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Chapter 3: Syntax Analysis

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Outline

- Introduction: Syntax and Parsers
- Context-Free Grammars
- Overview of Parsing Techniques
- Top-Down Parsing
- Bottom-Up Parsing
- Parser Generators (Lab)

Shift-Reduce Parsing (Revisit)

- Bottom-up parsing can be seen as a process of "reducing" a string ω to the start symbol of the grammar
- Shift-reduce parsing is a general style of bottom-up parsing in which:
  - A **stack** holds grammar symbols
  - An **input buffer** holds the rest of the string to be parsed
  - The stack content (from bottom to top) and the input buffer content form a right-sentential form (assuming no errors)

Shift-Reduce Parsing (Revisit)

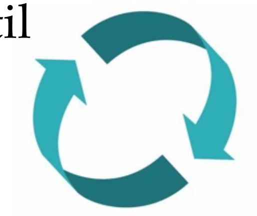
Initial status:

STACK	INPUT
$	$\omega$$

Actions:
Shift
Reduce
Accept
Error

Shift-reduce process:

- The parser **shifts** zero or more input symbols onto the stack, until it is ready to reduce a string β on top of the stack*
- **Reduce** β to the head of the appropriate production



The parser repeats the above cycle until it has detected an error or the stack contains the start symbol and input is empty

The Challenge (Revisit)

Parsing steps on input $\mathbf{id}_1 * \mathbf{id}_2$

STACK	INPUT	ACTION
$\$$	$\mathbf{id}_1 * \mathbf{id}_2 \$$	shift
$\$\,\mathbf{id}_1$	$* \mathbf{id}_2 \$$	reduce by $F \to \mathbf{id}$
$\$\,F$	$* \mathbf{id}_2 \$$	reduce by $T \to F$
$\$\,T$	$* \mathbf{id}_2 \$$	shift
$\$\,T *$	$\mathbf{id}_2 \$$	shift
$\$\,T * \mathbf{id}_2$	$\$$	reduce by $F \to \mathbf{id}$
$\$\,T * F$	$\$$	reduce by $T \to T * F$
$\$\,T$	$\$$	reduce by $E \to T$
$\$\,E$	$\$$	accept

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid \mathbf{id}$$

Why shifting * instead of reducing T ?

Generally, when to shift/reduce? How to reduce (choosing which production)?

Outline

- Introduction: Syntax and Parsers
- Context-Free Grammars
- Overview of Parsing Techniques
- Top-Down Parsing
- Bottom-Up Parsing
  - Simple LR (SLR)
  - Canonical LR (CLR)
  - Look-ahead LR (LALR)
  - Error Recovery (Lab)
- Parser Generators (Lab)

LR Parsing (LR语法分析技术)

- **LR(k) parsers:** the most prevalent type of bottom-up parsers
  - L: left-to-right scan of the input
  - R: construct a rightmost derivation in reverse
  - k: use k input symbols of lookahead in making parsing decisions

- LR(0) and LR(1) parsers are of practical interest
  - When $k \geq 2$, the parser becomes too complex to construct (parsing table will be too huge to manage)

Advantages of LR Parsers

- Table-driven (like non-recursive LL parsers) and powerful
  - Although it is too much work to construct an LR parser by hand, there are parser generators to construct parsing tables automatically
  - Comparatively, LL parsers tend to be easier to write by hand, but less powerful (handle fewer grammars)

- LR-parsing is the most general nonbacktracking shift-reduce parsing method known

- LR parsers can be constructed to recognize virtually all programming language constructs for which CFGs can be written

- LR grammars can describe more languages than LL grammars
  - Recall the stringent conditions for a grammar to be LL(1)

When to Shift/Reduce?

STACK	INPUT	ACTION
$\$$	$\mathbf{id}_1 * \mathbf{id}_2 \$$	shift
$\$\, \mathbf{id}_1$	$* \mathbf{id}_2 \$$	reduce by $F \rightarrow \mathbf{id}$
$\$\, F$	$* \mathbf{id}_2 \$$	reduce by $T \rightarrow F$
$\$\, T$	$* \mathbf{id}_2 \$$	shift
$\$\, T *$	$\mathbf{id}_2 \$$	shift
$\$\, T * \mathbf{id}_2$	$\$$	reduce by $F \rightarrow \mathbf{id}$
$\$\, T * F$	$\$$	reduce by $T \rightarrow T * F$
$\$\, T$	$\$$	reduce by $E \rightarrow T$
$\$\, E$	$\$$	accept

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{id}$$

Parsing input $\mathbf{id}_1 * \mathbf{id}_2$

How does a shift/reduce parser know that T on stack top is not a choice for reduction (the right action is to shift)?

LR(0) Items (LR(0) 项)

- An LR parser makes shift-reduce decisions by **maintaining states** to keep track of what have been seen during parsing

- An ***LR(0) item*** (item for short) is a production with a dot at some position of the body, indicating how much we have seen at a given point in the parsing process
  - $A \to \cdot XYZ \qquad A \to X \cdot YZ \qquad A \to XY \cdot Z \qquad A \to XYZ \cdot$
  - $A \to X \cdot YZ$: we have just seen on the input a string derivable from X and we hope to see a string derivable from YZ next
  - The production $A \to \epsilon$ generates only one item $A \to \cdot$

- **States:** sets of LR(0) items (LR(0) 项集)

Canonical LR(0) Collection

- One collection of states (i.e., sets of LR(0) items), called the ***canonical* LR(0) collection** (**LR(0)项集规范族**), provides the basis for constructing a DFA to make parsing decisions

- To construct canonical LR(0) collection for a grammar, we need to define:
  - An augmented grammar (增广文法)
  - Two functions: (1) CLOSURE of item sets (项集闭包) and (2) GOTO

Augmented Grammar

- Augmenting a grammar G with start symbol S
  - Introduce a new start symbol S' to replace S
  - Add a new production $S' \to S$

- Obviously, $L(G) = L(G')$

- **Benefit:** With the augmentation, acceptance occurs when and only when the parser is about to reduce by $S' \to S$
  - Otherwise, acceptance could occur at many points since there may be multiple S-productions

Closure of Item Sets

- If I is a set of items for a grammar G, then $\text{CLOSURE}(I)$ is the set of items constructed from I by the two rules

  1. Initially, add every item in I to $\text{CLOSURE}(I)$
  2. If $A \to \alpha \cdot B\beta$ is in $\text{CLOSURE}(I)$ and $B \to \gamma$ is a production, then add the item $B \to \cdot\, \gamma$ to $\text{CLOSURE}(I)$, if it is not already there. Apply this rule until no more new items can be added to $\text{CLOSURE}(I)$

- **Intuition:** $A \to \alpha \cdot B\beta$ indicates that we hope to see a substring derivable from $B\beta$. This substring will have a prefix derivable from B. Therefore, we add items for all B-productions.

Algorithm for CLOSURE(I)

// the earlier natural language description is already clear enough

SetOfItems $\textsc{closure}(I)$ {
    $J = I$;
    repeat
        for (each item $A \to \alpha \cdot B\beta$ in J)
            for (each production $B \to \gamma$ of G)
                if ($B \to \cdot\gamma$ is not in J)
                    add $B \to \cdot\gamma$ to J;
    until no more items are added to J on one round;
    return J;
}

Example

$E \to E + T \mid T$
$T \to T * F \mid F$
$F \to (E) \mid \textbf{id}$

- Augmented grammar
  - $E' \to E$ $\quad E \to E + T \mid T$ $\quad T \to T * F \mid F$ $\quad F \to (E) \mid \textbf{id}$
- Computing the closure of the item set $\{[E' \to \cdot\, E]\}$
  - Initially, $[E' \to \cdot\, E]$ is in the closure
  - Add $[E \to \cdot\, E + T]$ and $[E \to \cdot\, T]$ to the closure
  - Add $[T \to \cdot\, T * F]$ and $[T \to \cdot\, F]$ to the closure
  - Add $[F \to \cdot\, (E)]$ and $[F \to \cdot\, \textbf{id}]$ and reach fixed point

- $[E' \to \cdot\, E]$
- $[E \to \cdot\, E + T]$
- $[E \to \cdot\, T]$
- $[T \to \cdot\, T * F]$
- $[T \to \cdot\, F]$
- $[F \to \cdot\, (E)]$
- $[F \to \cdot\, \textbf{id}]$

The Function GOTO

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid \mathbf{id}$$

- **GOTO(I, X)**, where I is a set of items and X is a grammar symbol, is defined to be the closure of the set of all items $[A \to \alpha X \cdot \beta]$ such that $[A \to \alpha \cdot X\beta]$ is in I
  - $CLOSURE(\{[A \to \alpha X \cdot \beta] \mid [A \to \alpha \cdot X\beta] \in I\})$

- **Example:** Computing GOTO$(I, +)$ for $I = \{[E' \to E\,\cdot], [E \to E \cdot +T]\}$
  - There is only one item $[E \to E \cdot +T]$, in which $+$ follows $\cdot$
  - Then compute the CLOSURE$(\{[E \to E +\cdot\, T]\})$, which contains:
    - $[E \to E +\cdot\, T]$
    - $[T \to\cdot\, T * F]$, $[T \to\cdot\, F]$
    - $[F \to\cdot\, (E)]$, $[F \to\cdot\, \mathbf{id}]$

Constructing Canonical LR(0) Collection

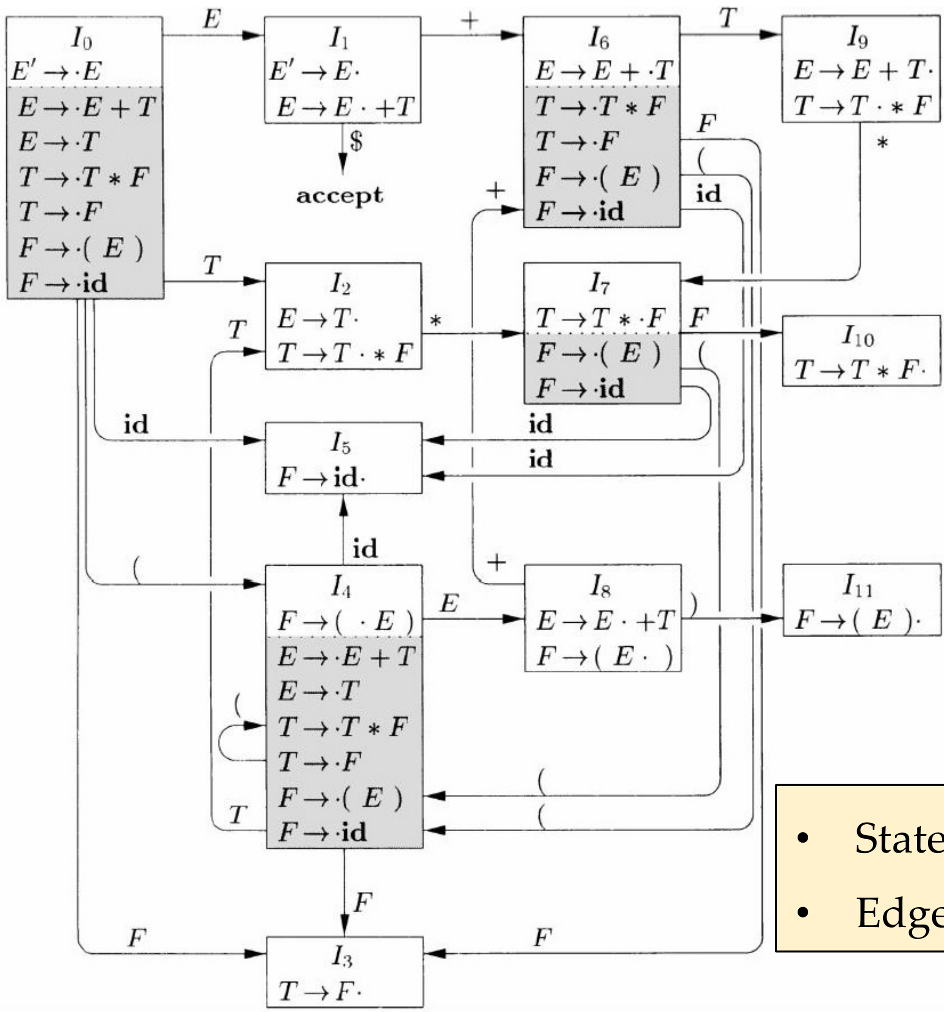
```
void items(G') {
    C = {CLOSURE({[S' → ·S]})};
    repeat
        for ( each set of items I in C )
            for ( each grammar symbol X )
                if ( GOTO(I, X) is not empty and not in C )
                    add GOTO(I, X) to C;
    until no new sets of items are added to C on a round;
}
```

Initial item set
(i.e., initial state)

Iteratively find all possible GOTO targets
(states in the automaton for parsing)

Example

The canonical LR(0) collection for the grammar below is $\{I_0, I_1, \ldots, I_{11}\}$



