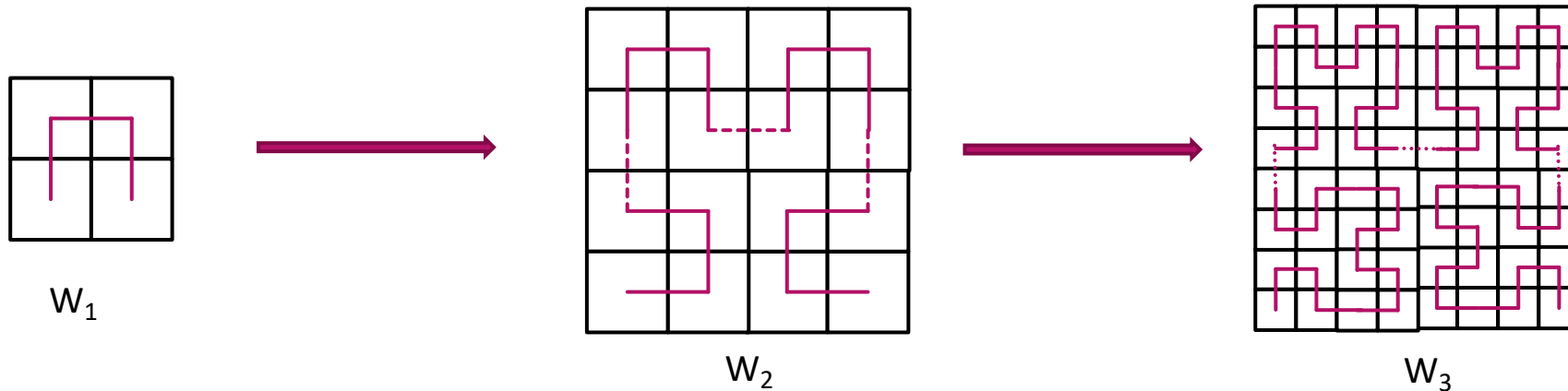


Problem analysis of Divide And Conquer

YAO ZHAO

Hilbert Curve



W_2 consists of four W_1 structures with the lower-left and the lower-right ones are 90 degree rotated clockwise and counter-clockwise, respectively; the upper ones have the same structure with W_1 . Connect the four structures with 3 unit lines.

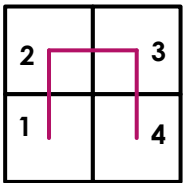
W_3 consists of four W_2 structures with the lower-left and the lower-right ones are 90 degree rotated clockwise and counter-clockwise, respectively; the upper ones have the same structure with W_2

Hilbert Curve

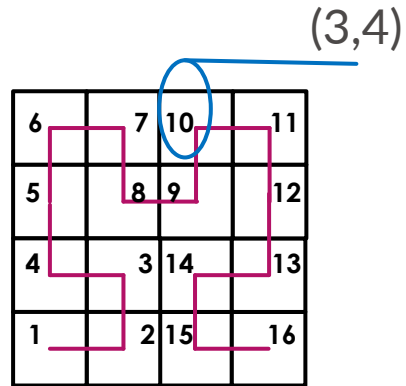
- ▶ This rule has been devised by a mathematical philosopher David Hilbert (1862 – 1943), and the resulting curve is usually called a Hilbert Curve named after him. He once talked about a space filling method using this kind of curve to fill up a square with 2^k sides.

For each vertex p on the Hilbert curve, we define the coordinates of p to be the location of the square of p in the squares matrix, and we define the serial number of p to be the vertices count on the curve from the beginning to p .

