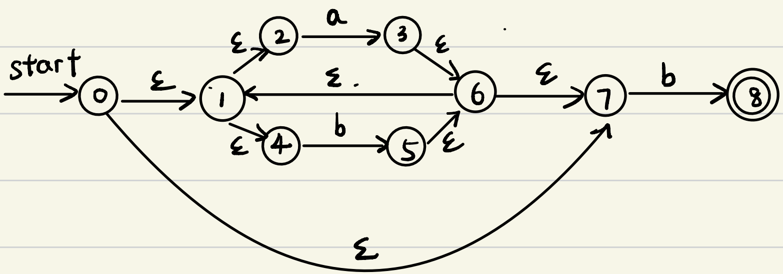
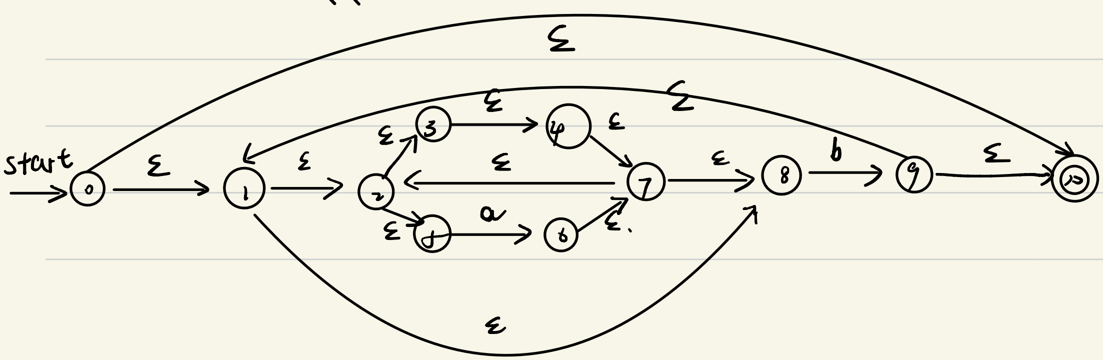
Exercise 1