(1) $E \to E + T$

(2) $E \to T$

(3) $T \to T * F$

(4) $T \to F$

(5) $F \to (E)$

(6) $F \to \mathbf{id}$

- States are constructed by CLOSURE function
- Edges are constructed by GOTO function

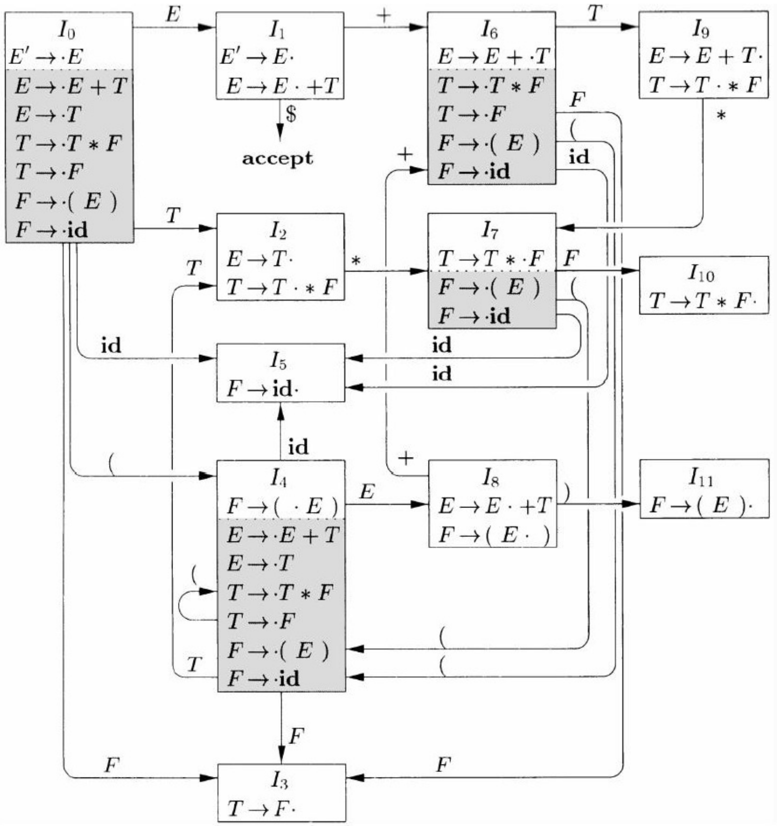
LR(0) Automaton

- The central idea behind "Simple LR", or SLR, is constructing the LR(0) automaton from the grammar
  - The states are the item sets in the canonical LR(0) collection
  - The transitions are given by the GOTO function
  - The start state is $\mathrm{CLOSURE}(\{S' \rightarrow \cdot\, S\})$

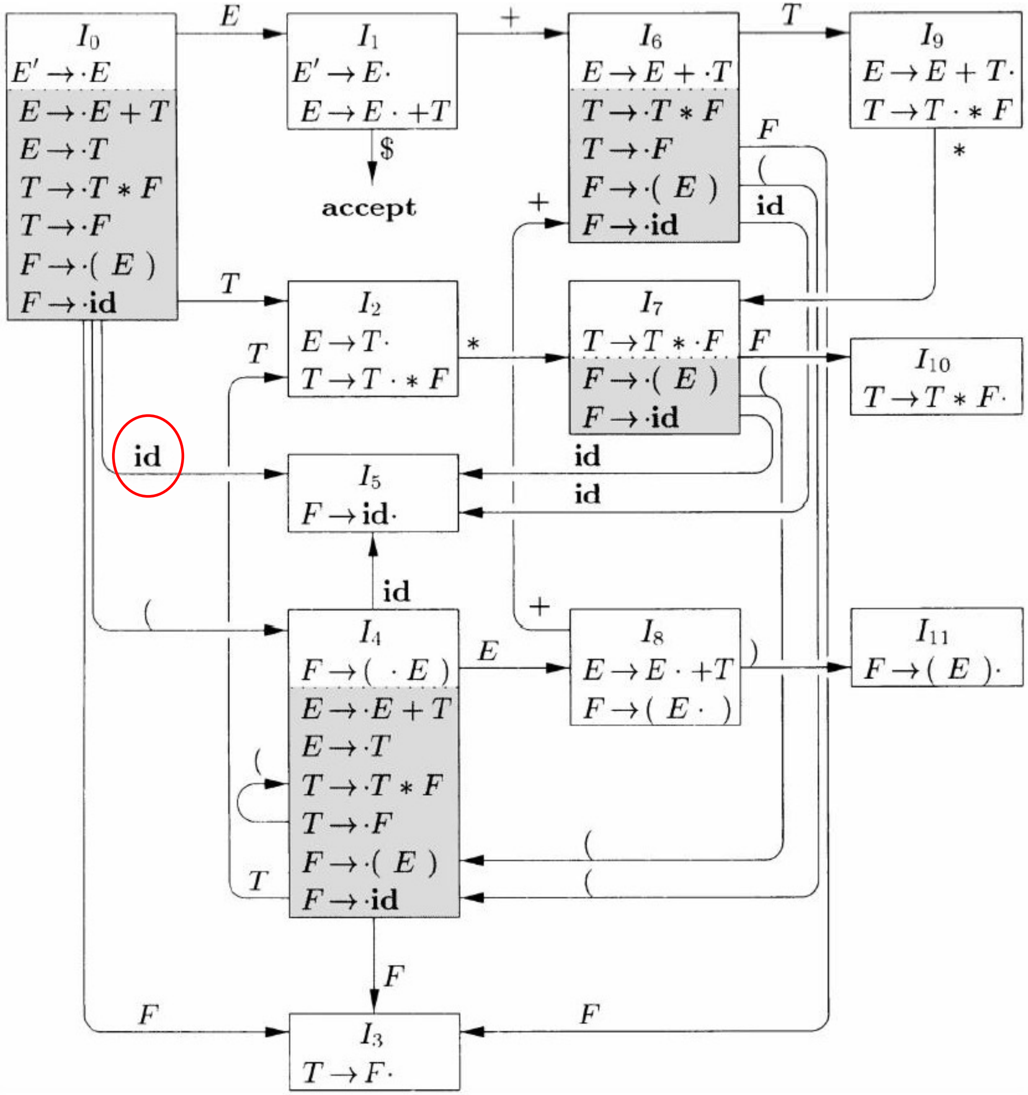
The Use of LR(0) Automaton

Helps make shift-reduce decisions:

- Suppose that the string γ of grammar symbols takes the automaton from the start state 0 to some state j
- **Shift** on next input symbol a if state j has a transition on a
- Otherwise, **reduce**; the items in state j will tell us which production to use



Example: Parsing id $*$ id



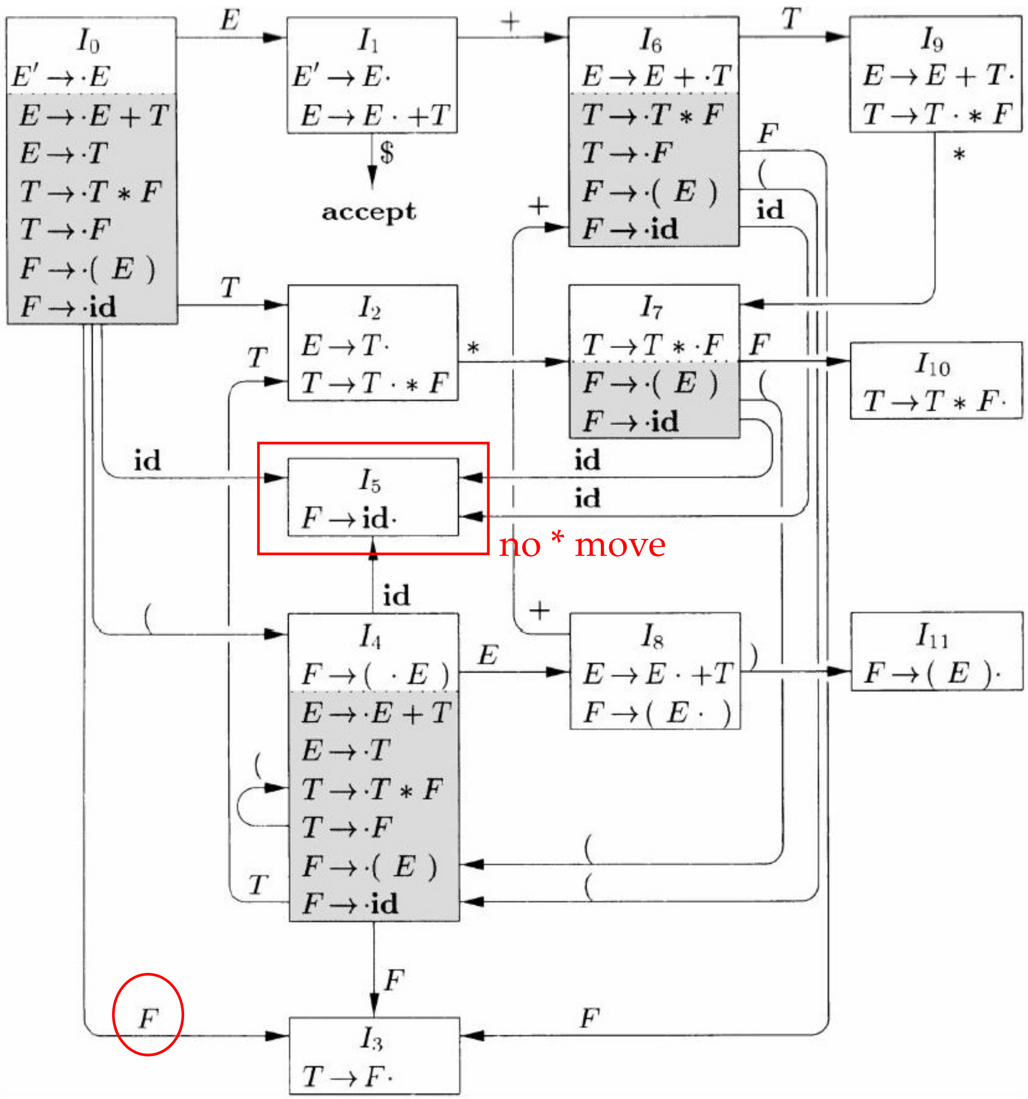
We only keep states in the stack; grammar symbols can be recovered from the states

Stack: $ 0 **Input:** id * id $

Gramar Symbols: $

Action: Shift to 5

Example: Parsing id $*$ id



We only keep states in the stack; grammar symbols can be recovered from the states

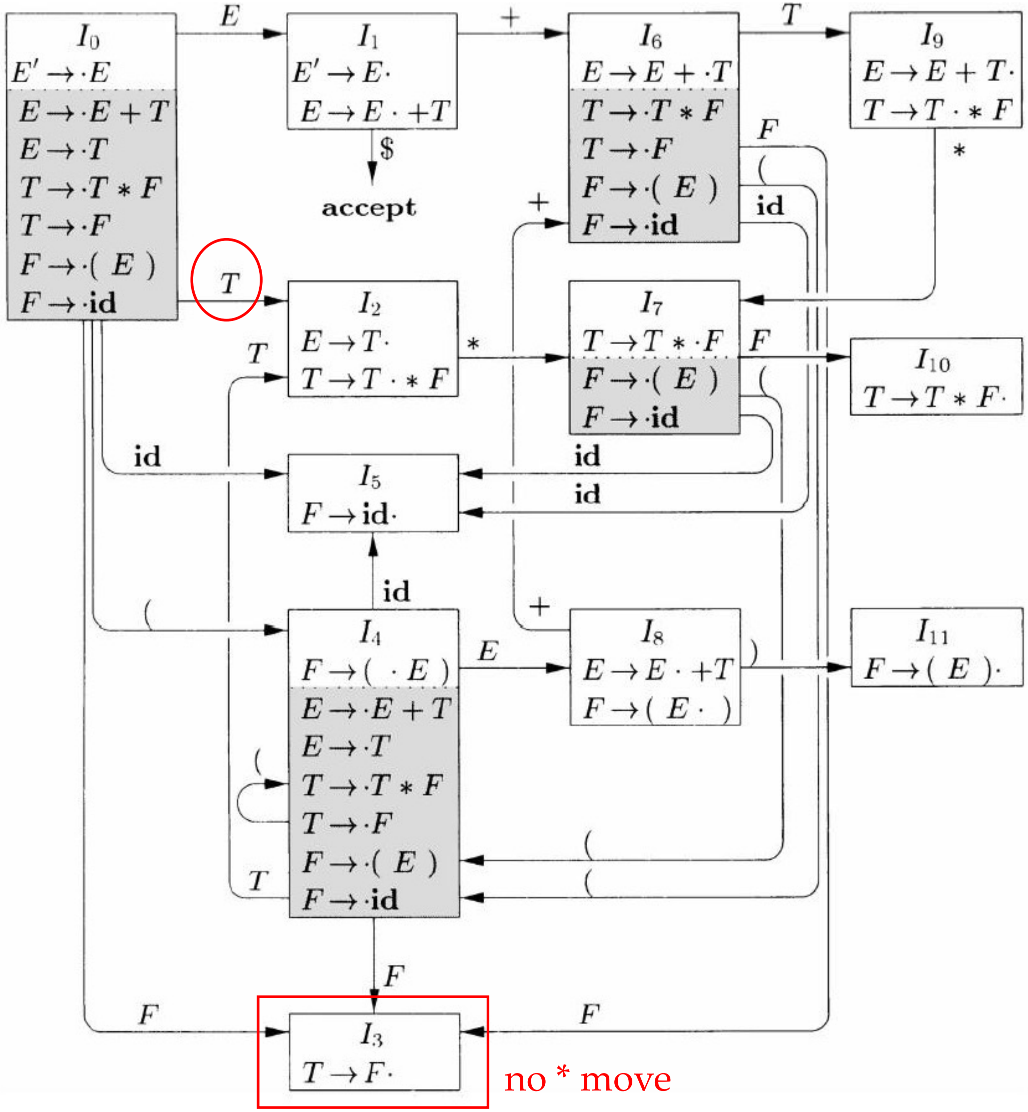
Stack: \$ 0 5 **Input:** * id \$

Gramar Symbols: \$ id

Action: Reduce by $F \to \mathbf{id}$

- Pop state 5 (one symbol corresponds to one state)
- Push state 3

Example: Parsing **id** $*$ **id**



no * move

We only keep states in the stack; grammar symbols can be recovered from the states

