Lab10 Solution

YAO ZHAO

Lab10.A:LGM

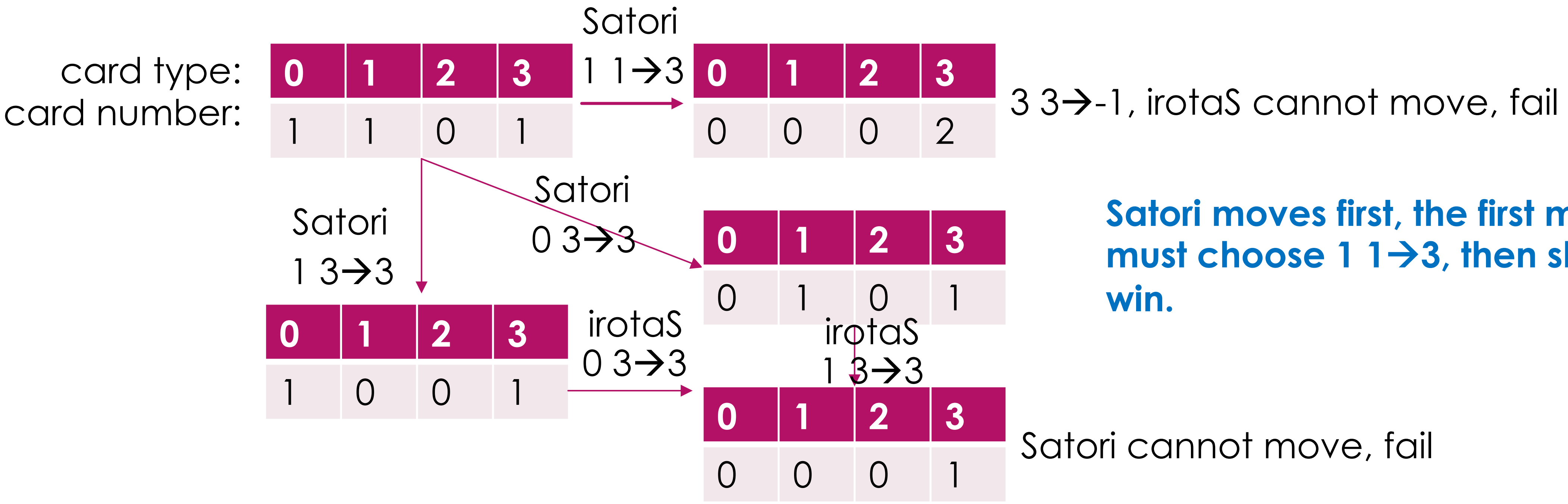
- Satori is an LGM, so she decided to play a card game with herself, irotaS.
- Satori found herself many cards. There are totally 4 types of cards. According to today's weather, both Satori and irotaS can consume 2 cards to produce 1 new card of certain type.
- Formally speaking, given a 4×4 symmetric matrix T whose entries are among $0, 1, 2, 3$ (the matrix index starts from 0). $T_{i,j} = k$ means that consuming one type i card and one type j card will produce a type k card. Some of the combinations are invalid and $k = -1$ for these cases.
- Satori and irotaS take turns to move. Satori moves first. In each move, the operator consumes two cards and produce a new one according to the matrix. The player who cannot move loses the game.
- Satori and irotaS decide to play Q rounds today. In round i, she has $a_{i,0}, a_{i,1}, a_{i,2}, a_{i,3}$ cards for each type, respectively. They wonder who will win the game in each round.

