CS215 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

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Euler's Formula

- **Theorem** (Euler's Formula) Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then $r = e - v + 2$.

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- **Corollary 1** If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$.

  Corollary 2 If G is a connected planar simple graph, then G has a vertex of degree not exceeding 5.

  Corollary 3 In a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$.

Graph Coloring

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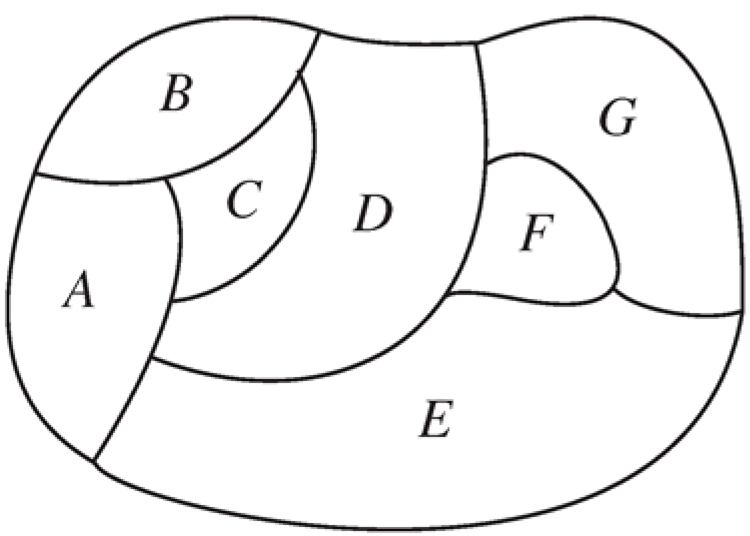

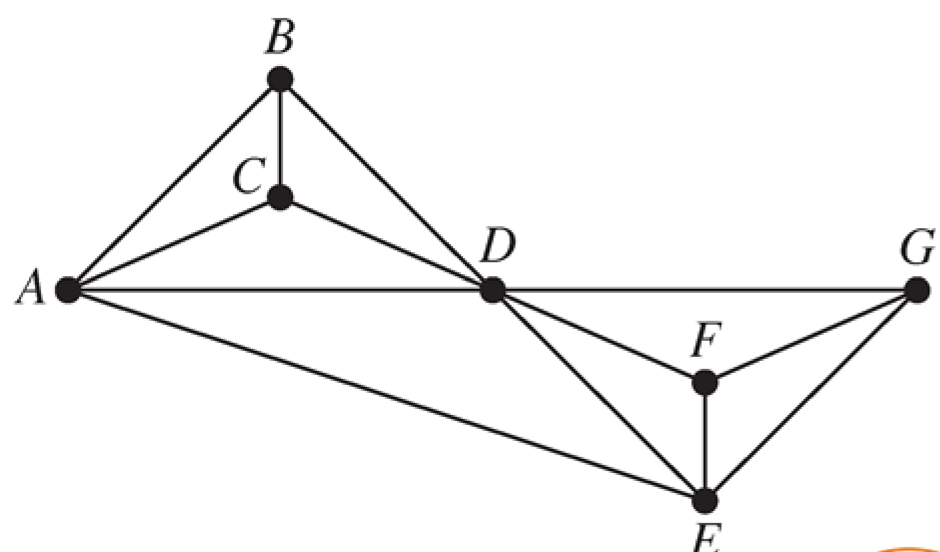

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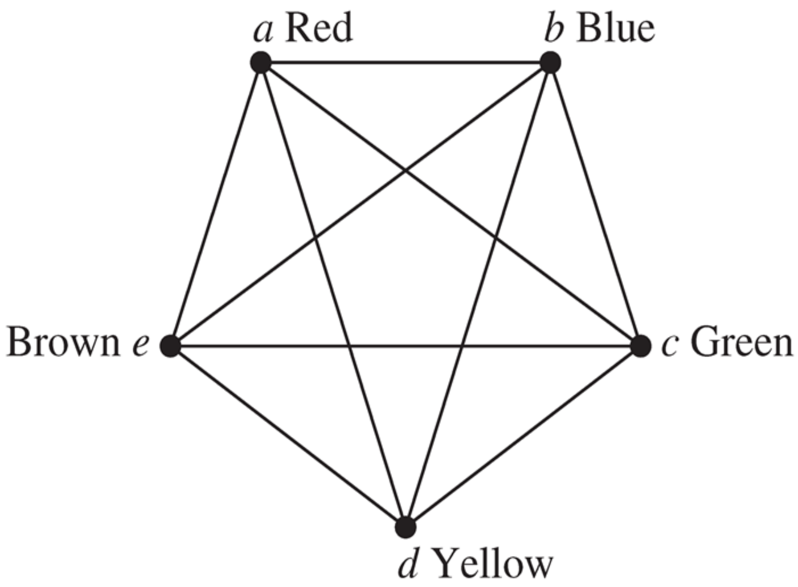

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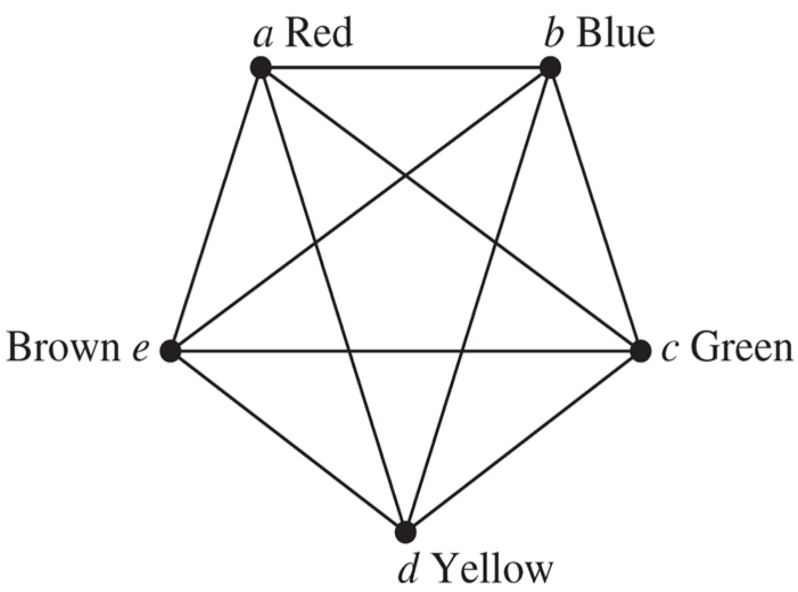

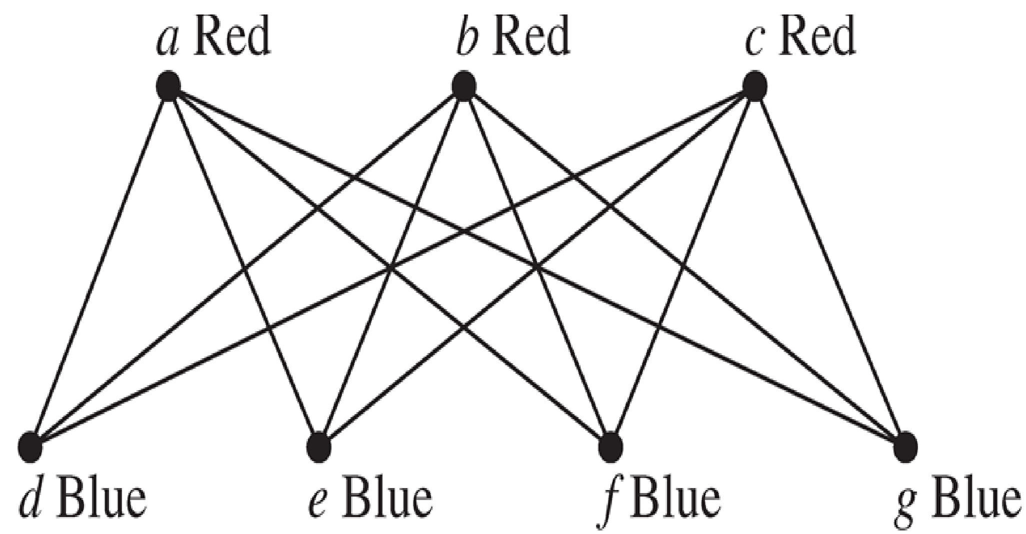

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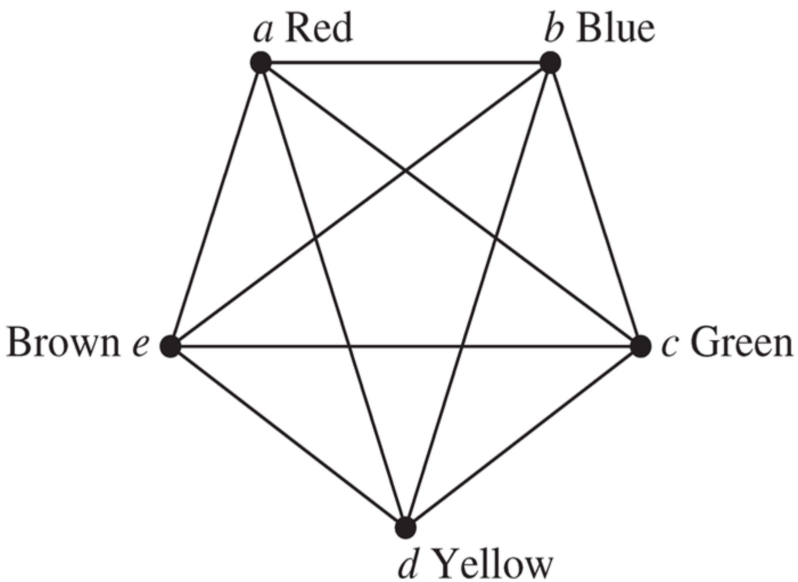

