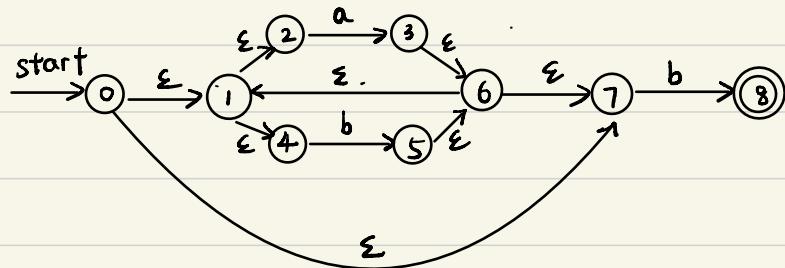
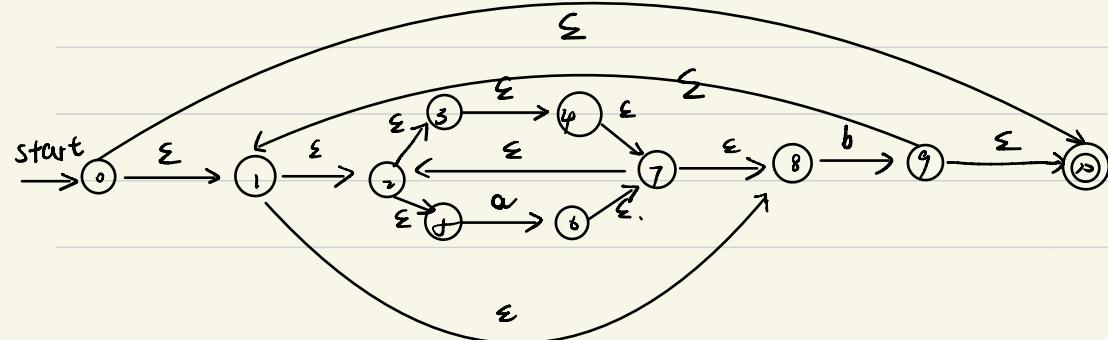


