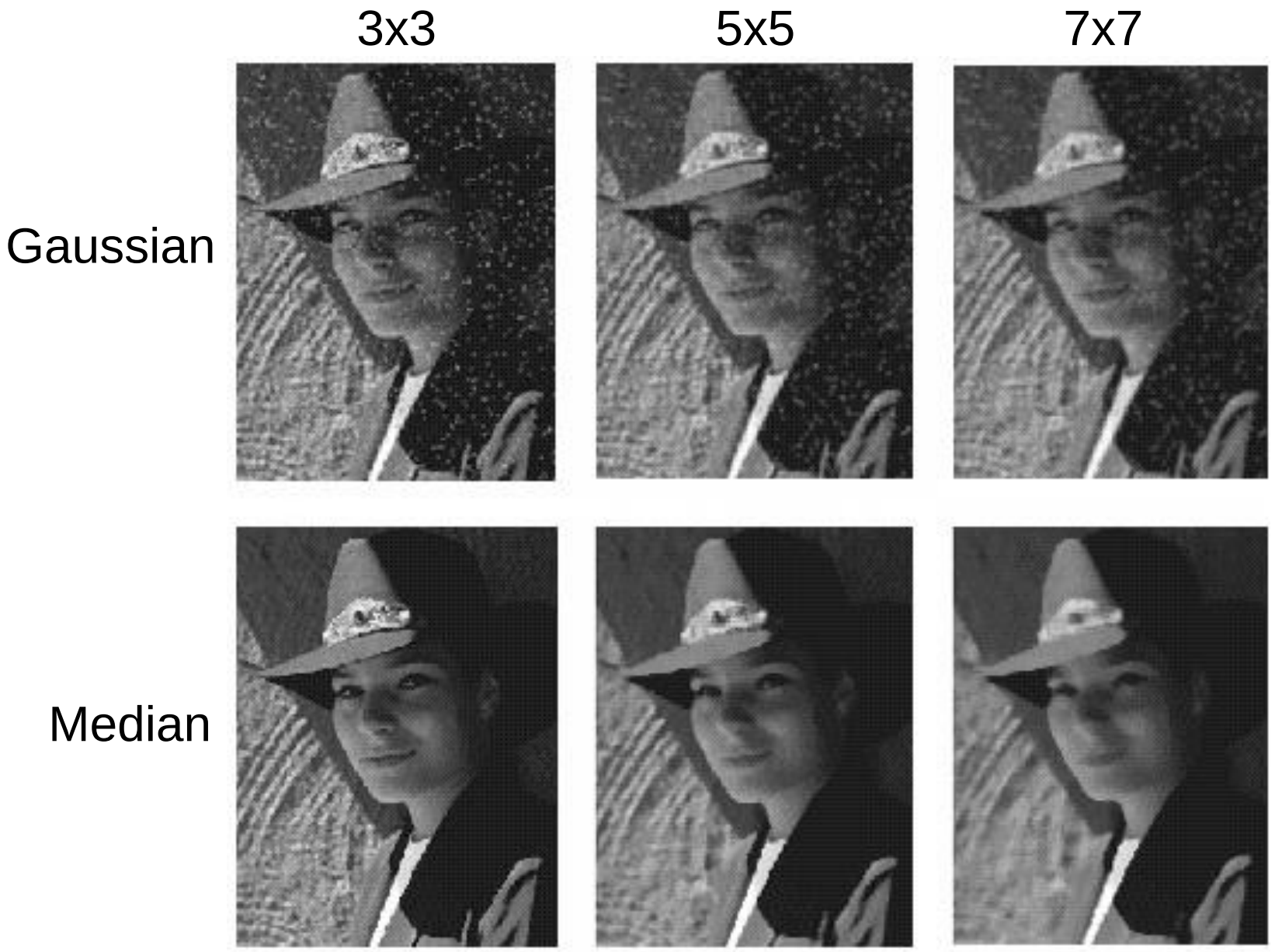




MATLAB: medfilt2(image, [h w])