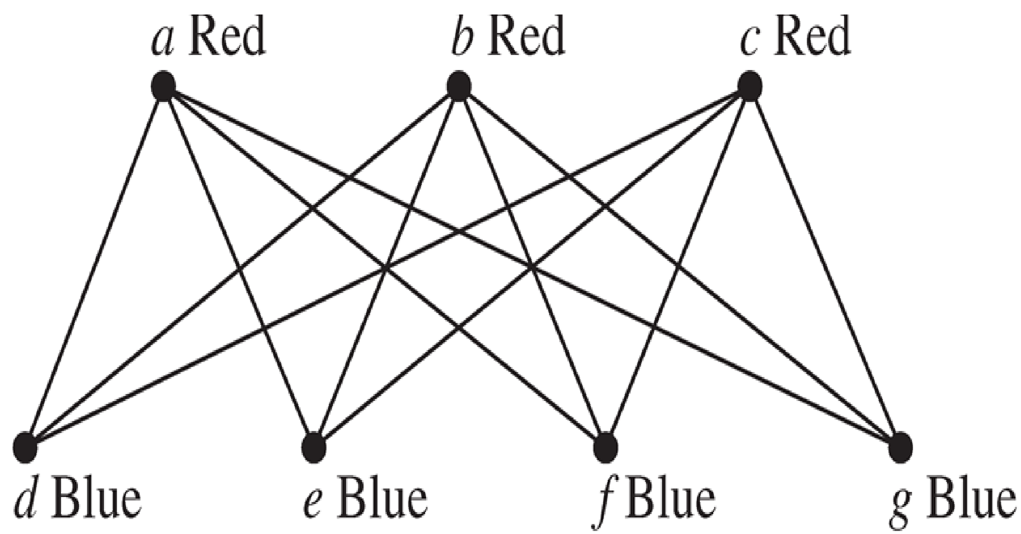

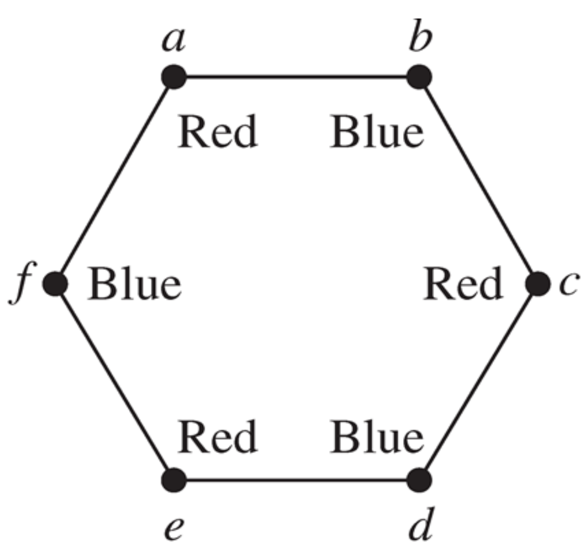

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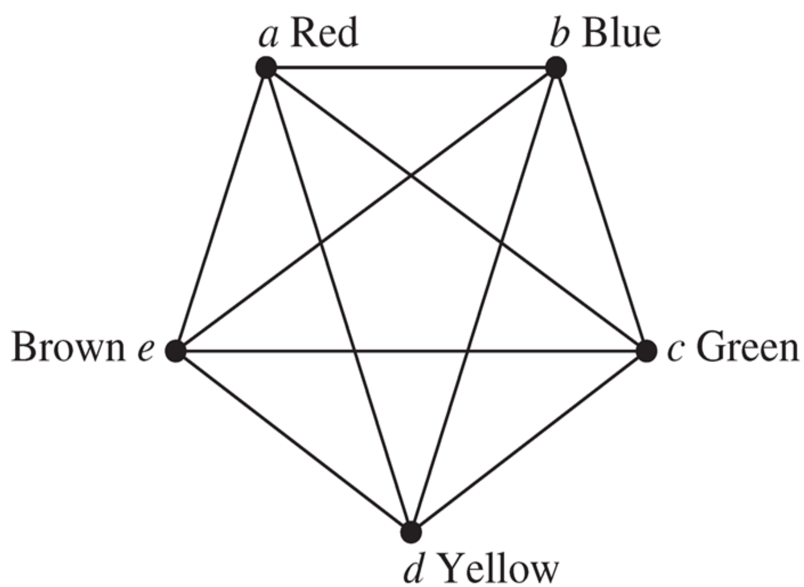
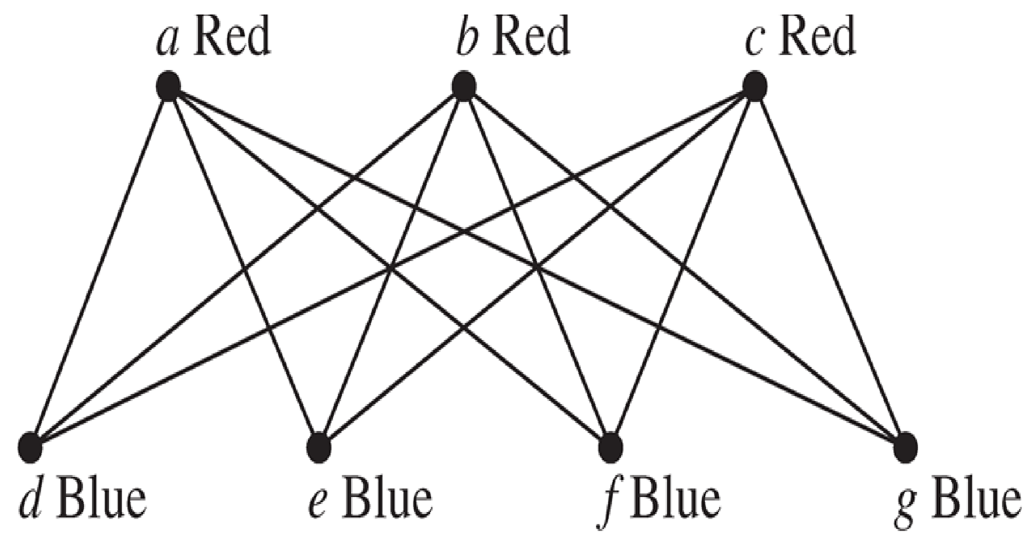
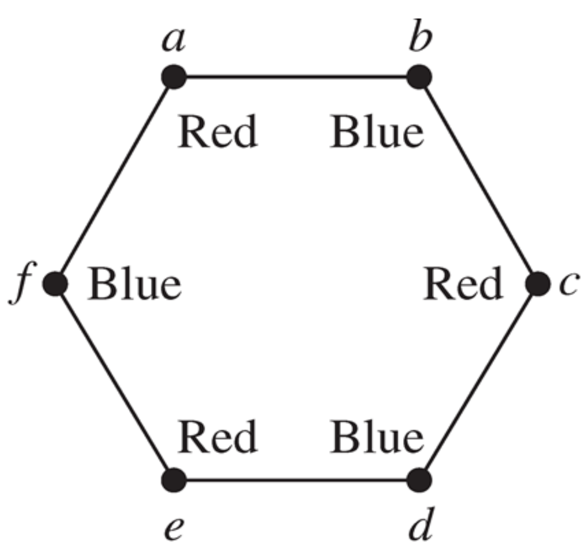
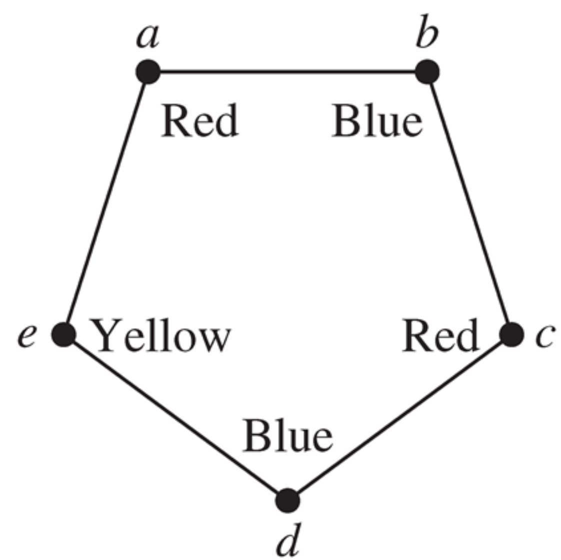




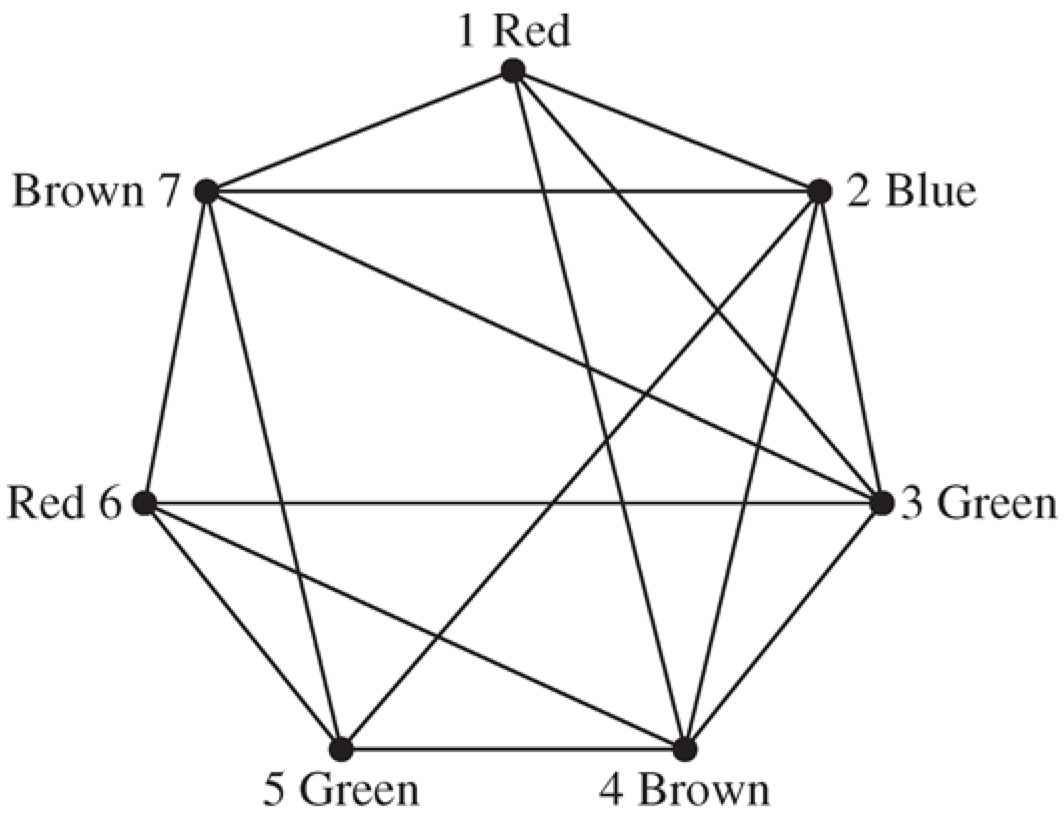




Applications of Graph Coloring

- Scheduling Final Exams

  Vertices represent courses, and there is an edge between two vertices if there is a common student in the courses.



Time Period	Courses
I	1, 6
II	2
III	3, 5
IV	4, 7

Applications of Graph Coloring

- ## Channel Assignments

  Television channels 2 through 13 are assigned to stations in North America so that no two stations within 150 miles can operate on the same channel . How can the assignment of channels be modeled by graph coloring?

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Trees

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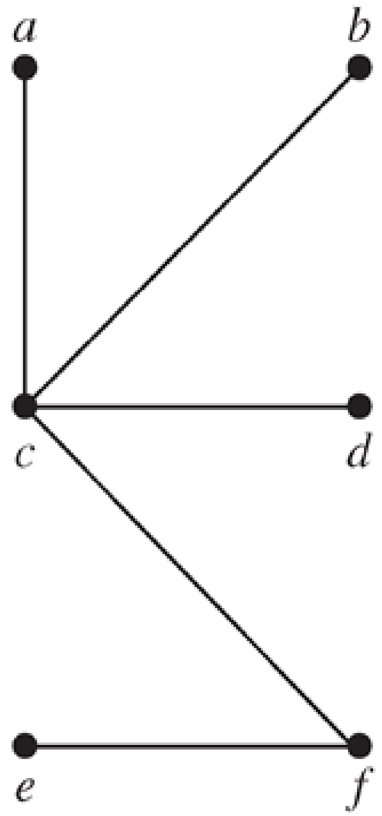

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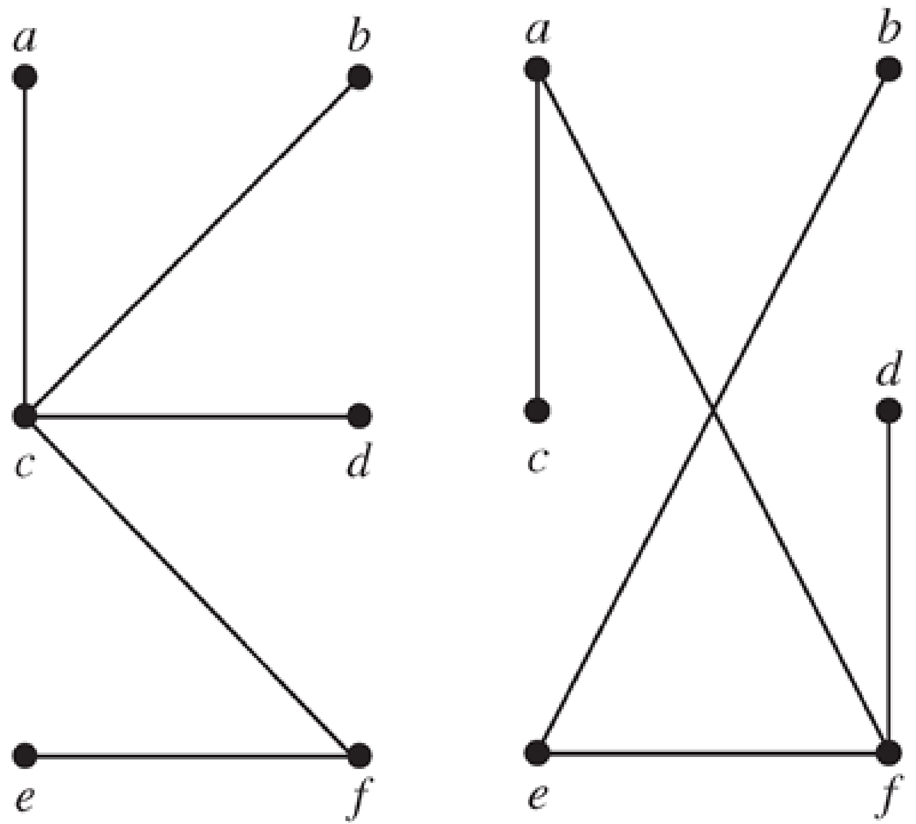

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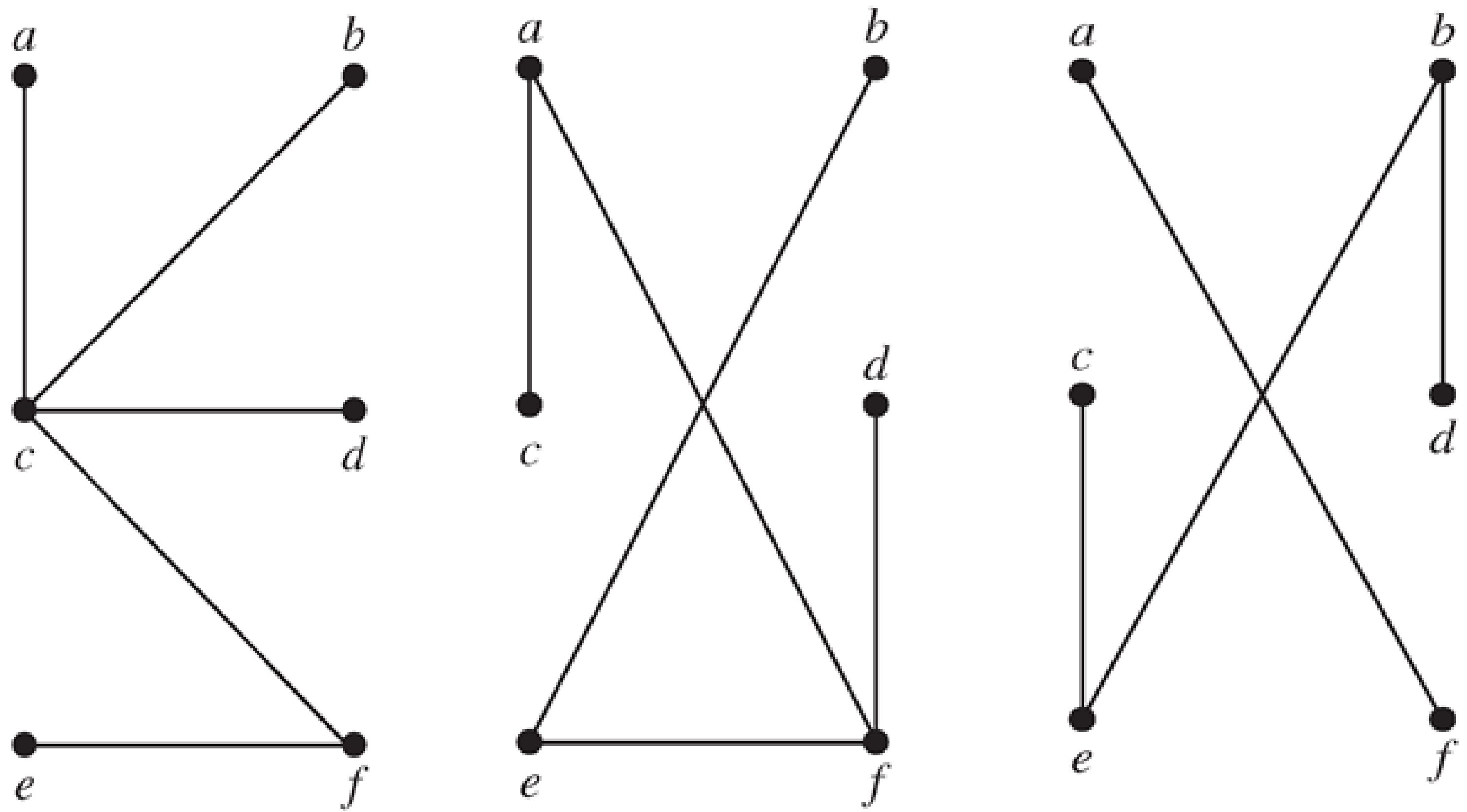