Exercise 1

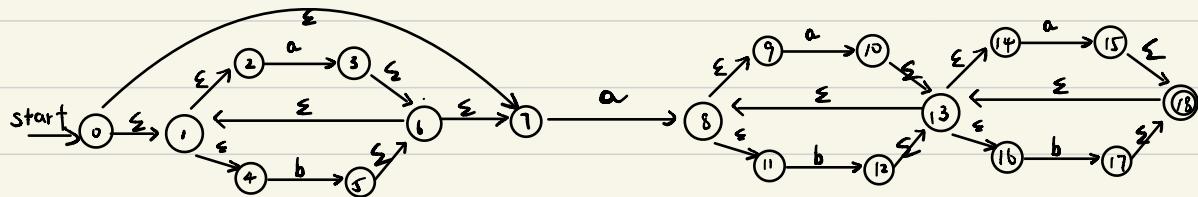
1. The NFA of $L((a|b)^*b)$ is



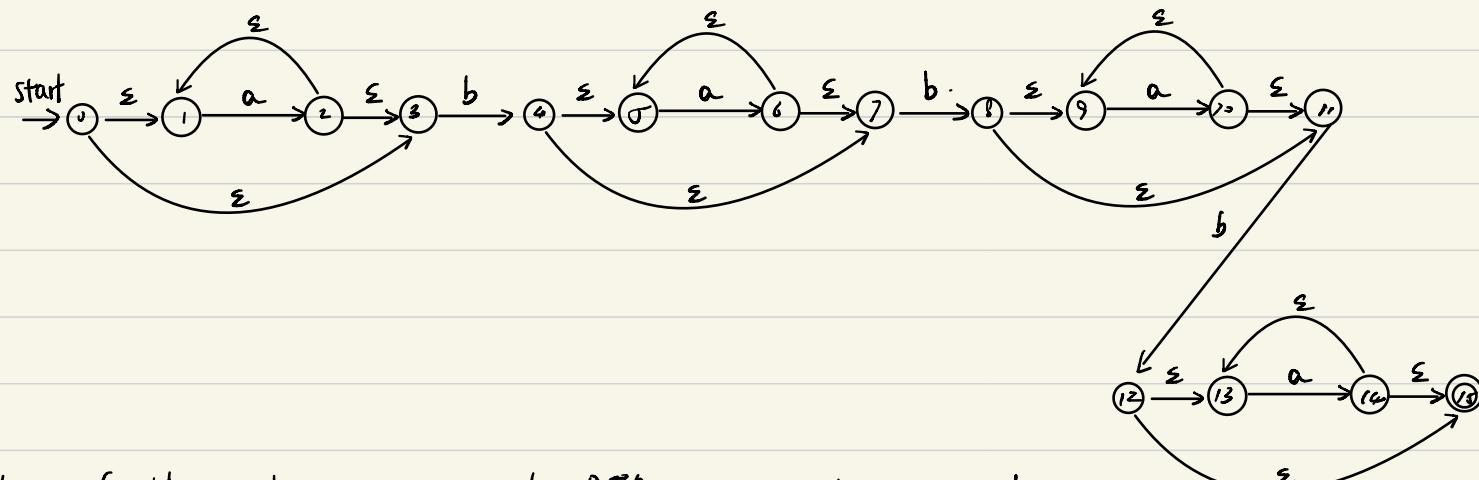
2. The NFA of $L(((\epsilon|a)^*b)^*)$ is



3. The NFA of $L((a|b)^*a(a|b)(a|b))$ is



4. The NFA of $L(a^*ba^*ba^*ba^*)$ is



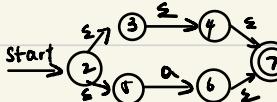
All of the above are not DFA since they have edges on input ϵ

Exercise 2

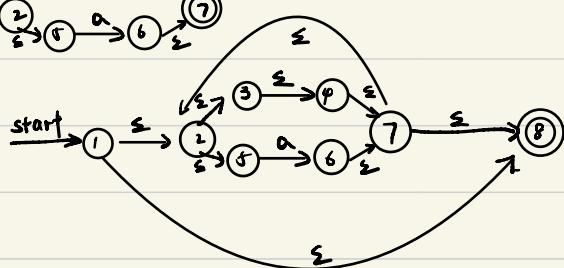
1. $((\varepsilon \cup a)^* b)^*$

① for $r_1 = \Sigma$, $r_2 = a$, we construct $\xrightarrow{\text{start}} 3 \xrightarrow{\varepsilon} 4$, $\xrightarrow{\text{start}} 5 \xrightarrow{a} 6$

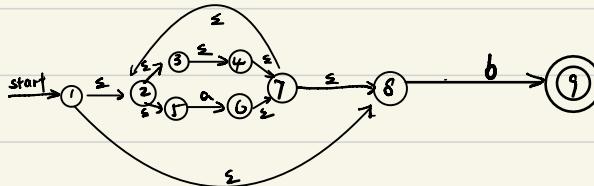
② for $r_3 = r_1 \cup r_2$, we construct



