



CS201H DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

Dr. QI WANG

Department of Computer Science and Engineering

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Course Information

- Instructor:

Dr. QI WANG

Department of Computer Science and Engineering

Office: Room413, CoE South Tower

Email: wangqi@sustech.edu.cn

- TA:

Assignment marker & problem collector

- Course webpage:

sakai.sustech.edu.cn → “[CS201 H fall2021](#)”

QQ chat group: [658439426](#)



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Course Information

- Lectures:

Teaching Building 1

Wednesday Rm405

19:00 – 20:50

every week

Friday Rm403

14:00 – 15:50

every two weeks



Course Information

- Lectures:

Teaching Building 1

Wednesday Rm405

19:00 – 20:50

every week

Friday Rm403

14:00 – 15:50

every two weeks

- Office hours:

Thu. 16:00-18:00 or send email for appointment



Course Information

- Lecture Notes (in progress)

CS201 Discrete Math for Computer Science¹

Qi Wang

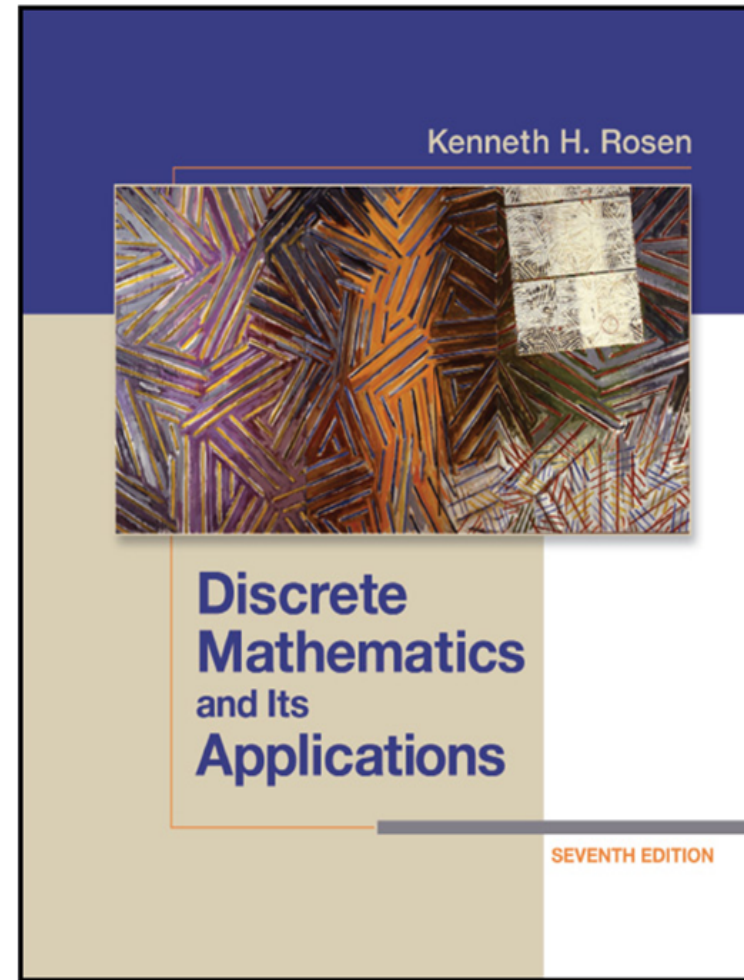
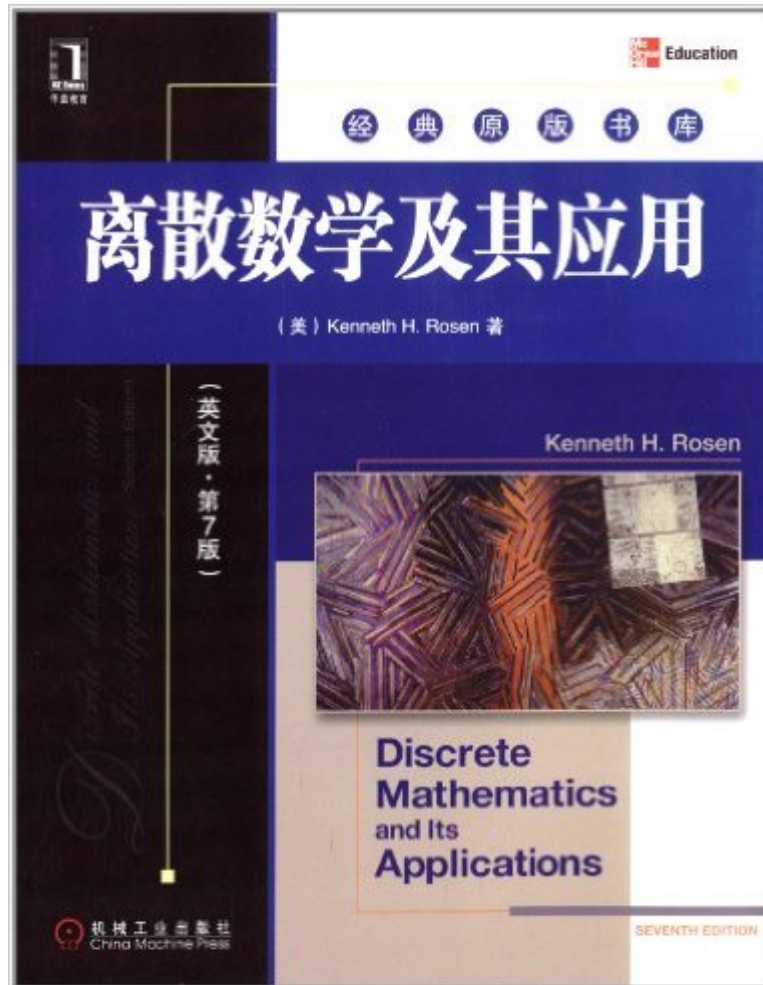
Department of Computer Science and Engineering
Southern University of Science and Technology
Spring 2019

