Computer Vision

CS308
Feng Zheng
SUSTech CS Vision Intelligence and Perception
Week 5





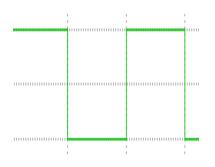
Brief Review

Points Detection

- Points Descriptor
- Points Matching

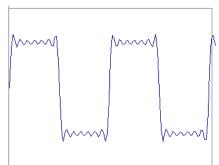
Brief Review

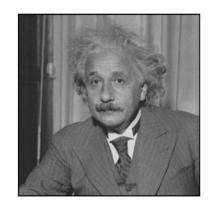


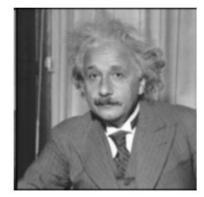


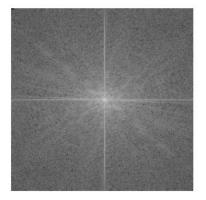
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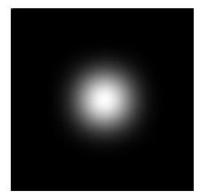


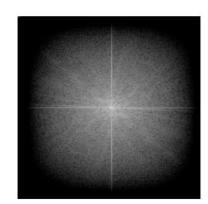






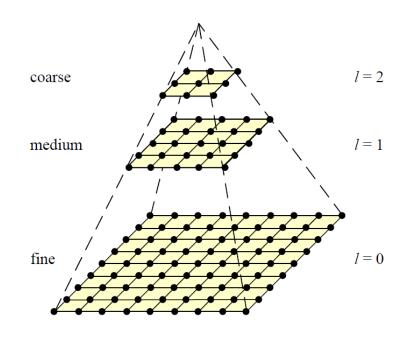


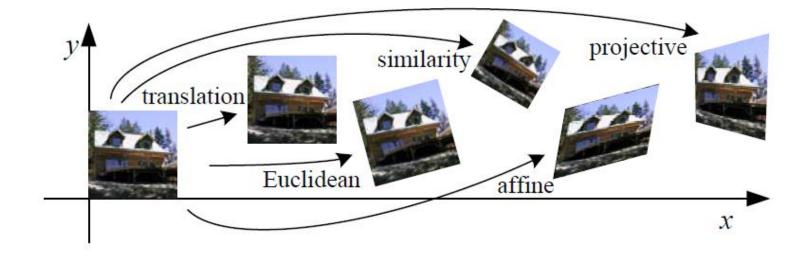




Smoothing







Points



Correspondence Across Views

 Correspondence: matching points, patches, edges, or regions across images



Example: structure from motion

How to confirm that the two images are similar with each other?



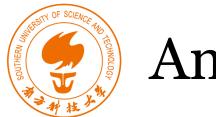
Applications

- Feature points are used for:
 - > Image alignment
 - > 3D reconstruction
 - > Motion tracking
 - > Indexing and database retrieval
 - > Object recognition









An Example

- Motivation: panorama stitching
 - > We have two images how do we combine them?







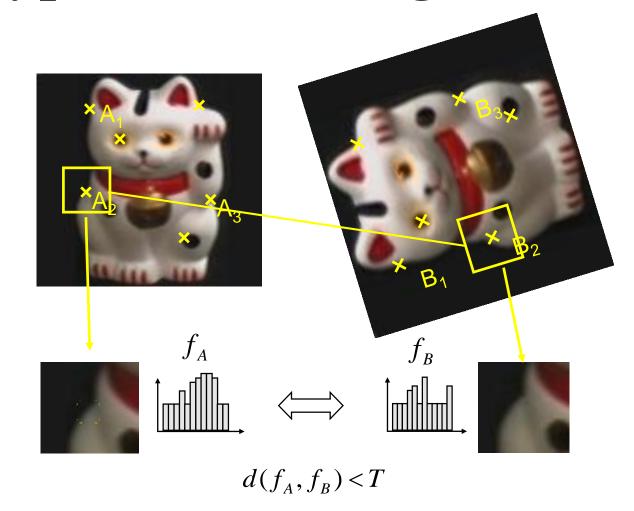
Overview of Keypoint Matching

Steps

- Find a set of distinctive keypoints
- Define a region around each keypoint
- Compute a local descriptor from the region
- Match local descriptors

• Goals

 Detect points that are repeatable and distinctive





Interesting Points

- Goal: interest operator repeatability
 - > Detect (at least some of) the same points in both images.
 - > Run the detection procedure independently per image

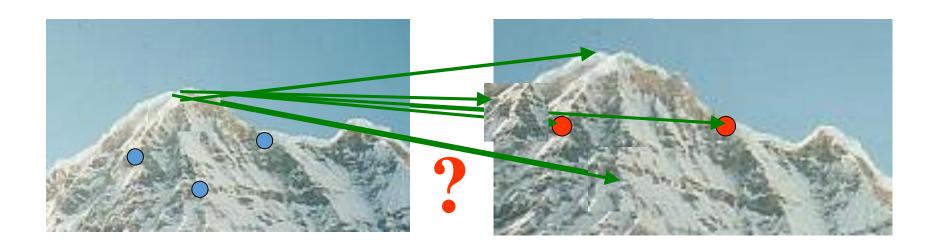


No chance to find true matches!