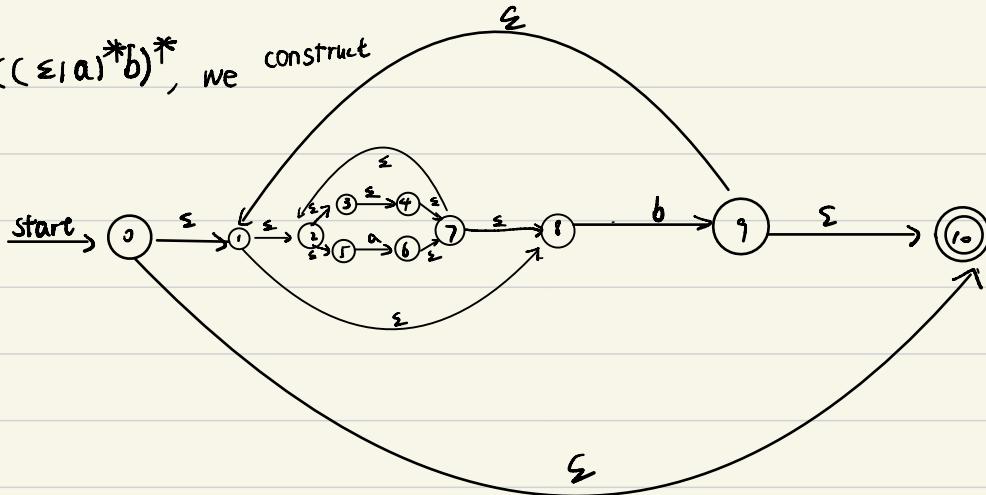
③ for $r_4 = (r_3)^*$, we construct



④. for $r_5 = r_4 b$, we construct



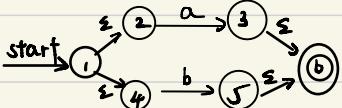
⑤ for $r_6 = (r_5)^*$, which is $((\varepsilon | a)^* b)^*$, we construct



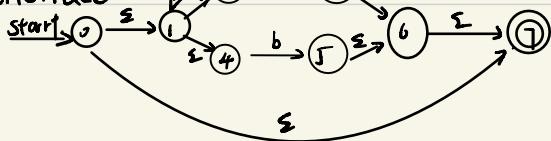
2. $(a|b)^* a(a|b)(a|b)$

① for $r_1 = a$, $r_2 = b$, we construct $\xrightarrow{\text{start}} 2 \xrightarrow{a} 3 \xrightarrow{\text{start}} 4 \xrightarrow{b} 1$

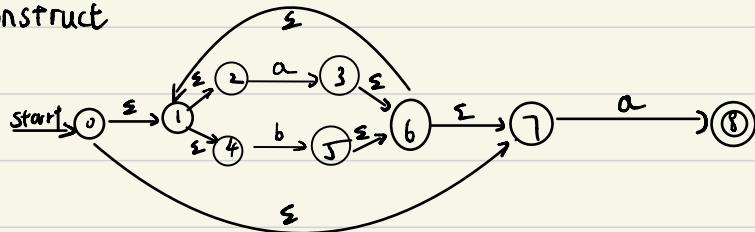
② for $r_3 = r_1 \cup r_2$, we construct



