CS323 Assignment4

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Exercise 1(Simple LR)

Q1

Augmented Grammar

$$\begin{gathered} S' \to S \ \ (1) \\ S \to aB \ \ (2) \\ B \to S * B \ \ (3) \\ B \to \epsilon \ \ (4) \end{gathered} \tag{1}$$

Calculations of closures and GOTO targets

For I_0:

$$\begin{gathered} S' \to \cdot S \\ S \to \cdot aB \end{gathered} \tag{2}$$

Let $I_1 = GOTO(I_0, S)$:

$$S' \to S\cdot \tag{3}$$

Let $I_2 = GOTO(I_0, a)$:

$$\begin{aligned} S &\to a \cdot B \\ B &\to \cdot S * B \\ B &\to \cdot \\ S &\to \cdot aB \end{aligned} \tag{4}$$

Let $I_3 = GOTO(I_2, B)$:

$$S \to aB\cdot \tag{5}$$

Let $I_4 = GOTO(I_2, S)$:

$$B \to S \cdot *B \tag{6}$$

$GOTO(I_2, a)$ will still be I_2

Let $I_5 = GOTO(I_4, *)$:

$$\begin{aligned} B &\to S * \cdot B \\ B &\to \cdot S * B \\ S &\to \cdot aB \\ B &\to \cdot \end{aligned} \tag{7}$$

$GOTO(I_5, S)$ will go to I_4

$GOTO(I_5, a)$ will go to I_2

Let $I_6 = GOTO(I_5, B)$:

$$B \to S * B\cdot \tag{8}$$

Calculation of FIRST/FOLLOW

$$\begin{aligned} FIRST(B) &= \{\epsilon, a\} \\ FIRST(S) &= \{a\} \\ FIRST(S') &= \{a\} \\ FOLLOW(S') &= \{\$, *\} \\ FOLLOW(S) &= \{\$, *\} \\ FOLLOW(B) &= \{\$, *\} \end{aligned} \tag{9}$$

SLR(1) Table

STATE	ACTION			GOTO	
	a	*****	**$**	**S**	**B**
0	s2			1	
1			acc		
2	s2	r4	r4	4	3
3		r2	r2		
4		s5			
5	s2	r4	r4	4	6
6		r3	r3		

Q2

Yes, since there is no conflict, it is an SLR(1) grammar.

Q3

Yes.

STACK	SYMBOL	INPUT	ACTION
0	$	aaaa***$	shift to 2
0 2	$a	aaa***$	shift to 2
0 2 2	$aa	aa***$	shift to 2
0 2 2 2	$aaa	a***$	shift to 2
0 2 2 2 2	$aaaa	***$	reduce by $B \rightarrow \epsilon$
0 2 2 2 2	$aaaaB	***$	GOTO 3
0 2 2 2 2 3	$aaaaB	***$	reduce by $S \rightarrow aB$
0 2 2 2 4	$aaaS	***$	shift to 5
0 2 2 2 4 5	$aaaS*	**$	reduce by $B \rightarrow \epsilon$
0 2 2 2 4 5	$aaaS*B	**$	GOTO 6
0 2 2 2 4 5 6	$aaaS*B	**$	reduce by $B \rightarrow S * B$
0 2 2 2 3	$aaaB	**$	reduce by $S \rightarrow aB$
0 2 2 4	$aaS	**$	shift to 5
0 2 2 4 5	$aaS*	*$	reduce by $B \rightarrow \epsilon$
0 2 2 4 5	$aaS*B	*$	GOTO 6
0 2 2 4 5 6	$aaS*B	*$	reduce by $B \rightarrow S * B$
0 2 2 3	$aaB	*$	reduce by $S \rightarrow aB$
0 2 4	$aS	*$	shift to 5
0 2 4 5	$aS*	$	reduce by $B \rightarrow \epsilon$
0 2 4 5	$aS*B	$	GOTO 6
0 2 4 5 6	$aS*B	$	reduce by $B \rightarrow S * B$
0 2 3	$aB	$	reduce by $S \rightarrow aB$
0 1	$S	$	acc

Exercise 2(Canonical LR)

Q1

Augmented Grammar

$$\begin{gathered} S' \to S \ \ (1) \\ S \to aB \ \ (2) \\ B \to S * B \ \ (3) \\ B \to \epsilon \ \ (4) \end{gathered} \tag{10}$$