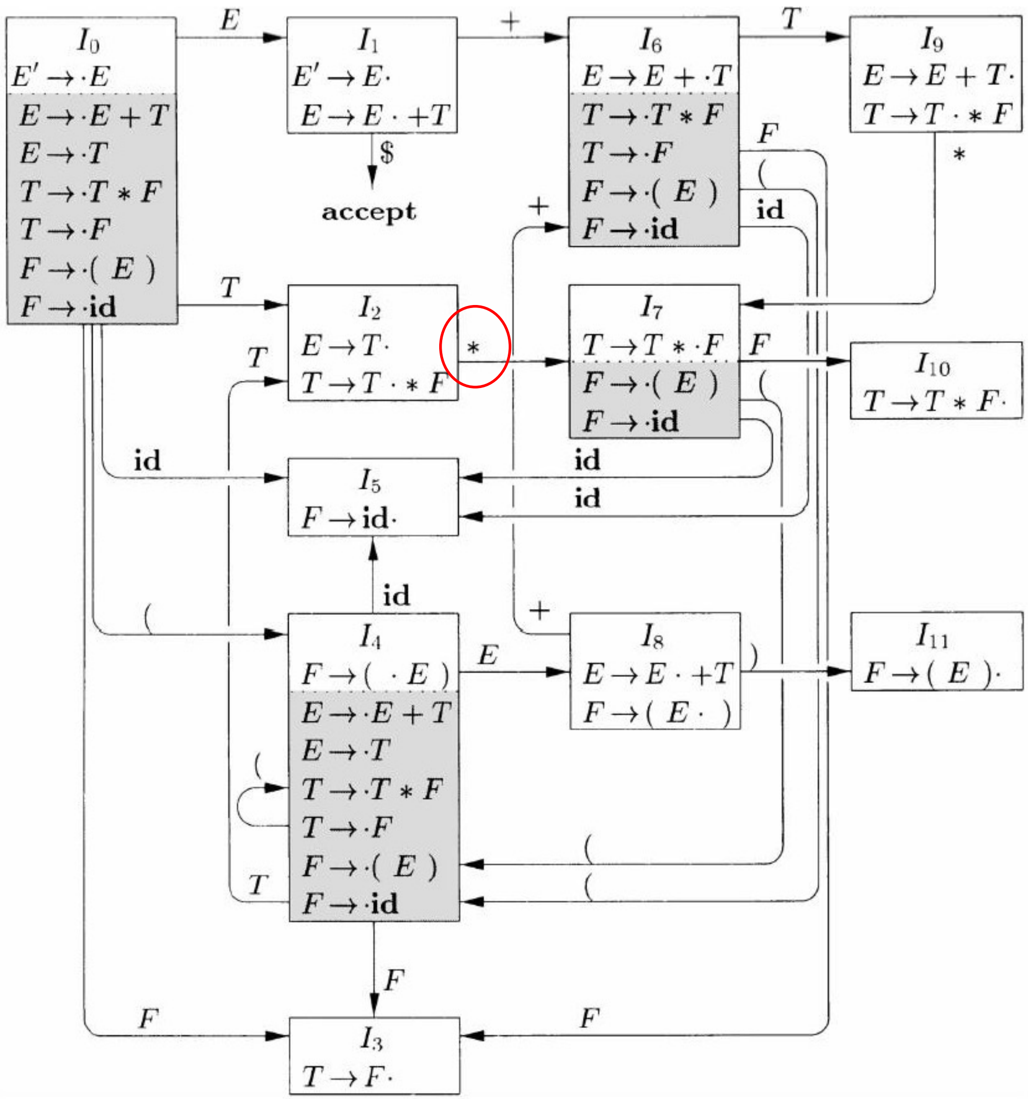
Stack: \$ 0 3 **Input:** * id \$

Gramar Symbols: \$ F

Action: Reduce by $T \rightarrow F$

- Pop state 3 (one symbol corresponds to one state)
- Push state 2

Example: Parsing id $*$ id



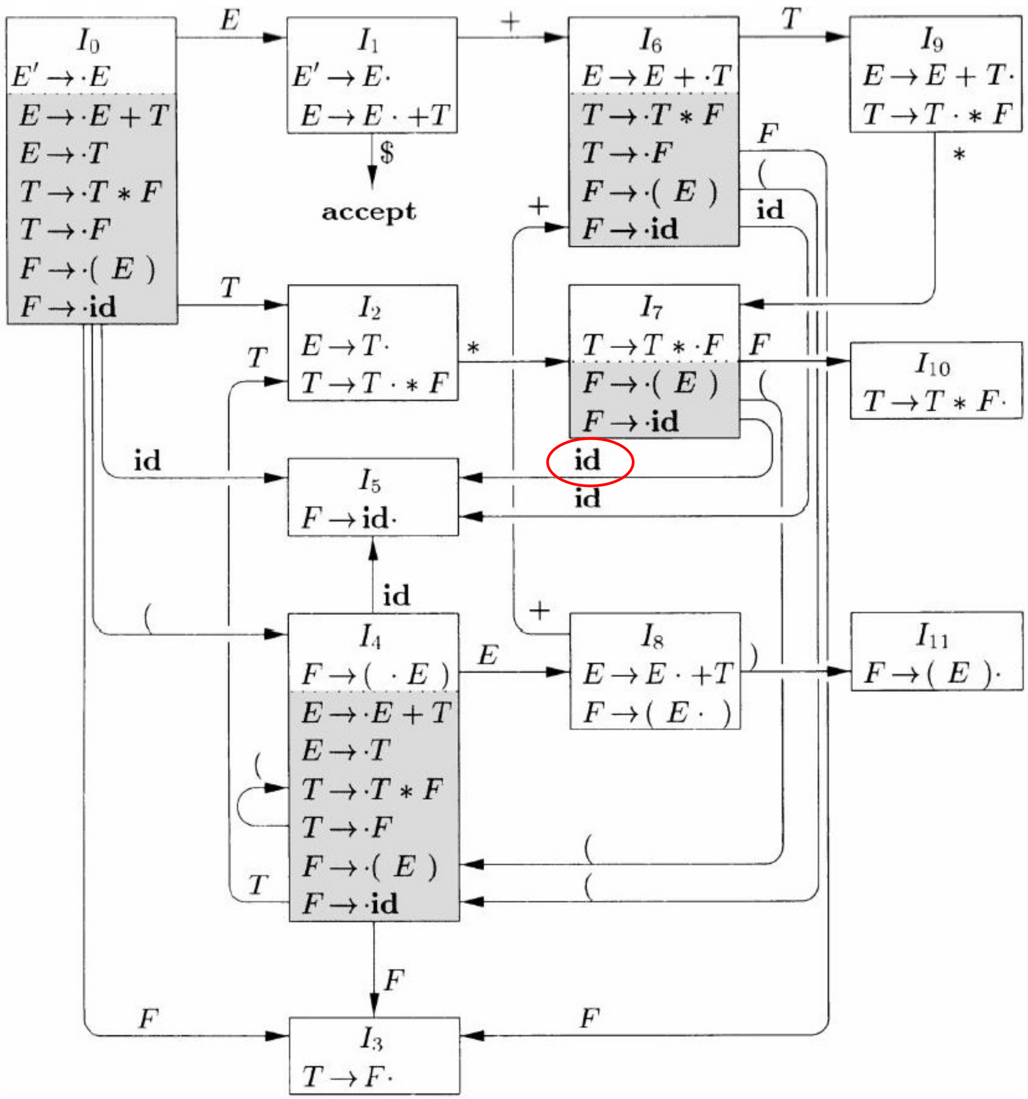
We only keep states in the stack; grammar symbols can be recovered from the states

Stack: $ 0 2 **Input:** * id $

Gramar Symbols: $ T

Action: Shift to 7

Example: Parsing id $*$ id



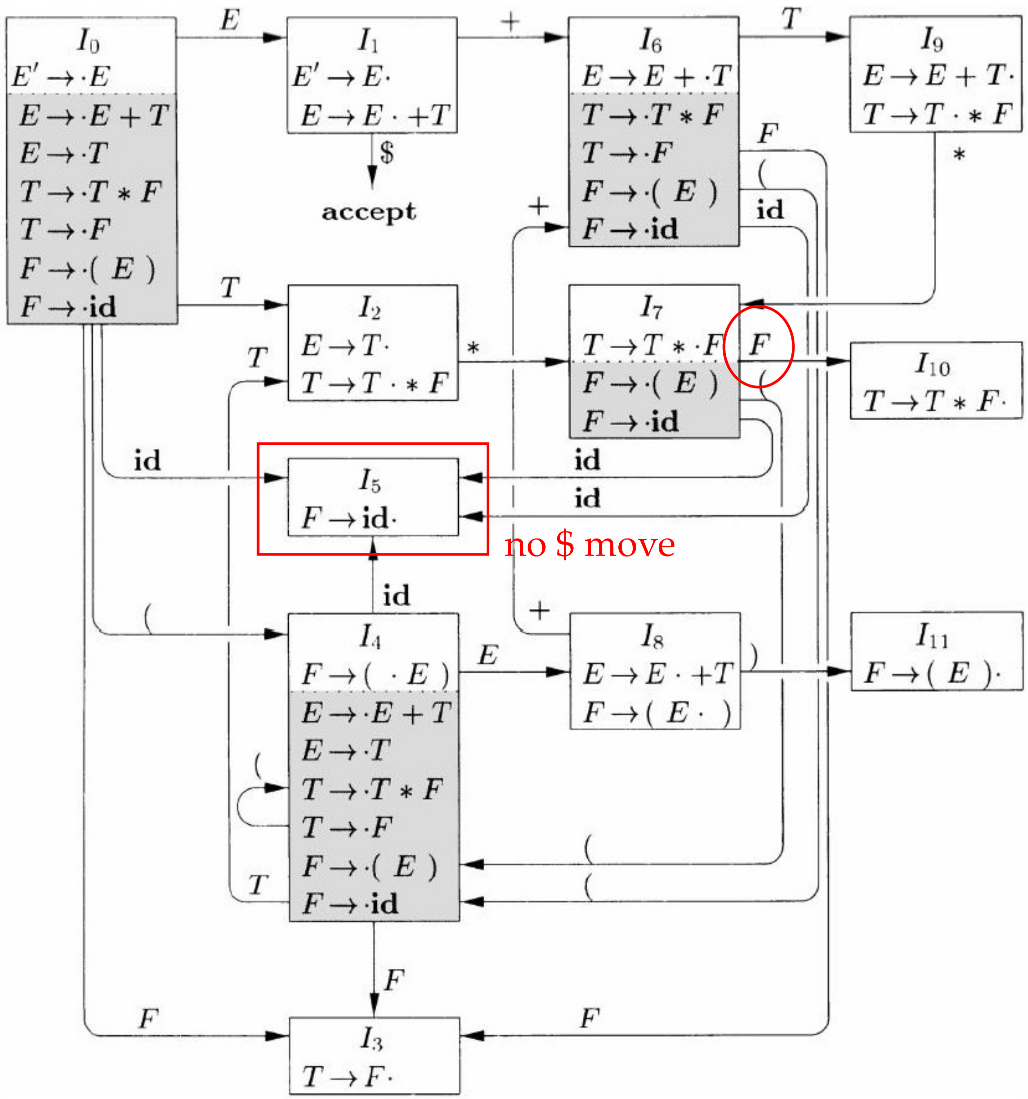
We only keep states in the stack; grammar symbols can be recovered from the states