Source: M. Hebert

Median vs. Gaussian filtering



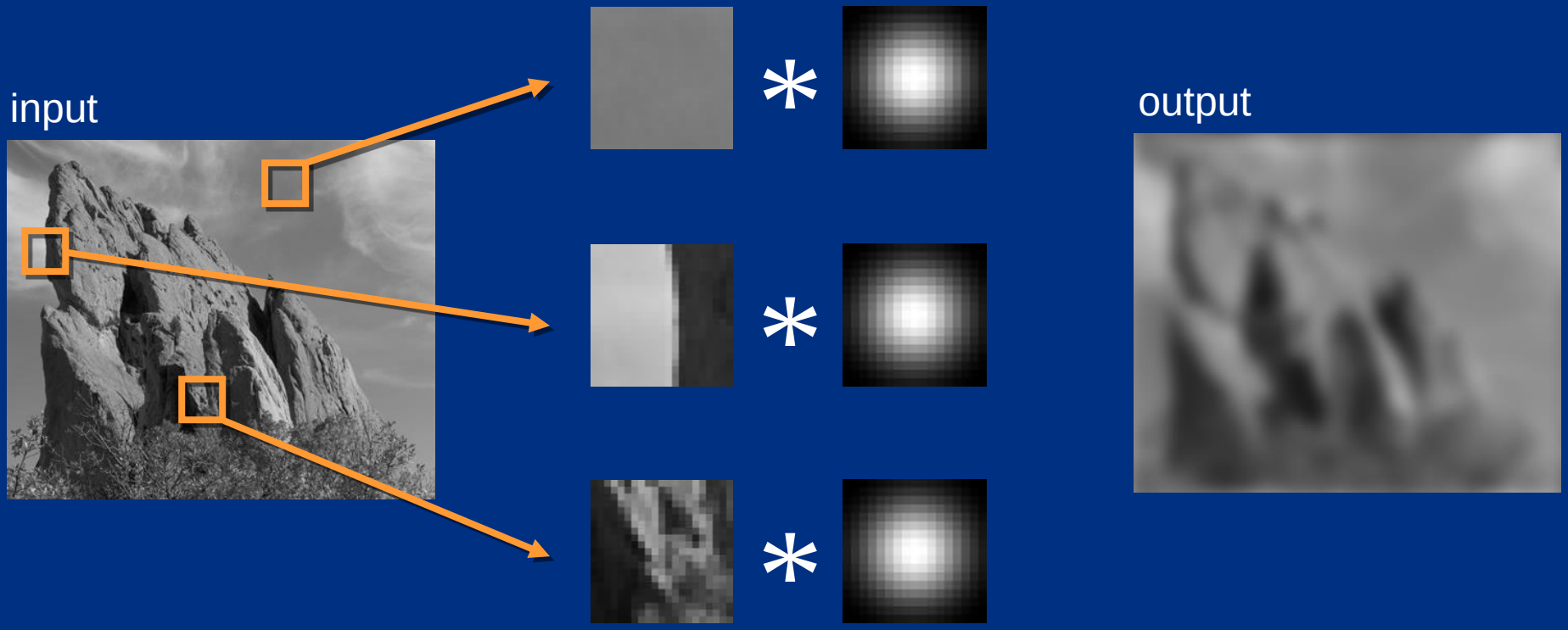
A Gentle Introduction
to Bilateral Filtering
and its Applications