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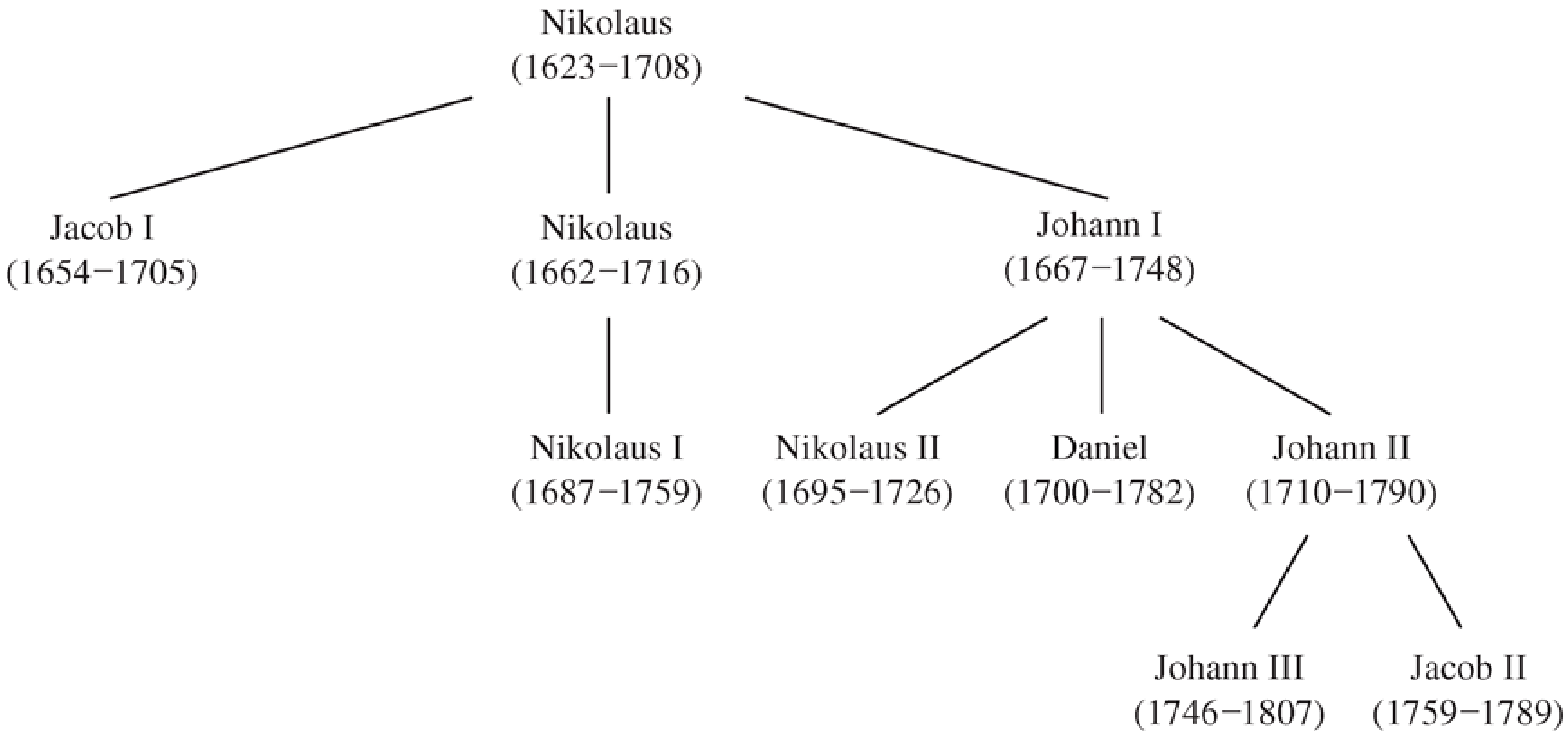

- **Theorem** An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

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Two properties of tree: connected, no circuit

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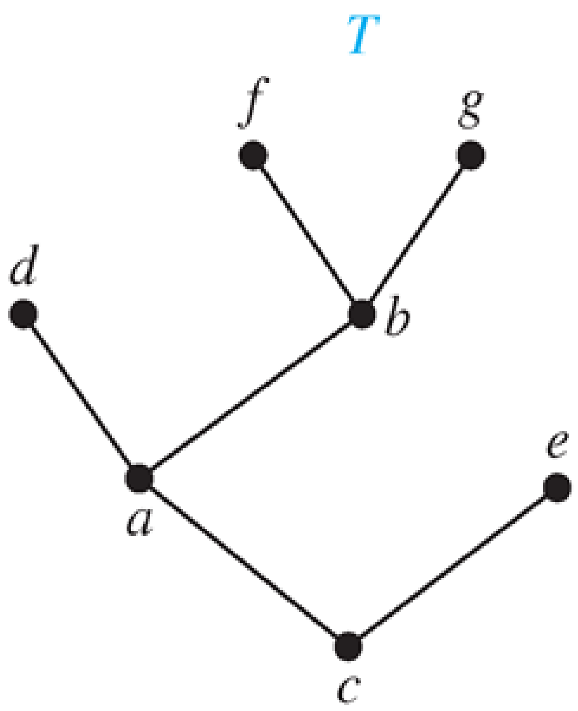

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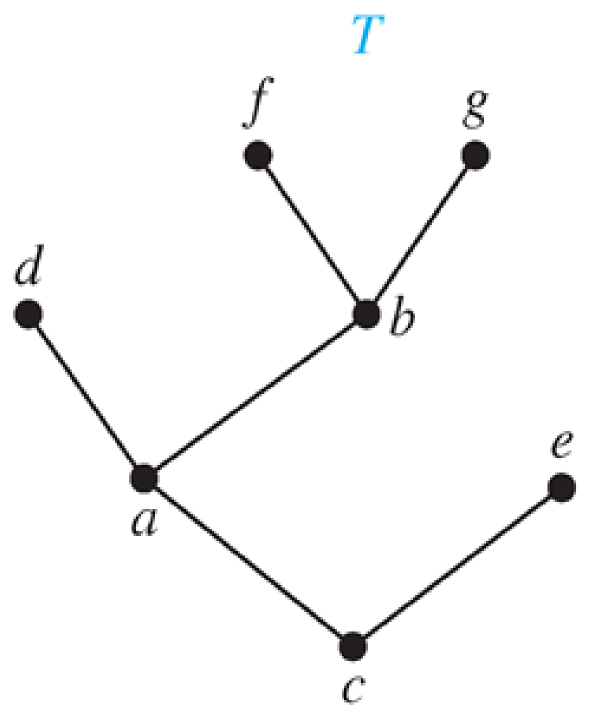

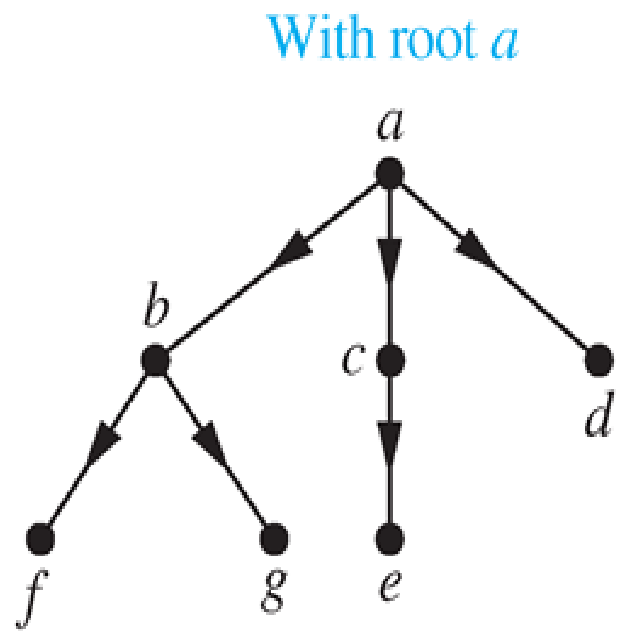

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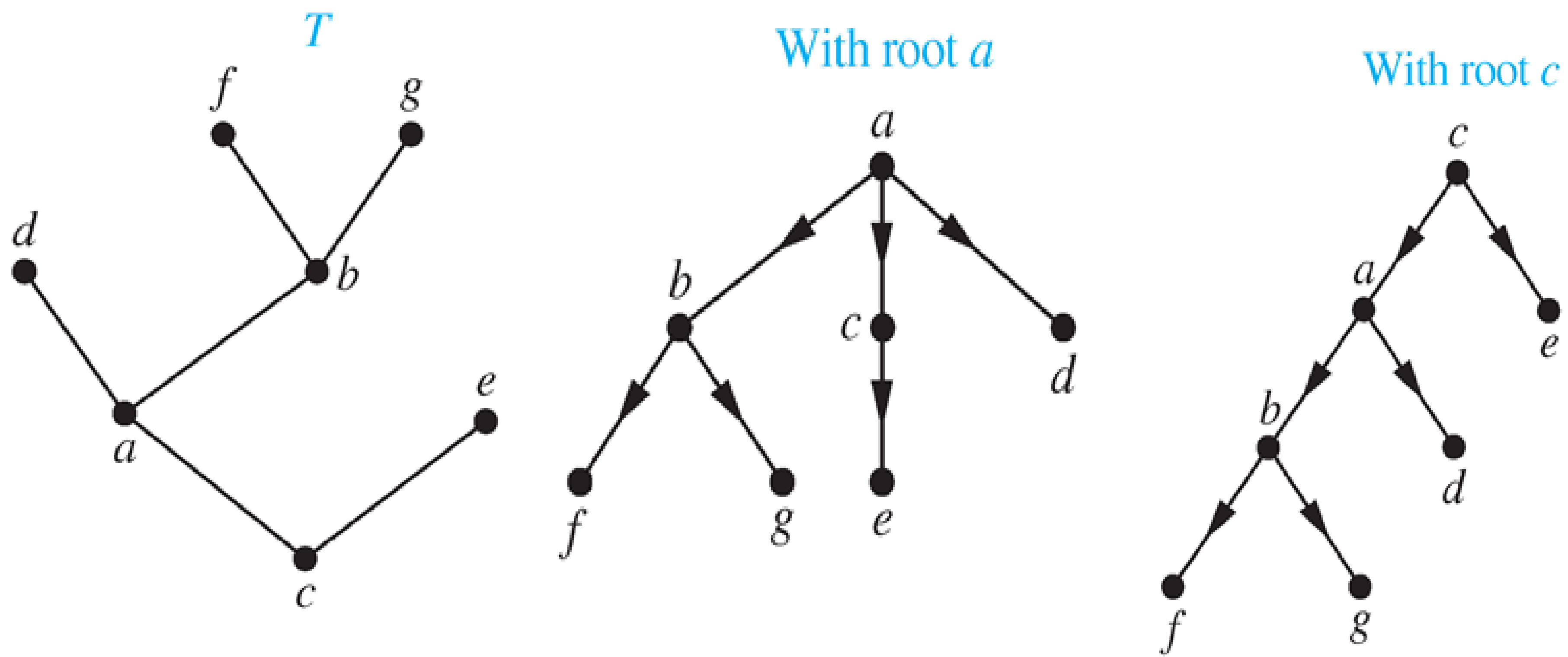

Rooted Trees

- *parent, child, sibling*

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  subtree with a as its root: consists of a and its descendants and all edges incident to these descendants

- **Definition** A rooted tree is called an *m-ary tree* if every internal vertex has no more than m children. The tree is called a *full m-ary tree* if every internal vertex has exactly m children. In particular, an m-ary tree with $m = 2$ is called a *binary tree*.

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  left subtree, *right subtree*

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  (i) n vertices, $i = (n-1)/m$, $\ell = [(m-1)n+1)]/m$
  (ii) i internal vertices, $n = mi + 1$, $\ell = (m-1)i + 1$
  (iii) ℓ leaves, $n = (m\ell - 1)/(m-1)$, $i = (\ell - 1)/(m-1)$

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  using $n = mi + 1$ and $n = i + \ell$

Level and Height

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  The *height* of a rooted tree is the maximum of the levels of the vertices.

  Definition A rooted m-ary tree of height h is *balanced* if all leaves are at levels h or $h-1$. (differ no greater than 1)

The Number of Leaves

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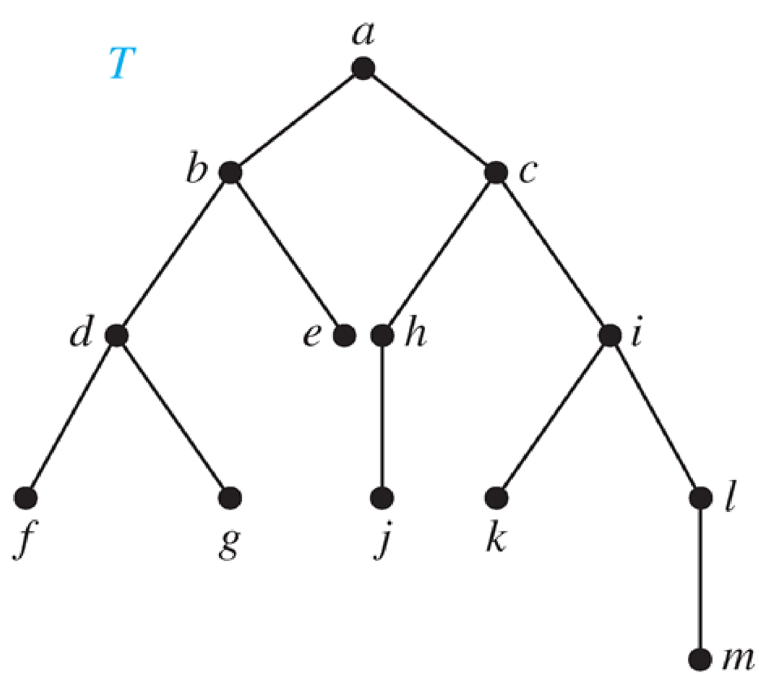

Tree Traversal

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  The three most commonly used traversals are *preorder traversal*, *inorder traversal*, *postorder traversal*.