Interesting Points

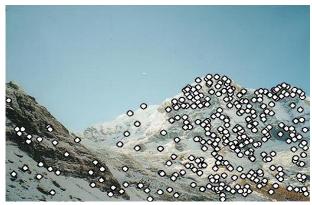
- Goal: descriptor distinctiveness
 - > Reliably determine which point goes with which
 - > Must provide some invariance to geometric and photometric differences between the two views





Interesting Points

- What are the characteristics of good features?
 - Repeatability
 - ✓ The same feature can be found in several images despite geometric and photometric transformations
 - > Saliency
 - ✓ Each feature is distinctive
 - > Compactness and efficiency
 - ✓ Many fewer features than image pixels
 - Locality
 - ✓ Relatively small area of the image
 - ✓ Robust to clutter and occlusion



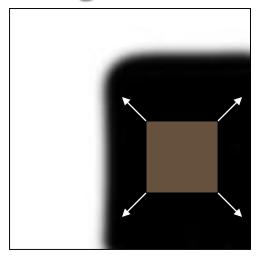


Points Detection

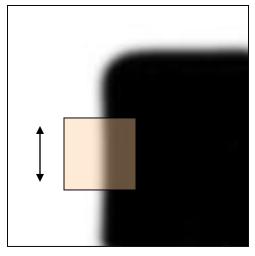


Corner Detection: Basic Idea

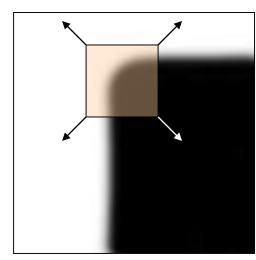
- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity



"flat" region: no change in all directions



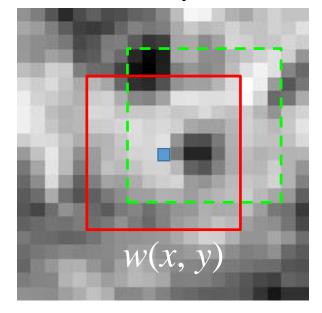
"edge": no change along the edge direction



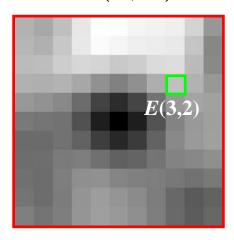
"corner": significant change in all directions



$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

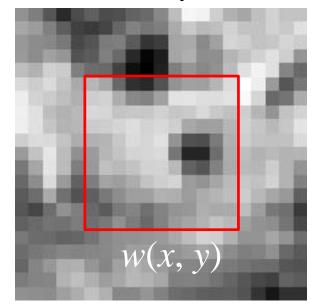


E(u, v)

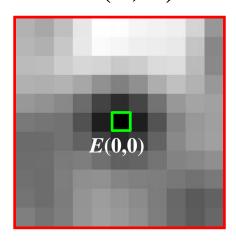




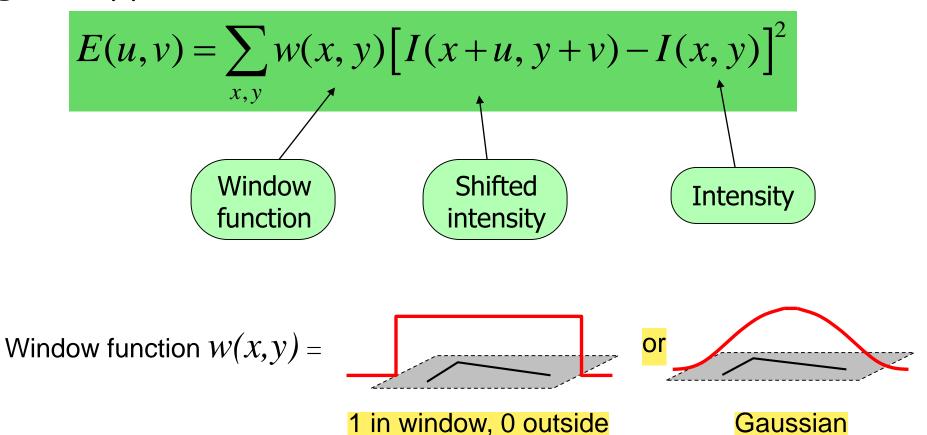
$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$



E(u, v)





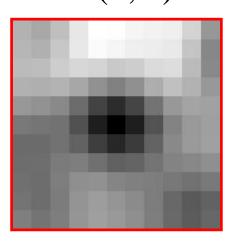




• Change in appearance of window w(x, y) for the shift [u, v]:

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

• We want to find out how this function behaves for small shifts E(u, v)





$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^2$$



- · We want to find out how this function behaves for small shifts
- But this is very slow to compute naively
 O(window_width² * shift_range² * image_width²)
 O(11² * 11² * 600²) = 5.2 billion of these

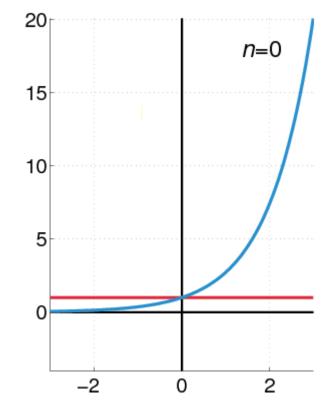
 - > 14.6 thousand per pixel in your image



 Recall Taylor series expansion. A function f can be approximated around point a as

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$
 15

Approximation of $f(x) = e^x$ centered at f(0)





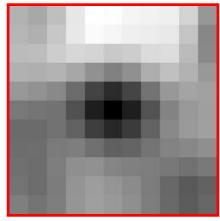
• Local quadratic approximation of E(u,v) in the neighborhood of (0,0) is given by the second-order Taylor expansion:

$$E(u,v) \approx E(0,0) + [u \ v] \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix} + \frac{1}{2} [u \ v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$
Always 0

First

derivative

is 0





• Second-order Taylor expansion of E(u, v) about (0, 0):

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E(u, v) \approx E(0,0)$$

$$+ \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix}$$

$$+ \frac{1}{2} \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y)I_{x}(x+u,y+v)I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y)[I(x+u,y+v)-I(x,y)]I_{xx}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y)[I(x+u,y+v)-I(x,y)]I_{xx}(x+u,y+v)$$

$$= \sum_{x,y} 2w(x,y)I_{y}(x+u,y+v)I_{x}(x+u,y+v)$$

$$E_{u}(u,v) = \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{x}(x+u,y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_{x}(x+u,y+v) I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{xx}(x+u,y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_{y}(x+u,y+v) I_{x}(x+u,y+v)$$

$$+ \sum_{x,y} 2w(x,y) [I(x+u,y+v) - I(x,y)] I_{xy}(x+u,y+v)$$



• Second-order Taylor expansion of E(u, v) about (0,0):

$$E(u,v) = \sum_{x,y} w(x,y) [I(x+u,y+v) - I(x,y)]^{2}$$

$$E(u, v) \approx E(0,0)$$

$$+ \left[u \quad v\right] \begin{bmatrix} E_{u}(0,0) \\ E_{v}(0,0) \end{bmatrix}$$

$$+ \frac{1}{2} \left[u \quad v\right] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_{u}(0,0) = 0$$

$$E_{v}(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y)I_{x}(x,y)I_{x}(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y)I_{y}(x,y)I_{y}(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y)I_{x}(x,y)I_{y}(x,y)$$



