

CS323 Assignment4

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Exercise 1(Simple LR)

Q1

Augmented Grammar

$$\begin{aligned} S' &\rightarrow S \quad (1) \\ S &\rightarrow aB \quad (2) \\ B &\rightarrow S * B \quad (3) \\ B &\rightarrow \epsilon \quad (4) \end{aligned}$$

Calculations of closures and GOTO targets

For I_0 :

$$\begin{aligned} S' &\rightarrow \cdot S \\ S &\rightarrow \cdot aB \end{aligned} \quad (2)$$

Let $I_1 = GOTO(I_0, S)$:

$$S' \rightarrow S \cdot \quad (3)$$

Let $I_2 = GOTO(I_0, a)$:

$$\begin{aligned}
S &\rightarrow a \cdot B \\
B &\rightarrow \cdot S * B \\
B &\rightarrow \cdot \epsilon \\
S &\rightarrow \cdot aB
\end{aligned} \tag{4}$$

Let $I_3 = GOTO(I_2, B)$:

$$S \rightarrow aB \cdot \tag{5}$$

Let $I_4 = GOTO(I_2, S)$:

$$B \rightarrow S \cdot * B \tag{6}$$

$GOTO(I_2, a)$ will still be I_2

Let $I_5 = GOTO(I_4, *)$:

$$\begin{aligned}
B &\rightarrow S * \cdot B \\
B &\rightarrow \cdot S * B \\
S &\rightarrow \cdot aB \\
B &\rightarrow \cdot \epsilon
\end{aligned} \tag{7}$$

$GOTO(I_5, S)$ will go to I_4

$GOTO(I_5, a)$ will go to I_2

Let $I_6 = GOTO(I_5, B)$:

$$B \rightarrow S * B \cdot \tag{8}$$

Calculation of FIRST/FOLLOW

$$\begin{aligned}
FIRST(B) &= \{\epsilon, a\} \\
FIRST(S) &= \{a\} \\
FIRST(S') &= \{a\} \\
FOLLOW(S') &= \{\$, *\} \\
FOLLOW(S) &= \{\$, *\} \\
FOLLOW(B) &= \{\$, *\}
\end{aligned} \tag{9}$$

SLR(1) Table

STATE	ACTION			GOTO	
	a	*	\$	S	B
0	s2			1	
1			acc		
2	s2	r4	r4	4	3
3		r2	r2		
4		s5			
5	s2	r4	r4	4	6
6		r3	r3		

Q2

Yes, since there is no conflict, it is an SLR(1) grammar.

Q3

Yes.

STACK	SYMBOL	INPUT	ACTION
0	\$	aaaa***\$	shift to 2
0 2	\$a	aaa***\$	shift to 2
0 2 2	\$aa	aa***\$	shift to 2
0 2 2 2	\$aaa	a***\$	shift to 2
0 2 2 2 2	\$aaaa	***\$	reduce by $B \rightarrow \epsilon$
0 2 2 2 2	\$aaaaB	***\$	GOTO 3
0 2 2 2 2 3	\$aaaaB	***\$	reduce by $S \rightarrow aB$
0 2 2 2 4	\$aaaS	***\$	shift to 5
0 2 2 2 4 5	\$aaaS*	**\$	reduce by $B \rightarrow \epsilon$
0 2 2 2 4 5	\$aaaS*B	**\$	GOTO 6
0 2 2 2 4 5 6	\$aaaS*B	**\$	reduce by $B \rightarrow S * B$
0 2 2 2 3	\$aaaB	**\$	reduce by $S \rightarrow aB$
0 2 2 4	\$aaS	**\$	shift to 5
0 2 2 4 5	\$aaS*	*\$	reduce by $B \rightarrow \epsilon$
0 2 2 4 5	\$aaS*B	*\$	GOTO 6
0 2 2 4 5 6	\$aaS*B	*\$	reduce by $B \rightarrow S * B$
0 2 2 3	\$aaB	*\$	reduce by $S \rightarrow aB$
0 2 4	\$aS	*\$	shift to 5
0 2 4 5	\$aS*	\$	reduce by $B \rightarrow \epsilon$
0 2 4 5	\$aS*B	\$	GOTO 6
0 2 4 5 6	\$aS*B	\$	reduce by $B \rightarrow S * B$
0 2 3	\$aB	\$	reduce by $S \rightarrow aB$
0 1	\$S	\$	acc

Exercise 2(Canonical LR)

Q1

Augmented Grammar

$$\begin{aligned}S' &\rightarrow S \quad (1) \\S &\rightarrow aB \quad (2) \\B &\rightarrow S * B \quad (3) \\B &\rightarrow \epsilon \quad (4)\end{aligned}\tag{10}$$