Preorder Traversal

- **Definition** Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *preorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees of r from left to right in T. The *preorder traversal* begins by visiting r, and continues by traversing T_1 in preorder, then T_2 in preorder, and so on, until T_n is traversed in preorder.

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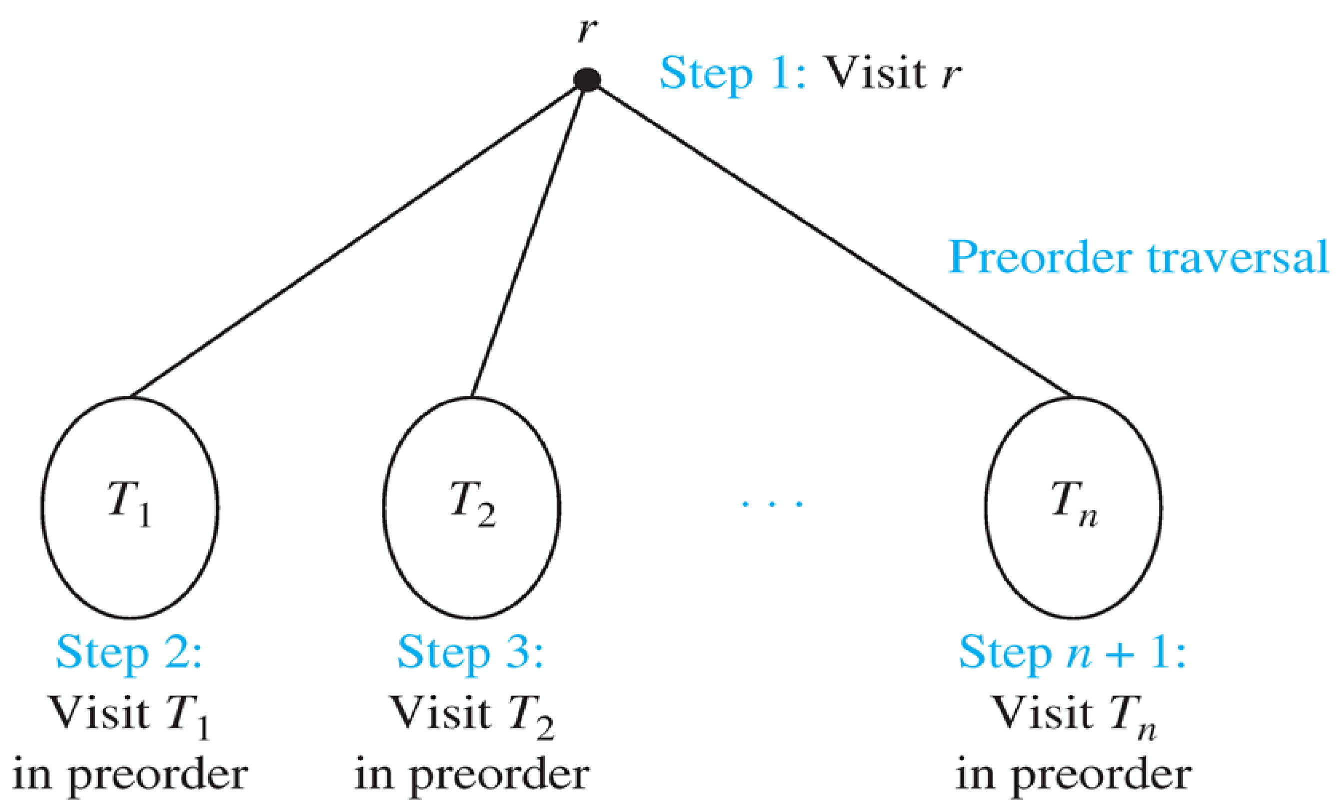
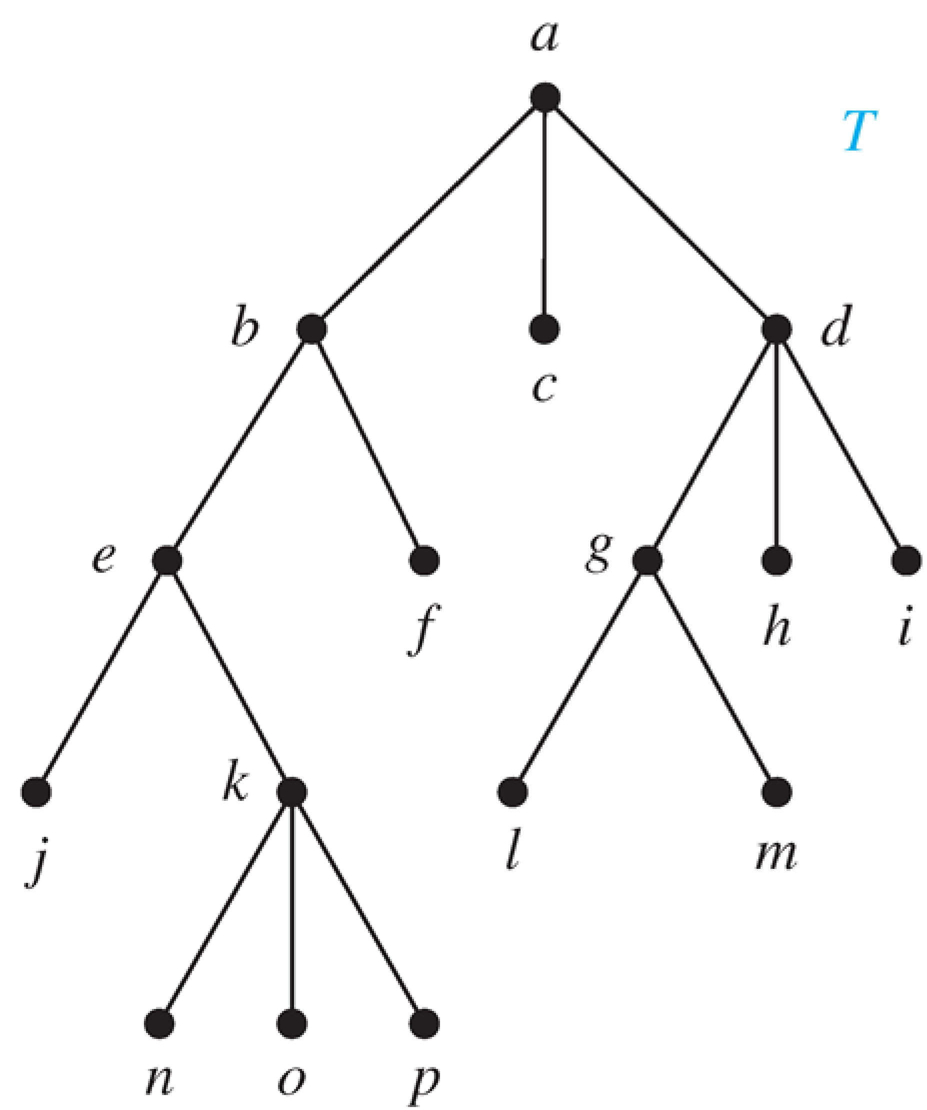


Preorder Traversal

- **Example**



Preorder Traversal

procedure *preorder* (T: ordered rooted tree)
r := root of T
list r
for each child c of r from left to right
    $T(c)$:= subtree with c as root
    preorder($T(c)$)

Inorder Traversal

- **Definition** Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *inorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees of r from left to right in T. The *inorder traversal* begins by traversing T_1 in inorder, then visiting r, and continues by traversing T_2 in inorder, and so on, until T_n is traversed in inorder.

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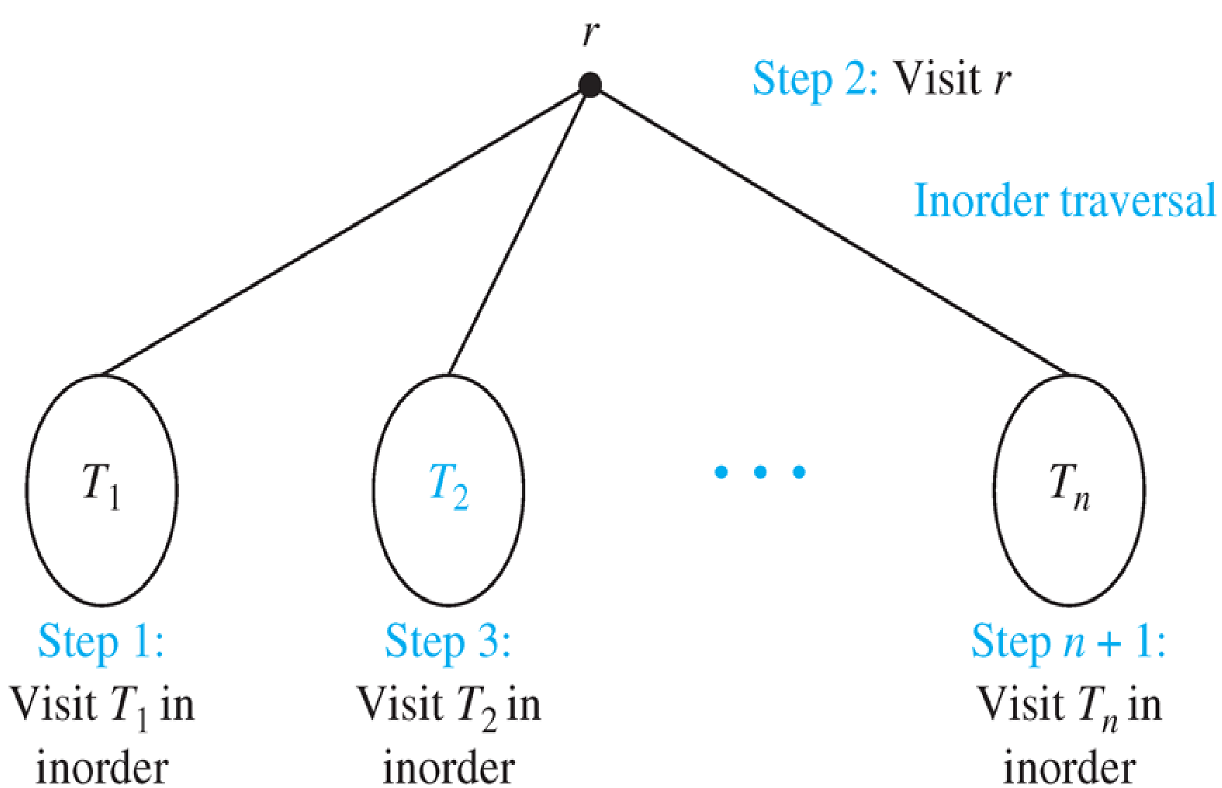

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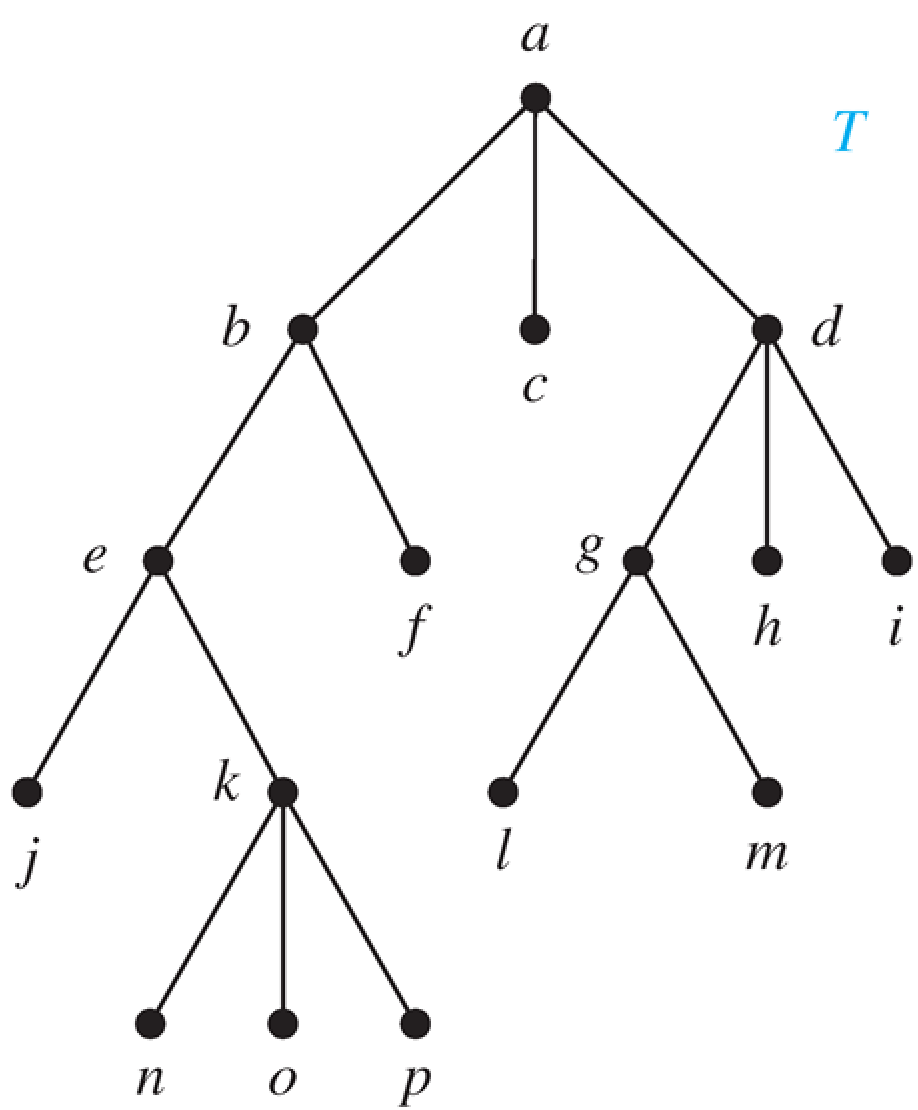

Inorder Traversal

procedure *inorder* (T: ordered rooted tree)
r := root of T
if r is a leaf **then** list r
else
  l := first child of r from left to right
  $T(l)$:= subtree with l as its root
  inorder($T(l)$)
  list(r)
  for each child c of r from left to right
    $T(c)$:= subtree with c as root
    inorder($T(c)$)

Postorder Traversal

- **Definition** Let T be an ordered rooted tree with root r. If T consists only of r, then r is the *postorder traversal* of T. Otherwise, suppose that $T_1, T_2, \ldots, T_n$ are the subtrees of r from left to right in T. The *postorder traversal* begins by traversing T_1 in postorder, then T_2 in postorder, and so on, after T_n is traversed in postorder, r is visited.