The quadratic approximation simplifies to

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} \sum_{x,y} w(x,y)I_x^2(x,y) & \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y) \\ \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y) & \sum_{x,y} w(x,y)I_y^2(x,y) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \quad E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

• M is a second moment matrix computed from image derivatives:

$$M = \begin{bmatrix} \sum_{I_x I_x} I_x & \sum_{I_x I_y} I_x I_y \\ \sum_{I_x I_y} I_y & \sum_{I_y I_y} I_y \end{bmatrix} = \sum_{I_x I_y} \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} = \sum_{I_x I_y} \nabla I(\nabla I)^T$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

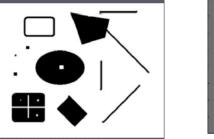


• Corners as distinctive interest points

$$M = \sum w(x, y) \begin{vmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{vmatrix}$$

• 2 x 2 matrix of image derivatives (averaged in neighborhood

of a point)



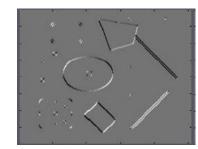
Notation:



$$I_x \Leftrightarrow \frac{\partial I}{\partial x}$$



$$I_y \Leftrightarrow \frac{\partial I}{\partial y}$$



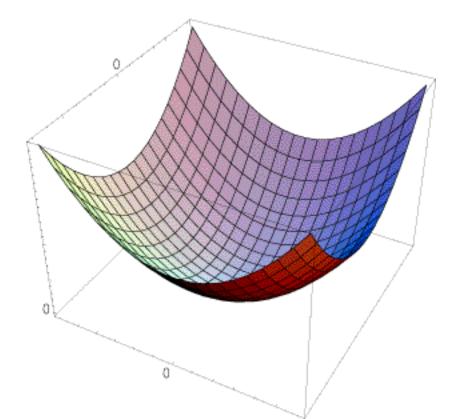
$$I_x I_y \Leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y}$$



- Interpreting the second moment matrix
 - \succ The surface E(u,v) is locally approximated by a quadratic form
 - > Let's try to understand its shape

$$E(u,v) \approx [u \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$





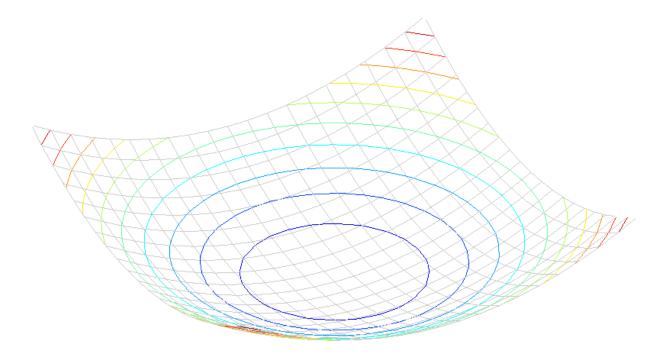
- Interpreting the second moment matrix

 - > This is the equation of an ellipse

Interpreting the second moment matrix

Consider a horizontal "slice" of
$$E(u, v)$$
:

This is the equation of an allinga





- Interpreting the second moment matrix
 - First, consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \quad M = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

> If either λ is close to 0, then this is not a corner, so look for locations where both are large

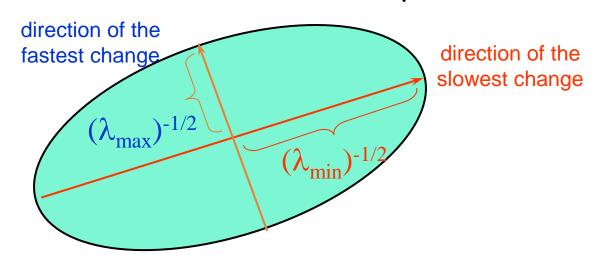


- Interpreting the second moment matrix
 - \triangleright Consider a horizontal "slice" of E(u, v):
 - > This is the equation of an ellipse
- Diagonalization of M:

 $\begin{bmatrix} u & v \end{bmatrix} M \begin{vmatrix} u \\ v \end{vmatrix} = \text{const}$

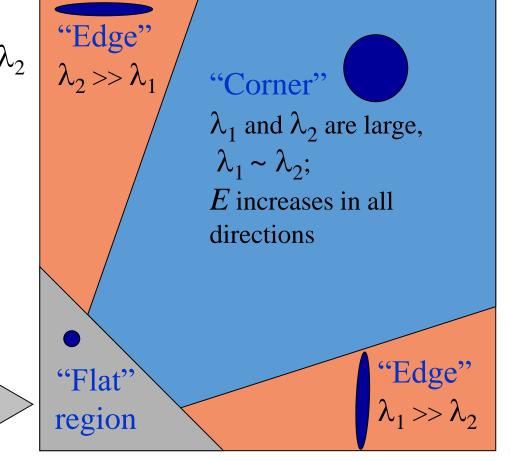
$$M = R^{-1} \begin{vmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{vmatrix} R$$

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$ The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined. and the orientation is determined by R





- Interpreting the second moment matrix
 - Classification of image points using eigenvalues of M:



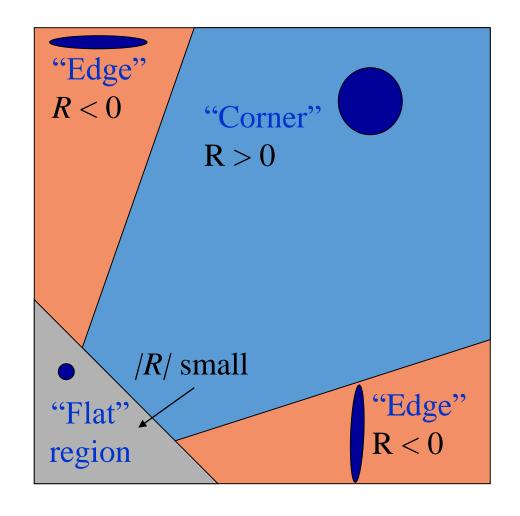
 λ_1 and λ_2 are small; E is almost constant in all directions



Corner response function

 α : constant (0.04 to 0.06)

$$R = \det(M) - \alpha \operatorname{trace}(M)^{2}$$
$$= \lambda_{1}\lambda_{2} - \alpha(\lambda_{1} + \lambda_{2})^{2}$$





Harris Corner Detector

Steps

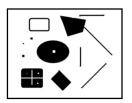
- Compute M matrix for each image window to get their cornerness scores
- Find points whose surrounding window gave large corner response (R> threshold)
- Take the points of local maxima, i.e., perform non-maximum suppression

C.Harris and M.Stephens. <u>"A Combined Corner and Edge Detector."</u>

Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.