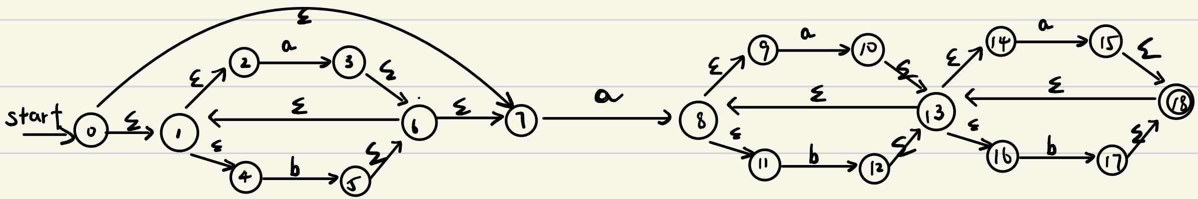
1. The NFA of $L((a|b)^*b)$ is



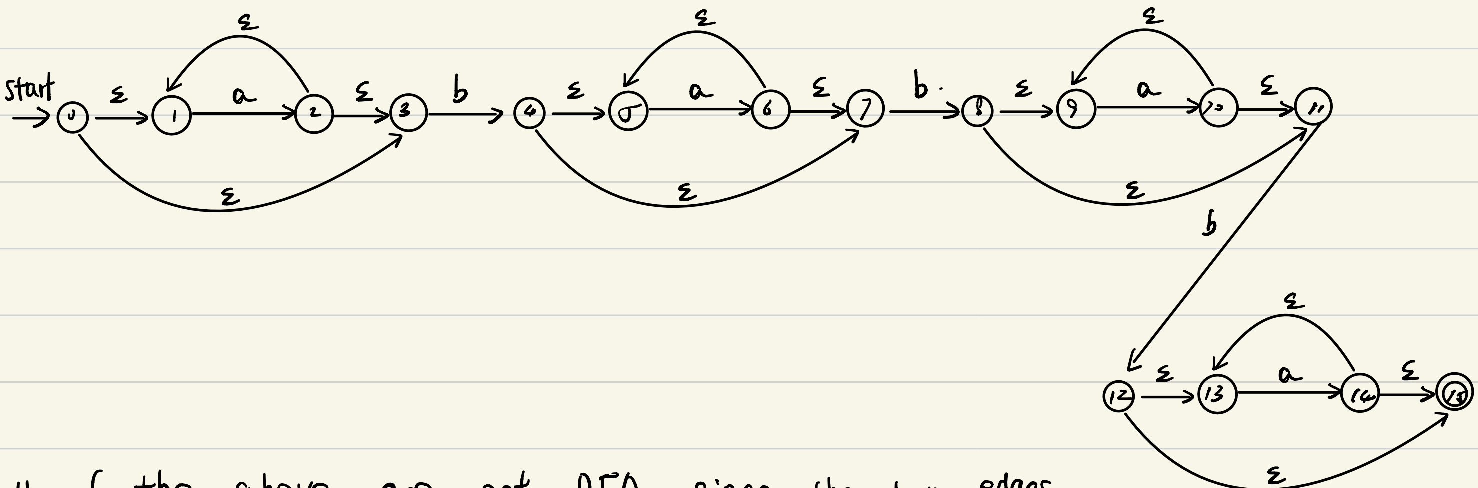
2. The NFA of $L(((\varepsilon|a)^*b)^*)$ is



3. The NFA of $L((a|b)^*a(a|b)(a|b))$ is



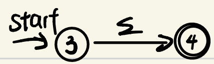
4. The NFA of $L(a^*ba^*ba^*ba^*)$ is

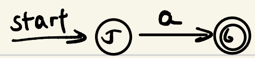


All of the above are not DFA. since they have edges on input ε

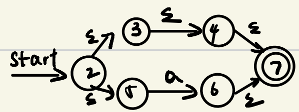
Exercise 2

1. $((\varepsilon|a)^*b)^*$

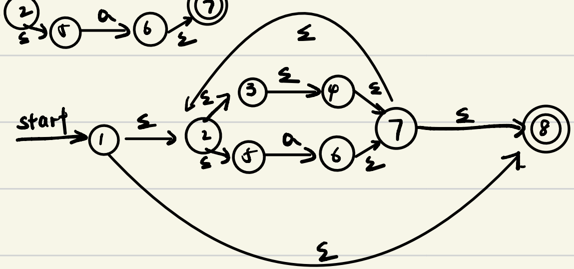
① for $r_1=\varepsilon$, $r_2=a$, we construct 