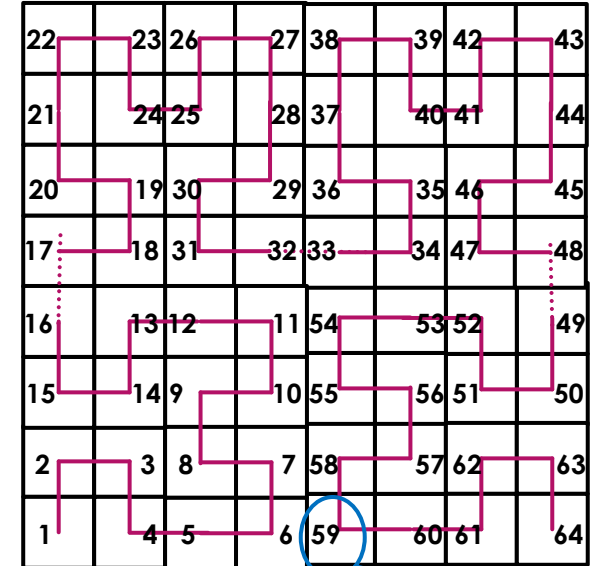
For example, when the coordinates of p is $(3,4)$ and the order is 2, the number of p is 10;
when the coordinates of p is $(5,1)$ and the order is 3, the number of p is 59.