Harris Corner Detector: Steps

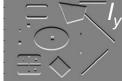


Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives (optionally, blur first)

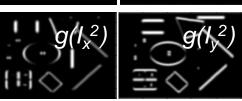




2. Square of derivatives



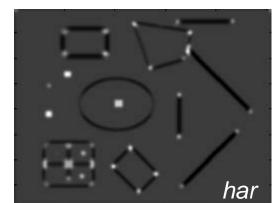






4. Cornerness function – both eigenvalues are strong

$$har = \det[\mu(\sigma_{I}, \sigma_{D})] - \alpha[\operatorname{trace}(\mu(\sigma_{I}, \sigma_{D}))^{2}] = g(I_{x}^{2})g(I_{y}^{2}) - [g(I_{x}I_{y})]^{2} - \alpha[g(I_{x}^{2}) + g(I_{y}^{2})]^{2}$$



5. Non-maxima suppression

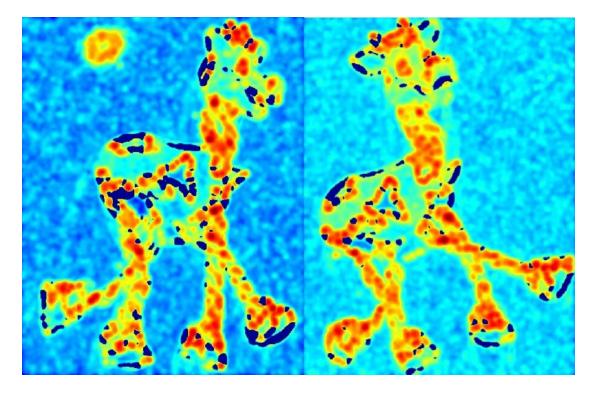


Harris Detector: Example

- Compute corner response R
 - $ightharpoonup R_harris(x,y) = det(M) a*trace(M)$

Repeatability







Harris Detector: Example

• Find points with large corner response: R>threshold







Invariance and Covariance

- We want corner locations to be invariant to photometric transformations and covariant to geometric transformations
 - > Invariance: image is transformed and corner locations do not change
 - Covariance: if we have two transformed versions of the same image, features should be detected in corresponding locations

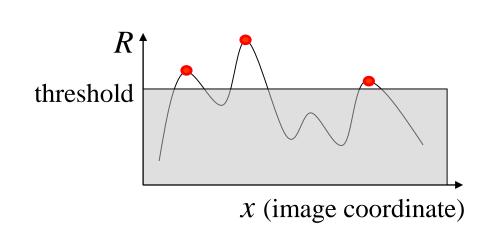




Affine Intensity Change

- Intensity change

- $I \rightarrow a I + b$
- \triangleright Only derivatives are used => invariance to intensity shift: $I \rightarrow I + b$
- \triangleright Intensity scaling: $I \rightarrow a I$



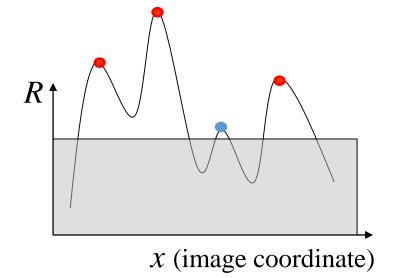
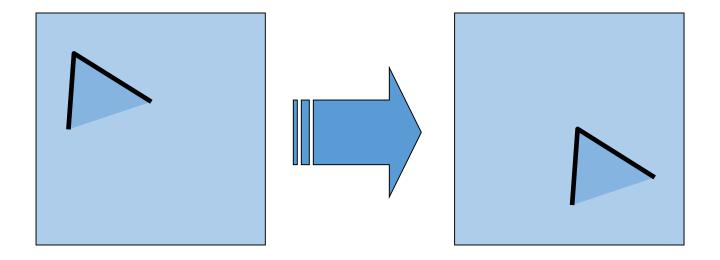




Image Translation

Derivatives and window function are shift-invariant

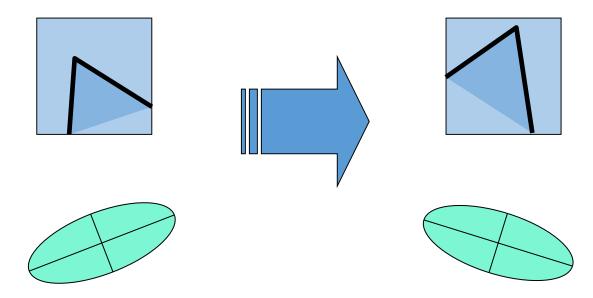


Corner location is covariant w.r.t. translation



Image Rotation

 Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

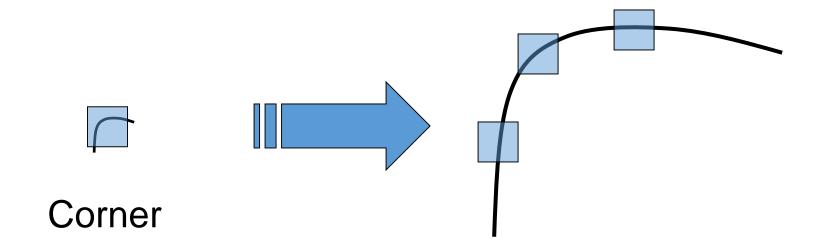


Corner location is covariant w.r.t. rotation



All points will be classified as edges

A Problem!