¹©Copyright, Qi Wang, 2019, Dept. of Computer Science and Engineering, Southern University of Science and Technology, No. 1088 Xueyuan Blvd., Xili Nanshan District, Shenzhen 518055, Guangdong, China. These lecture notes were prepared by Qi Wang for the course CS201 Discrete Math for Computer Science, at the Southern University of Science and Technology. Permission to use, copy, modify, and distribute these notes for educational purposes and without fee is hereby granted, provided that this copyright notice appears in all copies.



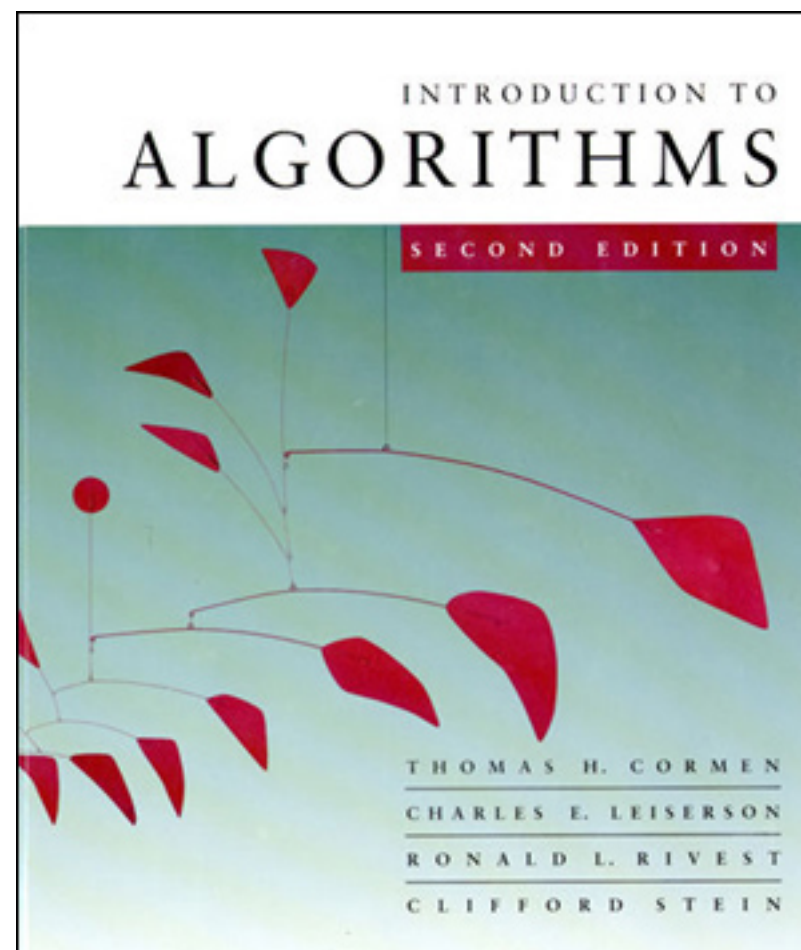
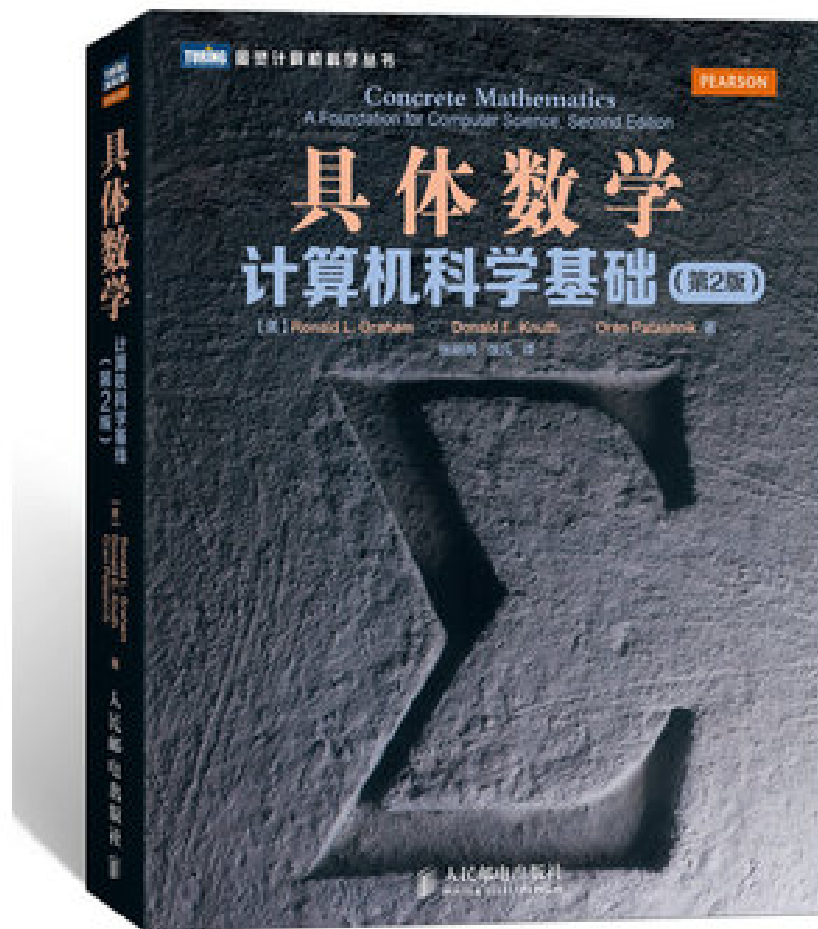
Course Information

■ Textbook:



Course Information

■ Reference Books:



Marking Scheme

- Quiz in class (10%)
- Homework assignments (20%)
 - ◇ assigned in class and posted on the course webpage
 - ◇ must be submitted **online before** due date
 - ◇ no extension policy with exceptions
- Midterm exam (30%)
 - ◇ To be announced
 - ◇ covers the first half of class material
- Final exam (40%)
 - ◇ covers the entire semester's material
- Project (5%, **optional**)

Important Messages to Students

- Main ideas will be covered in class but some details might be skipped. You are responsible for **all materials** in assigned sections of book, even if they are not taught in class.
- Homework assignments should be worked on and written up **individually**, though group discussions are allowed.
- Any unintellectual behavior and cheating on exams, homework assignments will be dealt with **severely**.
- If you get the main idea for a solution from someone else or a website, you **must acknowledge** that source in your submission.



Plagiarism Policy

- * If an undergraduate assignment is found to be plagiarized, the first time the score of the assignment will be 0.
- * The second time the score of the course will be 0.

As it may be difficult when two assignments are identical or nearly identical who actually wrote it, the policy will apply to BOTH students, unless one confesses having copied without the knowledge of the other.



What is OK and what is not OK?