W_1



W_2

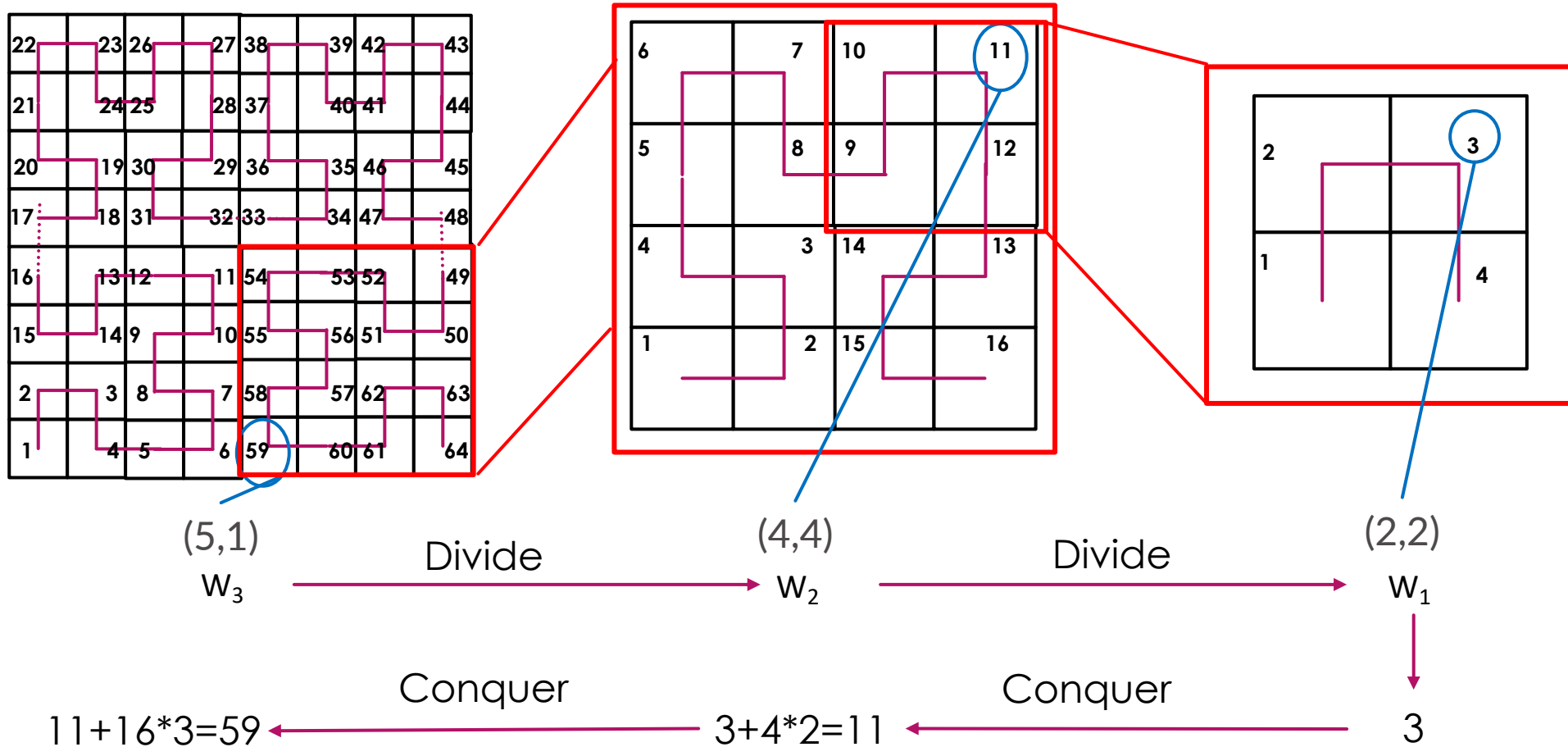


W_3

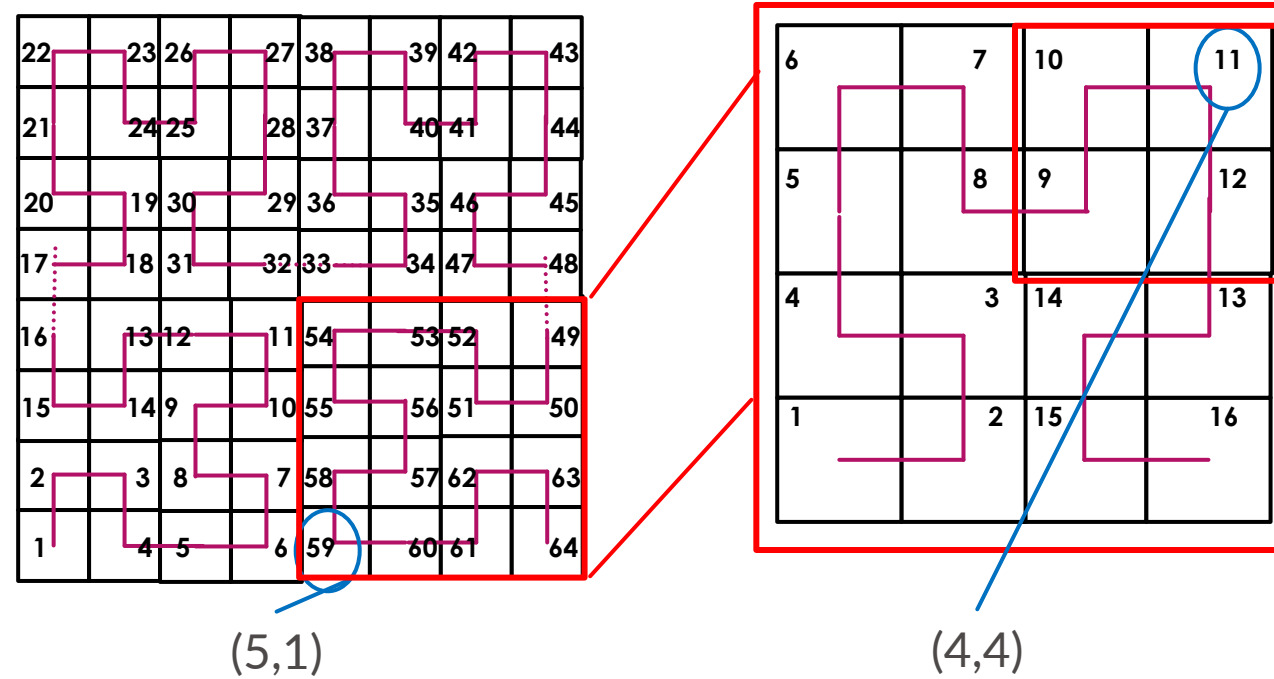
Given the order of the Hilbert curve and the coordinates of p , can you figure out the number of p ?