, 

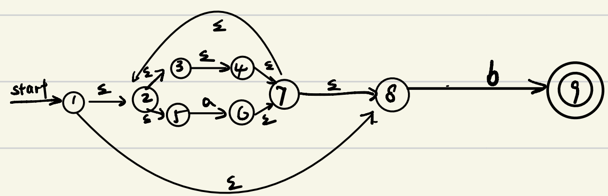
② for $r_3 = r_1 | r_2$, we construct



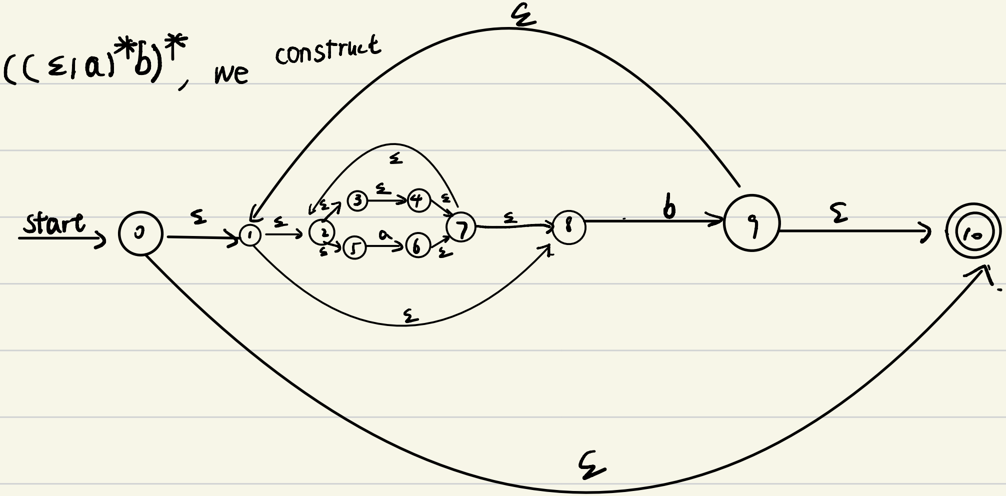
③ for $r_4 = (r_3)^*$, we construt



④. for $r_5 = r_4 b$, we construct

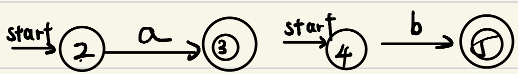


⑤ for $r_6 = (r_5)^*$, which is $((\varepsilon|a)^*b)^*$, we construct

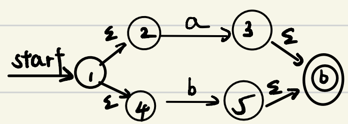


2. $(a|b)^* a(a|b)(a|b)$

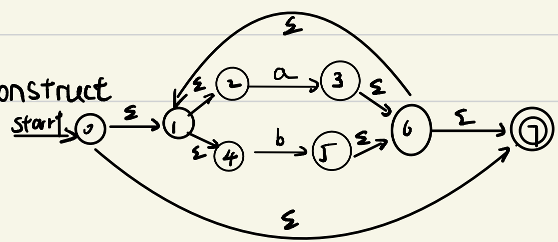
① for $r_1 = a$, $r_2 = b$, we construct