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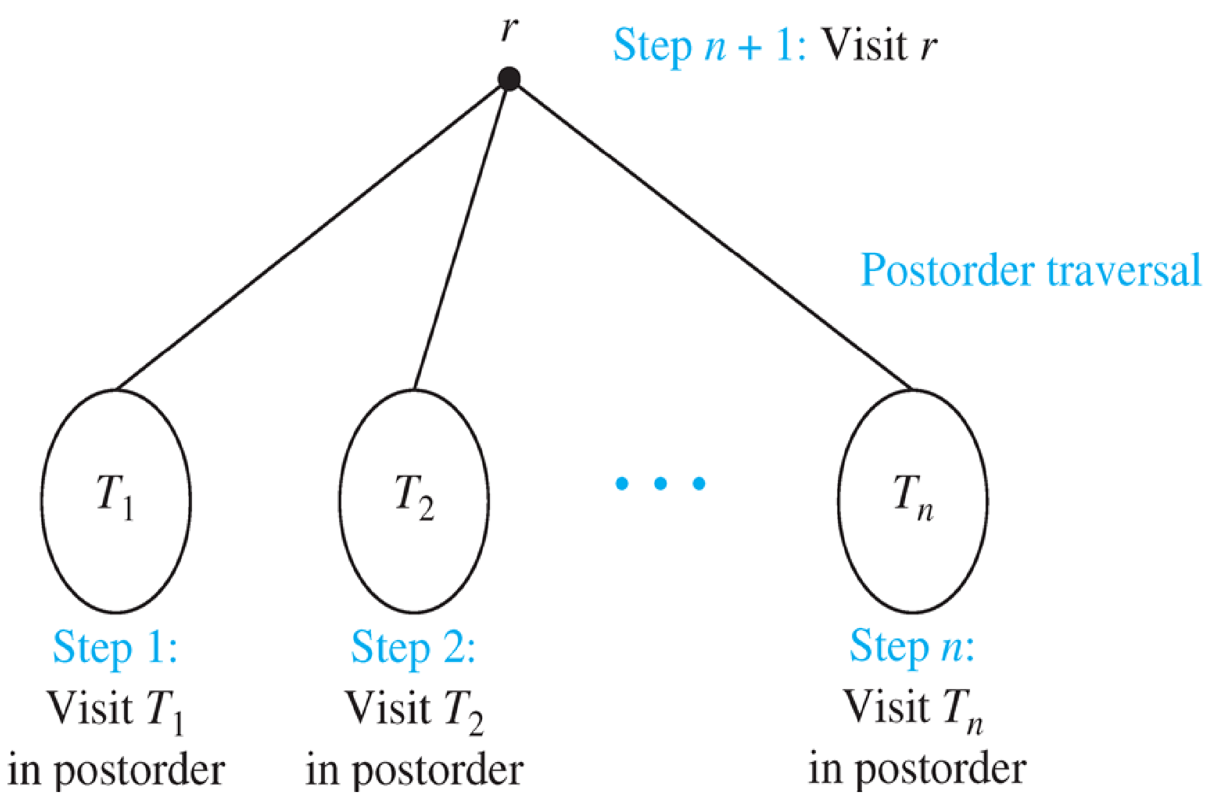

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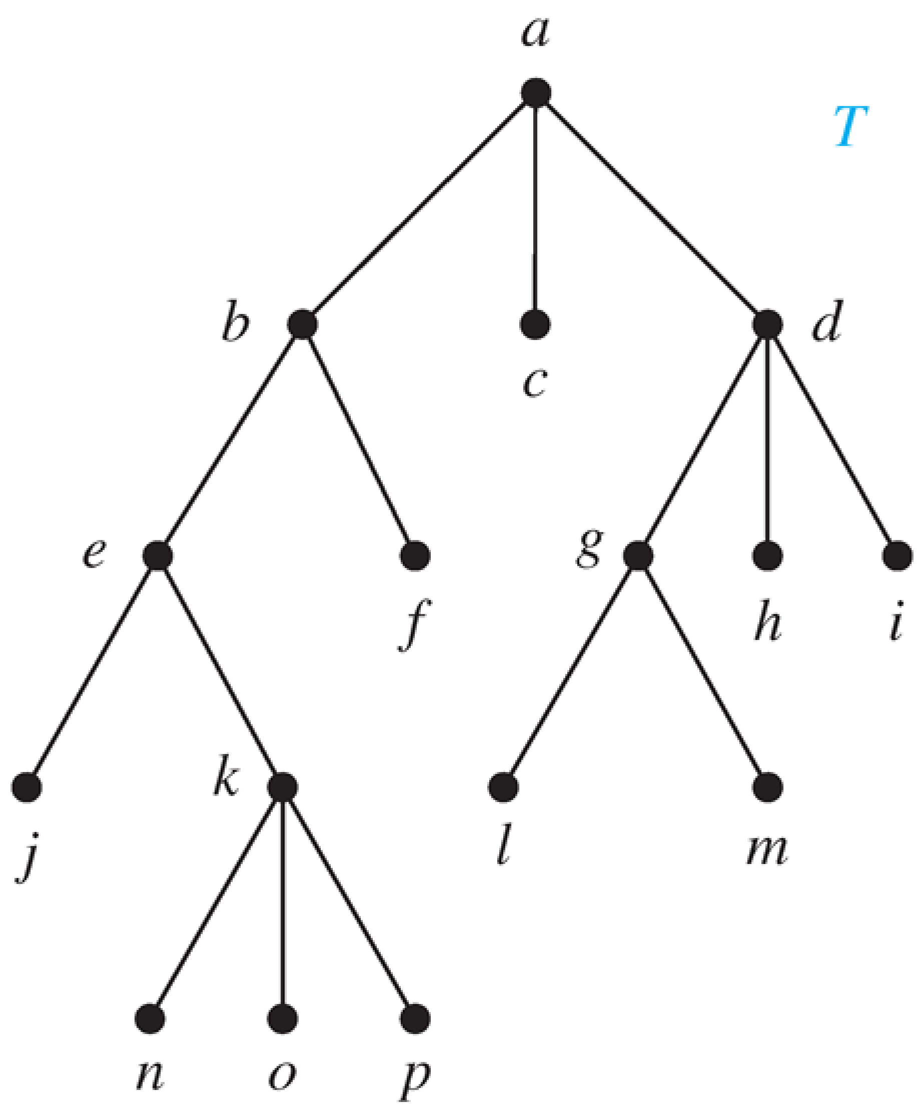
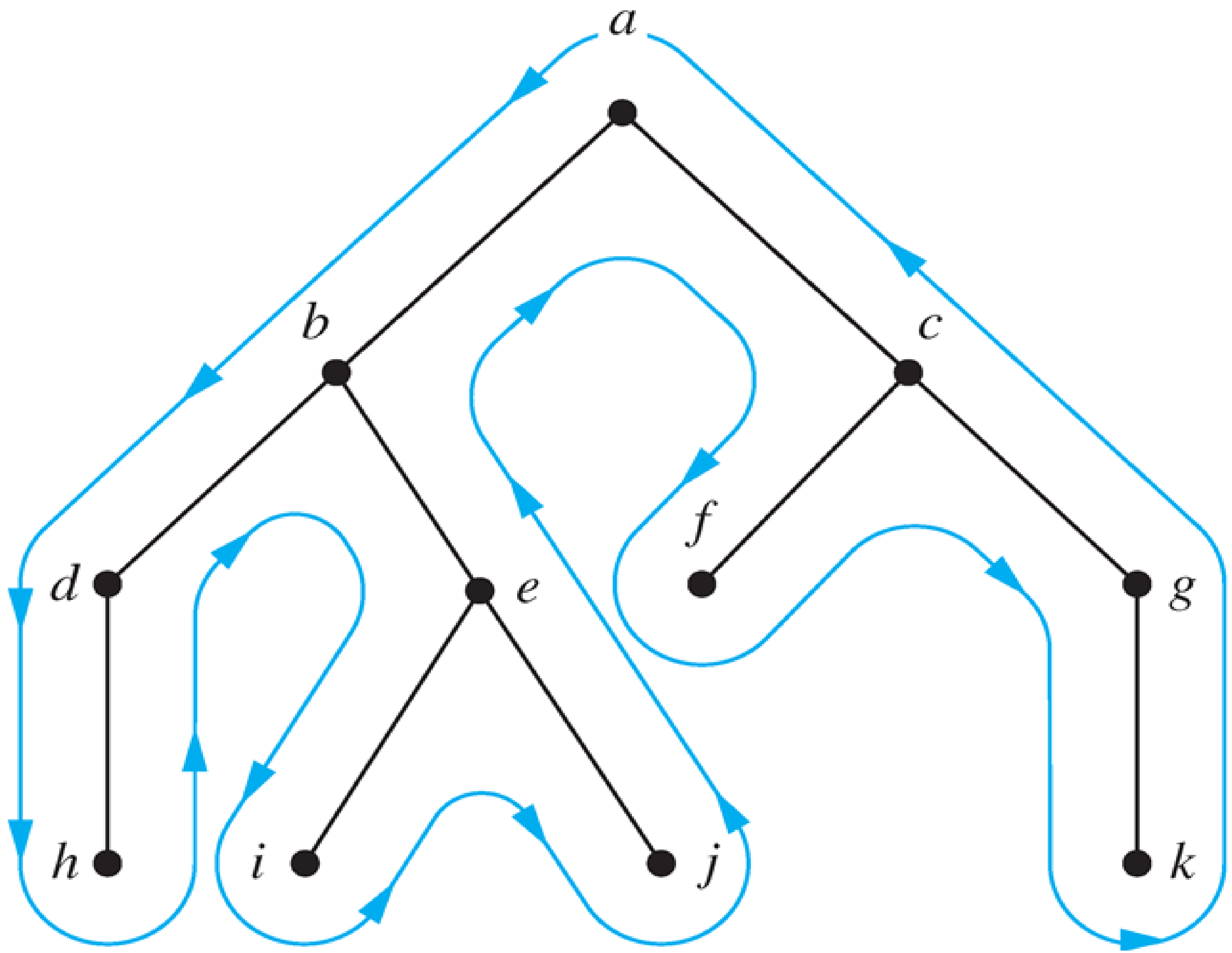


Postorder Traversal

procedure *postordered* (T: ordered rooted tree)
r := root of T
for each child c of r from left to right
    $T(c)$:= subtree with c as root
    postorder($T(c)$)
list r

Preorder, Inorder, Postorder Traversal



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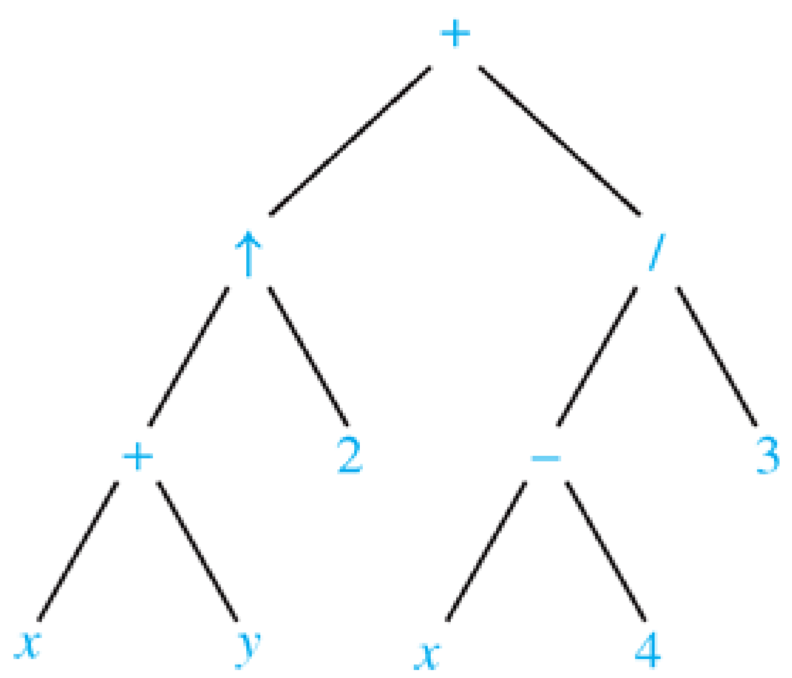