Problem analysis

W_n consists of four W_{n-1} structures

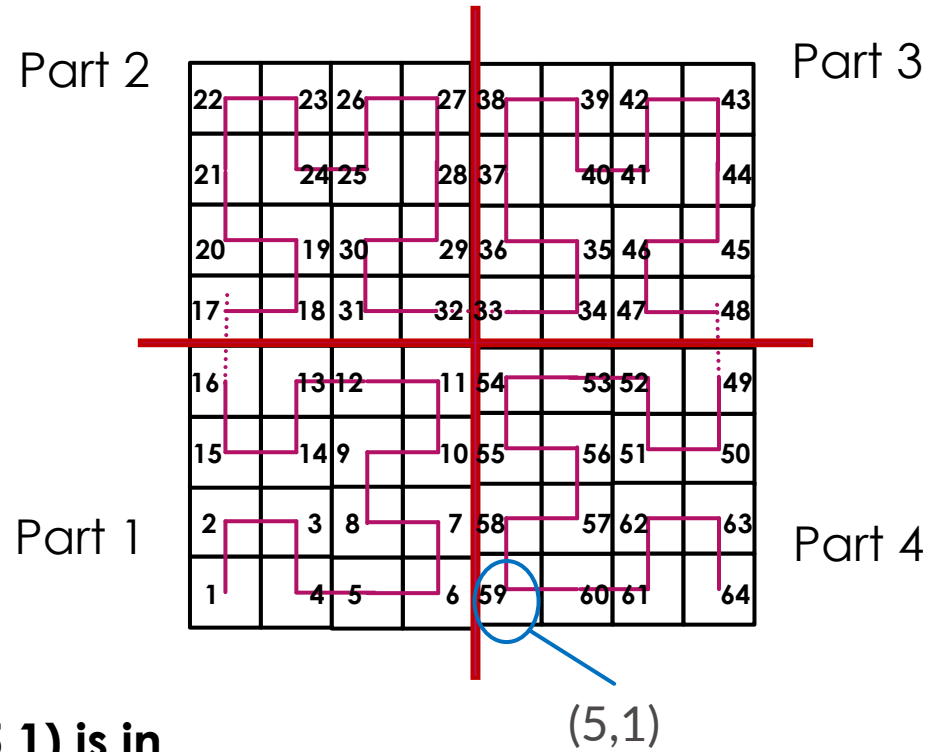


Problem analysis



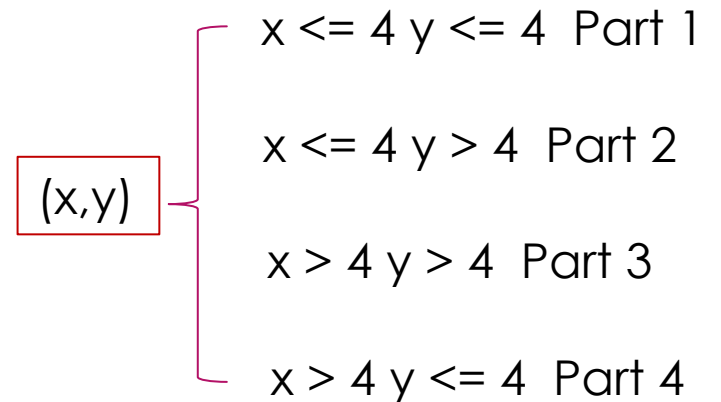
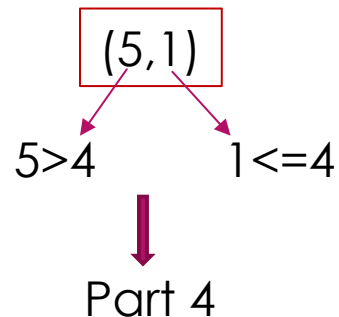
How to map (5,1) to (4,4)?