Calculation of FIRST/FOLLOW

$$\begin{gathered} FIRST(B) = \{\epsilon, a\} \\ FIRST(S) = \{a\} \\ FIRST(S') = \{a\} \\ FOLLOW(S') = \{\$, *\} \\ FOLLOW(S) = \{\$, *\} \\ FOLLOW(B) = \{\$, *\} \end{gathered} \tag{11}$$

Calculations of closures and GOTO targets

For I_0 :

Since $FIRST(\$) = \{\$\}, FIRST(S\$) = \{a\}$

$I_0 = CLOSURE([S' \to \cdot S, \$]) =$

$$\begin{gathered} [S' \to \cdot S, \$] \\ [S \to \cdot aB, \$] \end{gathered} \tag{12}$$

$I_1 = GOTO(I_0, S) = CLOSURE(\{[S' \to S\cdot, \$]\}) = \{[S' \to S\cdot, \$]\}$

Since $FIRST(B) = \{\epsilon, a\}, FIRST(*B\$) = \{*\},$
$I_2 = GOTO(I_0, a) = CLOSURE(\{[S \to a \cdot B, \$]\}) =$

$$
\begin{gathered}
[S \to a \cdot B, \$] \\
[B \to \cdot S * B, \$] \\
[B \to \cdot \epsilon, \$] \\
[S \to \cdot aB, *]
\end{gathered} \tag{13}
$$

$I_3 = GOTO(I_2, B) = CLOSURE(\{[S \to aB\cdot, \$]\}) = \{[S \to aB\cdot, \$]\}$

$I_4 = GOTO(I_2, S) = CLOSURE(\{[B \to S \cdot *B, \$]\}) = \{[B \to S \cdot *B, \$]\}$

$I_5 = GOTO(I_2, a) = CLOSURE(\{[S \to a \cdot B, *]\}) =$

$$
\begin{gathered}
[S \to a \cdot B, *] \\
[B \to \cdot S * B, *] \\
[B \to \cdot \epsilon, *] \\
[S \to \cdot aB, *]
\end{gathered} \tag{14}
$$

$I_6 = GOTO(I_4, *) = CLOSURE(\{B \to S * \cdot B, \$\}) =$

$$
\begin{gathered}
[B \to S * \cdot B, \$] \\
[B \to \cdot S * B, \$] \\
[B \to \cdot \epsilon, \$] \\
[S \to \cdot aB, *]
\end{gathered} \tag{15}
$$

$I_7 = GOTO(I_5, B) = CLOSURE(\{[S \to aB\cdot, *]\}) = \{[S \to aB\cdot, *]\}$

$I_8 = GOTO(I_5, S) = CLOSURE(\{[B \to S \cdot *B, *]\}) = \{[B \to S \cdot *B, *]\}$

Since $FIRST(*) = \{*\}, GOTO(I_5, a) = CLOSURE(\{[S \to a \cdot B, *]\}) = I_5$

$GOTO(I_6, S) = CLOSURE(\{[B \to S \cdot *B, \$]\}) = I_4$

$GOTO(I_6, a) = CLOSURE(\{[S \to a \cdot B, *]\}) = I_5$

$I_9 = GOTO(I_6, B) = CLOSURE(\{[B \to S * B\cdot, \$]\}) = \{[B \to S * B\cdot, \$]\}$

$I_{10} = GOTO(I_8, *) = CLOSURE(\{[B \to S * \cdot B, *]\}) =$

$$
\begin{gathered}
[B \rightarrow S * \cdot B, *] \\
[B \rightarrow \cdot S * B, *] \\
[B \rightarrow \cdot \epsilon, *] \\
[S \rightarrow \cdot a B, *]
\end{gathered} \tag{16}
$$

$GOTO(I_{10}, B) = CLOSURE(\{[B \rightarrow S * B\cdot, *]\}) = \{[B \rightarrow S * B\cdot, *]\} = I_{11}$

$GOTO(I_{10}, S) = CLOSURE(\{[B \rightarrow S \cdot * B, *]\}) = I_8$

$GOTO(I_{10}, a) = CLOSURE(\{[B \rightarrow a \cdot B, *]\}) = I_5$

CLR Table

STATE	ACTION			GOTO	
	a	*	$	S	B
0	s2			1	
1			acc		
2	s5		r4	4	3
3			r2		
4		s6			
5	s5	r4		8	7
6	s5		r4	4	9
7		r2			
8		s10			
9			r3		
10	s5	r4		8	11
11		r3			

Q2

Yes. Since there is no conflict in the table, it is an LR(1) grammar.

Q3

Yes.