Infix Notation

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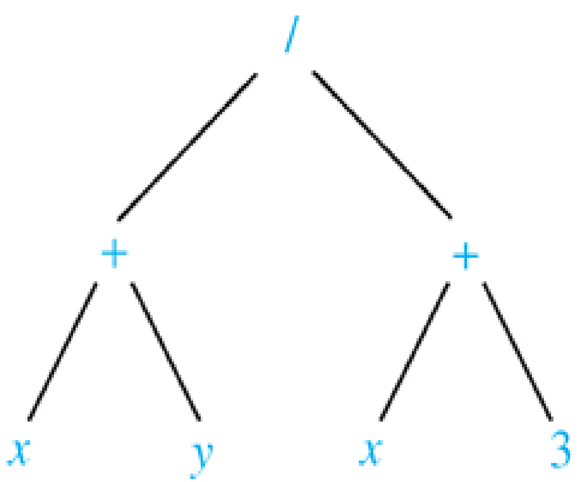

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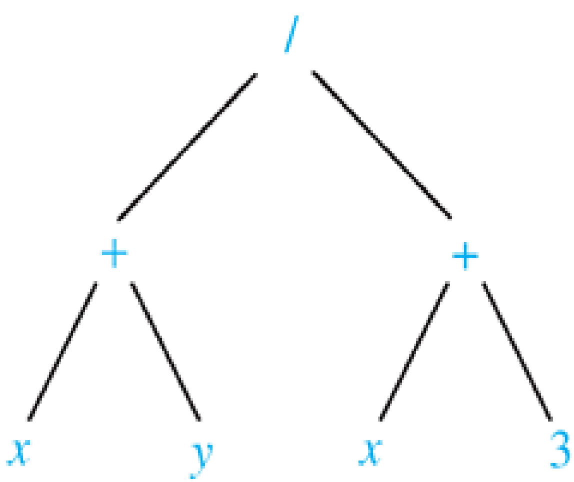

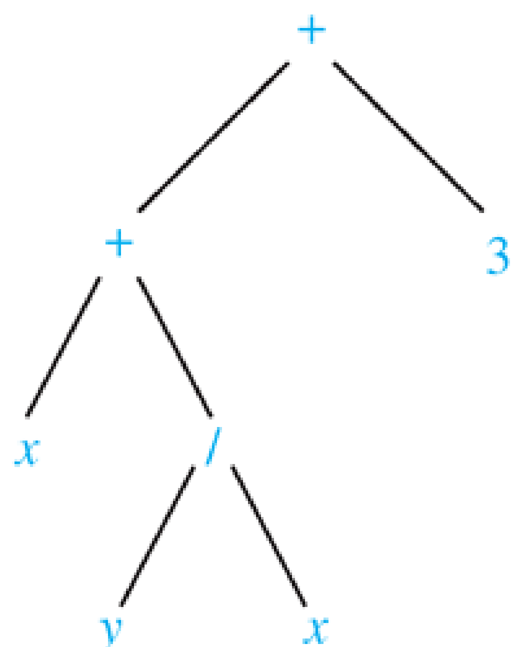

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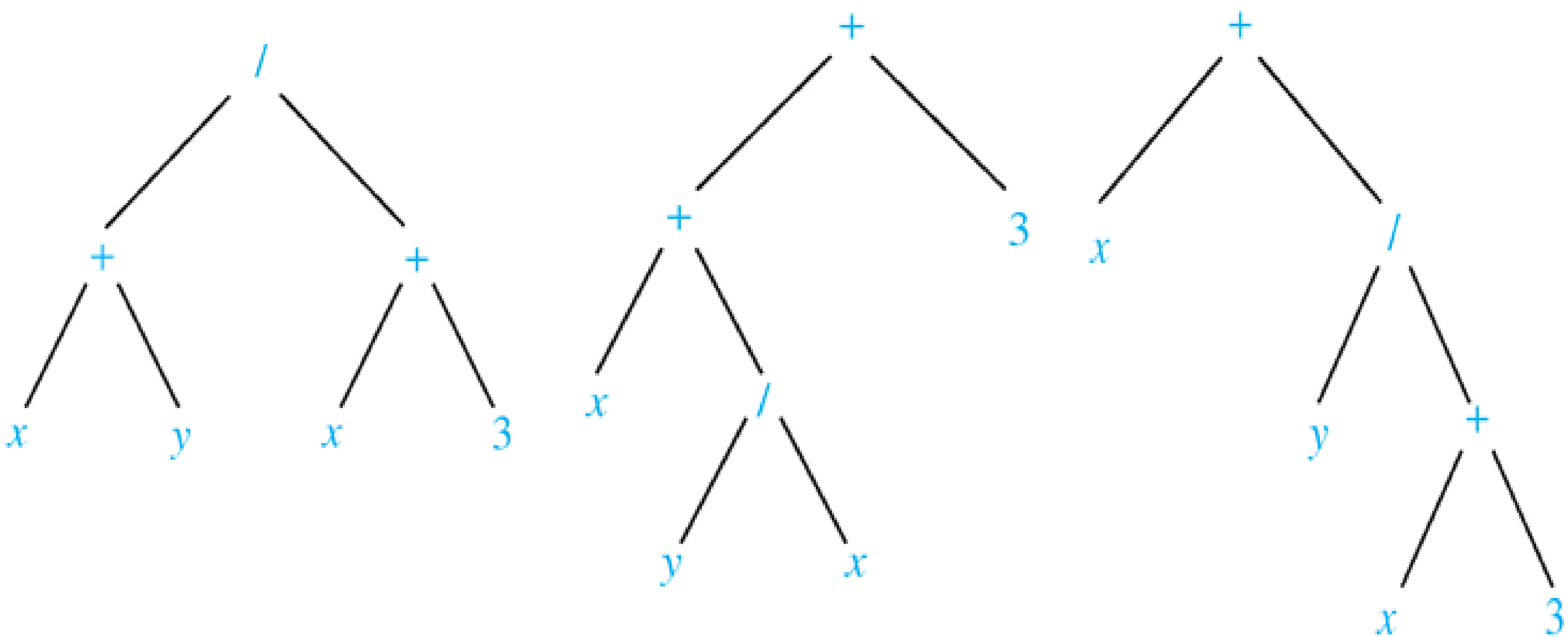

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  Operators precede their operands in the prefix notation. Parentheses are not needed as the representation is unambiguous.

  Prefix expressions are evaluated by working from right to left. When we encounter an operator, we perform the operation with the two operands to the right.

Prefix Notation

- **Example**

  $+ \; - \; * \, 2 \; 3 \; 5 \; / \; \uparrow \; 2 \; 3 \; 4$

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- **Example**

$$+ \; - \; * \; 2 \; 3 \; 5 \; / \; \uparrow \; 2 \; 3 \; 4$$

$$+ \quad - \quad * \quad 2 \quad 3 \quad 5 \quad / \quad \underbrace{\uparrow \quad 2 \quad 3}_{2 \uparrow 3 = 8} \quad 4$$

$$+ \quad - \quad * \quad 2 \quad 3 \quad 5 \quad \underbrace{/ \quad 8 \quad 4}_{8 / 4 = 2}$$

$$+ \quad - \quad \underbrace{* \quad 2 \quad 3}_{2 * 3 = 6} \quad 5 \quad 2$$

$$+ \quad \underbrace{- \quad 6 \quad 5}_{6 - 5 = 1} \quad 2$$

$$\underbrace{+ \quad 1 \quad 2}_{1 + 2 = 3}$$

Postfix Notation

- The postorder traversal of expression trees leads to the *postfix form* of the expression (*reverse Polish notation*).

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  Postfix expressions are evaluated by working from left to right. When we encounter an operator, we perform the operation with the two operands to the left.

Postfix Notation

- **Example**

  $7\ 2\ 3\ *\ -\ 4\ \uparrow\ 9\ 3\ /\ +$

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$$\underbrace{7\ 2\ 3\ *}_{2*3=6}\ -\ 4\ \uparrow\ 9\ 3\ /\ +$$

$$\underbrace{7\ 6\ -}_{7-6=1}\ 4\ \uparrow\ 9\ 3\ /\ +$$

$$\underbrace{1\ 4\ \uparrow}_{1^4=1}\ 9\ 3\ /\ +$$

$$1\ \underbrace{9\ 3\ /}_{9/3=3}\ +$$

$$\underbrace{1\ 3\ +}_{1+3=4}$$

Spanning Trees

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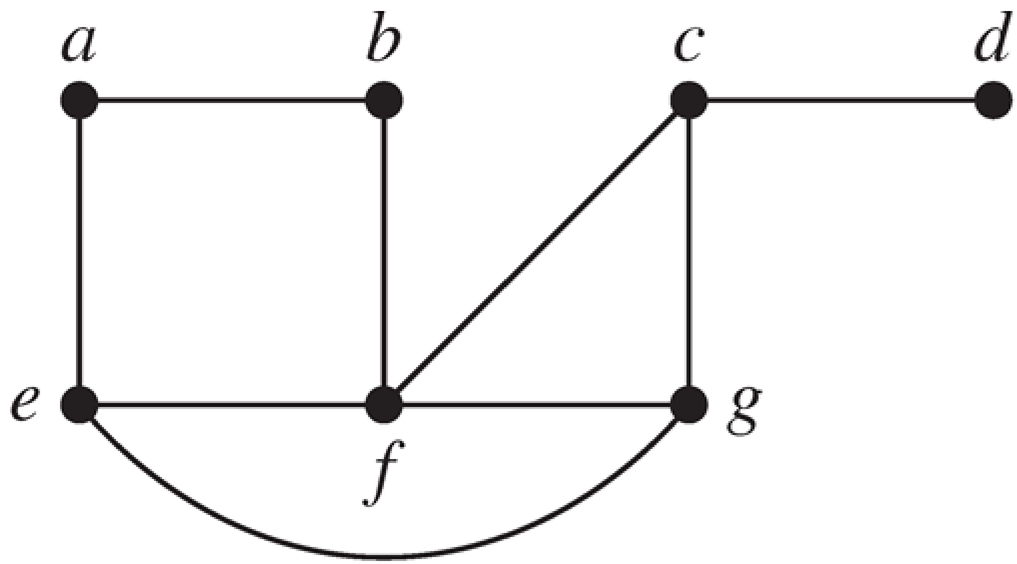

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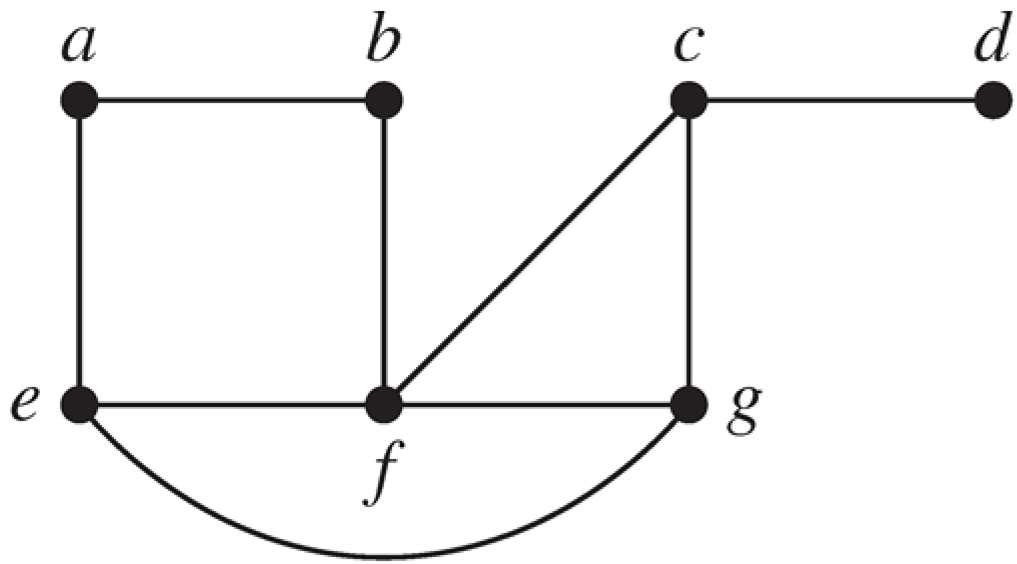

remove edges to avoid circuits

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  Continue adding to this path as long as possible.