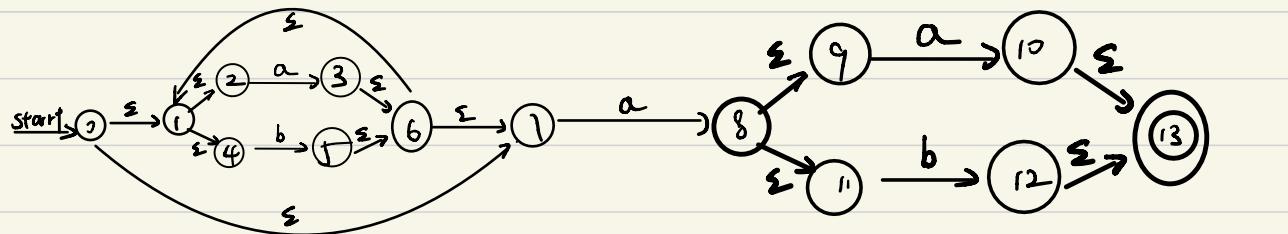
③ for $r_4 = (r_3)^*$, we construct



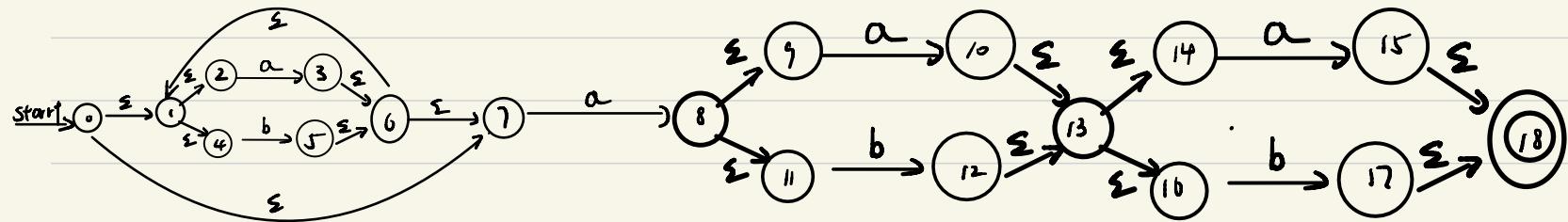
④ for $r5 = r4a$, we construct



⑤ for $r6 = r5(a \cup b)$, we construct

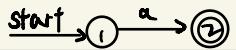


⑥ for $r_7 = r_6(a \cup b)$, we construct

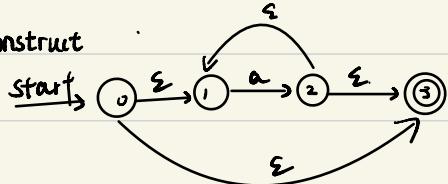


3. $a^*ba^*ba^*ba^*$

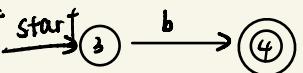
① for $r_1=a$, we construct



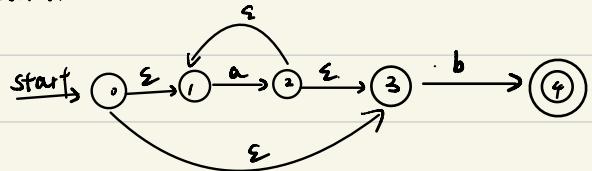
② for $r_2=(r_1)^*$, we construct



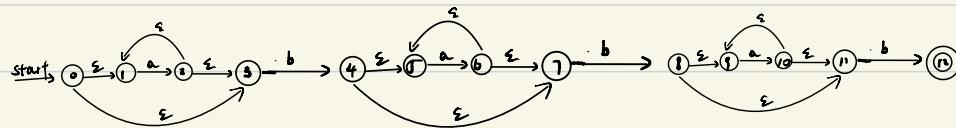
③ for $r_3=b$, we construct



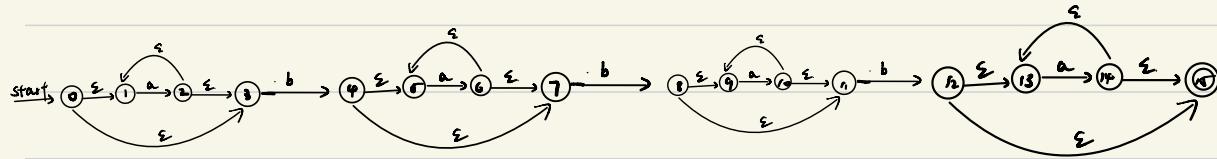
④ for $r_4 = r_2 r_3$, we construct



⑤ for $r_5 = r_4 r_4 r_4$, we construct

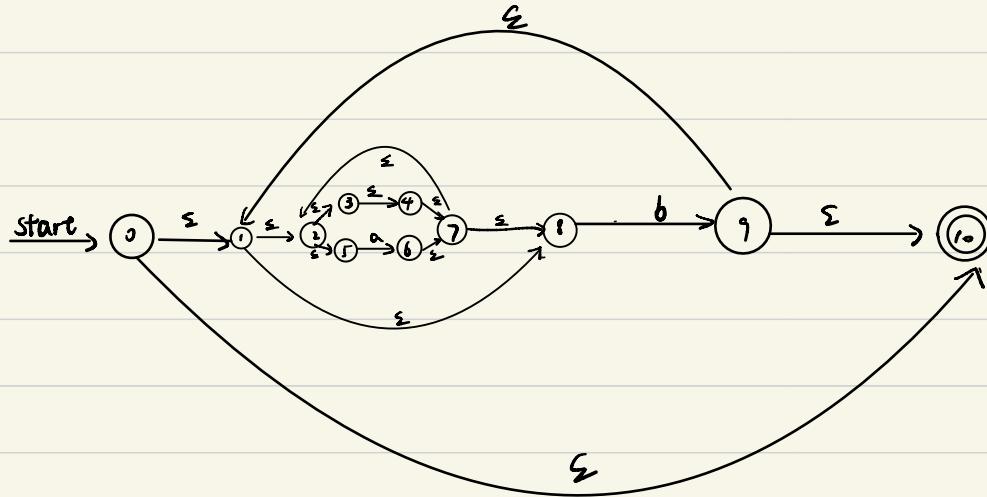


⑥ for $r_6 = r_5 r_2$, which is $a^*ba^*ba^*ba^*$ we construct



Exercise 3.