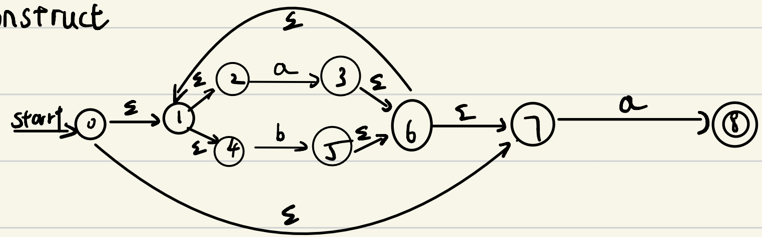
② for $r_3 = r_1 | r_2$, we construct



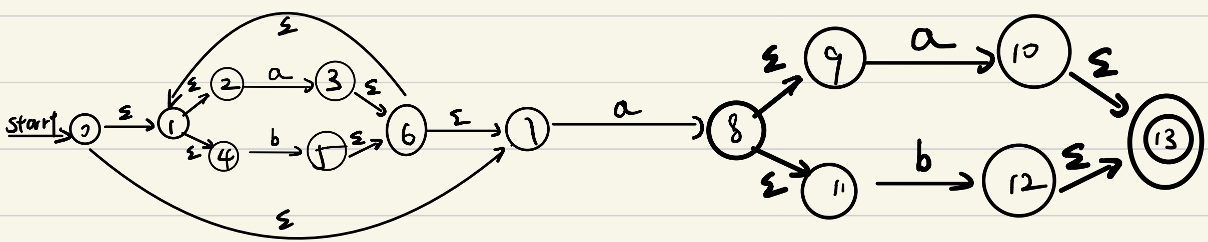
③ for $r_4 = (r_3)^*$, we construct



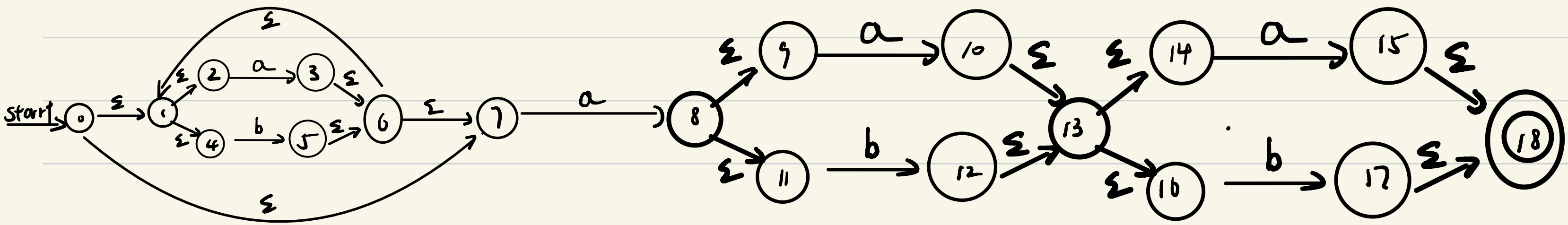
④ for $r5 = r4\,a$, we construct



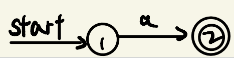
⑤ for $r6 = r5\,(a|b)$, we construct



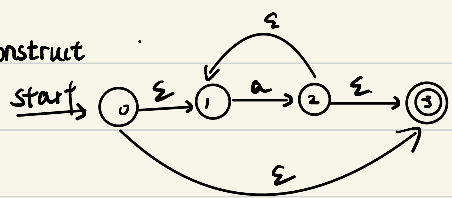
⑥ for $r_7 = r_6 (a|b)$, we construct

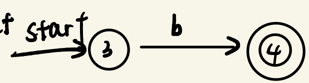


3. $a^*ba^*ba^*ba^*$

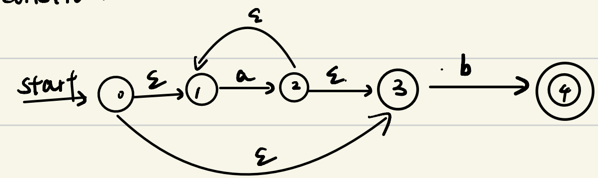
① for $r_1 = a$, we construct 

② for $r_2 = (r_1)^*$, we construct

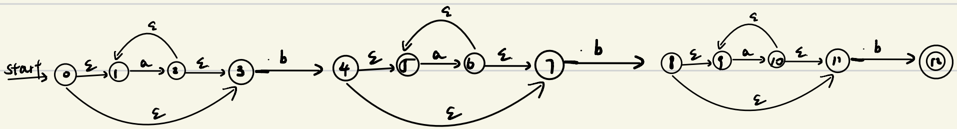


③ for $r_3 = b$, we construct 

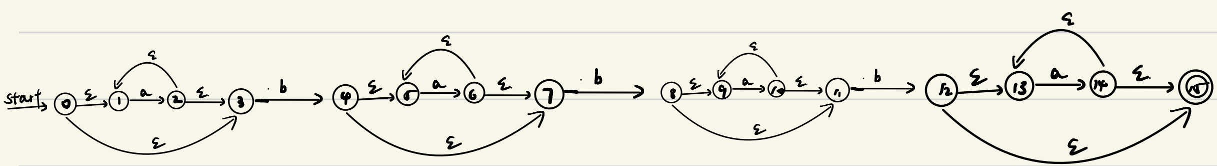
④ for $r_4 = r_2 r_3$. we construct



⑤ for $r_5 = r_4 r_4 r_4$, we construct

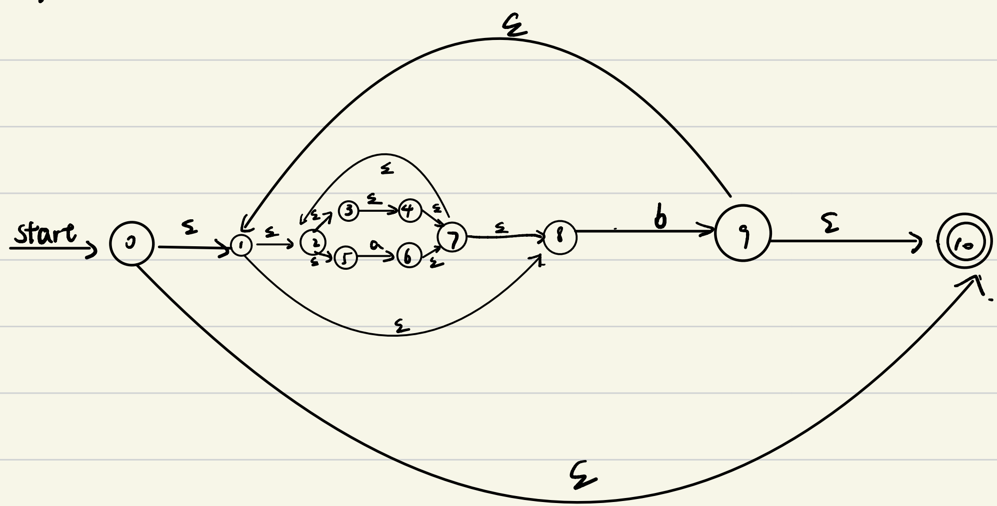


⑥ for $r_6 = r_5 r_2$, which is $a^*ba^*ba^*ba^*$ we construct.