Sample 1 Input

3 3 3 3
3 3 3 3
3 3 0 3
3 3 3 -1
3
1 1 0 1
0 1 2 1
0 0 0 2

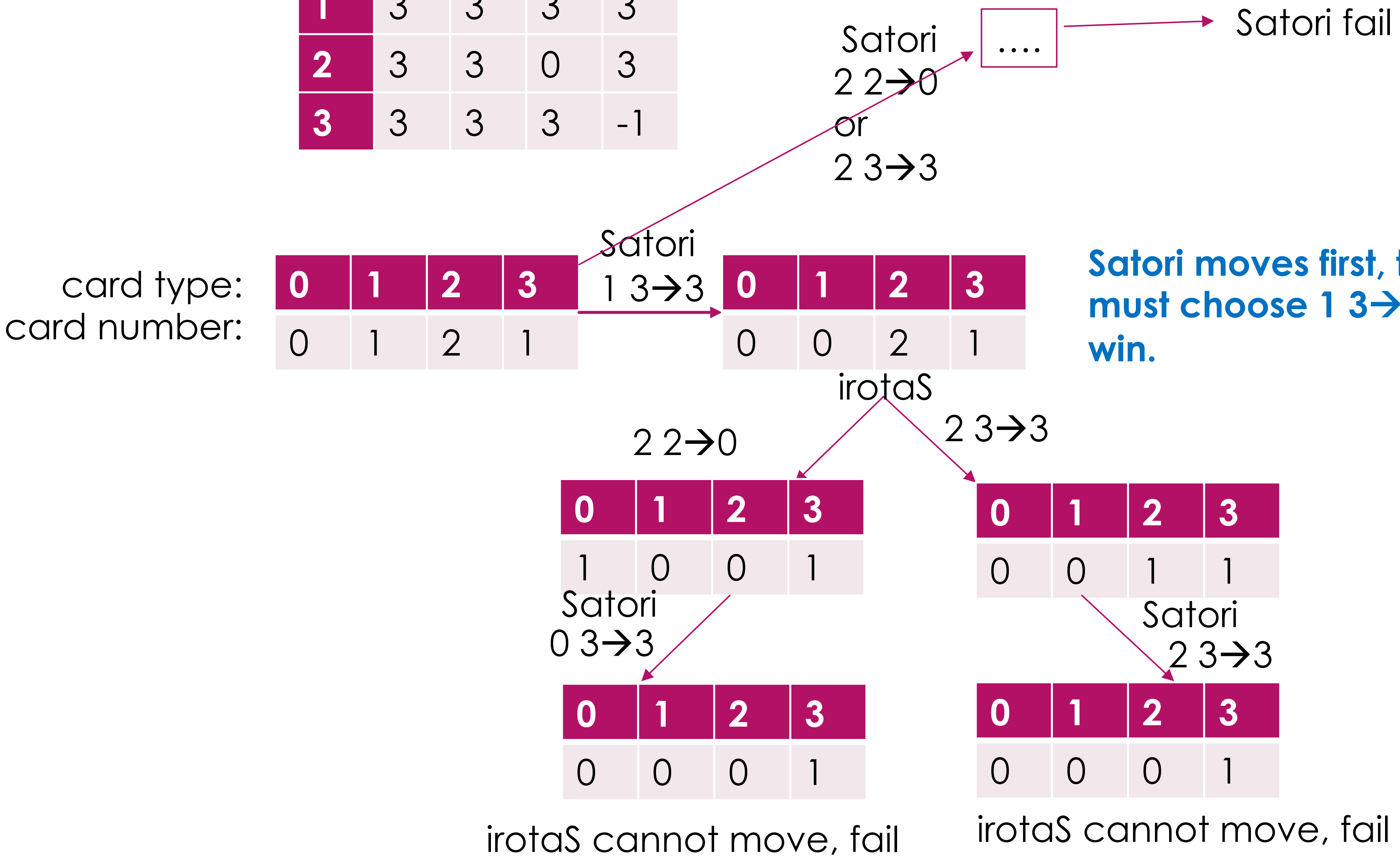
4×4 symmetric matrix T

	0	1	2	3
0	3	3	3	3
1	3	3	3	3
2	3	3	0	3
3	3	3	3	-1



Satori moves first, the first move must choose 1 1→3, then she will win.

	0	1	2	3
0	3	3	3	3
1	3	3	3	3
2	3	3	0	3
3	3	3	3	-1



Satori moves first, the first move must choose 1 3→3, then she will win.