Step 1: Divide the W_3 to 4 parts



Step 2: Find which part (5,1) is in

W_3 is a $8*8$ square matrix



Step 3: get offset value according part number

Part 4 case

Part 2

22	23	26	27	38	39	42	43
21	24	25	28	37	40	41	44
20	19	30	29	36	35	46	45
17	18	31	32	33	34	47	48
16	13	12	11	54	53	52	49
15	14	9	10	55	56	51	50
2	3	8	7	58	57	62	63
1	4	5	6	59	60	61	64

Part 1

Part 3

(5,1) is in part4

Offset = 3*16 = 48

Each element-48

coordinates x-4

Part 4

6	5	4	1
7	8	3	2
10	9	14	15
11	12	13	16

(5,1)

(1,1)

(4,4)

6	7	10	11
5	8	9	12
4	3	14	13
1	2	15	16

W₂

y = -x

Observe the part4 and W₂

The coordinates and the number are flipped symmetrically by the line y = -x

$$x' = 4 + 1 - y$$

$$y' = 4 + 1 - x$$

(5,1) → (1,1) → (4,4)
offset flip

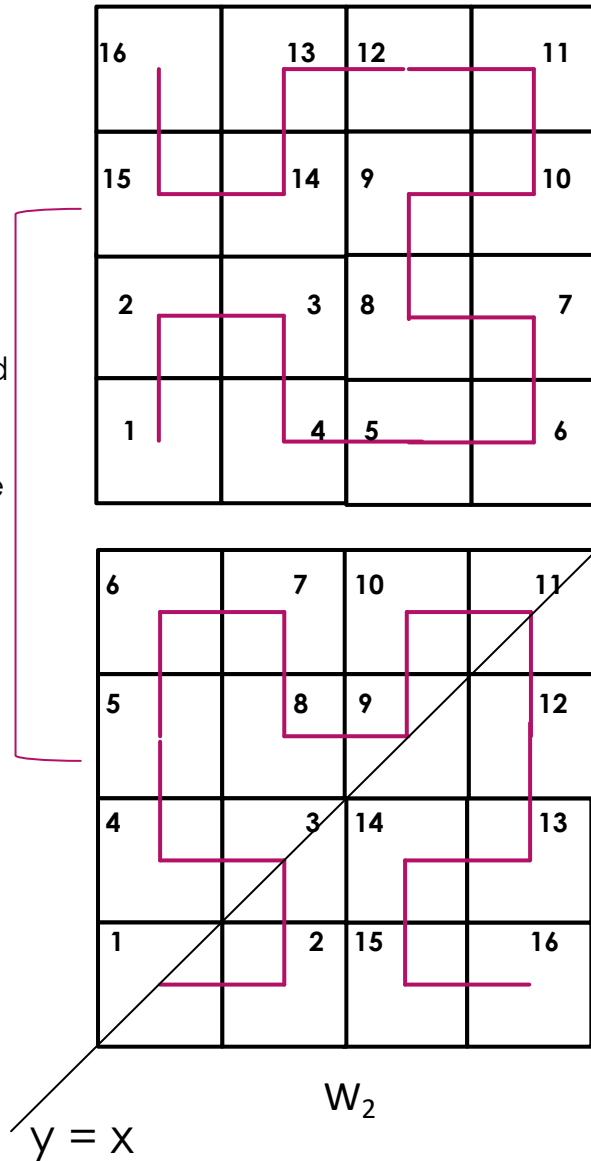
Step 3: Part 1 case

Observe the part1 and W_2