Calculation of FIRST/FOLLOW

$$\begin{aligned}FIRST(B) &= \{\epsilon, a\} \\FIRST(S) &= \{a\} \\FIRST(S') &= \{a\} \\FOLLOW(S') &= \{\$, *\} \\FOLLOW(S) &= \{\$, *\} \\FOLLOW(B) &= \{\$, *\}\end{aligned}\tag{11}$$

Calculations of closures and GOTO targets

For I_0 :

Since $FIRST(\$) = \{\$\}$, $FIRST(S\$) = \{a\}$

$$I_0 = CLOSURE([S' \rightarrow \cdot S, \$]) =$$

$$\begin{aligned}[S' &\rightarrow \cdot S, \$] \\[S &\rightarrow \cdot aB, \$]\end{aligned}\tag{12}$$

$$I_1 = GOTO(I_0, S) = CLOSURE(\{[S' \rightarrow S\cdot, \$]\}) = \{[S' \rightarrow S\cdot, \$]\}$$

Since $FIRST(B) = \{\epsilon, a\}$, $FIRST(*B\$) = \{*\}$,

$$I_2 = GOTO(I_0, a) = CLOSURE(\{[S \rightarrow a\cdot B, \$]\}) =$$

$$\begin{aligned}
& [S \rightarrow a \cdot B, \$] \\
& [B \rightarrow \cdot S * B, \$] \\
& [B \rightarrow \cdot \epsilon, \$] \\
& [S \rightarrow \cdot aB, *]
\end{aligned} \tag{13}$$

$$I_3 = GOTO(I_2, B) = CLOSURE(\{[S \rightarrow aB\cdot, \$]\}) = \{[S \rightarrow aB\cdot, \$]\}$$

$$I_4 = GOTO(I_2, S) = CLOSURE(\{[B \rightarrow S \cdot *B, \$]\}) = \{[B \rightarrow S \cdot *B, \$]\}$$

$$I_5 = GOTO(I_2, a) = CLOSURE(\{[S \rightarrow a \cdot B, *]\}) =$$

$$\begin{aligned}
& [S \rightarrow a \cdot B, *] \\
& [B \rightarrow \cdot S * B, *] \\
& [B \rightarrow \cdot \epsilon, *] \\
& [S \rightarrow \cdot aB, *]
\end{aligned} \tag{14}$$

$$I_6 = GOTO(I_4, *) = CLOSURE(\{[B \rightarrow S * \cdot B, \$]\}) =$$

$$\begin{aligned}
& [B \rightarrow S * \cdot B, \$] \\
& [B \rightarrow \cdot S * B, \$] \\
& [B \rightarrow \cdot \epsilon, \$] \\
& [S \rightarrow \cdot aB, *]
\end{aligned} \tag{15}$$

$$I_7 = GOTO(I_5, B) = CLOSURE(\{[S \rightarrow aB\cdot, *]\}) = \{[S \rightarrow aB\cdot, *]\}$$

$$I_8 = GOTO(I_5, S) = CLOSURE(\{[B \rightarrow S \cdot *B, *]\}) = \{[B \rightarrow S \cdot *B, *]\}$$

$$\text{Since } FIRST(*) = \{*\}, GOTO(I_5, a) = CLOSURE(\{[S \rightarrow a \cdot B, *]\}) = I_5$$

$$GOTO(I_6, S) = CLOSURE(\{[B \rightarrow S \cdot *B, \$]\}) = I_4$$

$$GOTO(I_6, a) = CLOSURE(\{[S \rightarrow a \cdot B, *]\}) = I_5$$

$$I_9 = GOTO(I_6, B) = CLOSURE(\{[B \rightarrow S * B\cdot, \$]\}) = \{[B \rightarrow S * B\cdot, \$]\}$$

$$I_{10} = GOTO(I_8, *) = CLOSURE(\{[B \rightarrow S * \cdot B, *]\}) =$$

$$\begin{aligned}
&[B \rightarrow S * \cdot B, *] \\
&[B \rightarrow \cdot S * B, *] \\
&[B \rightarrow \cdot \epsilon, *] \\
&[S \rightarrow \cdot aB, *]
\end{aligned}
\tag{16}$$

$$GOTO(I_{10}, B) = CLOSURE(\{[B \rightarrow S * B \cdot, *]\}) = \{[B \rightarrow S * B \cdot, *]\} = I_{11}$$

$$GOTO(I_{10}, S) = CLOSURE(\{[B \rightarrow S \cdot * B, *]\}) = I_8$$

$$GOTO(I_{10}, a) = CLOSURE(\{[B \rightarrow a \cdot B, *]\}) = I_5$$

CLR Table

STATE	ACTION			GOTO	
	a	*	\$	S	B
0	s2			1	
1			acc		
2	s5		r4	4	3
3			r2		
4		s6			
5	s5	r4		8	7
6	s5		r4	4	9
7		r2			
8		s10			
9			r3		
10	s5	r4		8	11
11		r3			

Q2

Yes. Since there is no conflict in the table, it is an LR(1) grammar.

Q3

Yes.