SIGGRAPH2007

“Fixing the Gaussian Blur”: the Bilateral Filter

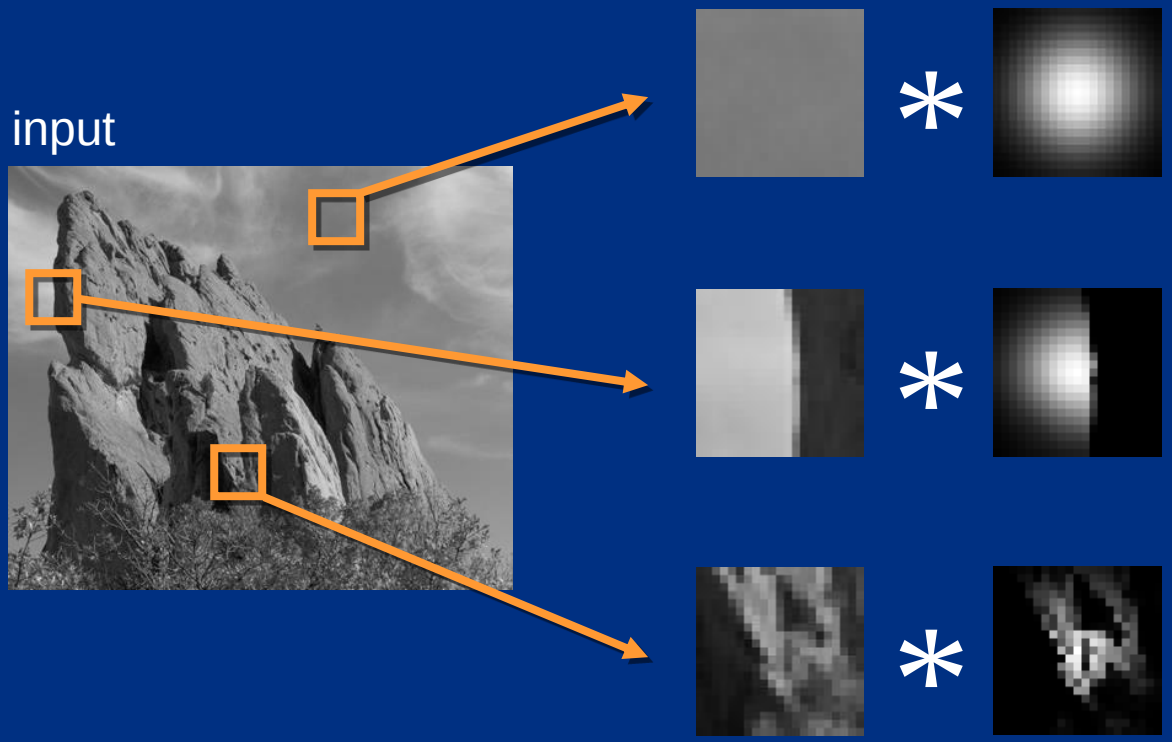
Sylvain Paris – MIT CSAIL

Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

Bilateral Filter [Aurich 95, Smith 97, Tomasi 98]
No Averaging across Edges



The kernel shape depends on the image content.

Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels.**

new

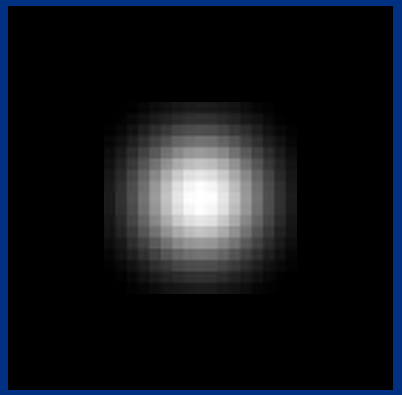
not new

new

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

normalization factor

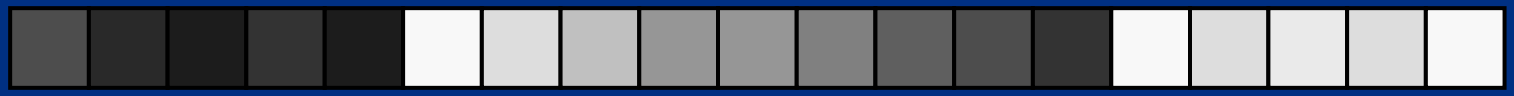
space weight



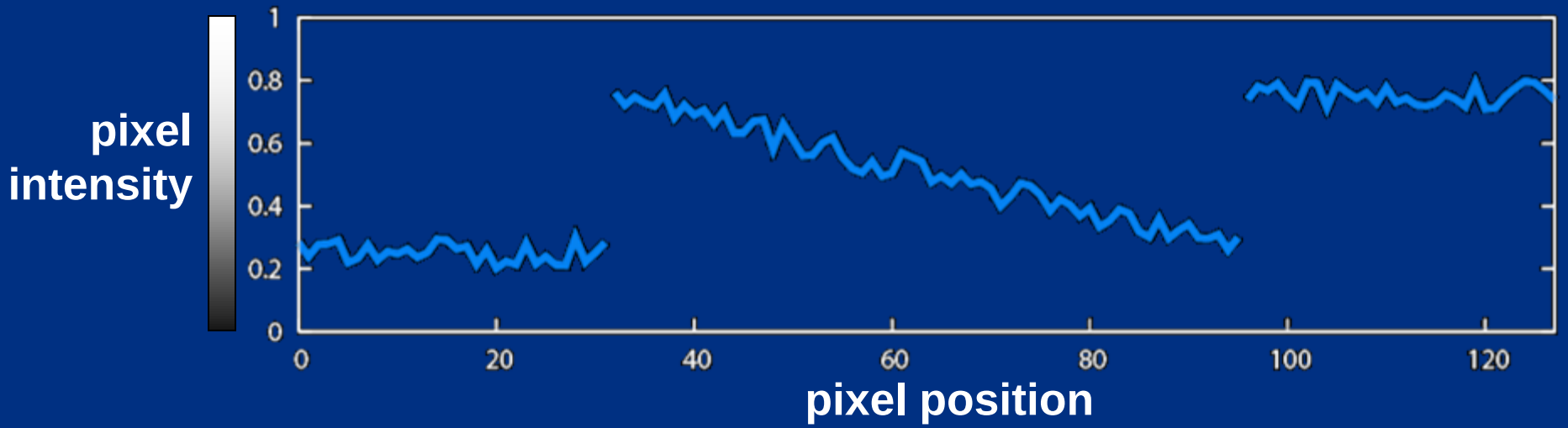
range weight

Illustration a 1D Image

- 1D image = line of pixels

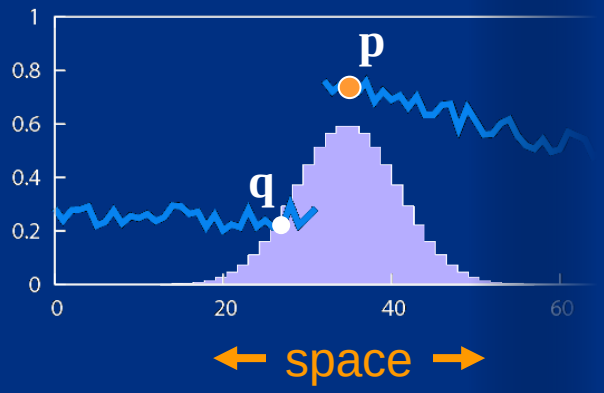


- Better visualized as a plot



Gaussian Blur and Bilateral Filter

Gaussian blur

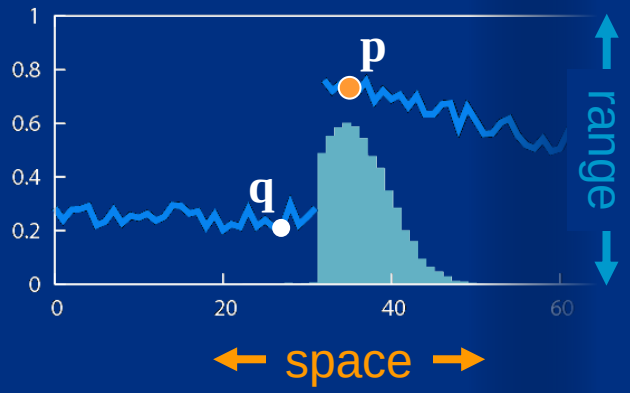


$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|) I_{\mathbf{q}}$$

space

Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]