1. For $((\Sigma|oy)^* b)^*$



Let's define A as ϵ -closure (0), which is the start state of equivalent DFA , $A = \{0, 1, 2, 3, 4, 5, 7, 8, 10\}$,

since the input alphabet is {a,b}. define B as ϵ -closure (move(A,a)) =

$$\{2, 3, 4, 5, 6, 7, 8\}$$

\downarrow
 $\{6\}$

{q₉}

define C as Σ -closure (move (A, b)) = {1, 2, 3, 4, 5, 7, 8, 9, 10}

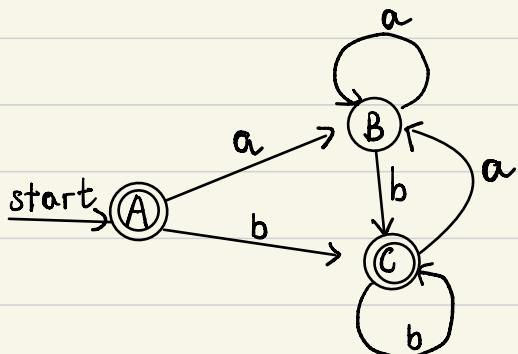
Σ -closure (move (B, a)) = {2, 3, 4, 5, 6, 7, 8}, which is the same as B

Σ -closure (move (B, b)) = C.

Σ -closure (move (C, a)) = B

Σ -closure (move (C, b)) = C.

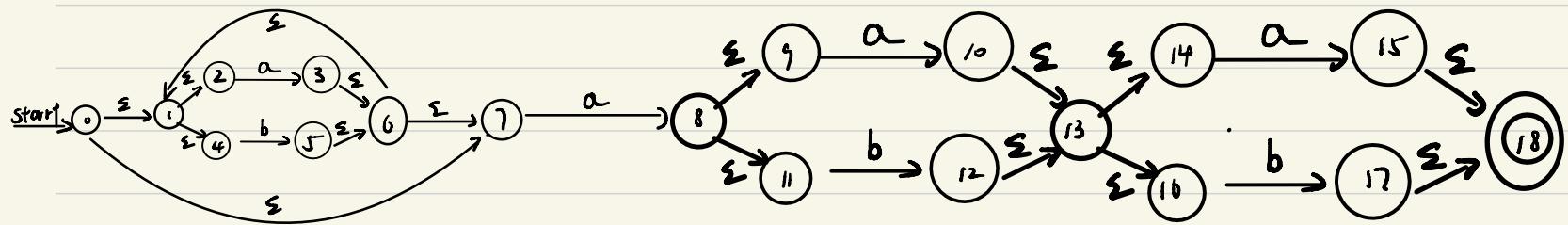
As a result, the DFA is



the transition table is

NFA's state	DFA's state	a	b
{0, 1, 2, 3, 4, 5, 7, 8, 10},	A	B	C.
{2, 3, 4, 5, 6, 7, 8}	B	B	C.
{1, 2, 3, 4, 5, 7, 8, 9, 10}	C.	B	C.