Stack: $ 0 2 7 **Input:** id $

Gramar Symbols: $ T *

Action: Shift to 5

Example: Parsing id $*$ id



We only keep states in the stack; grammar symbols can be recovered from the states

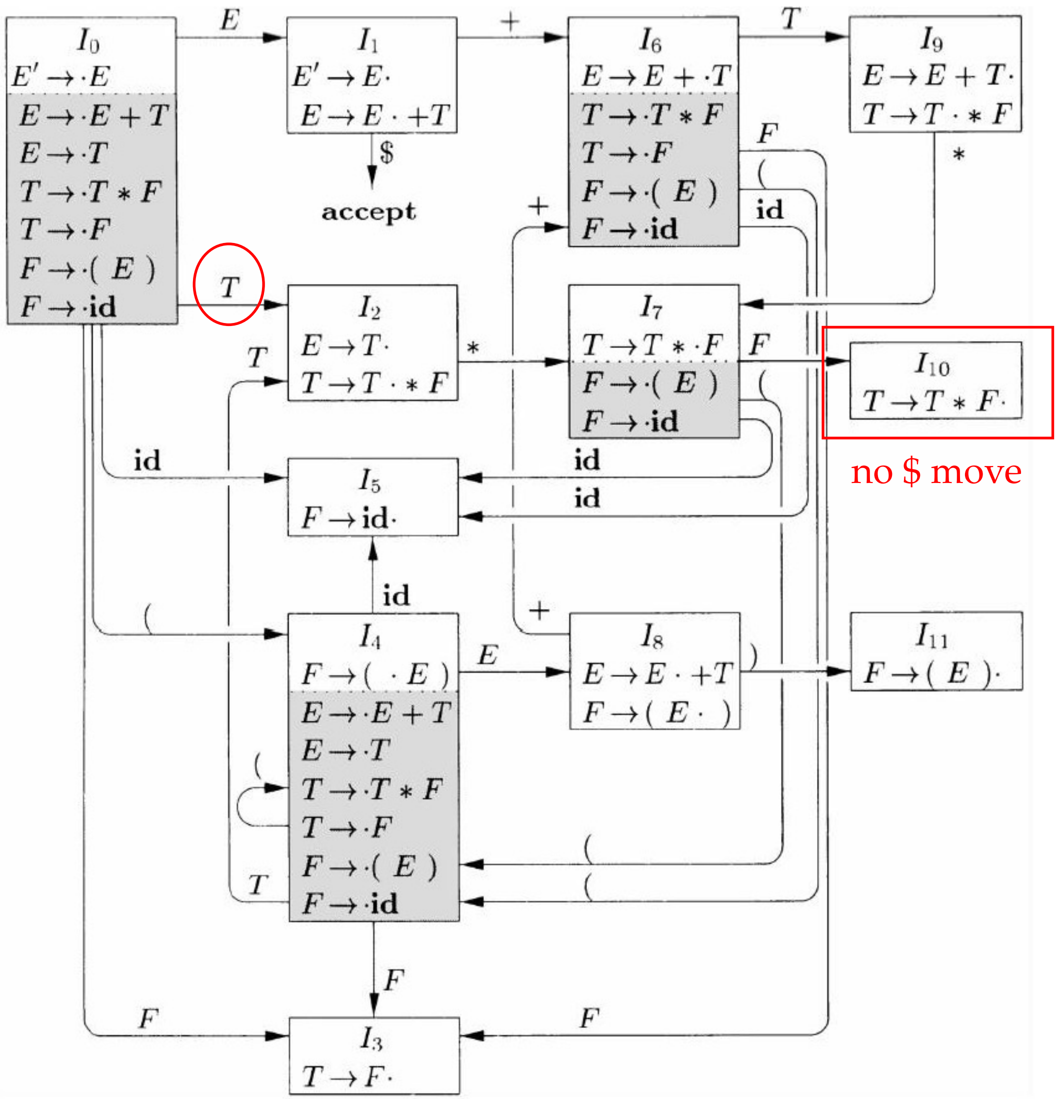
Stack: $ 0 2 7 5 **Input:** $

Gramar Symbols: $ T * id

Action: Reduce by $F \to \mathbf{id}$

- Pop state 5 (one symbol corresponds to one state)
- Push state 10

Example: Parsing id $*$ id



We only keep states in the stack; grammar symbols can be recovered from the states

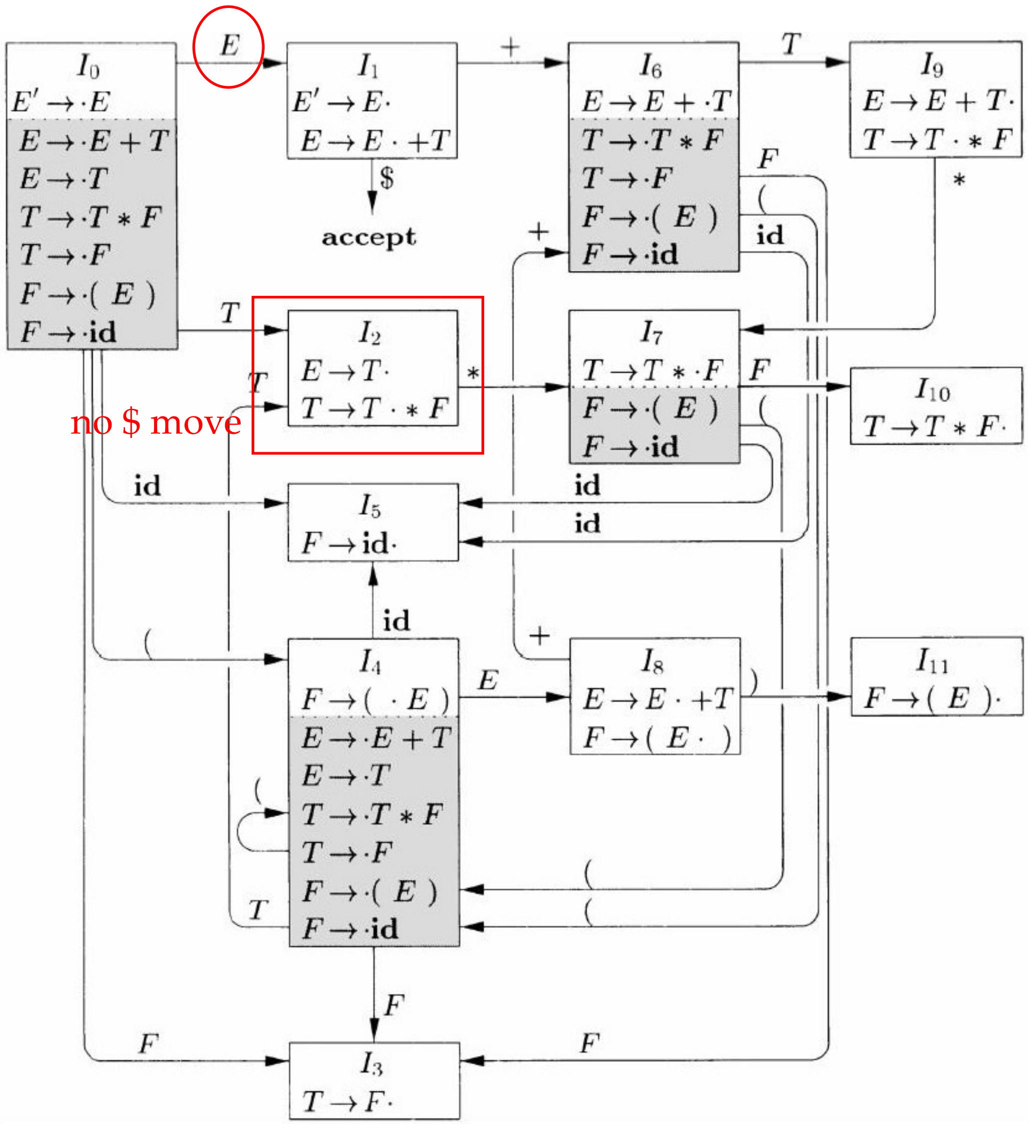
Stack: $ 0 2 7 10 **Input:** $

Gramar Symbols: $ T * F

Action: Reduce by $T \to T * F$

- Pop states 2, 7, 10 (one symbol corresponds to one state)
- Push state 2

Example: Parsing id $*$ id



We only keep states in the stack; grammar symbols can be recovered from the states

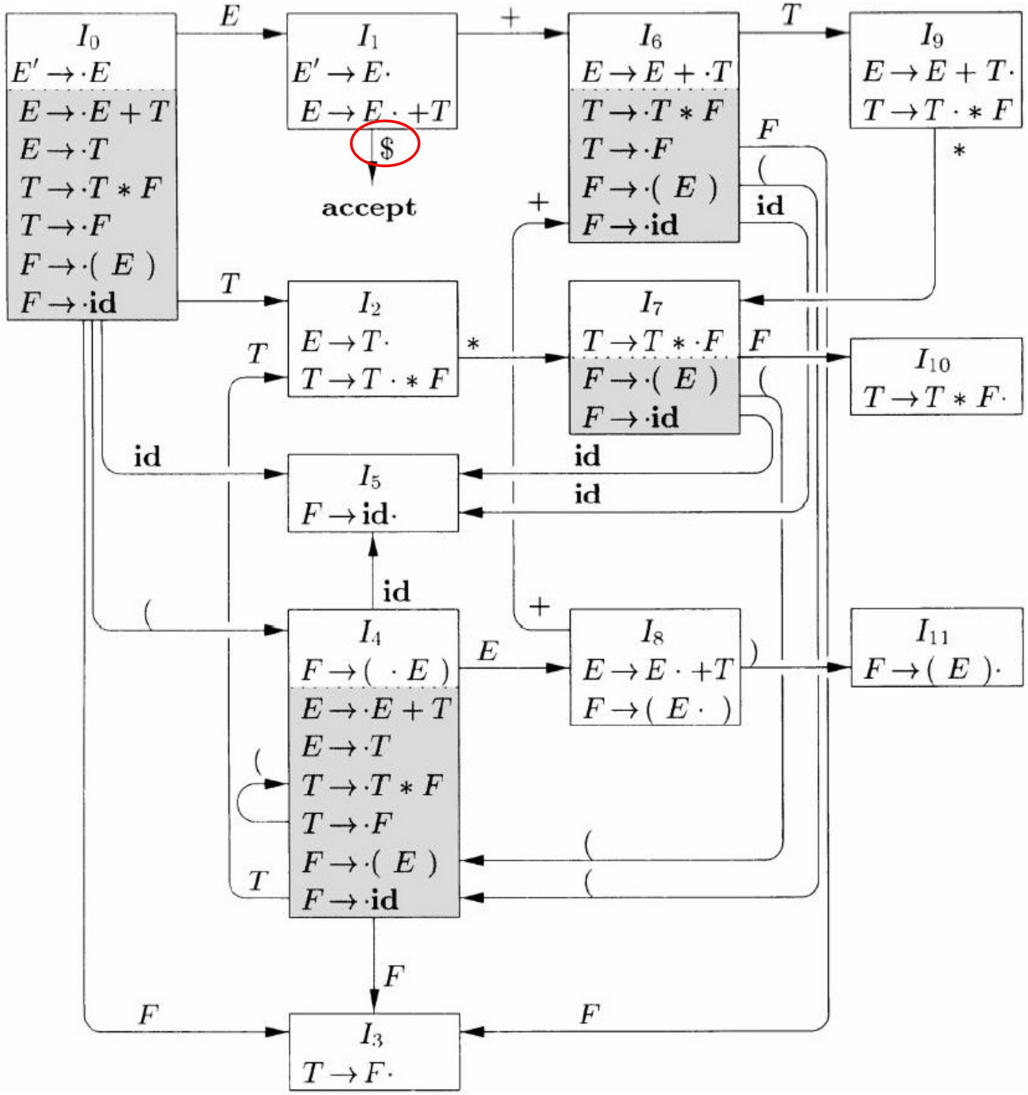
Stack: $ 0 2 **Input:** $

Gramar Symbols: $ T

Action: Reduce by $E \rightarrow T$

- Pop states 2 (one symbol corresponds to one state)
- Push state 1

Example: Parsing id $*$ id



We only keep states in the stack; grammar symbols can be recovered from the states