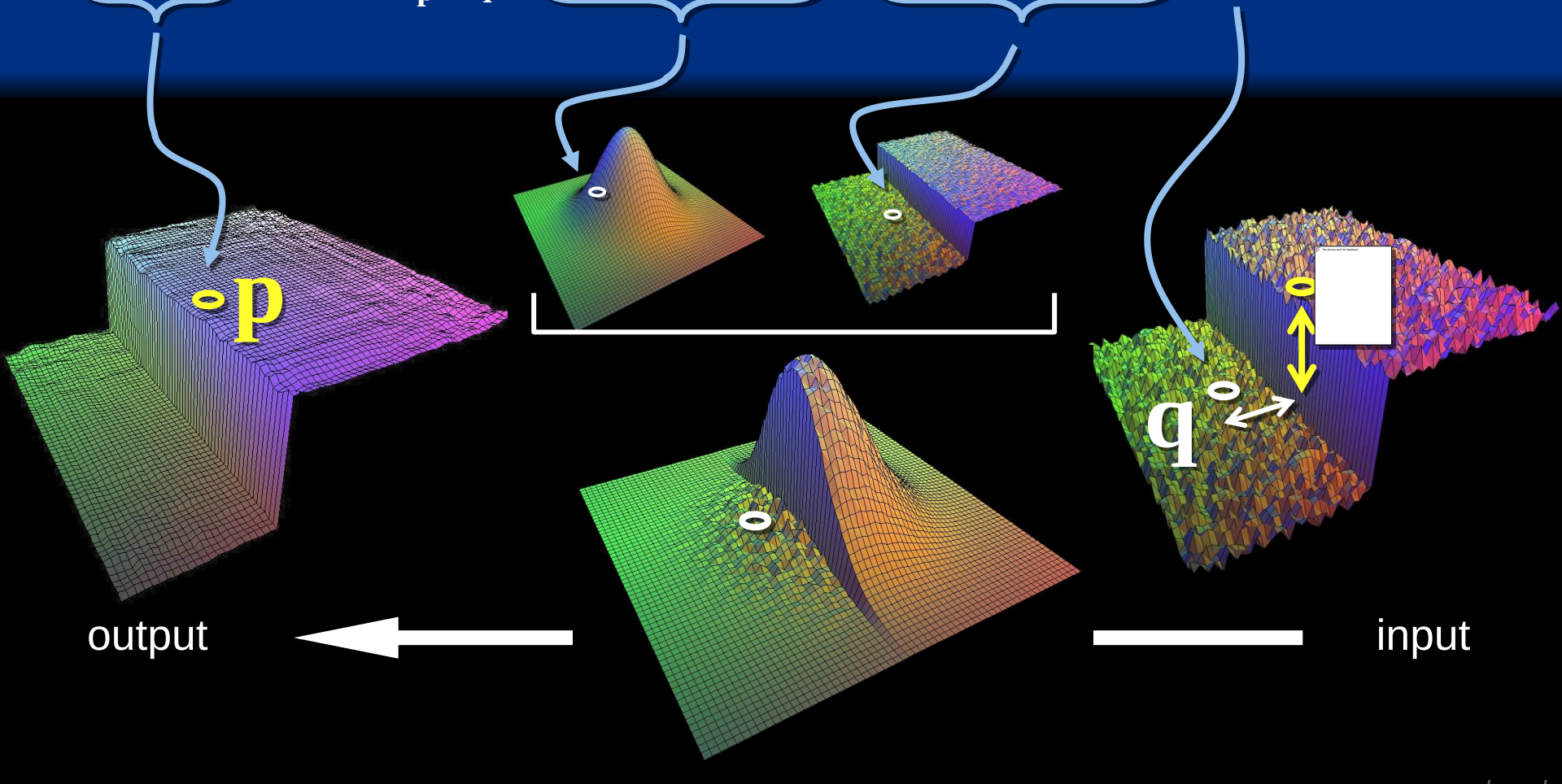
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$

space range

normalization

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$



reproduced from [Durand 02]