Exercise 3.

1. For $((\varepsilon|a)^* b)^*$



Let's define A as ε-closure (0), which is the start state of equivalent DFA, A = {0, 1, 2, 3, 4, 5, 7, 8, 10};

since the input alphabet is {a, b}. define B as ε-closure (move(A, a)) =

{2, 3, 4, 5, 6, 7, 8}

{6}

$\{9\}$

define C as $\varepsilon\text{-closure}(move(A,b)) = \{1,2,3,4,5,7,8,9,10\}$

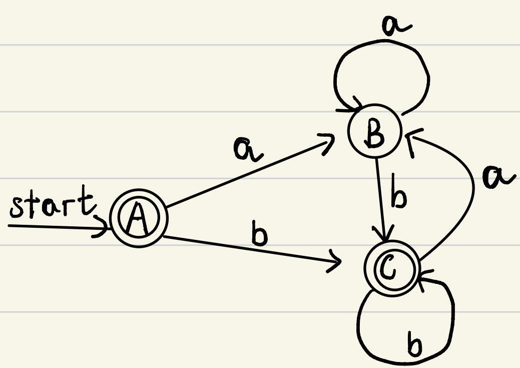
$\varepsilon\text{-closure}(move(B,a)) = \{2,3,4,5,6,7,8\}$, which is the same as B

$\varepsilon\text{-closure}(move(B,b)) = C$.

$\varepsilon\text{-closure}(move(C,a)) = B$

$\varepsilon\text{-closure}(move(C,b)) = C$.

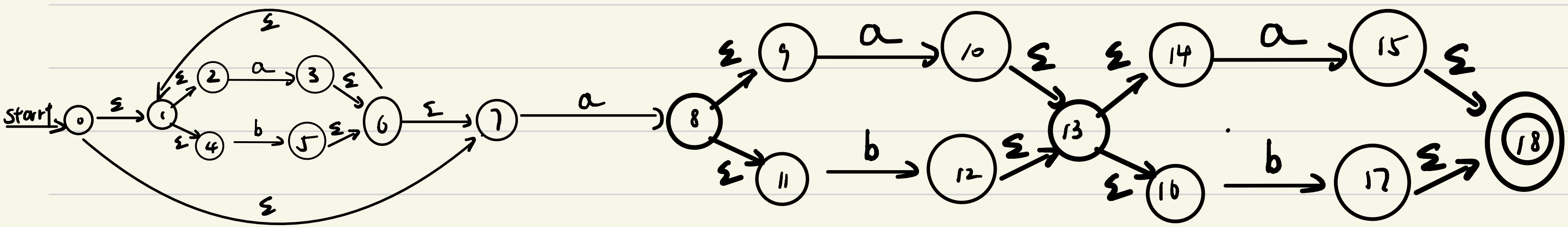
As a result, the DFA is



the transition table is

NFA's state	DFA's state	a	b
$\{0,1,2,3,4,5,7,8,10\}$	A	B	C.
$\{2,3,4,5,6,7,8\}$	B	B	C.
$\{1,2,3,4,5,7,8,9,10\}$	C.	B	C.