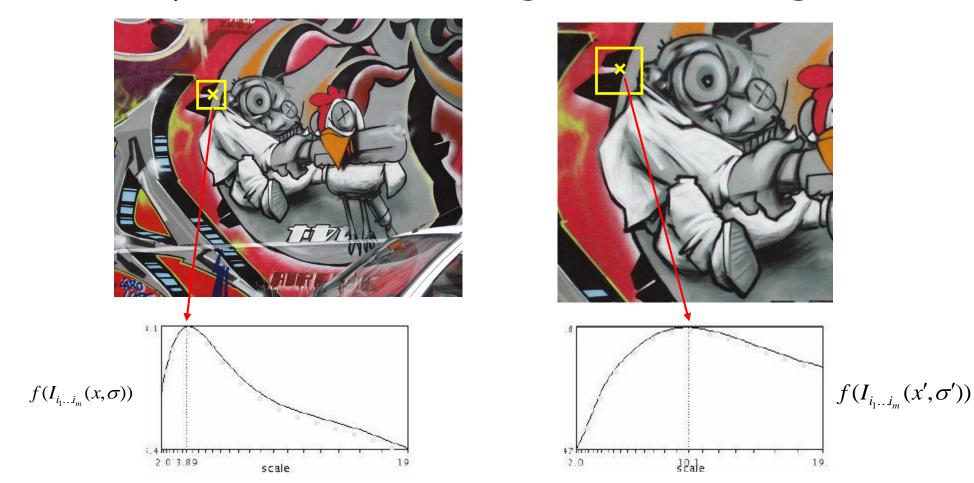


Corner location is not covariant to scaling!



Automatic Scale Selection

• Function responses for increasing scale (scale signature)





Difference-of-Gaussian (DoG)

Laplace Operator

$$\triangle = \nabla \cdot \nabla = \nabla^2 = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{vmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{vmatrix} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

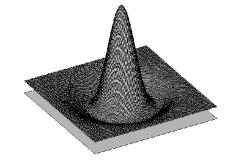
Laplacian of Gaussian (LoG)

$$\triangle[G_{\sigma}(x,y)*f(x,y)] = [\triangle G_{\sigma}(x,y)]*f(x,y) = LoG*f(x,y) \qquad LoG \stackrel{\triangle}{=} \triangle G_{\sigma}(x,y) = \frac{\partial^{2}}{\partial x^{2}}G_{\sigma}(x,y) + \frac{\partial^{2}}{\partial y^{2}}G_{\sigma}(x,y) = \frac{x^{2} + y^{2} - 2\sigma^{2}}{\sigma^{4}}e^{-(x^{2} + y^{2})/2\sigma^{2}}e^{-(x^{2} + y^{2})/2\sigma^{2}}e^$$

Difference of Gaussian (DoG)

$$g_1(x,y) - g_2(x,y) = G_{\sigma_1} * f(x,y) - G_{\sigma_2} * f(x,y) = (G_{\sigma_1} - G_{\sigma_2}) * f(x,y) = DoG * f(x,y)$$

$$DoG \stackrel{\triangle}{=} G_{\sigma_1} - G_{\sigma_2} = \frac{1}{\sqrt{2\pi}} \left(\frac{1}{\sigma_1} e^{-(x^2 + y^2)/2\sigma_1^2} - \frac{1}{\sigma_2} e^{-(x^2 + y^2)/2\sigma_2^2} \right)$$

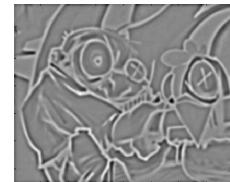


$$g_1(x,y) = G_{\sigma_1}(x,y) * f(x,y)$$

$$g_2(x,y) = G_{\sigma_2}(x,y) * f(x,y)$$

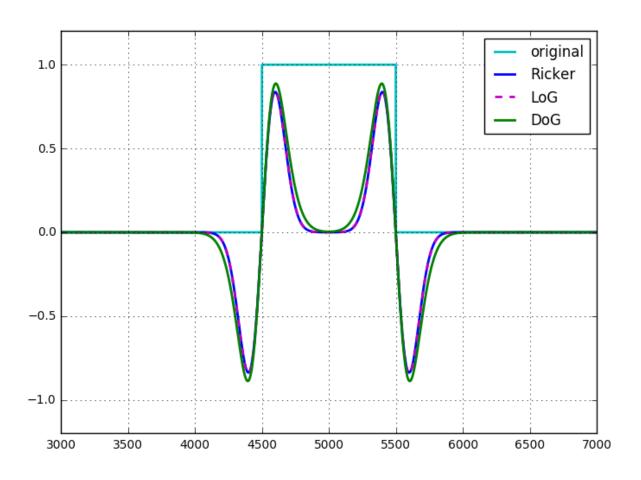








Difference-of-Gaussian (DoG)

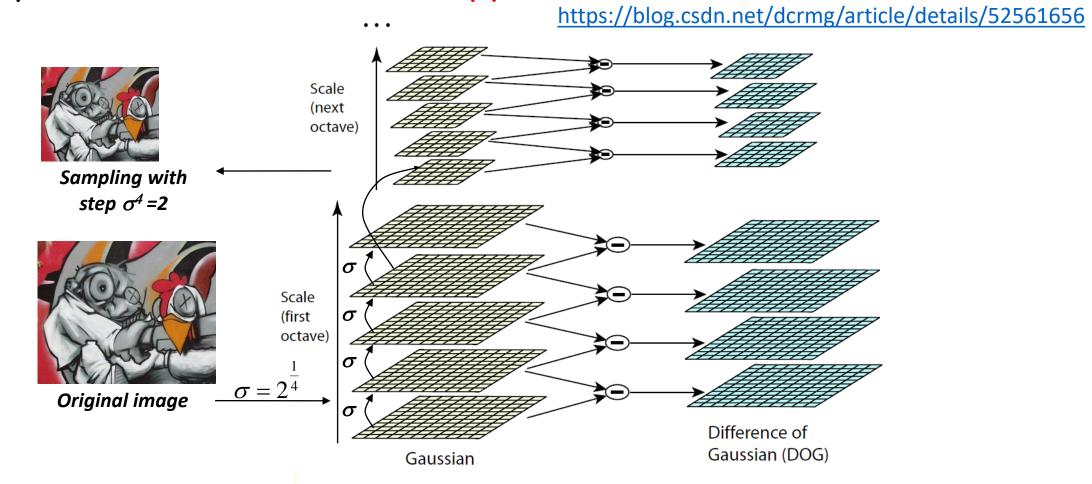


https://dsp.stackexchange.com/questions/37673/what-is-the-difference-between-difference-of-gaussian-laplace-of-gaussian-and