STACK	SYMBOL	INPUT	ACTION
0	\$	aaaa***\$	shift to 2
0 2	\$a	aaa***\$	shift to 5
0 2 5	\$aa	aa***\$	shift to 5
0 2 5 5	\$aaa	a***\$	shift to 5
0 2 5 5 5	\$aaaa	***\$	reduce by $B \rightarrow \epsilon$
0 2 5 5 5 7	\$aaaaB	***\$	reduce by $S \rightarrow aB$
0 2 5 5 8	\$aaaS	***\$	shift to 10
0 2 5 5 8 10	\$aaaS*	**\$	reduce by $B \rightarrow \epsilon$
0 2 5 5 8 10 11	\$aaaS*B	**\$	reduce by $B \rightarrow S * B$
0 2 5 5 7	\$aaaB	**\$	reduce by $S \rightarrow aB$
0 2 5 8	\$aaS	**\$	shift to 10
0 2 5 8 10	\$aaS*	*\$	reduce by $B \rightarrow \epsilon$
0 2 5 8 10 11	\$aaS*B	*\$	reduce by $B \rightarrow S * B$
0 2 5 7	\$aaB	*\$	reduce by $S \rightarrow aB$
0 2 4	\$aS	*\$	shift to 6
0 2 4 6	\$aS*	\$	reduce by $B \rightarrow \epsilon$
0 2 4 6 9	\$aS*B	\$	reduce by $B \rightarrow S * B$
0 2 3	\$aB	\$	reduce by $S \rightarrow aB$
0 1	\$S	\$	acc

Exercise 3(LALR)

Q1

Augmented Grammar

$$\begin{aligned} S' &\rightarrow S \quad (1) \\ S &\rightarrow aB \quad (2) \\ B &\rightarrow S * B \quad (3) \\ B &\rightarrow \epsilon \quad (4) \end{aligned} \tag{17}$$

There are five pairs that can be merged in the augmented grammar.

We can replace the I_2, I_5 with the union of them:

$$I_{25} =$$

$$\begin{aligned} &[S \rightarrow a \cdot B, \$/*] \\ &[B \rightarrow \cdot S * B, \$/*] \\ &[B \rightarrow \cdot \epsilon, \$/*] \\ &[S \rightarrow \cdot aB, *] \end{aligned} \tag{18}$$

We can replace the I_6, I_{10} with the union of them, too:

$$I_{610} =$$

$$\begin{aligned} &[B \rightarrow S * \cdot B, \$/*] \\ &[B \rightarrow \cdot S * B, \$/*] \\ &[B \rightarrow \cdot \epsilon, \$/*] \\ &[S \rightarrow \cdot aB, *] \end{aligned} \tag{19}$$

$$I_{37} = \{[S \rightarrow aB \cdot, \$/*]\}$$

$$I_{48} = \{[B \rightarrow S \cdot * B, \$/*]\}$$

$$I_{911} = \{[B \rightarrow S * B \cdot, \$/*]\}$$

LALR Table

STATE	ACTION			GOTO	
	a	*	\$	S	B
0	s25			1	
1			acc		
25	s25	r4	r4	I_{48}	I_{37}
37		r2	r2		
48		s610			
610	s25	r4	r4	I_{48}	I_{911}
911		r3	r3		

Q2

Yes, since there is no conflict in the table.

Q3

Yes

STACK	SYMBOL	INPUT	ACTION
0	\$	aaaa***\$	shift to 25
0 25	\$a	aaa***\$	shift to 25
0 25 25	\$aa	aa***\$	shift to 25
0 25 25 25	\$aaa	a***\$	shift to 25
0 25 25 25 25	\$aaaa	***\$	reduce by $B \rightarrow \epsilon$
0 25 25 25 25 37	\$aaaaB	***\$	reduce by $S \rightarrow aB$
0 25 25 25 48	\$aaaS	***\$	shift to 610
0 25 25 25 48 610	\$aaaS*	**\$	reduce by $B \rightarrow \epsilon$
0 25 25 25 48 610 911	\$aaaS*B	**\$	reduce by $B \rightarrow S * B$
0 25 25 25 37	\$aaaB	**\$	reduce by $S \rightarrow aB$
0 25 25 48	\$aaS	**\$	shift to 610
0 25 25 48 610	\$aaS*	*\$	reduce by $B \rightarrow \epsilon$
0 25 25 48 610 911	\$aaS*B	*\$	reduce by $B \rightarrow S * B$
0 25 25 37	\$aaB	*\$	reduce by $S \rightarrow aB$
0 25 48	\$aS	*\$	shift to 610
0 25 48 610	\$aS*	\$	reduce by $B \rightarrow \epsilon$
0 25 48 610 911	\$aS*B	\$	reduce by $B \rightarrow S * B$
0 25 37	\$aB	\$	reduce by $S \rightarrow aB$
0 1	\$S	\$	acc