2. For $(a|b)^*a(a|b)(a|b)$, the NFA is as below



Let's define the start state A = ε-closure(0).

which is {0, 1, 2, 4, 7}, define B = ε-closure(move(A, a)) = {1, 2, 3, 4, 6, 7, 8, 9, 11}

{3, 8}

define C = ε-closure(move(A, b)) = {1, 2, 4, 5, 6, 7}

{5}

define D = ε-closure(move(B, a)) = {1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 16}

{3, 8, 10}

define $E = \varepsilon\text{-closure}(\underbrace{move(B,b)}_{\{5,12\}}) = \{1,2,4,5,6,7,12,13,14,16\}$

Notice that $\varepsilon\text{-closure}(\underbrace{move(C,a)}_{\{3,8\}}) = B$, $\varepsilon\text{-closure}(\underbrace{move(C,b)}_{\{5,\}}) = C$.

define $F = \varepsilon\text{-closure}(\underbrace{move(D,a)}_{\{3,8,10,15\}}) = \{1,2,3,4,6,7,8,9,11,13,14,15,16,18\}$

define $G = \varepsilon\text{-closure}(\underbrace{move(D,b)}_{\{5,12,17\}}) = \{1,2,4,5,6,7,12,13,14,16,17,18\}$

define $H = \varepsilon\text{-closure}(\underbrace{move(E,a)}_{\{3,8,15\}}) = \{1,2,3,4,6,7,8,9,11,15,18\}$

define $I = \varepsilon\text{-closure}(\underbrace{move(E,b)}_{\{5,17\}}) = \{1,2,4,5,6,7,17,18\}$

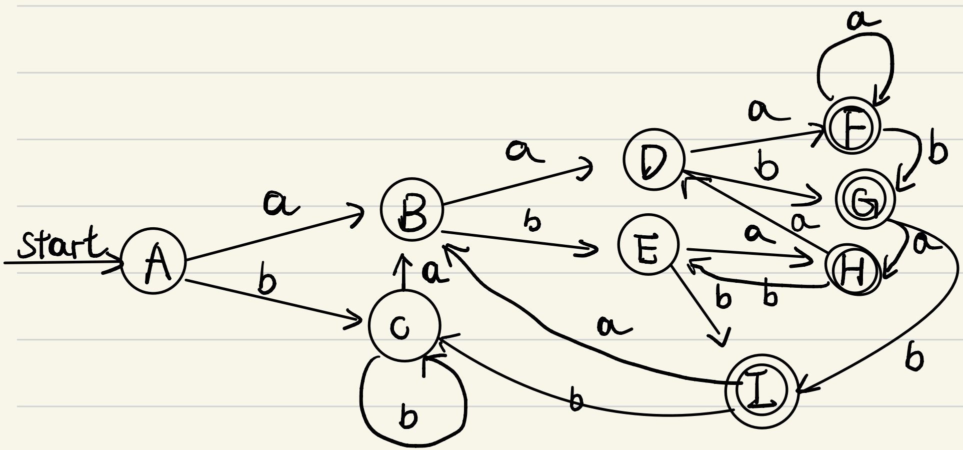
$\varepsilon\text{-closure}(\underbrace{move(F,a)}_{\{3,8,10,15\}}) = F$, $\varepsilon\text{-closure}(\underbrace{move(F,b)}_{\{5,12,17\}}) = G$

$\varepsilon\text{-closure}(\underbrace{move(G,a)}_{\{3,8,15\}}) = H$, $\varepsilon\text{-closure}(\underbrace{move(G,b)}_{\{5,17\}}) = I$

$\varepsilon\text{-closure}(\underbrace{move(H,a)}_{\{3,8,10\}}) = D$, $\varepsilon\text{-closure}(\underbrace{move(H,b)}_{\{5,12\}}) = E$