	0	1	2	3
0	3	3	3	3
1	3	3	3	3
2	3	3	0	3
3	3	3	3	-1

	0	1	2	3
card type:	0	1	2	3
card number:	0	0	0	2

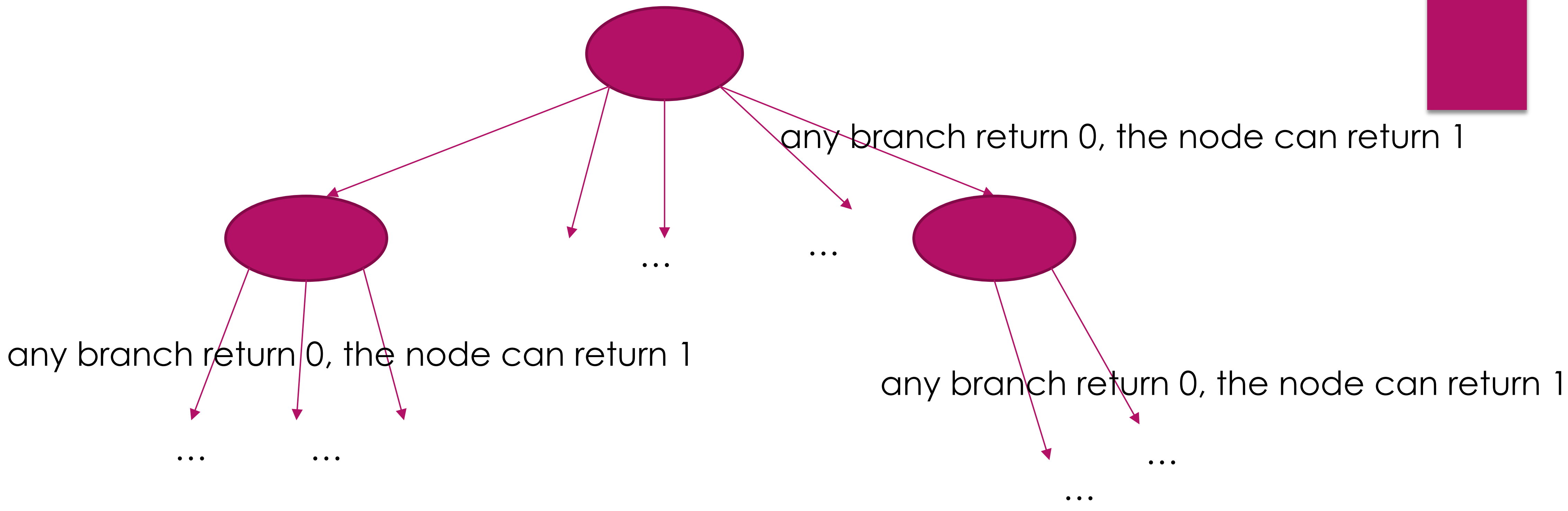
3 3→-1, Satori cannot move, fail

Sample 1 Output

1 1 0 1
0 1 2 1
0 0 0 2

→

Satori
Satori
irotaS


any branch return 0, the node can return 1
...
...
any branch return 0, the node can return 1
any branch return 0, the node can return 1
...
...
...
...

Lab10.B: Let there be love

- FluffyBunny wants to fly N balloons which are initially tied on the ground. She will manually untie balloons one at a time. At any time, if balloon $i-1$ and $i+1$ are both untied, balloon i will untie and fly away automatically.
- Now FluffyBunny wonders the number of ways to fly all the N balloons. Two ways are considered different if either the set of balloons she fly manually is different or the order of balloons she manually fly is different.
- As the answer might be extremely large, please output the number modulo M.

Sample 1 Input

3 100000007

1	2	3

m:{1, 3} auto:{2}
m:{3, 1} auto:{2}
m:{1, 2, 3} auto:ø
m:{3, 2, 1} auto:ø
m:{2, 1, 3} auto:ø
m:{2, 3, 1} auto:ø

Sample 1 Output

6

1	2	3	4

Sample 2 Input

4 100000007

m:{1, 3, 4} auto:{2}
m:{4, 1, 3} auto:{2}
m:{4, 3, 1} auto:{2}
m:{3, 1, 4} auto:{2}
m:{1, 4, 3} auto:{2}
m:{3, 4, 1} auto:{2}
m:{1, 2, 4} auto:{3}
m:{2, 4, 1} auto:{3}
m:{4, 2, 1} auto:{3}
m:{1, 4, 2} auto:{3}
m:{1, 3, 4} auto:{3}
m:{4, 3, 1} auto:{3}

m:{1, 2, 3, 4} auto:ø
m:{2, 3, 4, 1} auto:ø
m:{2, 3, 1, 4} auto:ø
m:{2, 1, 3, 4} auto:ø
m:{3, 4, 2, 1} auto:ø
m:{3, 2, 1, 4} auto:ø
m:{3, 2, 4, 1} auto:ø
m:{4, 3, 2, 1} auto:ø

Sample 2 Output

20

auto:ø ways number

1	2	3

m:{1, 2, 3} auto:ø
m:{3, 2, 1} auto:ø
m:{2, 1, 3} auto:ø
m:{2, 3, 1} auto:ø

1	2	3	4

m:{1, 2, 3, 4} auto:ø
m:{2, 3, 4, 1} auto:ø
m:{2, 3, 1, 4} auto:ø
m:{2, 1, 3, 4} auto:ø
m:{3, 4, 2, 1} auto:ø
m:{3, 2, 1, 4} auto:ø
m:{3, 2, 4, 1} auto:ø
m:{4, 3, 2, 1} auto:ø

If manually fly balloon 1 first, must fly balloon 2, 3, 4, …n in order.
If manually fly balloon 2 first, must fly balloon 3, 4, …n in order, fly balloon 1 can insert somewhere.