DoG – Efficient Computation

Computation in Gaussian scale pyramid

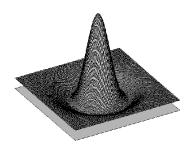


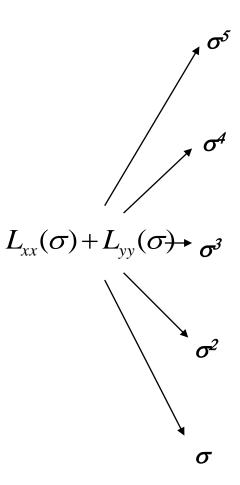


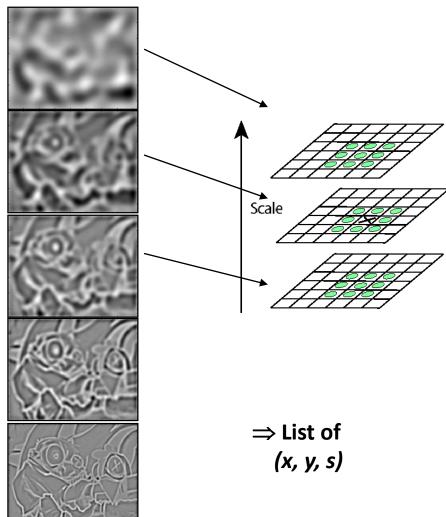
DoG – Efficient Computation

 Find local maxima in position-scale space of Difference-of-Gaussian











DoG – Efficient Computation

- Keypoint Localization
 - > The approach is similar to the one used in the Harris Corner Detector for removing edge features.
 - Reject flats:

$$|D(\hat{\mathbf{x}})| < 0.03$$

Reject edges:

$$\mathbf{H} = \left[\begin{array}{cc} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{array} \right]$$

$$Tr(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$Det(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

Let
$$r = \alpha/\beta$$
.
So $\alpha = r\beta$

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r}, \quad \frac{(r+1)^2/r \text{ is at a min when the}}{r}$$

2 eigenvalues are equal.

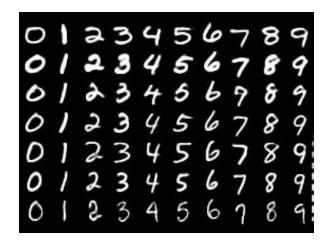
Points Descriptor



Image Representations

- Templates
 - > Intensity, gradients, etc.





- Histograms
 - > Color, texture, SIFT descriptors, etc.

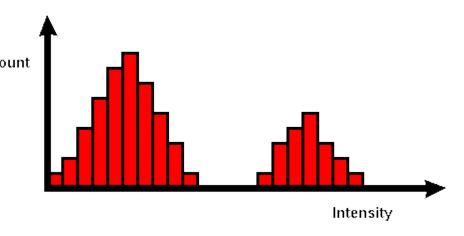




Image Representations: Histograms

- Global histogram
 - > Represent distribution of features
 - > Color, texture, depth, ...

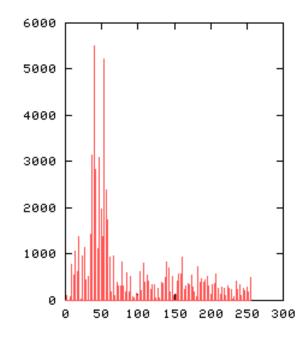
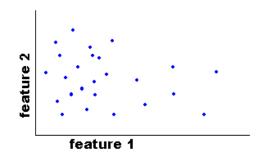


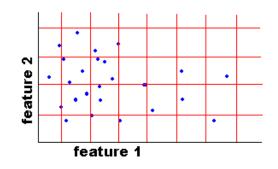




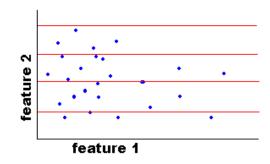
Image Representations: Histograms

- · Histogram: probability or count of data in each bin
- Joint histogram
 - > Requires lots of data
 - Loss of resolution to avoid empty bins





- Marginal histogram
 - > Require independent features
 - More data/bin than joint histogram



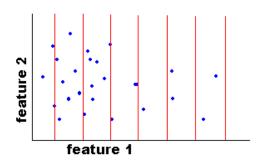




Image Representations

What kind of things do we compute histograms of?

Color

Hue

Value

HSV color space

HSV color space

> Texture (filter banks or HOG over regions)



Histogram of Oriented Gradients

- Gaussian-smoothed image at the keypoint's scale $L(x, y, \sigma)$
 - > Every pixel in a neighboring region around the keypoint
 - ✓ Magnitude
 - ✓ Direction

$$m\left(x,y
ight) = \sqrt{\left(L\left(x+1,y
ight) - L\left(x-1,y
ight)
ight)^{2} + \left(L\left(x,y+1
ight) - L\left(x,y-1
ight)
ight)^{2}}$$
 $heta\left(x,y
ight) = an2\left(L\left(x,y+1
ight) - L\left(x,y-1
ight), L\left(x+1,y
ight) - L\left(x-1,y
ight)
ight)$

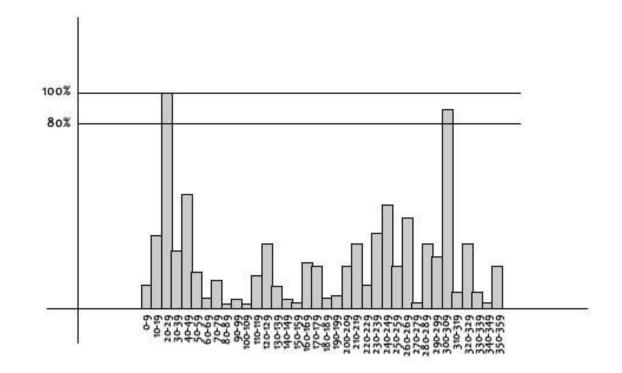


- Each sample in the neighboring window added to a histogram bin is weighted by its gradient magnitude and by a Gaussian-weighted circular window
- > The peaks in this histogram correspond to dominant orientations