Stack: \$ 0 1 **Input:** \$

Gramar Symbols: \$ *E*

Action: Accept

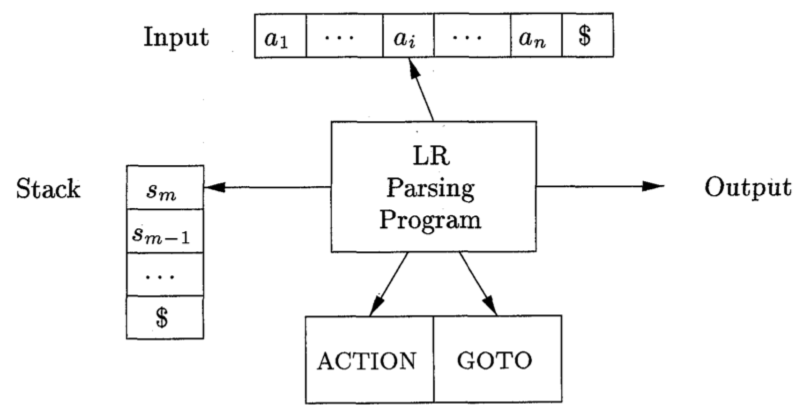
Example: Parsing id $*$ id

- The complete parsing steps

LINE	STACK	SYMBOLS	INPUT	ACTION
(1)	0	$\$$	$\mathbf{id} * \mathbf{id}\ \$$	shift to 5
(2)	0 5	$\$\ \mathbf{id}$	$* \mathbf{id}\ \$$	reduce by $F \to \mathbf{id}$
(3)	0 3	$\$\ F$	$* \mathbf{id}\ \$$	reduce by $T \to F$
(4)	0 2	$\$\ T$	$* \mathbf{id}\ \$$	shift to 7
(5)	0 2 7	$\$\ T *$	$\mathbf{id}\ \$$	shift to 5
(6)	0 2 7 5	$\$\ T * \mathbf{id}$	$\$$	reduce by $F \to \mathbf{id}$
(7)	0 2 7 10	$\$\ T * F$	$\$$	reduce by $T \to T * F$
(8)	0 2	$\$\ T$	$\$$	reduce by $E \to T$
(9)	0 1	$\$\ E$	$\$$	accept

LR Parser Structure

- An LR parser consists of an input, an output, a stack, a driver program, and a parsing table (ACTION + GOTO)
- The driver program is the same for all LR parsers; only the parsing table changes from one parser to another
- The stack holds a sequence of states
  - In SLR, the stack holds states from the LR(0) automaton
- The parser decides the next action based on (1) the state at the top of the stack and (2) the terminal read from the input buffer



Parsing Table: ACTION + GOTO

- The **ACTION** function takes two arguments: (1) a state i and (2) a terminal a (or \$)
- **ACTION[i, a]** can have one of the four forms of values:
  - **Shift j:** shift input a to the stack, but uses state j to represent a
  - **Reduce $A \rightarrow \beta$:** reduce β on the top of the stack to head A
  - **Accept:** The parser accepts the input and finishes parsing
  - **Error:** syntax errors exist
- The **GOTO** function is obtained from the one defined on sets of items: if GOTO(I_i, A) = I_j, then GOTO(i, A) = j

Parsing Table Example

STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

(1) $E \to E + T$
(2) $E \to T$
(3) $T \to T * F$

(4) $T \to F$
(5) $F \to (E)$
(6) $F \to \mathbf{id}$

- *s*5: shift by pushing state 5 *r*3: reduce using production No. 3
- GOTO entries for terminals are not listed, can be checked in ACTION part

LR Parser Configurations (态势)

- "**Configuration**" is notation for representing the complete state of the parser (stack status + input status). A *configuration* is a pair:

$$(\underbrace{s_0 s_1 \dots s_m}_{\text{Stack contents (top on the right)}}, \underbrace{a_i a_{i+1} \dots a_n \$}_{\text{Remaining input}})$$

- By construction, each state (except s_0) in an LR parser corresponds to a set of items and a grammar symbol (the symbol that leads to the state transition, i.e., the symbol on the incoming edge)
  - Suppose X_i is the grammar symbol for state s_i
  - Then $X_0 X_1 \dots X_m a_i a_{i+1} \dots a_n$ is a right-sentential form (assume no errors)

Behavior of the LR Parser

- For the configuration $(s_0 s_1 \dots s_m, a_i a_{i+1} \dots a_n \$)$, the LR parser checks ACTION$[s_m, a_i]$ in the parsing table to decide the parsing action
  - shift s: shift the next state s onto the stack, entering the configuration $(s_0 s_1 \dots s_m s, a_{i+1} \dots a_n \$)$
  - reduce $A \rightarrow \beta$: execute a reduce move, entering the configuration $(s_0 s_1 \dots s_{m-r} s, a_i a_{i+1} \dots a_n \$)$, where r = the length of β, and $s = \text{GOTO}(s_{m-r}, A)$
  - accept: parsing is completed
  - error: the parser has found an error and calls an error recovery routine

LR-Parsing Algorithm

- **Input:** The parsing table for a grammar G and an input string ω
- **Output:** If ω is in $L(G)$, the reduction steps of a bottom-up parse for ω; otherwise, an error indication
- **Initial configuration:** $(s_0, \omega\$)$

```
let a be the first symbol of w$;
while(1) { /* repeat forever */
        let s be the state on top of the stack;
        if ( ACTION[s, a] = shift t ) {
                push t onto the stack;
                let a be the next input symbol;
        } else if ( ACTION[s, a] = reduce A → β ) {
                pop |β| symbols off the stack;
                let state t now be on top of the stack;
                push GOTO[t, A] onto the stack;
                output the production A → β;
        } else if ( ACTION[s, a] = accept ) break; /* parsing is done */
        else call error-recovery routine;
}
```

Constructing SLR-Parsing Tables

- The SLR-parsing table for a grammar G can be constructed based on the LR(0) item sets and LR(0) automaton

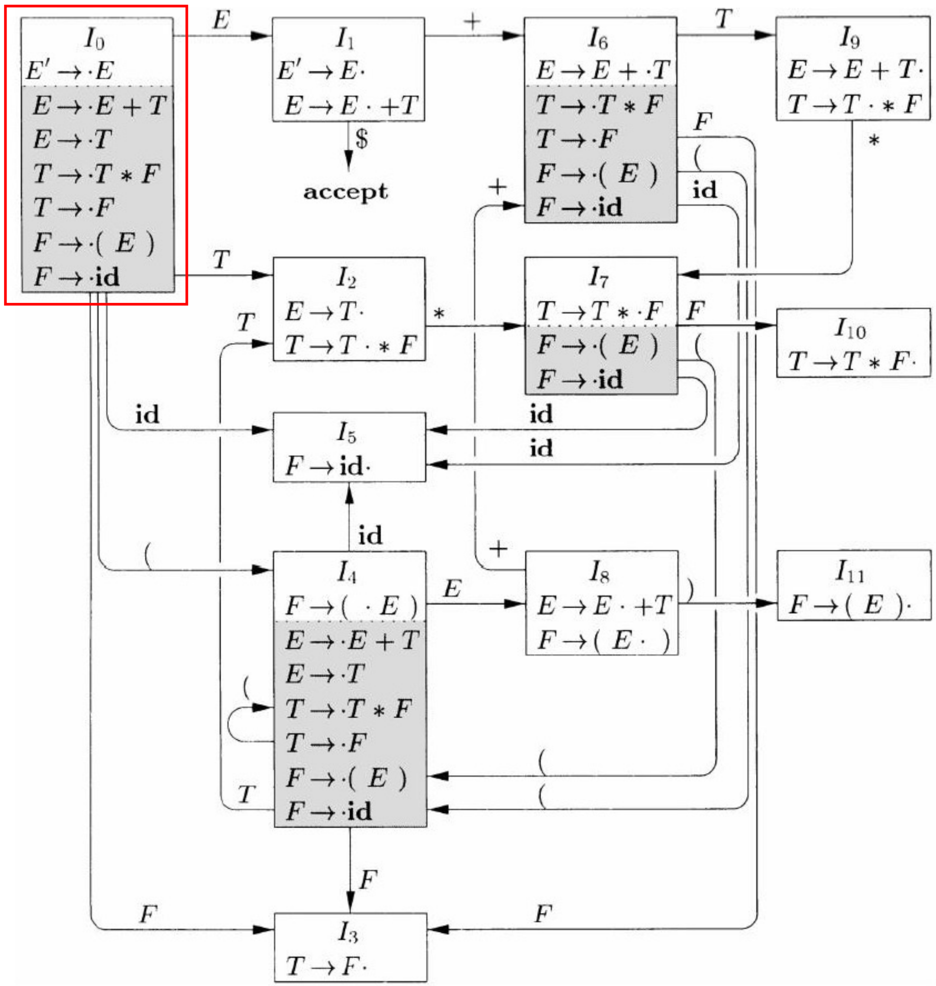
1. Construct the canonical LR(0) collection $\{I_0, I_1, \dots, I_n\}$ for the augmented grammar G'
2. State i is constructed from I_i. ACTION can be determined as follows:
   - If $[A \to \alpha \cdot a\beta]$ is in I_i and $\mathtt{GOTO}[I_i, a] = I_j$, then set $\mathtt{ACTION}[i, a]$ to "shift j" (here a must be a terminal)
   - If $[A \to \alpha \cdot]$ is in I_i, then set $\mathtt{ACTION}[i, a]$ to "reduce $A \to \alpha$" for **all a in $\mathtt{FOLLOW}(A)$**; here A may not be S'
   - If $[S' \to S \cdot]$ is in I_i, then set $\mathtt{ACTION}[i, \$]$ to "accept"
3. The goto transitions for state i are constructed for all nonterminals A using the rule: If $\mathtt{GOTO}(I_i, A) = I_j$, then $\mathtt{GOTO}(i, A) = j$

Constructing SLR-Parsing Tables

4. All entries not defined in steps 2 and 3 are set to "error"
5. Initial state is the one constructed from the item set containing $[S' \rightarrow \cdot\, S]$

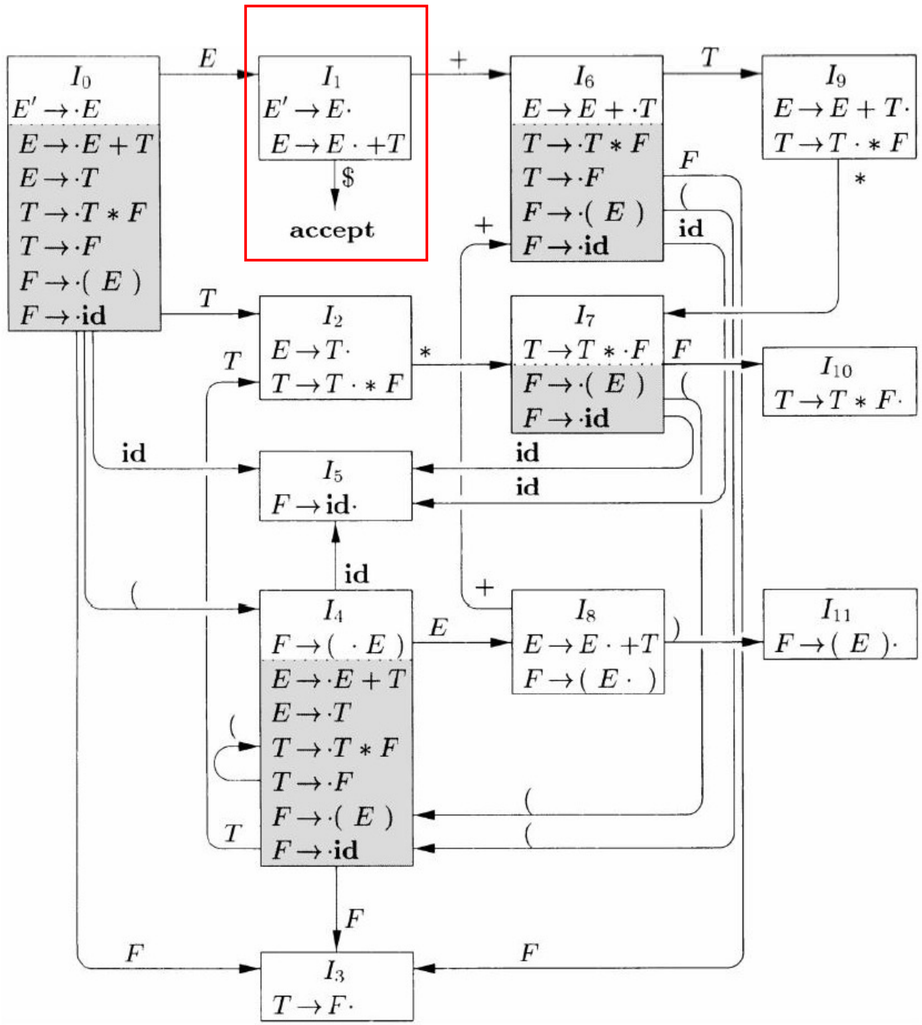
If there is no conflict during the parsing table construction (i.e., multiple actions for a table entry), the grammar is **SLR(1)**