$\varepsilon\text{-closure}(\underbrace{move(I,a)}_{\{3,8\}}) = B$, $\varepsilon\text{-closure}(\underbrace{move(I,b)}_{\{5\}}) = C$

The DFA is shown as below

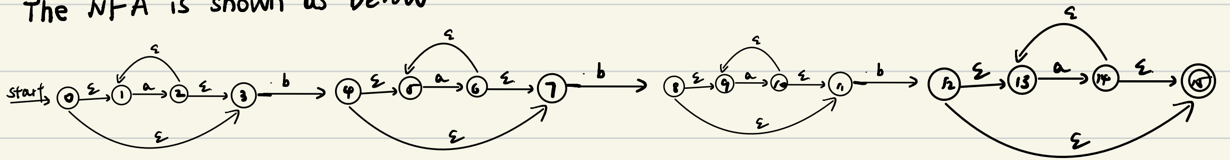


Transition Table

NFA's state	DFA's state	a	b
{0,1,2,4,7}	A	B	C
{1,2,3,4,6,7,8,9,11}	B	D	E
{1,2,4,5,6,7}	C	B	C
{1,2,3,4,6,7,8,9,10,11,13,14,16}	D	F	G
{1,2,4,5,6,7,12,13,14,16}	E	H	I
{1,2,3,4,6,7,8,9,11,13,14,15,16,18}	F	F	G
{1,2,4,5,6,7,12,13,14,16,17,18}	G	H	I
{1,2,3,4,6,7,15,18}	H	D	E
{1,2,4,5,6,7,17,18}	I	B	C

(3) For $a^* b a^* b a^* b a^*$

The NFA is shown as below



Define A as the start state, which is ε-closure(0) = {0, 1, 3}

$B = \varepsilon\text{-closure}(\underbrace{\text{move}(A, a)}_{\{2\}}) = \{1, 2, 3\}$

$C = \varepsilon\text{-closure}(\underbrace{\text{move}(A, b)}_{\{4\}}) = \{4, 5, 7\}$

$\varepsilon\text{-closure}(\underbrace{\text{move}(B, a)}_{\{2\}}) = B$ $\quad$ $\varepsilon\text{-closure}(\text{move}(B, b)) = C$.

$D = \varepsilon\text{-closure}(\underbrace{\text{move}(C, a)}_{\{6\}}) = \{5, 6, 7\}$ $\quad$ $E = \varepsilon\text{-closure}(\underbrace{\text{move}(C, b)}_{\{8\}}) = \{8, 9, 11\}$

$\varepsilon\text{-closure}(\underbrace{\text{move}(D, a)}_{\{6\}}) = D$ $\quad$ $\varepsilon\text{-closure}(\underbrace{\text{move}(D, b)}_{\{8\}}) = E$

$F = \varepsilon\text{-closure}(\underbrace{move(E, a)}_{\{10\}}) = \{9, 10, 11\}$ $G = \varepsilon\text{-closure}(\underbrace{move(F, b)}_{\{12\}}) = \{12, 13, 15\}$

ition Table $\varepsilon\text{-closure}(\underbrace{move(F, a)}_{\{10\}}) = F$ $\varepsilon\text{-closure}(\underbrace{move(F, b)}_{\{12\}}) = G$

FA 's state DFA's state a b

$H = \varepsilon\text{-closure}(\underbrace{move(G, a)}_{\{14\}}) = \{13, 14, 15\}$

$\varepsilon\text{-closure}(move(G, b)) = \emptyset$

$\varepsilon\text{-closure}(\underbrace{move(H, a)}_{\{14\}}) = H$ $\varepsilon\text{-closure}(move(H, b)) = \emptyset$

Transition Table

NFA's state	DFA's state	a	b
{0,1,3}	A	B	C
{1,2,3}	B	B	C
{4,5,7}	C	D	E
{5,6,7}	D	D	E
{8,9,11}	E	F	G
{9,10,11}	F	F	G
{12,13,15}	G	H	∅
{13,14,15}	H	H	∅

The DFA is