Histogram of Oriented Gradients

- An orientation histogram with 36 bins covering 360 degrees is created
 - > A histogram bin is weighted by its gradient magnitude





Orientation Normalization

- Compute orientation histogram
- Select dominant orientation
 - Normalize: rotate to fixed orientation

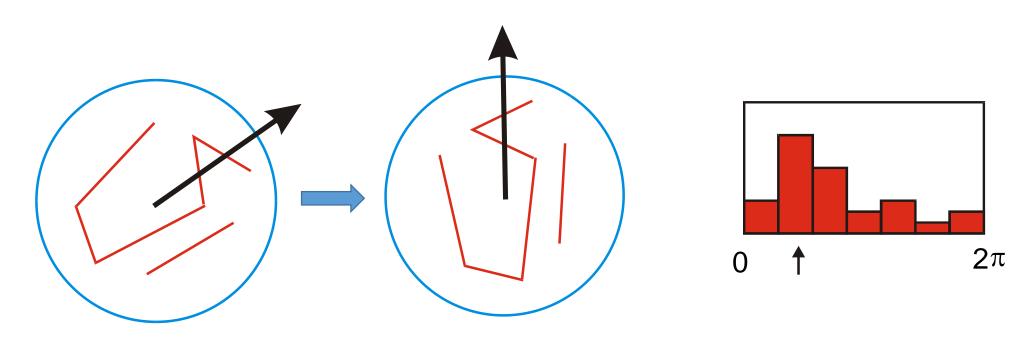
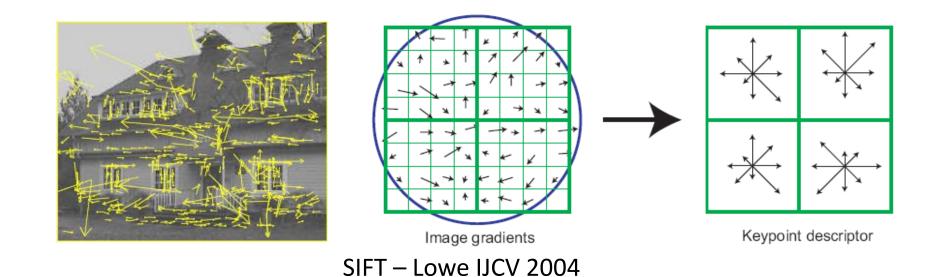




Image Representations

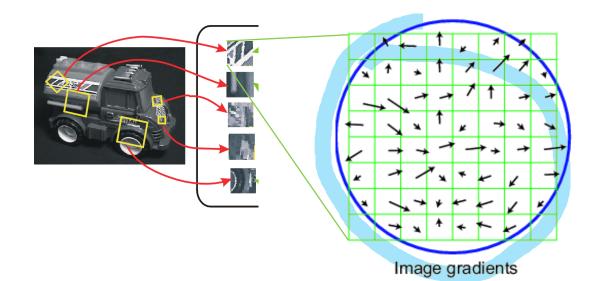
- At this point, each keypoint has a location, scale, orientation
- What kind of things do we compute histograms of?





SIFT Vector Formation

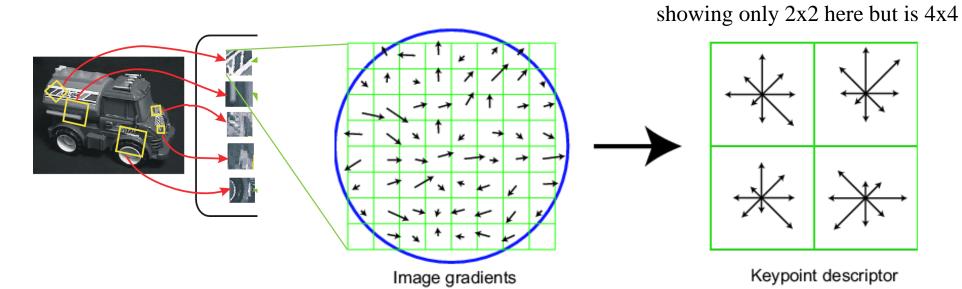
- Computed on rotated and scaled version of window according to computed orientation & scale
 - > Resample the window
- Based on gradients weighted by a Gaussian of variance half the window (for smooth falloff)





SIFT Vector Formation

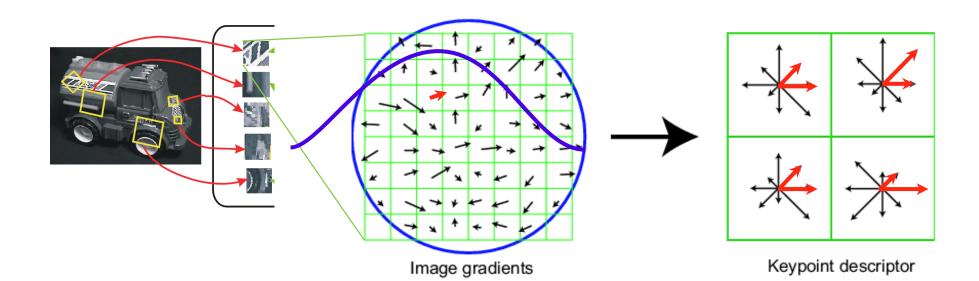
- 4x4 array of gradient orientation histogram weighted by magnitude
- 8 orientations \times 4×4 array = 128 dimensions
- Motivation: some sensitivity to spatial layout, but not too much.