Example



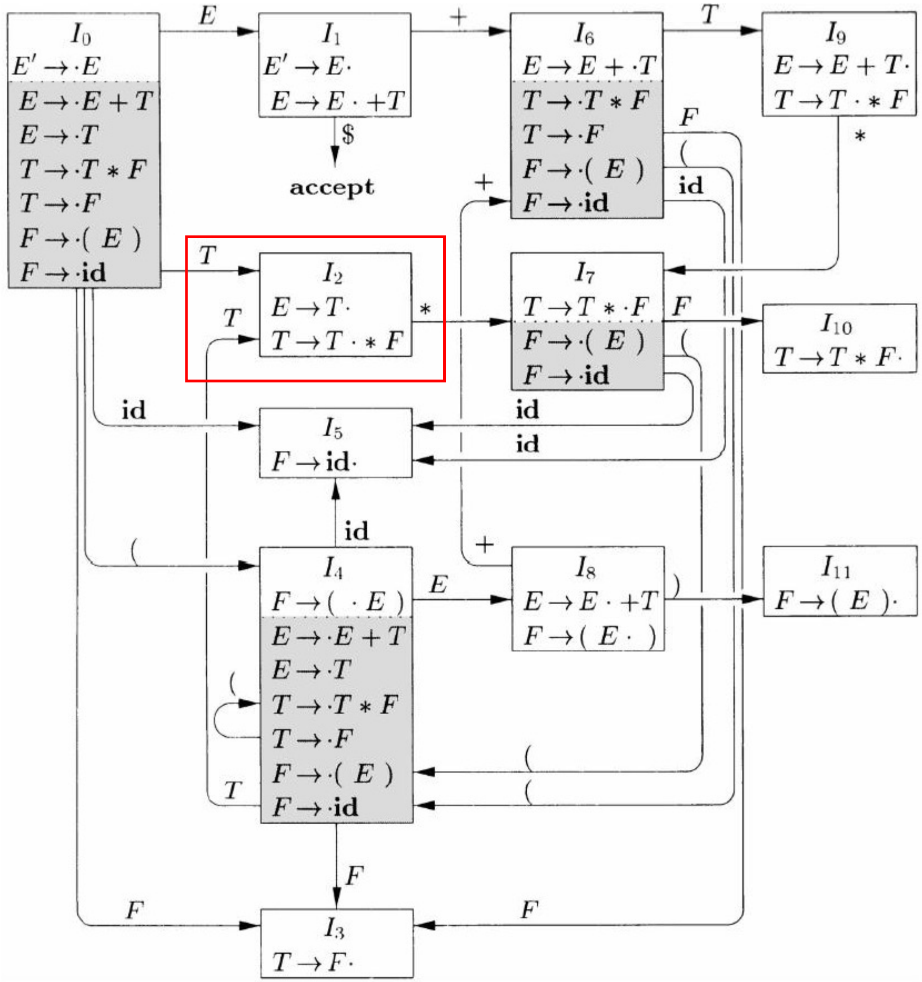
- ACTION(0, () = s4 (shift 4)
- ACTION(0, **id**) = s5
- $\mathrm{GOTO}[0, E] = 1$
- $\mathrm{GOTO}[0, T] = 2$
- $\mathrm{GOTO}[0, F] = 3$

Example



- $\text{ACTION}(1, +) = \text{s}6$
- $\text{ACTION}(1, \$) = \text{accept}$

Example



- ACTION(2,$*$) = s7
- ACTION(2, $) = reduce $E \to T$
- ACTION(2, +) = reduce $E \to T$
- ACTION(2,)) = reduce $E \to T$

FOLLOW(E) = {$, +,)}

Non-SLR Grammar

- Grammar
  - $S \rightarrow L = R \mid R$
  - $L \rightarrow * R \mid \mathbf{id}$
  - $R \rightarrow L$
- For item set I_2:
  - According to item #1: ACTION[2, =] is "s6"
  - According to item #2: ACTION[2, =] is "reduce $R \rightarrow L$" (FOLLOW(R) contains =)

I_0: $S' \to \cdot S$
$S \to \cdot L = R$
$S \to \cdot R$
$L \to \cdot * R$
$L \to \cdot \mathbf{id}$
$R \to \cdot L$

I_1: $S' \to S \cdot$

I_2: $\boxed{\begin{array}{l} S \to L \cdot = R \\ R \to L \cdot \end{array}}$

I_3: $S \to R \cdot$

I_4: $L \to * \cdot R$
$R \to \cdot L$
$L \to \cdot * R$
$L \to \cdot \mathbf{id}$

I_5: $L \to \mathbf{id} \cdot$

I_6: $S \to L = \cdot R$
$R \to \cdot L$
$L \to \cdot * R$
$L \to \cdot \mathbf{id}$

I_7: $L \to * R \cdot$

I_8: $R \to L \cdot$

I_9: $S \to L = R \cdot$

This grammar is not ambiguous

CLR and LALR will succeed on a larger collection of grammars, including the above one. However, there exist unambiguous grammars for which every LR parser construction method will encounter conflicts.

Weakness of the SLR Method

- In SLR, the state i calls for reduction by $A \to \alpha$ if (1) the item set I_i contains item $[A \to \alpha \cdot]$ and (2) input symbol a is in $\mathtt{FOLLOW}(A)$

- In some situations, after reduction, the content $\beta\alpha$ on stack top would become βA that cannot be followed by a in any right-sentential form* (i.e., only requiring "a is in $\mathtt{FOLLOW}(A)$" is not enough)

* Although SLR algorithm requires a to belong to FOLLOW(A), it is still too casual as the stack content below A is not considered.

Example: Parsing id = id

- $S \rightarrow L = R \mid R$
- $L \rightarrow * R \mid \mathbf{id}$
- $R \rightarrow L$

I_0: $S' \rightarrow \cdot S$
$S \rightarrow \cdot L = R$
$S \rightarrow \cdot R$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$
$R \rightarrow \cdot L$

I_1: $S' \rightarrow S \cdot$

I_2: $S \rightarrow L \cdot = R$
$R \rightarrow L \cdot$

I_3: $S \rightarrow R \cdot$

I_4: $L \rightarrow * \cdot R$
$R \rightarrow \cdot L$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$

I_5: $L \rightarrow \mathbf{id} \cdot$

I_6: $S \rightarrow L = \cdot R$
$R \rightarrow \cdot L$
$L \rightarrow \cdot * R$
$L \rightarrow \cdot \mathbf{id}$

I_7: $L \rightarrow * R \cdot$

I_8: $R \rightarrow L \cdot$

I_9: $S \rightarrow L = R \cdot$

Stack	Symbols	Input	Action
$0		id = id	Shift 5
$05	id	= id	Reduce by L→id
$02	L	= id	Suppose reduce by R→L
$03	R	= id	**Error!**

Cannot shift, cannot reduce since FOLLOW(S) = {$}

Problem: SLR reduces too casually

How to know if a reduction is a good move?
Utilize the next input symbol to precisely determine whether to call for a reduction.

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LR(1) Item

- **Idea:** Carry more information in the state to rule out some invalid reductions (splitting LR(0) states)

- General form of an LR(1) item: $[A \rightarrow \alpha \cdot \beta, a]$
  - $A \rightarrow \alpha\beta$ is a production and a is a terminal or \$
  - "1" refers to the length of the 2^{nd} component: the *lookahead* (向前看字符)*
  - The lookahead symbol has no effect if β is not ϵ since it only helps determine whether to reduce (a will be inherited during state transitions)
  - An item of the form $[A \rightarrow \alpha \cdot, a]$ calls for a reduction by $A \rightarrow \alpha$ only if the next input symbol is a (the set of such a's is a **subset** of $\text{FOLLOW}(A)$)

*: LR(0) items do not have lookahead symbols, and hence they are called LR(0)

Constructing LR(1) Item Sets (1)

- Constructing the collection of LR(1) item sets is essentially the same as constructing the canonical collection of LR(0) item sets. The only differences lie in the CLOSURE and GOTO functions.

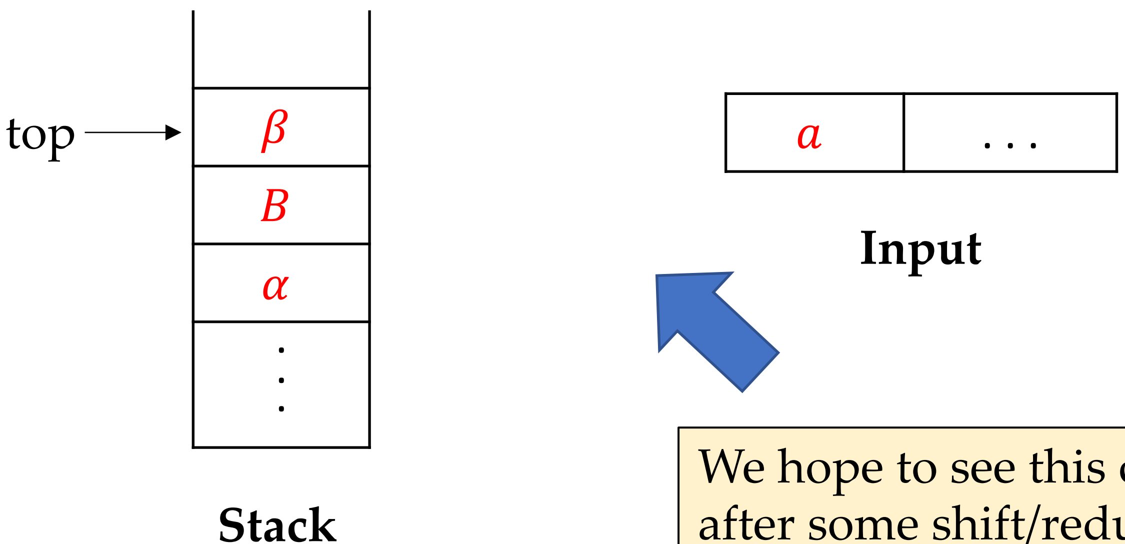
```
SetOfItems CLOSURE(I) {
    J = I;
    repeat
        for ( each item A → α·Bβ in J )
            for ( each production B → γ of G )
                if ( B → ·γ is not in J )
                    add B → ·γ to J;
    until no more items are added to J on one round;
    return J;
}
```

```
SetOfItems CLOSURE(I) {
    repeat
        for ( each item [A → α·Bβ, a] in I )
            for ( each production B → γ in G′ )
                for ( each terminal b in FIRST(βa) )
                    add [B → ·γ, b] to set I;
    until no more items are added to I;
    return I;
}
```

It only generates the new item $[B \to \cdot\, \gamma, b]$ from $[A \to \alpha \cdot B\beta, a]$ if b is in $\text{FIRST}(\beta a)$

Why b should be in FIRST(βa)?

- The item $[A \to \alpha \cdot B\beta, a]$ will derive $[A \to \alpha B\beta \cdot, a]$, which calls for reduction when the stack top contains $\alpha B\beta$ and the next input symbol is a

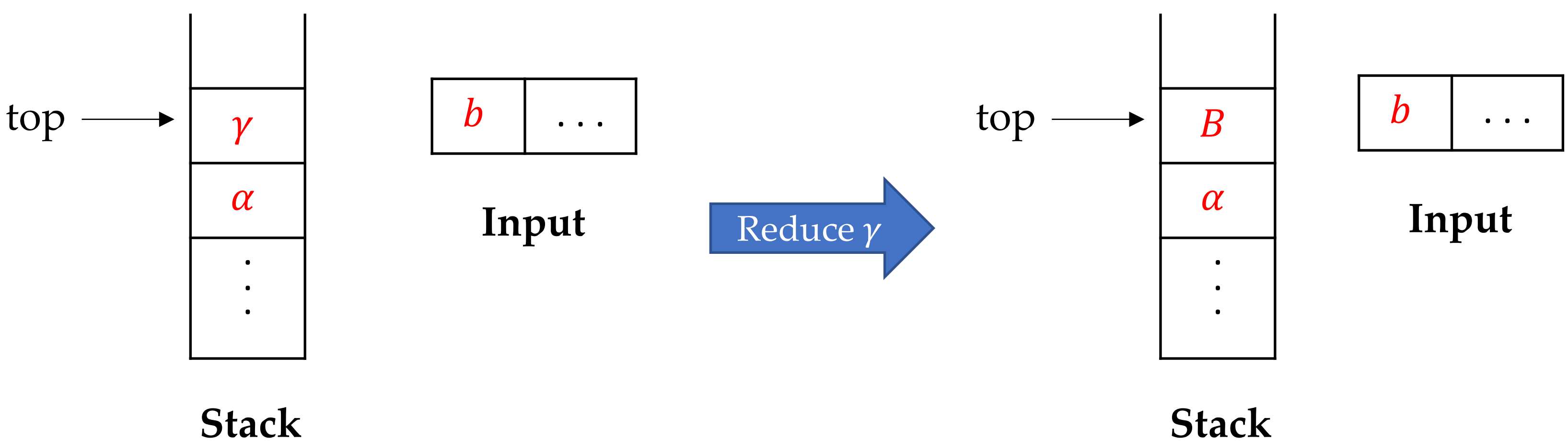


We hope to see this configuration after some shift/reduce steps.