The coordinates and the number are flipped symmetrically by the line $y = x$

$$x' = y$$

$$y' = x$$



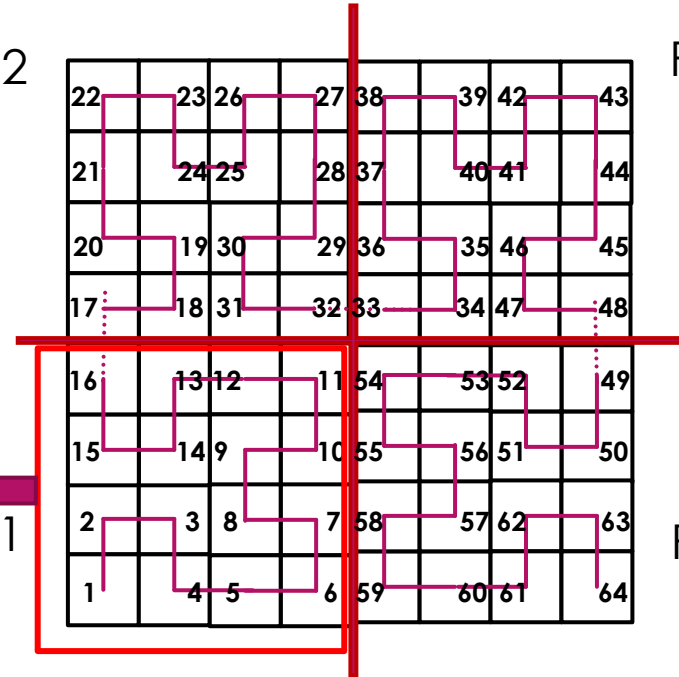
Offset = 0

Part 2

Part 3

Part 1

Part 4

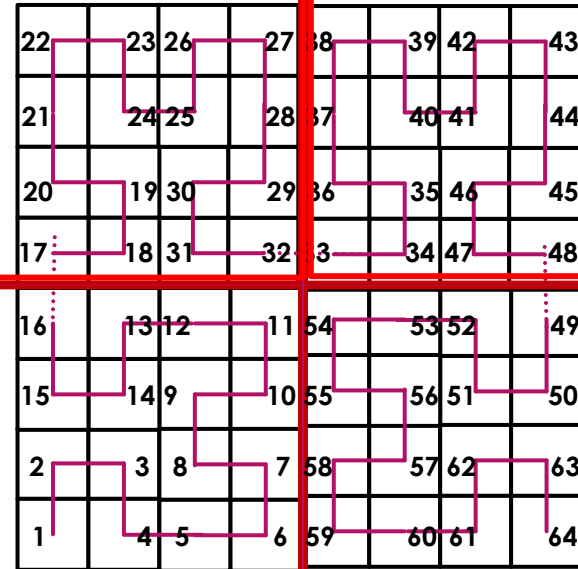


1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 26



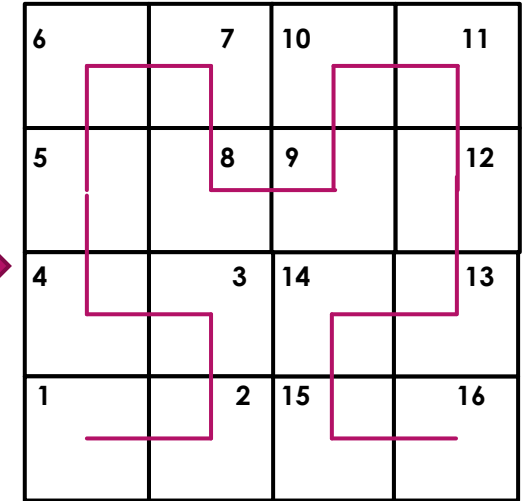
Each element-16
coordinates y-4

Part 3



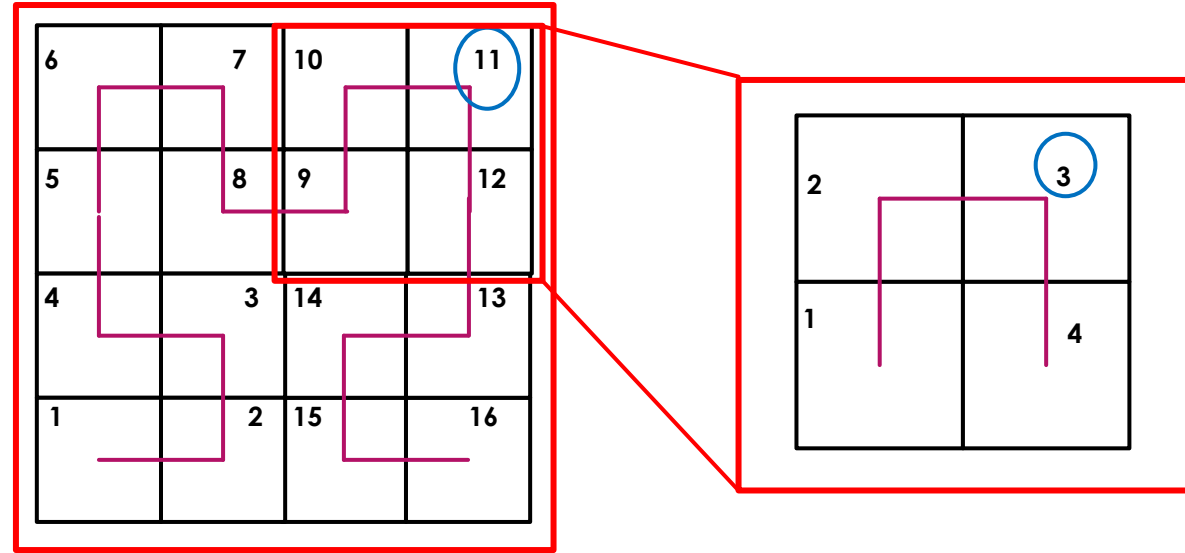
Part 4

Each element-32
coordinates x-4 y-4



same

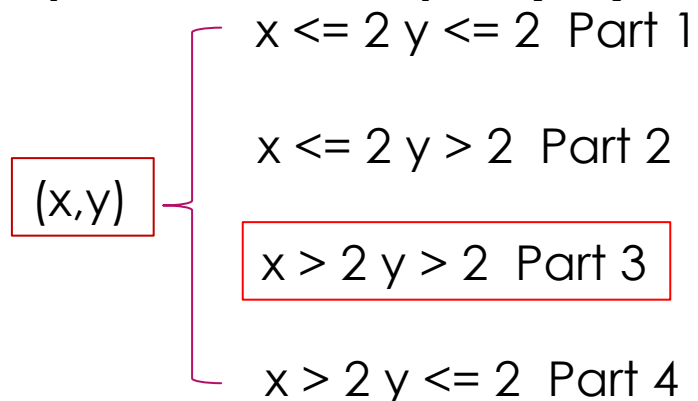
Further analysis: W_2



How to map (4,4) to (2,2)?

Step 1: Divide the W_2 to 4 parts

Step 2: Find which part (4,4) is in



Step 3: get offset value according part number

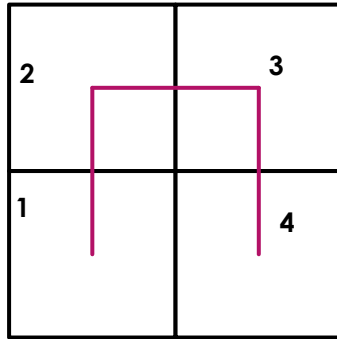
Part 3 **Offset = $2*4 = 8$**

Each element-8

coordinates $x-2$ $y-2$

$(4,4) \rightarrow (2,2)$

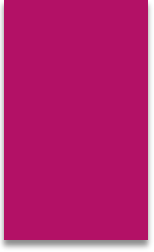
Base code: W_1



coordinates

number

(1,1)	1
(1,2)	2
(2,2)	3
(2,1)	4



```
w[3][3] = {{0,0,0},  
           {0,1,4},  
           {0,2,3}};
```

```
public static long HilbertNumber(int n, int x, int y) {  
    if(n==1)  
        return w[x][y];  
    int m = 1<<(n-1);  
    if(x<=m) {  
        if(y<=m)  
            return HilbertNumber(n-1, y, x); //Part 1, x' = y y'=x  
        else  
            return m*m + HilbertNumber(n-1, x, y-m); //Part 2, coordinates offset: y-m  
    } else {  
        if(y>m)  
            return 2*m*m + HilbertNumber(n-1, x-m, y-m); //Part 3, coordinates offset: x-m y-m  
        else  
            return 3*m*m + HilbertNumber(n-1, m+1-y, m+1-(x-m)); //Part4, coordinates offset: x=x-m  
                                                                    //x' = m+1-y, y' = m+1-x  
    }  
}
```

See: [HilbertCurve.java](#)