3, 4, …n
…

$$C_{n-2+1}^{1}$$

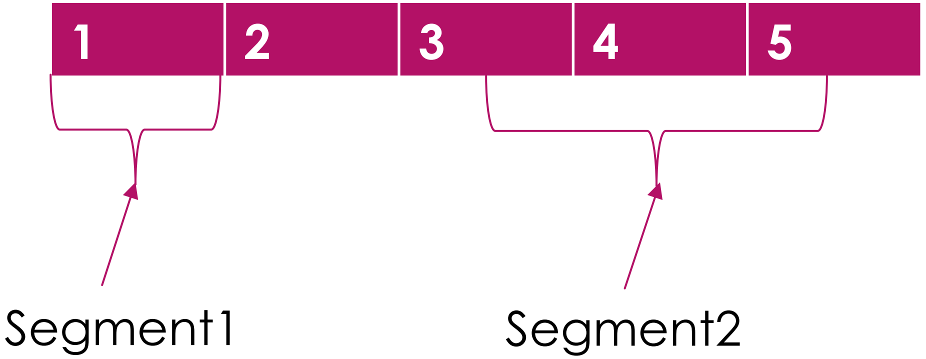
If manually fly balloon 3 first, must fly balloon 4, …n in order, fly balloon 2, 1 can insert somewhere.
2, 1 also should insert in order

$$C_{n-3+2}^{2}$$

4, …n

$$C_{n-1}^{0} + C_{n-1}^{1} + \cdots C_{n-1}^{n-1} = 2^{n-1}$$

Each balloon is untied either manually or automatically. An automatically untied balloon cannot be fl untied by other automatically untied balloons. For any sequence of manually fly operations, all balloons must be one segment manual + one automatic + one segment manual + one automatic +... etc.



only let balloon 2 to be auto flied:

All balloons in Segment1 should be manually untied
All balloons in Segment2 should be manually untied

Let OPT[len][j] present the number of ways to untie len – 1 balloons that contains j manual operations, the last one (the balloon len)will be untied automatically.

To transition, iterate the length k of a new segment of manual untie balloons to add and calculate the number of ways with formula:
OPT[len + k+1][j+k] +=OPT[len][j]* $C_{j+k}^{k} * 2^{k-1}$ answer = $\sum_{j=1}^{n} opt[n+1][j]$

	1	2	3	4	5
1	0				
2	1	0			
3	0	2	0		
4	0	0	4	0	
5	0	0	0	8	0
6	0	0	0	0	16

initial all OPT[i+1][i] to 2^{i-1}

the pre length no auto

	1	2	3	4	5
1	0				
2	1	0			
3		2	0		
4		2	4	0	
5			6	8	0
6				16	16

OPT[2][1]=1

k = 1: OPT[4][2]+=1*$C_2^1 * 2^0 = 2$

k = 2: OPT[5][3]+=1*$C_3^2 * 2^1 = 6$

k = 3: OPT[6][4]+=1*$C_4^3 * 2^2 = 16$

the balloon 2 can be auto untied

	1	2	3	4	5
1	0				
2	1	0			
3		2	0		
4		2	4	0	
5			12	8	0
6				40	16

OPT[3][2]=2

k=1: OPT[5][3] +=2* $C_3^1 * 2^0$=6+6 = 12

k=2: OPT[6][4]+=2*$C_4^2 * 2^1$ = 16+24 = 40

the balloon 3 can be auto untied

	1	2	3	4	5
1	0				
2	1	0			
3		2	0		
4		2	4	0	
5			12	8	0
6			6	56	16

OPT[4][2]=2

k=1: OPT[6][3] +=2* $C_3^1 * 2^0$=0+6=6

OPT[4][3]=4

k=1: OPT[6][4] += 4* $C_4^1 * 2^0$=40+16=56

the balloon 4 can be auto untied