Space and Range Parameters

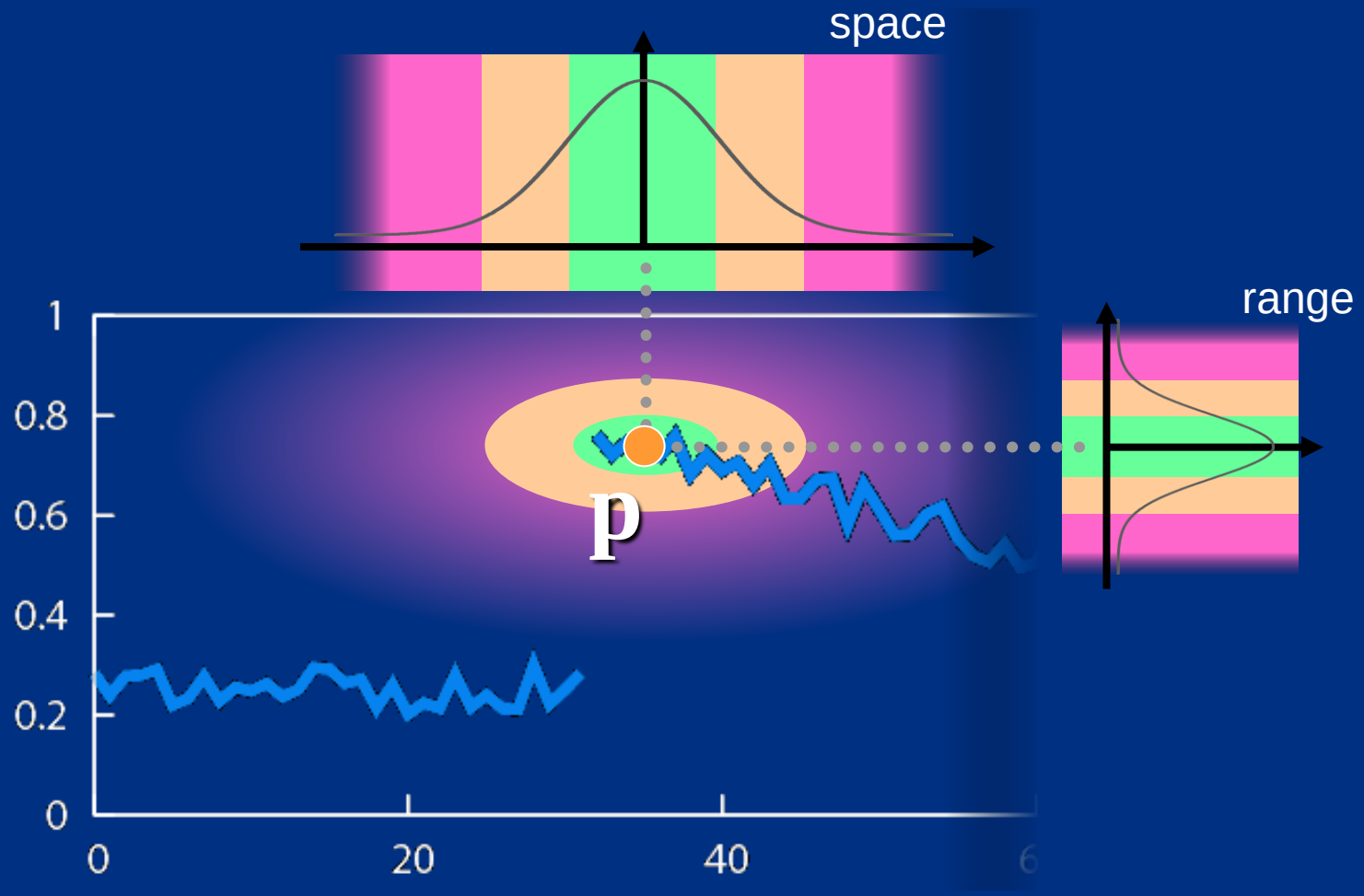
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.

- range σ_r : “minimum” amplitude of an edge

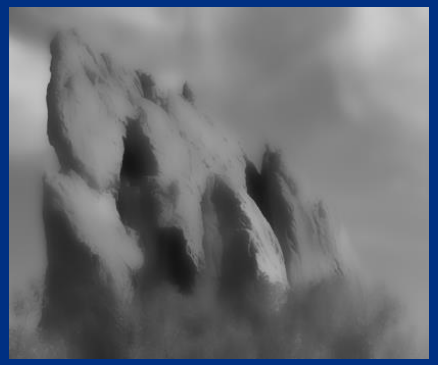
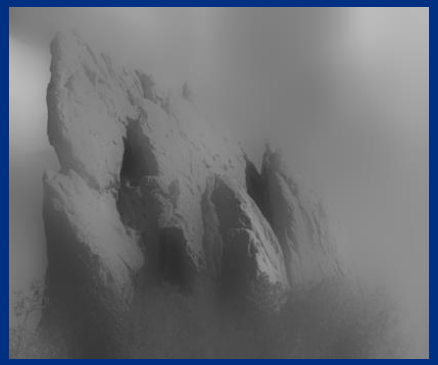
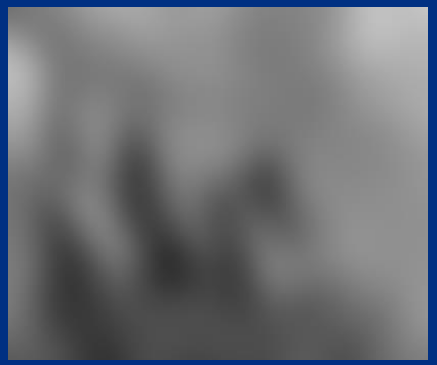
Influence of Pixels

Only pixels close in space and in range are considered.