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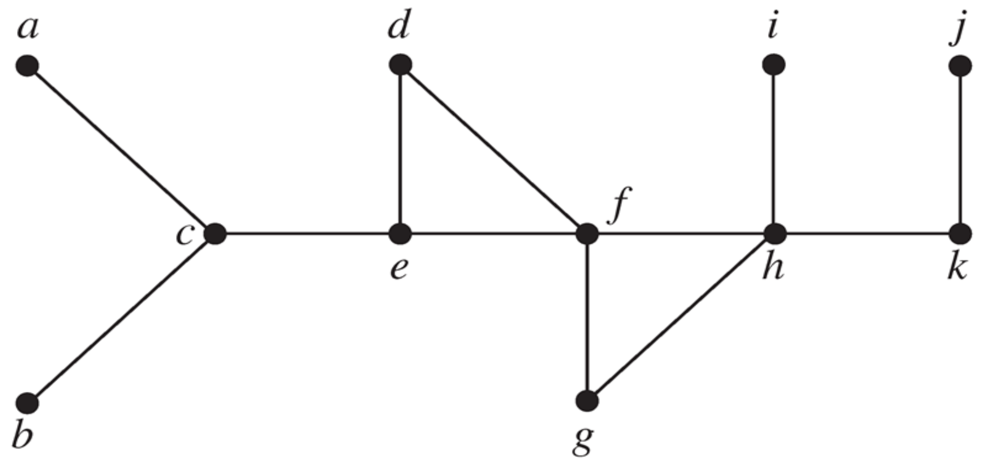
  ◇ Otherwise, move back to some vertex to repeat this procedure (*backtracking*)

- **Example**



Depth-First Search

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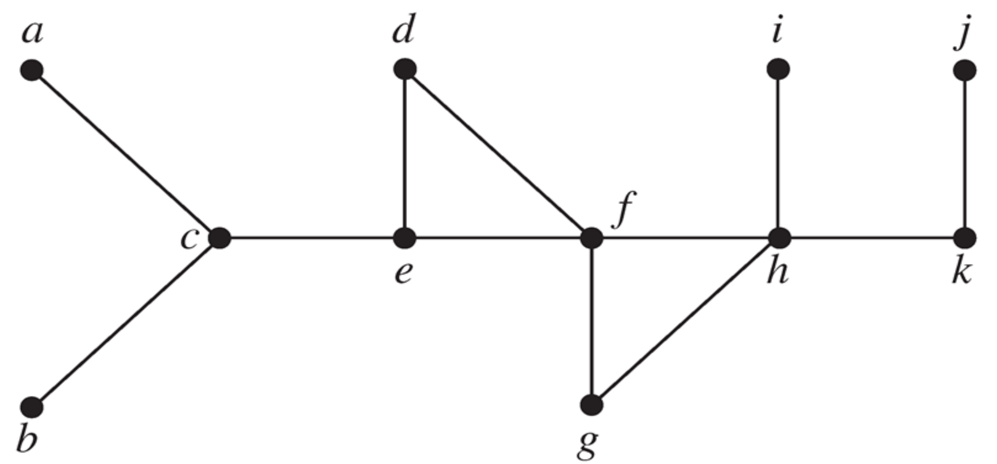

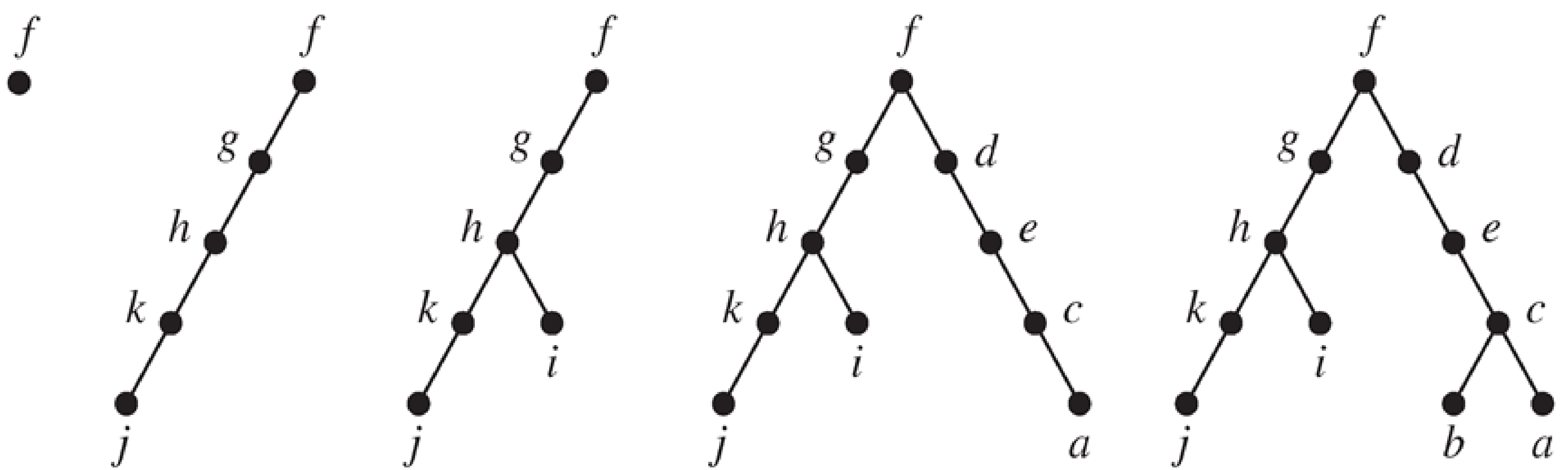

Depth-First Search Algorithm

procedure *DFS*(G: connected graph with vertices $v_1, v_2, \ldots, v_n$)
T := tree consisting only of the vertex v_1
visit(v_1)

procedure *visit*(v: vertex of G)
for each vertex w adjacent to v and not yet in T
  add vertex w and edge $\{v, w\}$ to T
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time complexity: $O(e)$

Breadth-First Search

- This is the second algorithm that we build up spanning trees by successively adding edges.

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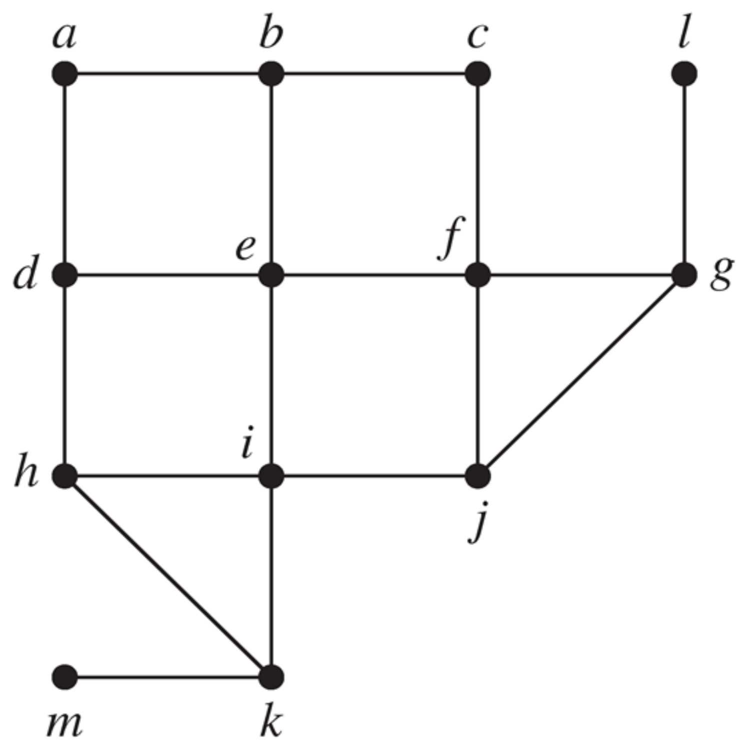
  ◇ First arbitrarily choose a vertex of the graph as the root.

  ◇ Form a path by adding all edges incident to this vertex and the other endpoint of each of these edges

  ◇ For each vertex added at the previous level, add edge incident to this vertex, as long as it does not produce a simple circuit.

  ◇ Continue in this manner until all vertices have been added.

- **Example**



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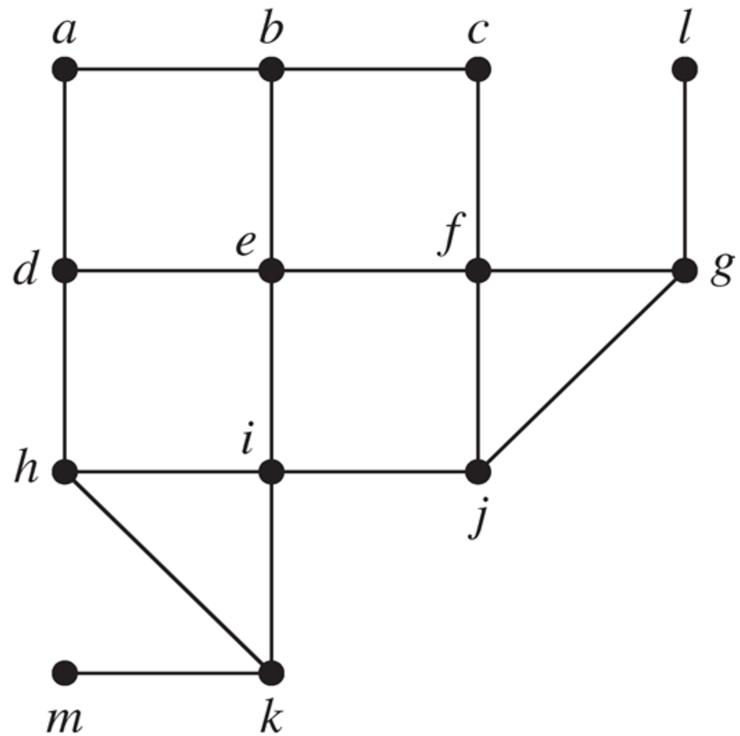

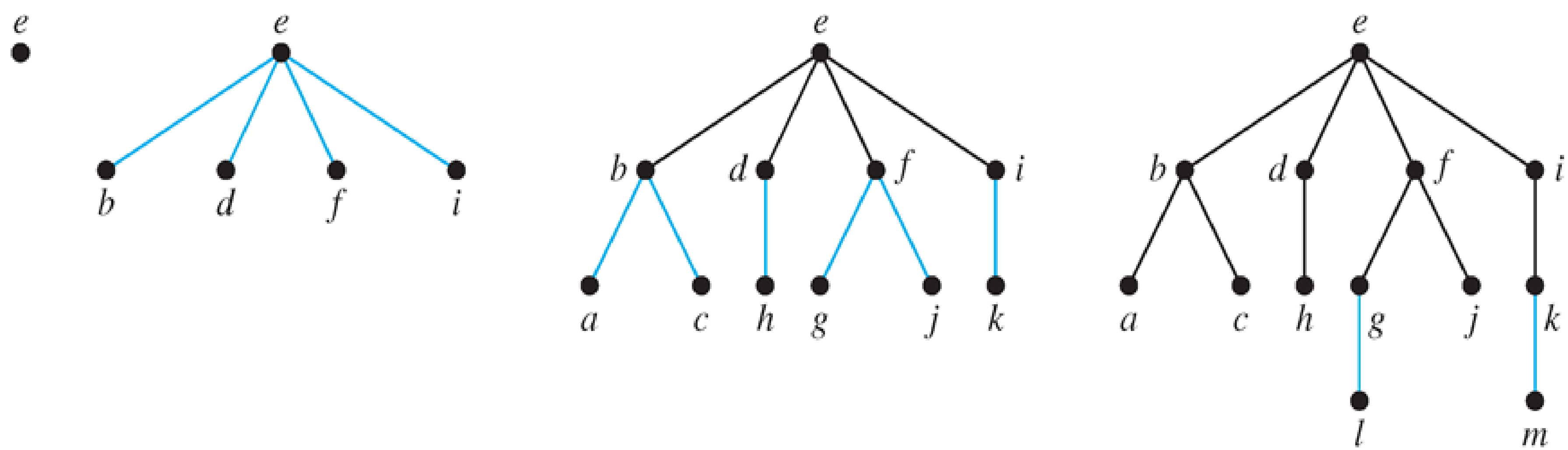

Breadth-First Search

procedure *BFS*(G: connected graph with vertices $v_1, v_2, \ldots, v_n$)
T := tree consisting only of the vertex v_1
L := empty list *visit*(v_1)
put v_1 in the list L of unprocessed vertices
while L is not empty
    remove the first vertex, v, from L
    for each neighbor w of v
        if w is not in L and not in T **then**
            add w to the end of the list L
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Applications of DFS, BFS

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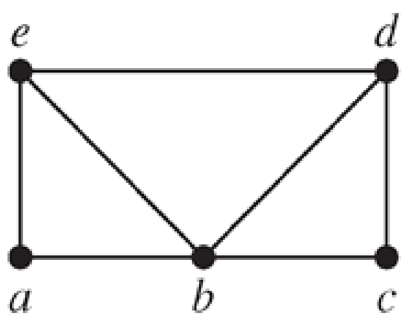
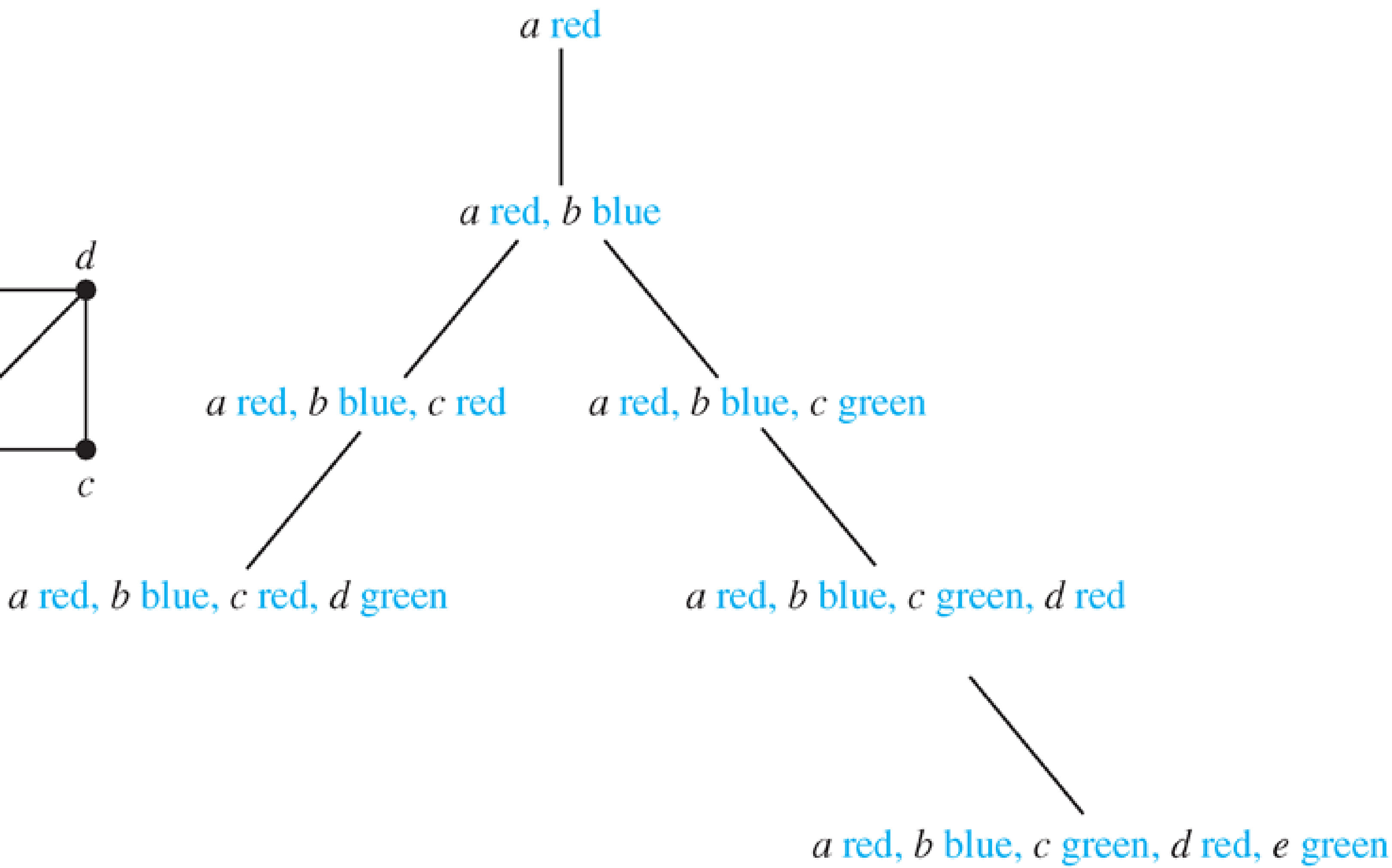
  graph coloring, sums of subsets, ...