Why b should be in FIRST(βa)?

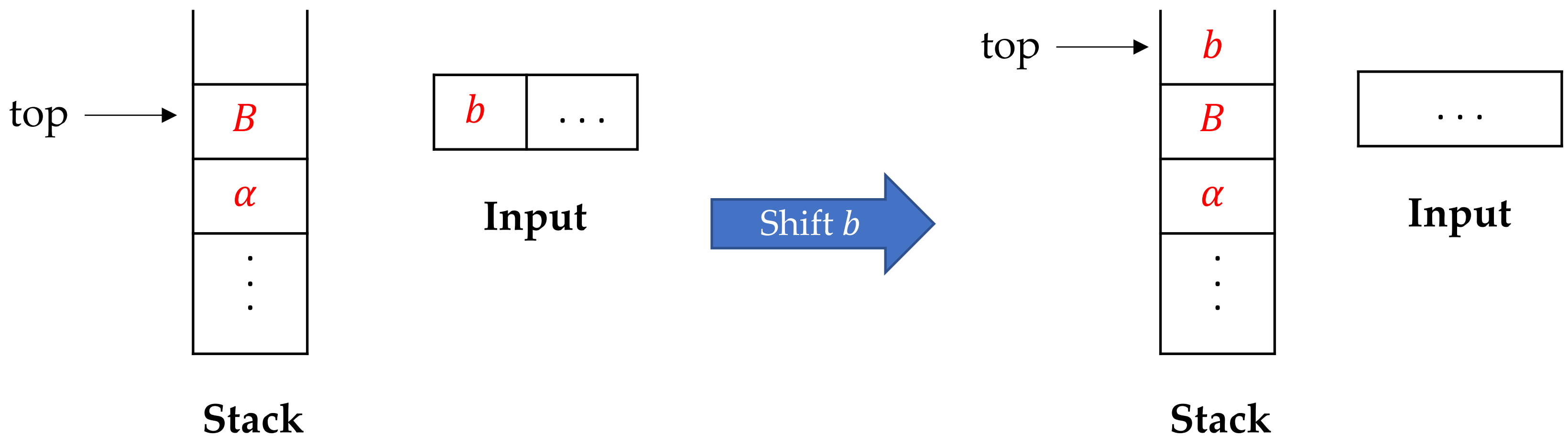
- When generating the item $[B \rightarrow \cdot\, \gamma, b]$ from $[A \rightarrow \alpha \cdot B\beta, a]$, suppose we allow that b is not in $FIRST(\beta a)$

- We add the item $[B \rightarrow \cdot\, \gamma, b]$ because we hope that at certain time point during parsing, when we see γ on stack top and b as the next input symbol, we can first reduce γ to B so that in some later step the stack top would contain $\alpha B\beta$ (then we can further reduce it to A)



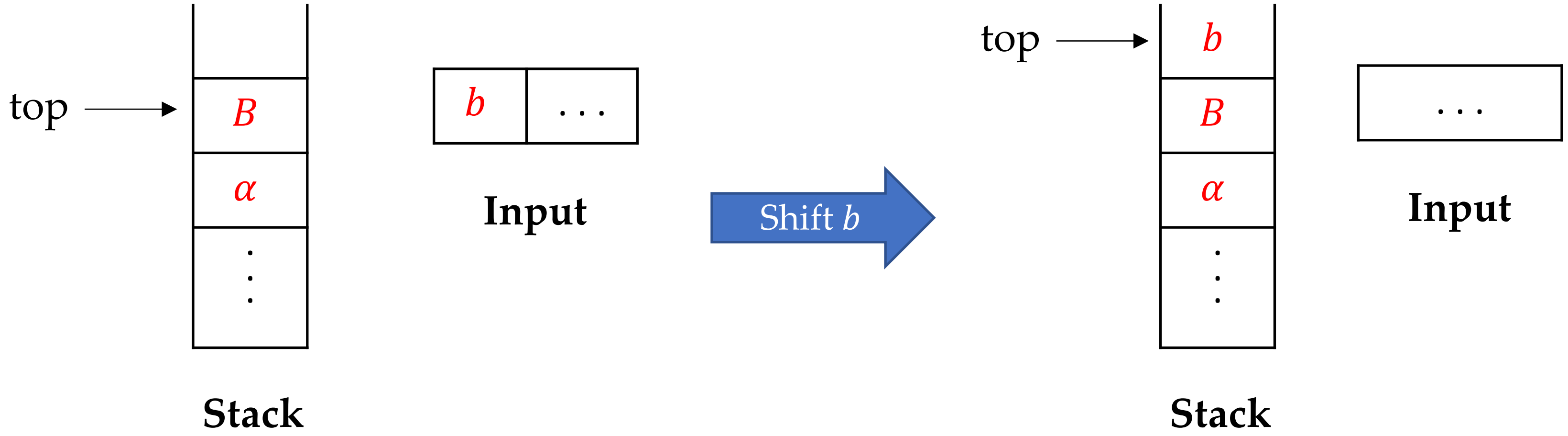
Why b should be in FIRST(βa)?

- If we reduce γ to B, the next action would be "shift b to the stack"
  - Because the production $A \rightarrow \alpha B \beta$ tells us that we are ready for reduction only when we see $\alpha B \beta$ on stack top



Why b should be in FIRST(βa)?

- Since b is not in $FIRST(\beta a)$, the stack top will never become the form $\alpha B\beta$, which means we will never be able to reduce $\alpha B\beta$ to A
- Then why should we generate $[B \rightarrow \cdot\, \gamma, b]$ from $[A \rightarrow \alpha \cdot B\beta, a]$ in the first palce???



Constructing LR(1) Item Sets (2)

- Constructing the collection of LR(1) item sets is essentially the same as constructing the canonical collection of LR(0) item sets. The only differences lie in the CLOSURE and GOTO functions.

```
SetOfItems GOTO(I, X) {
    initialize J to be the empty set;
    for ( each item [A → α·Xβ, a] in I )
        add item [A → αX·β, a] to set J;
    return CLOSURE(J);
}
```

GOTO(I, X) in LR(0) item sets:

The closure of the set of all items $[A \to \alpha X \cdot \beta]$ such that $[A \to \alpha \cdot X\beta]$ is in I.

The lookahead symbols are passed to new items from existing items

Constructing LR(1) Item Sets (3)

```
void items(G') {
    C = {CLOSURE({[S' → ·S]})};
    repeat
        for ( each set of items I in C )
            for ( each grammar symbol X )
                if ( GOTO(I, X) is not empty and not in C )
                    add GOTO(I, X) to C;
    until no new sets of items are added to C on a round;
}
```

Constructing the collection of LR(0) item sets

```
void items(G') {
    initialize C to {CLOSURE({[S' → ·S, $]})};
    repeat
        for ( each set of items I in C )
            for ( each grammar symbol X )
                if ( GOTO(I, X) is not empty and not in C )
                    add GOTO(I, X) to C;
    until no new sets of items are added to C;
}
```

Constructing the collection of LR(1) item sets

LR(1) Item Sets Example

- Augmented grammar:
  - $S' \to S \qquad S \to CC \qquad C \to cC \mid d$
- Constructing I_0 item set and GOTO function:
  - $I_0 = \text{CLOSURE}([S' \to \cdot S, \$]) =$
    - $\{[S' \to \cdot S, \$], [S \to \cdot CC, \$], [C \to \cdot cC, c/d], [C \to \cdot d, c/d]\}$

FIRST($) = {$}

FIRST(C$) = {c, d}

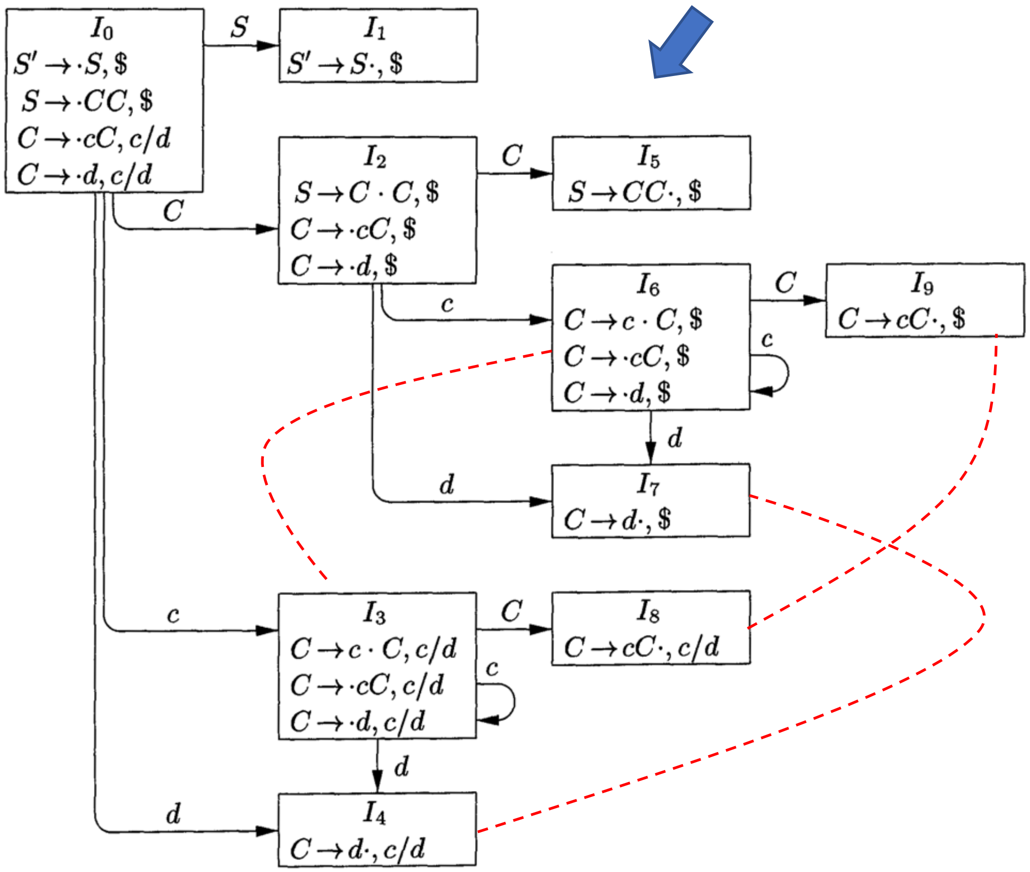
  - $\text{GOTO}(I_0, S) = \text{CLOSURE}(\{[S' \to S \cdot, \$]\}) = \{[S' \to S \cdot, \$]\}$
  - $\text{GOTO}(I_0, C) = \text{CLOSURE}(\{[S \to C \cdot C, \$]\}) =$
    - $\{[S \to C \cdot C, \$], [C \to \cdot cC, \$], [C \to \cdot d, \$]\}$

FIRST($) = {$}

  - $\text{GOTO}(I_0, c) = \text{CLOSURE}(\{[C \to c \cdot C, c/d]\}) =$
    - $\{[C \to c \cdot C, c/d], [C \to \cdot cC, c/d], [C \to \cdot d, c/d]\}$
  - $\text{GOTO}(I_0, d) = \text{CLOSURE}(\{[C \to d \cdot, c/d]\}) = \{[C \to d \cdot, c/d]\}$

The GOTO Graph Example

LR(1) Automaton



10 states in total

These states are equivalent if we ignore the lookahead symbols (SLR makes no such distinctions of states):

- I_3 and I_6
- I_4 and I_7
- I_8 and I_9

Constructing Canonical LR(1) Parsing Tables

1. Construct $C' = \{I_0, I_1, \dots, I_n\}$, the collection of LR(1) item sets for the augmented grammar G'

2. State i of the parser is constructed from I_i. Its parsing action is determined as follows:

   - If $[A \to \alpha \cdot a\beta, b]$ is in I_i and $\mathrm{GOTO}(I_i, a) = I_j$, then set $\mathrm{ACTION}[i, a]$ to "*shift j*." Here, a must be a terminal.
   - If $[A \to \alpha \cdot, a]$ is in I_i, $A \neq S'$, then set $\mathrm{ACTION}[i, a]$ to "*reduce $A \to \alpha$*"
   - If $[S' \to S \cdot, \$]$ is in I_i, then set $\mathrm{ACTION}[i, \$]$ to "*accept*"

More restrictive than SLR

If any conflicting actions result from the above rules, we say the grammar is not LR(1)