2. For $(a+b)^*a(a+b)(a+b)$, the NFA is as below



Let's define the start state $A = \epsilon\text{-closure}(0)$.

which is $\{0, 1, 2, 4, 7\}$, $B = \epsilon\text{-closure}(\underline{\text{move}(A, a)}) = \{1, 2, 3, 4, 6, 7, 8, 9, 11\}$

define

define $C = \epsilon\text{-closure}(\underline{\text{move}(A, b)}) = \{1, 2, 4, 5, 6, 7\}$

$\{5\}$

define $D = \epsilon\text{-closure}(\underline{\text{move}(B, a)}) = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 16\}$

$\{3, 8, 10\}$

define $E = \varepsilon\text{-closure}(\underline{\text{move}(B, b)}) = \{ 1, 2, 4, 5, 6, 7, 12, 13, 14, 16 \}$

\downarrow
 $\{ 7, 12 \}$

Notice that $\varepsilon\text{-closure}(\underline{\text{move}(C, a)}) \subseteq B$, $\varepsilon\text{-closure}(\underline{\text{move}(C, b)}) \subseteq C$.

\downarrow
 $\{ 3, 8 \}$

\downarrow
 $\{ 5, 9 \}$

define $F = \varepsilon\text{-closure}(\underline{\text{move}(D, a)}) = \{ 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 15, 16, 18 \}$

\downarrow
 $\{ 3, 8, 10, 15 \}$

define $G = \varepsilon\text{-closure}(\underline{\text{move}(D, b)}) = \{ 1, 2, 4, 5, 6, 7, 12, 13, 14, 16, 17, 18 \}$

\downarrow
 $\{ 5, 12, 17 \}$

define $H = \varepsilon\text{-closure}(\underline{\text{move}(E, a)}) = \{ 1, 2, 3, 4, 6, 7, 8, 9, 11, 15, 18 \}$

\downarrow
 $\{ 3, 8, 15 \}$

define $I = \Sigma\text{-closure}(\underline{\text{move}(E, b)}) = \{ 1, 2, 4, 5, 6, 7, 17, 18 \}$

\downarrow
 $\{ 5, 17 \}$

$\Sigma\text{-closure}(\underline{\text{move}(F, a)}) = F$, $\Sigma\text{-closure}(\underline{\text{move}(F, b)}) = G$

\downarrow
 $\{ 3, 8, 10, 15 \}$

\downarrow
 $\{ 5, 12, 17 \}$

$\Sigma\text{-closure}(\underline{\text{move}(G, a)}) = H$, $\Sigma\text{-closure}(\underline{\text{move}(G, b)}) = I$

\downarrow
 $\{ 3, 8, 15 \}$

\downarrow
 $\{ 5, 17 \}$

$\Sigma\text{-closure}(\underline{\text{move}(H, a)}) = D$, $\Sigma\text{-closure}(\underline{\text{move}(H, b)}) = E$

\downarrow
 $\{ 3, 8, 10 \}$

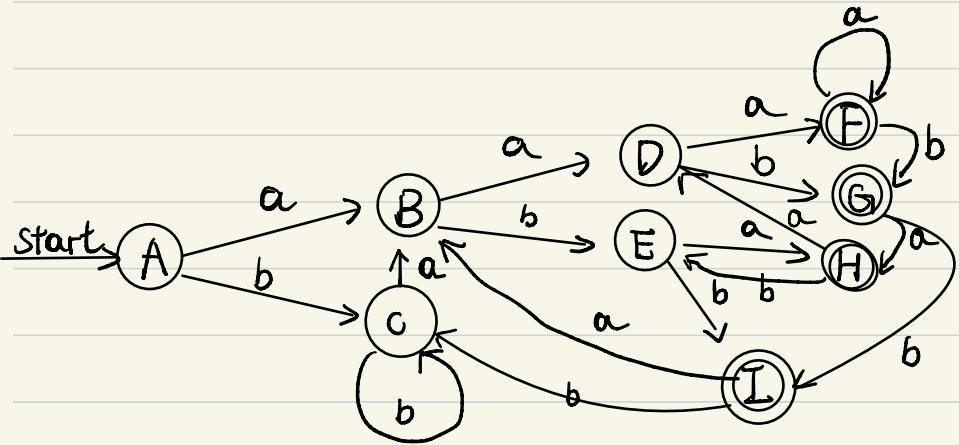
\downarrow
 $\{ 5, 12 \}$

$\Sigma\text{-closure}(\underline{\text{move}(I, a)}) = B$, $\Sigma\text{-closure}(\underline{\text{move}(I, b)}) = C$