Ensure Smoothness

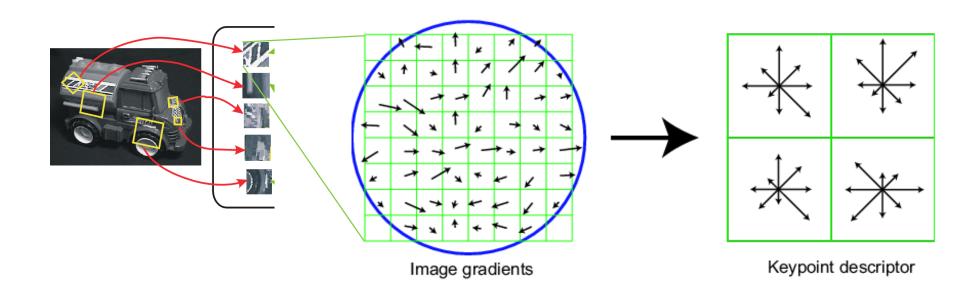
- Gaussian weight
- Interpolation
 - A given gradient contributes to 8 bins: 4 in space times 2 in orientation





Reduce Effect of Illumination

- 128-dim vector normalized to 1
- Threshold gradient magnitudes to avoid excessive influence of high gradients
 - \triangleright After normalization, clamp gradients > 0.2
 - > Renormalize





- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
 - > Robust
 - Distinctive
 - > Compact
 - > Efficient
- Most available descriptors focus on edge/gradient information
 - > Capture texture information
 - Color rarely used

Points Matching



- Simplest approach: Pick the nearest neighbor
- Threshold on absolute distance
- Problem: Lots of self similarity in many photos



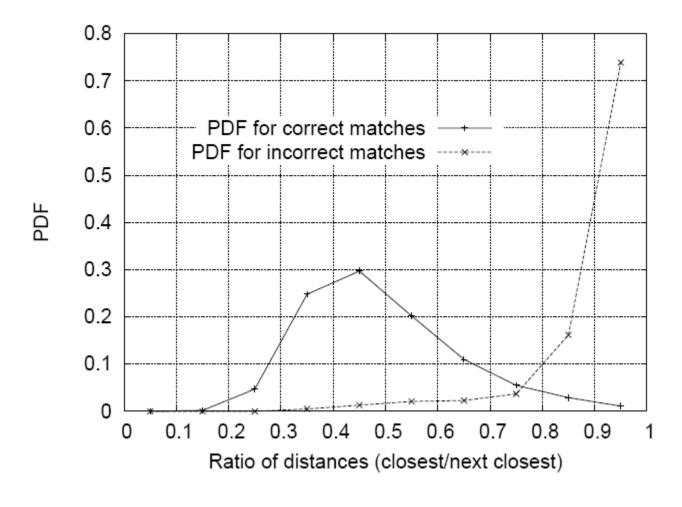


Distance: 0.34, 0.30, 0.40 Distance: 0.61, 1.22



Nearest Neighbor Distance Ratio

- NN1 where NN1 is the distance to the first nearest neighbor and NN2 is the distance to the second nearest neighbor
- Sorting by this ratio puts matches in order of confidence



Conclusions



Choosing a Detector

- What do you want it for?
 - > Precise localization in x-y: Harris
 - > Good localization in scale: Difference of Gaussian
- Best choice often application dependent
 - Harris-/Hessian-Laplace/DoG work well for many natural categories
- There have been extensive evaluations/comparisons
 - > [Mikolajczyk et al., IJCV'05, PAMI'05]
 - > All detectors/descriptors shown here work well



Comparison of Keypoint Detectors

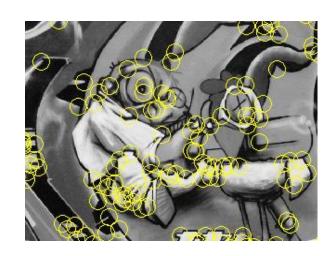
Table 7.1 Overview of feature detectors.

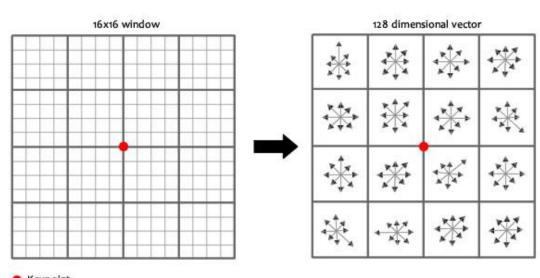
				Rotation	Scale	Affine		Localization		
Feature Detector	Corner	$_{\mathrm{Blob}}$	Region	invariant	invariant	invariant	Repeatability	accuracy	Robustness	Efficiency
Harris	√			√			+++	+++	+++	++
Hessian		\checkmark		√			++	++	++	+
SUSAN	\checkmark			√			++	++	++	+++
Harris-Laplace	√	(√)		√	√		+++	+++	++	+
Hessian-Laplace	(√)	\checkmark		√	\checkmark		+++	+++	+++	+
DoG	(√)	\checkmark		√	$\sqrt{}$		++	++	++	++
SURF	(√)	\checkmark		√	\checkmark		++	++	++	+++
Harris-Affine	√	(√)		√	√	√	+++	+++	++	++
Hessian-Affine	(√)	\checkmark		√	\checkmark	\checkmark	+++	+++	+++	++
Salient Regions	(√)	\checkmark		√	\checkmark	(√)	+	+	++	+
Edge-based	\checkmark			√	\checkmark	\checkmark	+++	+++	+	+
MSER			√	√	√	√	+++	+++	++	+++
Intensity-based			\checkmark	\checkmark	\checkmark	\checkmark	++	++	++	++
Superpixels			\checkmark	√	(√)	(√)	+	+	+	+

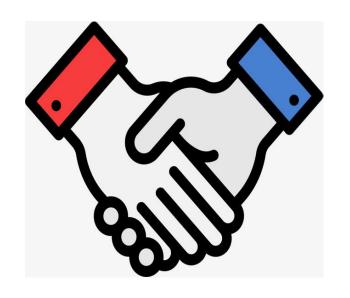


Things to Remember

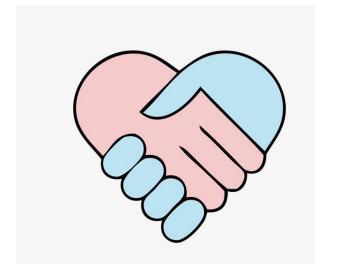
- Keypoint detection: repeatable and distinctive
 - Corners, blobs, stable regions
 - > Harris, DoG
- Descriptors: robust and selective
 - > Spatial histograms of orientation
 - > SIFT







Thanks



zhengf@sustc.edu.cn