Constructing Canonical LR(1) Parsing Tables

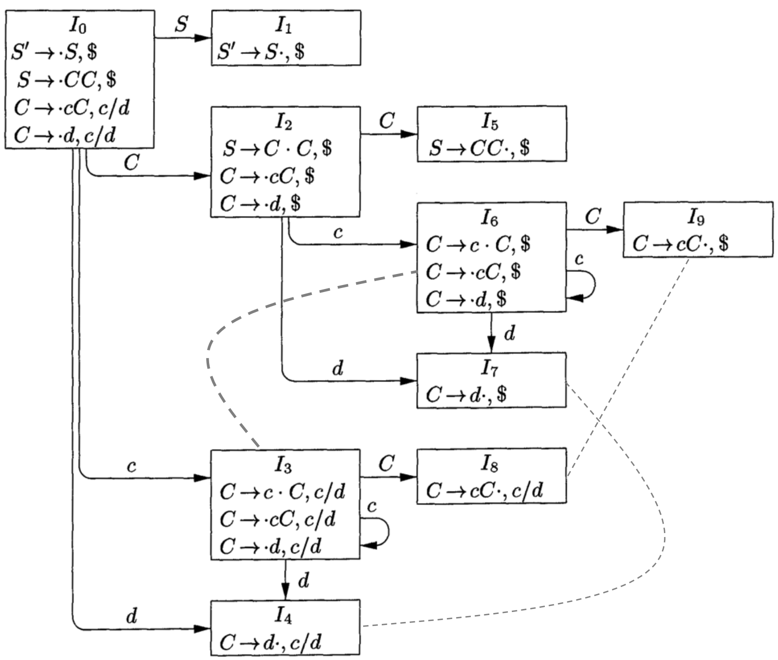
3. The goto transitions for state i are constructed from all nonterminals A using the rule: If $\mathtt{GOTO}(I_i, A) = I_j$, then $\mathtt{GOTO}(i, A) = j$

4. All entries not defined in steps (2) and (3) are made "error"

5. The initial state of the parser is the one constructed from the set of items containing $[S' \rightarrow \cdot\, S, \$]$

LR(1) Parsing Table Example

Grammar: $S' \to S$ $\quad S \to CC \quad C \to cC \mid d$



STATE	ACTION			GOTO	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Three pairs of states can be seen as being split from the corresponding LR(0) states:

(3, 6) (4, 7) (8, 9)

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Lookahead LR (LALR) Method

- SLR(1) is not powerful enough to handle a large collection of grammars (recall the previous unambiguous grammar)
- LR(1) has a huge set of states in the parsing table (states are too fine-grained)
- LALR(1) is often used in practice
  - Keeps the lookahead symbols in the items
  - Its number of states is the same as that of SLR(1)
  - Can deal with most common syntactic constructs of modern programming languages

Merging States in LR(1) Parsing Tables

- **State 4:**
  - Reduce by $C \to d$ if the next input symbol is c or d
  - Error if \$
- **State 7:**
  - Reduce by $C \to d$ if the next input symbol is \$
  - Error if c or d

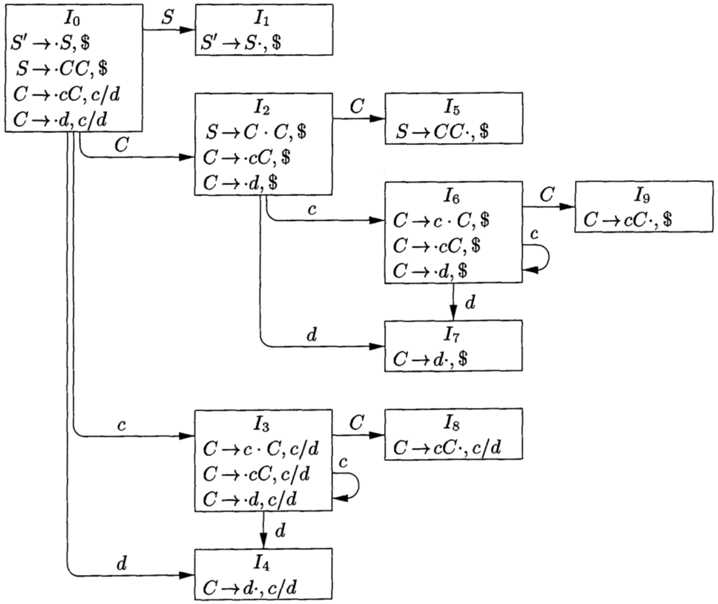
STATE	ACTION			GOTO	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Can we merge states 4 and 7 so that the parser can reduce for all input symbols?

- $I_4: C \to d \cdot, c/d$
- $I_7: C \to d \cdot, \$$

The Basic Idea of LALR

- Look for sets of LR(1) items with the same ***core***
  - The core of an LR(1) item set is the set of the first components
    - The core of I_4 and I_7 is $\{C \rightarrow d\ \cdot\}$
    - The core of I_3 and I_6 is $\{C \rightarrow c \cdot C, C \rightarrow \cdot\ cC, C \rightarrow \cdot\ d\}$

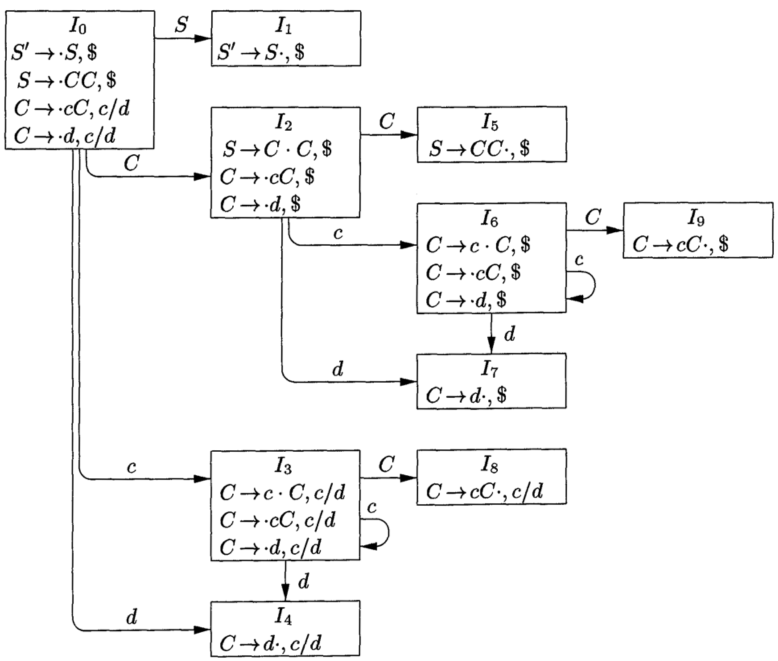


The Basic Idea of LALR Cont.

- Look for sets of LR(1) items with the same ***core***
  - The core of an LR(1) item set is the set of the first components
    - The core of I_4 and I_7 is $\{C \rightarrow d \cdot\}$
    - The core of I_3 and I_6 is $\{C \rightarrow c \cdot C, C \rightarrow \cdot cC, C \rightarrow \cdot d\}$
  - In general, a core is a set of LR(0) items
- We may merge the LR(1) item sets with common cores into one set of items

The Basic Idea of LALR Cont.

- Since the core of $\mathtt{GOTO}(I, X)$ depends only on the core of I, the goto targets of merged sets also have the same core and hence can be merged



Consider I_3 and I_6:

- The core $\{C \to c \cdot C, C \to \cdot cC, C \to \cdot d\}$ determines state transition targets
- Before merging, GOTO(I_3, C) = I_9, GOTO(I_6, C) = I_8
- After merging, I_3 and I_6 become I_{36}, I_8 and I_9 become I_{89}, and GOTO(I_{36}, C) = I_{89}

Conflicts Caused by State Merging

- Merging states in an LR(1) parsing table may cause conflicts
- Merging does not cause shift/reduce conflicts
  - Suppose after merging there is shift/reduce conflict on lookahead a
    - There is an item $[A \to \alpha \cdot, a]$ in a merged set calling for a reduction by $A \to \alpha$
    - There is another item $[B \to \beta \cdot a\gamma, ?]$ in the set calling for a shift
  - Since the cores of the sets to be merged are the same, there must be a set containing both $[A \to \alpha \cdot, a]$ and $[B \to \beta \cdot a\gamma, ?]$ before merging
  - Then before merging, there is already a shift/reduce conflict on a according to LR(1) parsing table construction algorithm. The grammar is not LR(1). **Contradiction!!!**
- Merging states may cause reduce/reduce conflicts

Example of Conflicts

- An LR(1) grammar:
  - $S' \to S \qquad S \to aAd \mid bBd \mid aBe \mid bAe \qquad A \to c \qquad B \to c$
- Language: $\{acd, bcd, ace, bce\}$
- One set of valid LR(1) items
  - $\{[A \to c\,\cdot, d], [B \to c\,\cdot, e]\}$
- Another set of valid LR(1) items
  - $\{[B \to c\,\cdot, d], [A \to c\,\cdot, e]$
- After merging, the new item set: $\{[A \to c\,\cdot, d/e], [B \to c., d/e]\}$
  - **Conflict:** reduce c to A or B when the next input symbol is d/e?

Constructing LALR Parsing Table

- Construct $C = \{I_0, I_1, \ldots, I_n\}$, the collection of sets of LR(1) items
- For each core present among a set of LR(1) items, find all sets having that core, and replace these sets by their union
- Let $C' = \{J_0, J_1, \ldots, J_m\}$ be the resulting collection after merging.
  - The parsing actions for state i are constructed from J_i following the LR(1) parsing table construction algorithm.
  - If there is a conflict, this algorithm fails to produce a parser and the grammar is not LALR(1)

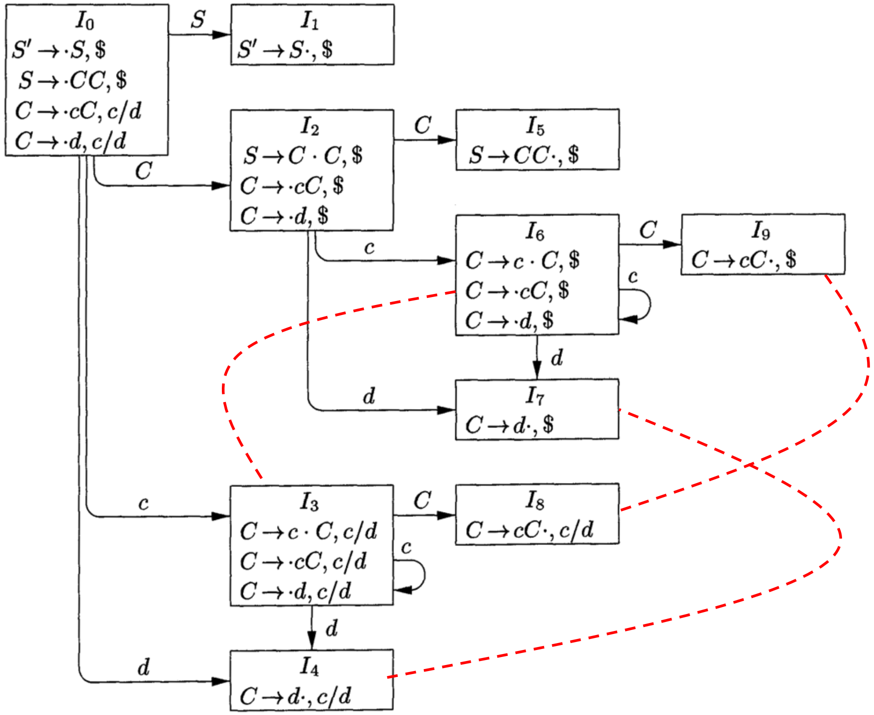
Basic idea: Merging states in LR(1) parsing table; If there is no reduce-reduce conflict, the grammar is LALR(1), otherwise not LALR(1).

Constructing LALR Parsing Table

- Construct the GOTO table as follows:
  - If J is the union of one or more sets of LR(1) items, that is $J = I_1 \cup I_2 \cup \cdots \cup I_k$, then the cores of $\mathtt{GOTO}(I_1, X)$, $\mathtt{GOTO}(I_2, X)$, …, $\mathtt{GOTO}(I_k, X)$ are the same, since $I_1, I_2, \dots, I_k$ all have the same core.
  - Let K be the union of all sets of items having the same core as $\mathtt{GOTO}(I_1, X)$
  - $\mathtt{GOTO}(J, X) = K$

LALR Parsing Table Example

- Merging item sets
  - I_{36}: $[C \rightarrow c \cdot C, c/d/\$], [C \rightarrow \cdot cC, c/d/\$], [C \rightarrow \cdot d, c/d/\$]$
  - I_{47}: $[C \rightarrow d \cdot, c/d/\$]$
  - I_{89}: $[C \rightarrow cC \cdot, c/d/\$]$
- $\mathrm{GOTO}(I_{36}, C) = I_{89}$



LALR Parsing Table Example

STATE	ACTION			GOTO	
	c	d	\$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

STATE	ACTION			GOTO	
	c	d	\$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

Comparisons Among LR Parsers

- The languages (grammars) that can be handled
  - CLR > LALR > SLR
- # states in the parsing table
  - CLR > LALR = SLR
- Driver programs
  - SLR = CLR = LALR