\downarrow
 $\{ 3, 8 \}$

\downarrow
 $\{ 5 \}$

The DFA is shown as below

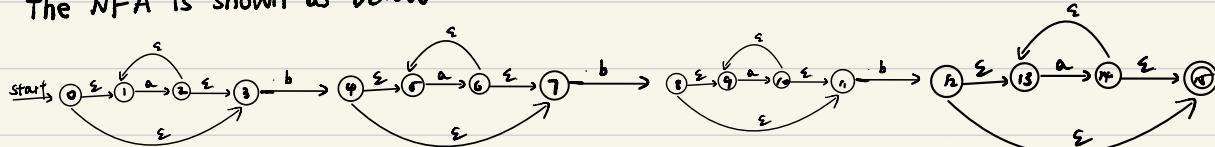


Transition Table

NFA's state	DFA's state	a	b
{0,1,2,4,7}	A	B	C.
{1,2,3,4,6,7,8,9,11}	B	D	E
{1,2,4,5,6,7}	C	B	C.
{1,2,3,4,6,7,8,9,10,11,13,14,16}	D	F	G
{1,2,4,5,6,7,12,13,14,16}	E	H	I
{1,2,3,4,6,7,8,9,11,13,14,15,16,18}	F	F	G
{1,2,4,5,6,7,12,13,14,16,17,18}	G	H	I
{1,2,3,4,6,7,8,9,11,15,18}	H	D	E
{1,2,4,5,6,7,17,18}	I	B	C

(3) For $a^*ba^*ba^*ba^*$

The NFA is shown as below



Define A as the start state, which is ϵ -closure(0) = {0, 1, 3}

$$B = \epsilon\text{-closure}(\text{move}(A, a)) = \{1, 2, 3\}$$

{2, 3}

$$C = \epsilon\text{-closure}(\text{move}(A, b)) = \{4, 5, 7\}$$

{4, 7}

$$\epsilon\text{-closure}(\text{move}(B, a)) = B$$

{2, 3}

$$\epsilon\text{-closure}(\text{move}(B, b)) = C$$

$$D = \epsilon\text{-closure}(\text{move}(C, a)) = \{5, 6, 7\}$$

{6, 7}

$$E = \epsilon\text{-closure}(\text{move}(C, b)) = \{8, 9, 11\}$$

{8, 9}

$$\epsilon\text{-closure}(\text{move}(D, a)) = D$$

{6, 7}

$$\epsilon\text{-closure}(\text{move}(D, b)) = E$$

{8, 9}

$$F = \Sigma\text{-closure}(\text{move}(E, a)) = \{9, 10, 11\}$$

\downarrow

$\{10\}$

$$G = \Sigma\text{-closure}(\text{move}(F, b)) = \{12, 13, 15\}$$

\downarrow

$\{12\}$

ition Table $\Sigma\text{-closure}(\text{move}(F, a)) = F$ $\Sigma\text{-closure}(\text{move}(F, b)) = G.$

FA's state DFA's state

a	b
$\{13, 14, 15\}$	$\{12\}$

$H = \Sigma\text{-closure}(\text{move}(G, a)) = \{13, 14, 15\}$ $\Sigma\text{-closure}(\text{move}(G, b)) = \emptyset$

$\Sigma\text{-closure}(\text{move}(H, a)) = H$ $\Sigma\text{-closure}(\text{move}(H, b)) = \emptyset$

\downarrow

$\{14\}$

Transition Table

NFA's state	DFA's state	a	b
$\{0, 1, 3\}$	A	B	C
$\{1, 2, 3\}$	B	B	C
$\{4, 5, 7\}$	C	D	E
$\{5, 6, 7\}$	D	D	E
$\{8, 9, 11\}$	E	F	G
$\{9, 10, 11\}$	F	F	G
$\{12, 13, 15\}$	G	H	\emptyset
$\{13, 14, 15\}$	H	H	\emptyset

The DFA is