Applications of DFS, BFS





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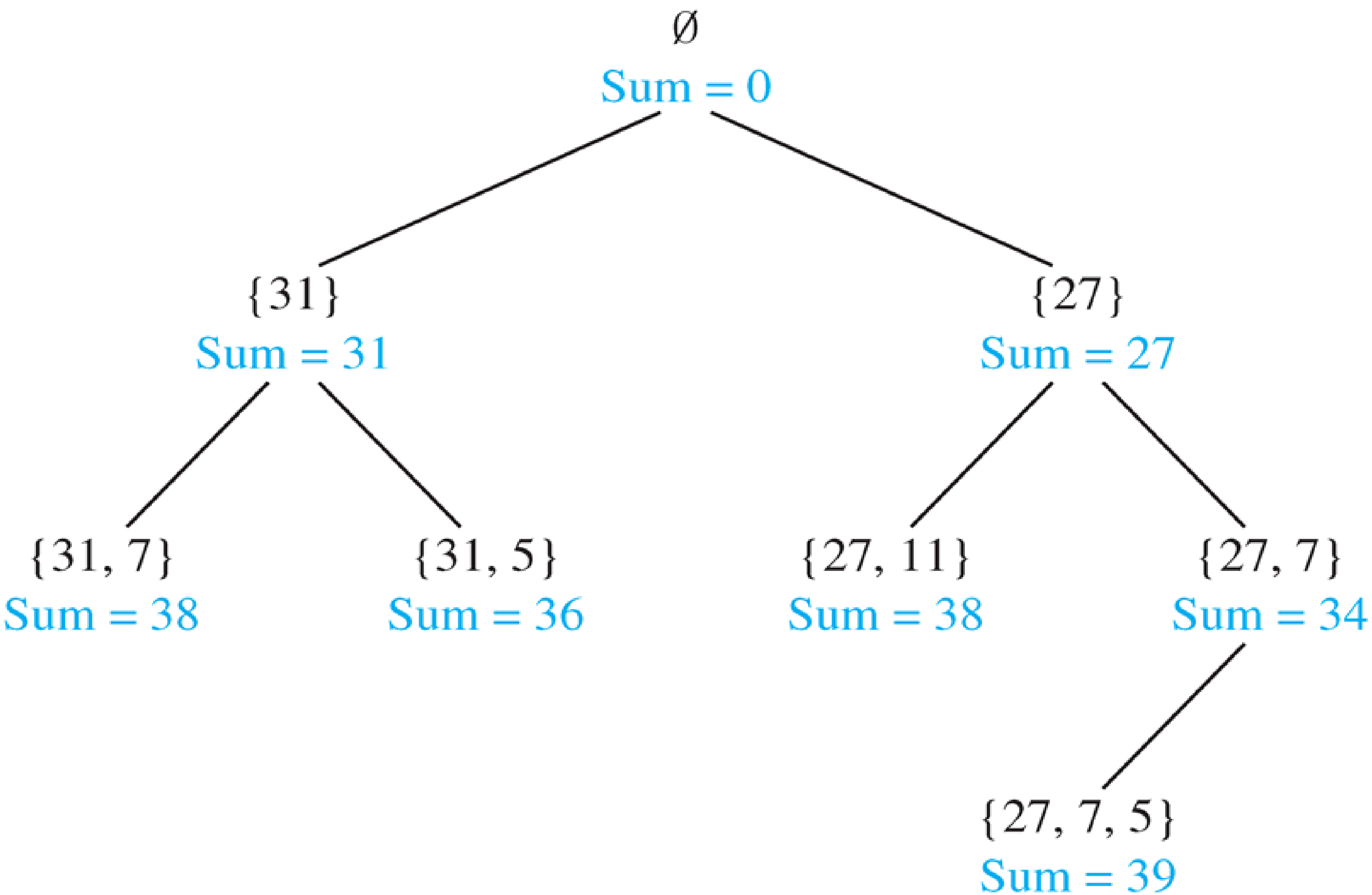
  find

  grap



find a subset of $\{31, 27, 15, 11, 7, 5\}$ with the sum 39

Minimum Spanning Trees

- **Definition** A *minimum spanning tree* in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

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  two greedy algorithms: Prim's Algorithm, Kruscal's Algorithm

Prim's Algorithm

ALGORITHM 1 Prim's Algorithm.

procedure *Prim*(G: weighted connected undirected graph with n vertices)
T := a minimum-weight edge
for $i := 1$ **to** $n - 2$
    e := an edge of minimum weight incident to a vertex in T and not forming a simple circuit in T if added to T
    $T := T$ with e added
return T {T is a minimum spanning tree of G}

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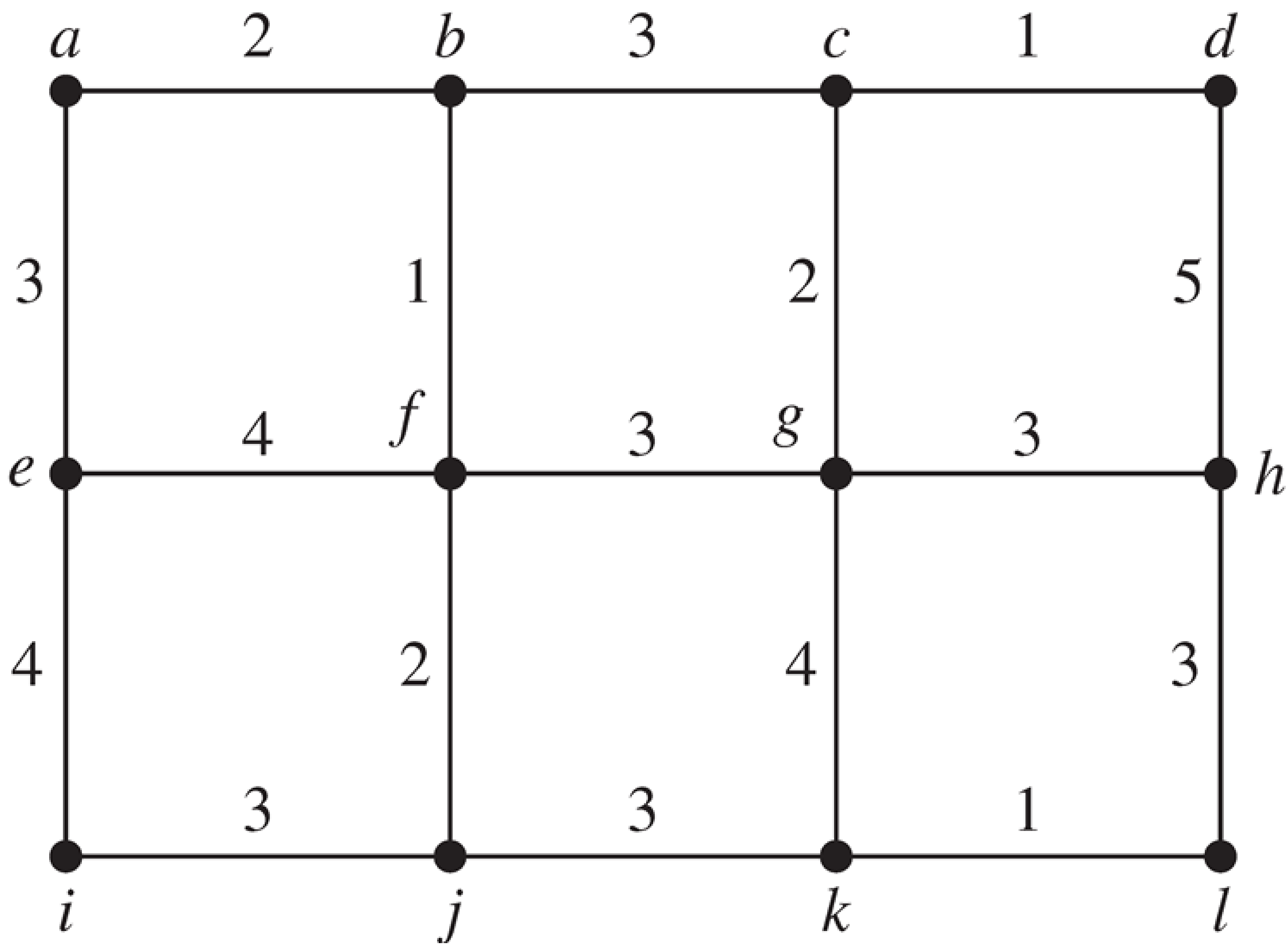
time complexity: $e \log v$

- **Example**



Kruscal's Algorithm

ALGORITHM 2 Kruskal's Algorithm.

procedure *Kruskal*(G: weighted connected undirected graph with n vertices)
T := empty graph
for $i := 1$ **to** $n - 1$
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see *CLRS* / *Algorithm Design*, J. Kleinberg, E. Tardos

Kruscal's Algorithm

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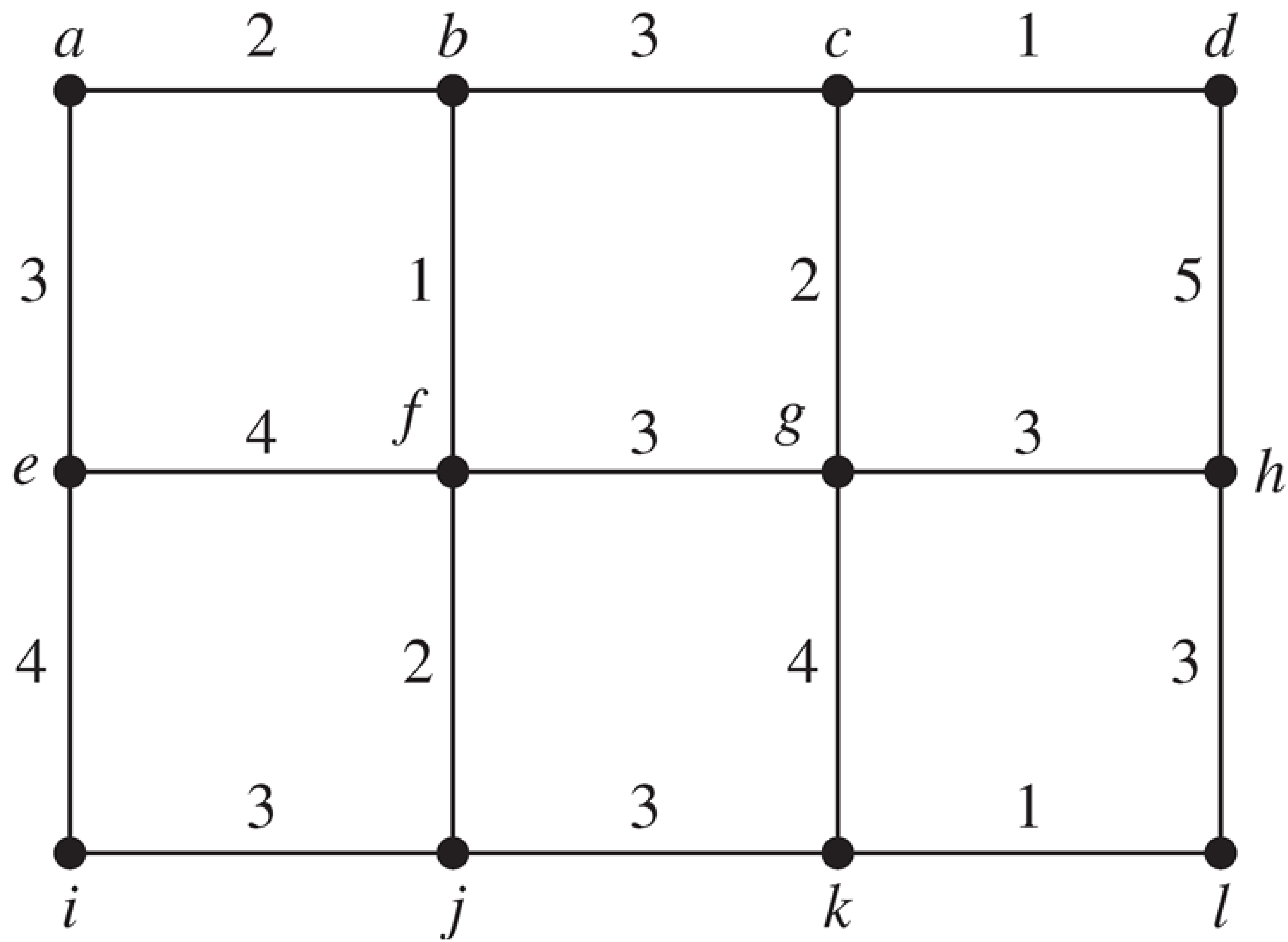

Next Lecture

- course review ...