Exploring the Parameter Space

input

	$\sigma_r = 0.1$	$\sigma_r = 0.25$	$\sigma_r = \infty$ (Gaussian blur)
$\sigma_s = 2$			
$\sigma_s = 6$			
$\sigma_s = 18$			

Varying the Range Parameter

input

$\sigma_r = 0.1$ $\sigma_r = 0.25$ $\sigma_r = \infty$ (Gaussian blur)

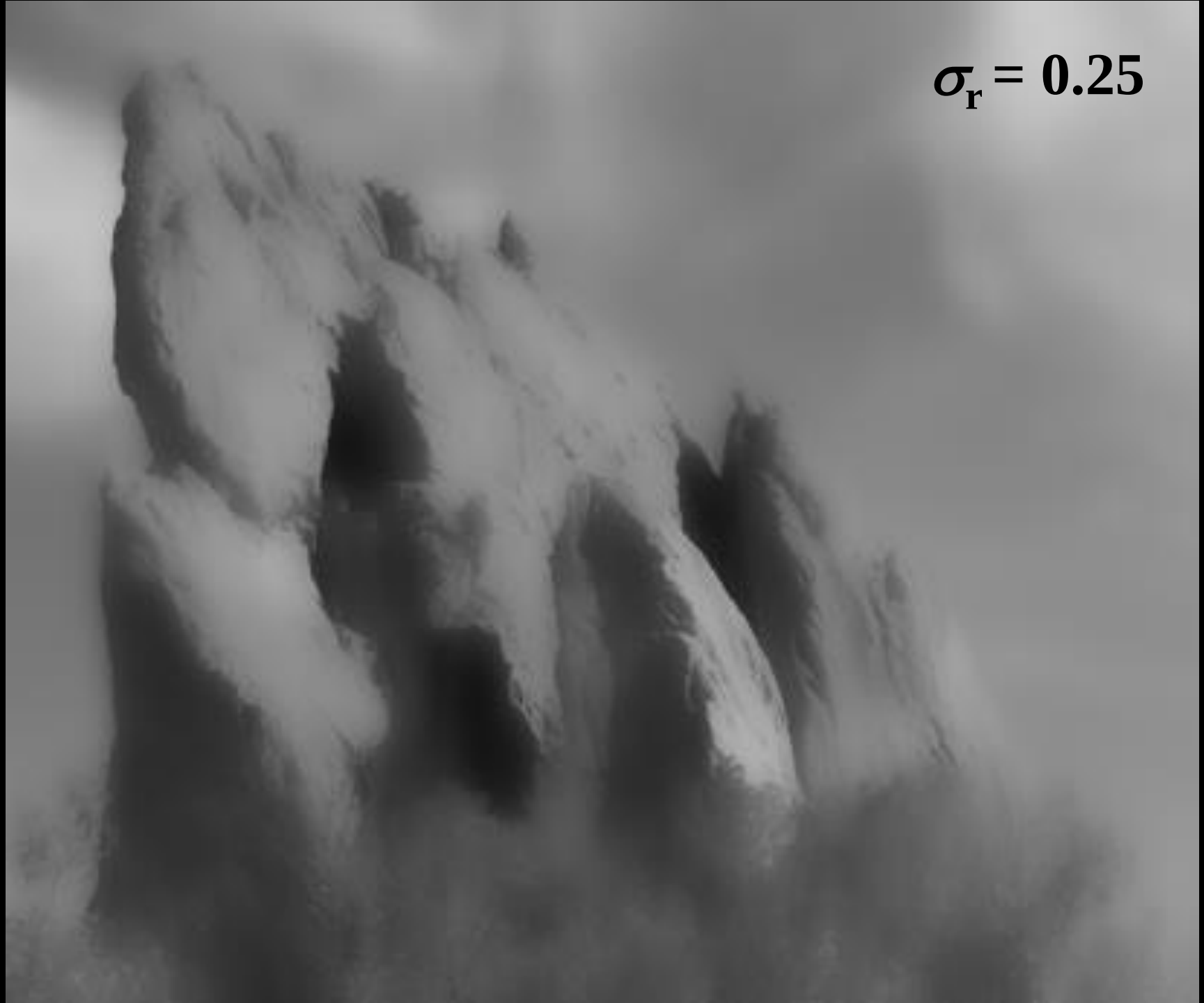
$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_r = 0.1$


$\sigma_r = 0.25$


$\sigma_r = \infty$
(Gaussian blur)

Varying the Space Parameter

input

$\sigma_r = 0.1$ | $\sigma_r = 0.25$ | $\sigma_r = \infty$ (Gaussian blur)

$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$