STACK	SYMBOL	INPUT	ACTION
0	$	aaaa***$	shift to 2
0 2	$a	aaa***$	shift to 5
0 2 5	$aa	aa***$	shift to 5
0 2 5 5	$aaa	a***$	shift to 5
0 2 5 5 5	$aaaa	***$	reduce by $B \rightarrow \epsilon$
0 2 5 5 5 7	$aaaaB	***$	reduce by $S \rightarrow aB$
0 2 5 5 8	$aaaS	***$	shift to 10
0 2 5 5 8 10	$aaaS*	**$	reduce by $B \rightarrow \epsilon$
0 2 5 5 8 10 11	$aaaS*B	**$	reduce by $B \rightarrow S * B$
0 2 5 5 7	$aaaB	**$	reduce by$S \rightarrow aB$
0 2 5 8	$aaS	**$	shift to 10
0 2 5 8 10	$aaS*	*$	reduce by $B \rightarrow \epsilon$
0 2 5 8 10 11	$aaS*B	*$	reduce by $B \rightarrow S * B$
0 2 5 7	$aaB	*$	reduce by $S \rightarrow aB$
0 2 4	$aS	*$	shift to 6
0 2 4 6	$aS*	$	reduce by$B \rightarrow \epsilon$
0 2 4 6 9	$aS*B	$	reduce by$B \rightarrow S * B$
0 2 3	$aB	$	reduce by$S \rightarrow aB$
0 1	$S	$	acc

Exercise 3(LALR)

Q1

Augmented Grammar

$$
\begin{gathered}
S' \to S \ \ (1) \\
S \to aB \ \ (2) \\
B \to S * B \ \ (3) \\
B \to \epsilon \ \ (4)
\end{gathered}
\tag{17}
$$

There are five pairs that can be merged in the augmented grammar.

We can replace the I_2, I_5 with the union of them:

$I_{25} =$

$$
\begin{gathered}
[S \to a \cdot B, \$/*] \\
[B \to \cdot S * B, \$/*] \\
[B \to \cdot \epsilon, \$/*] \\
[S \to \cdot aB, *]
\end{gathered}
\tag{18}
$$

We can replace the I_6, I_{10} with the union of them, too:

$I_{610} =$

$$
\begin{gathered}
[B \to S * \cdot B, \$/*] \\
[B \to \cdot S * B, \$/*] \\
[B \to \cdot \epsilon, \$/*] \\
[S \to \cdot aB, *]
\end{gathered}
\tag{19}
$$

$I_{37} = \{[S \to aB \cdot, \$/*]\}$

$I_{48} = \{[B \to S \cdot * B, \$/*]\}$

$I_{911} = \{[B \to S * B \cdot, \$/*]\}$

LALR Table

STATE	ACTION			GOTO	
	a	*	$	S	B
0	s25			1	
1			acc		
25	s25	r4	r4	I_{48}	I_{37}
37		r2	r2		
48		s610			
610	s25	r4	r4	I_{48}	I_{911}
911		r3	r3		

Q2

Yes, since there is no conflict in the table.

Q3

Yes

STACK	SYMBOL	INPUT	ACTION
0	$	aaaa***$	shift to 25
0 25	$a	aaa***$	shift to 25
0 25 25	$aa	aa***$	shift to 25
0 25 25 25	$aaa	a***$	shift to 25
0 25 25 25 25	$aaaa	***$	reduce by $B \rightarrow \epsilon$
0 25 25 25 25 37	$aaaaB	***$	reduce by $S \rightarrow aB$
0 25 25 25 48	$aaaS	***$	shift to 610
0 25 25 25 48 610	$aaaS*	**$	reduce by $B \rightarrow \epsilon$
0 25 25 25 48 610 911	$aaaS*B	**$	reduce by$B \rightarrow S * B$
0 25 25 25 37	$aaaB	**$	reduce by$S \rightarrow aB$
0 25 25 48	$aaS	**$	shift to 610
0 25 25 48 610	$aaS*	*$	reduce by $B \rightarrow \epsilon$
0 25 25 48 610 911	$aaS*B	*$	reduce by $B \rightarrow S * B$
0 25 25 37	$aaB	*$	reduce by $S \rightarrow aB$
0 25 48	$aS	*$	shift to 610
0 25 48 610	$aS*	$	reduce by $B \rightarrow \epsilon$
0 25 48 610 911	$aS*B	$	reduce by $B \rightarrow S * B$
0 25 37	$aB	$	reduce by $S \rightarrow aB$
0 1	$S	$	acc