- It's OK to work on an assignment with a friend, and think together about the program structure, share ideas and even the global logic. At the time of actually writing the code, you should write it alone.
- It's OK to use in an assignment a piece of code found on the web, as long as you indicate in a comment where it was found and don't claim it as your own work.
- It's OK to help friends debug their programs (you'll probably learn a lot yourself by doing so).
- It's OK to show your code to friends to explain the logic, as long as the friends write their code on their own later.

It's NOT OK to take the code of a friend, make a few cosmetic changes (comments, some variable names) and pass it as your own work.



What is OK and what is not OK?



南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

计算机科学与工程系
Department of Computer Science and Engineering

Undergraduate Students Assignment Declaration Form

This is _____ (student ID: _____, who has enrolled in _____ course, originated the Department of Computer Science and Engineering. I have read and understood the regulations on plagiarism in assignments and theses according to "Regulations on Academic Misconduct in Assignments for Undergraduate Students in the SUSTech Department of Computer Science and Engineering". I promise that I will follow these regulations during the study of this course.

Signature:

Date:



Important Messages to Students

- Please ask questions in class



If you're having trouble understanding something, then at least 50% of the class is also having trouble: they'll *be happy* if you ask for more explanation.

- The lecture slides are still *in progress*



If you find mistakes or just think that something's confusing, please email me. Your classmates and future students will *thank you*.



Discrete Mathematics

■ What is discrete mathematics?

◇ *Discrete mathematics* is the study of *mathematical structures* that are fundamentally discrete rather than continuous.

For example, integers, graphs, statements in logic, etc.

◇ Discrete mathematics therefore excludes topics in “**continuous mathematics**” such as *Calculus* and *Analysis*.



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■ Some applications

◇ *Computer science*: algorithms, programming languages, cryptography, artificial intelligence, ...

◇ *Electronic engineering*: digital communications, signal processing, information theory, coding theory, ...



Discrete Mathematics

The Abel Prize Laureates 2021

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2021 to

László Lovász

of Alfréd Rényi Institute of Mathematics (ELKH, MTA Institute of Excellence) and Eötvös Loránd University in Budapest, Hungary, and

Avi Wigderson

of the Institute for Advanced Study, Princeton, USA



“For their foundational contributions to theoretical computer science and discrete mathematics, and their leading role in shaping them into **central fields** of modern mathematics.”



Topics of This Course

- Propositional and Predicate Logic
- Mathematical Proofs
- Sets
- Functions
- Complexity of Algorithms
- Number Theory
- Groups, Rings, and Fields
- Cryptography
- Mathematical Induction
- Recursion
- Counting
- Discrete Probability*
- Relations
- Graphs
- Trees
- Finite Fields



Lecture Schedule (Tentatively)

- | | |
|------------------------------|------------------------------|
| 01. Logic | 09. Mathematical Induction |
| 02. Mathematical Proofs | 10. Recursion |
| 03. Sets and Functions | 11. Counting |
| 04. Complexity of Algorithms | 12. Discrete Probability |
| 05. Number Theory | 13. Relations |
| 06. Groups, Rings and Fields | 14. Graphs I |
| 07. Cryptography | 15. Graphs II |
| 08. Midterm Exam | 16. Trees and Review Lecture |



Learning Objectives

- Be able to read, understand, and construct **mathematical arguments and proofs**
- Understand the formulation of common problems in several areas of discrete mathematics, including *counting, graphs, number theory, cryptography, logic and proof, recursions, probability theory, finite fields*, etc.
- Learn a number of **discrete mathematical** tools
- Apply discrete mathematical tools to solve certain problems in **computer science** and **electronic engineering**



Acknowledgements

- Some of slides are based on lecture material used in the following institutions:



Problem I

- **Q. 1** Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.



Problem I

- **Q. 1** Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

Proof I (*direct proof*)

Note that $3n + 3$ is even, and so is $n + 1$. It then follows that n is odd.



Problem I

- **Q. 1** Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

Proof II (*proof by contrapositive*)

It is *equivalent* to prove that “If n is even , then $3n + 2$ is even.” – **Why?**

w.l.o.g. suppose that $n = 2k$ for some integer k , then

$$3n + 2 = 6k + 2,$$

which is even.



Problem I

- **Q. 1** Prove that “For an integer n , if $3n + 2$ is odd, then n is odd”.

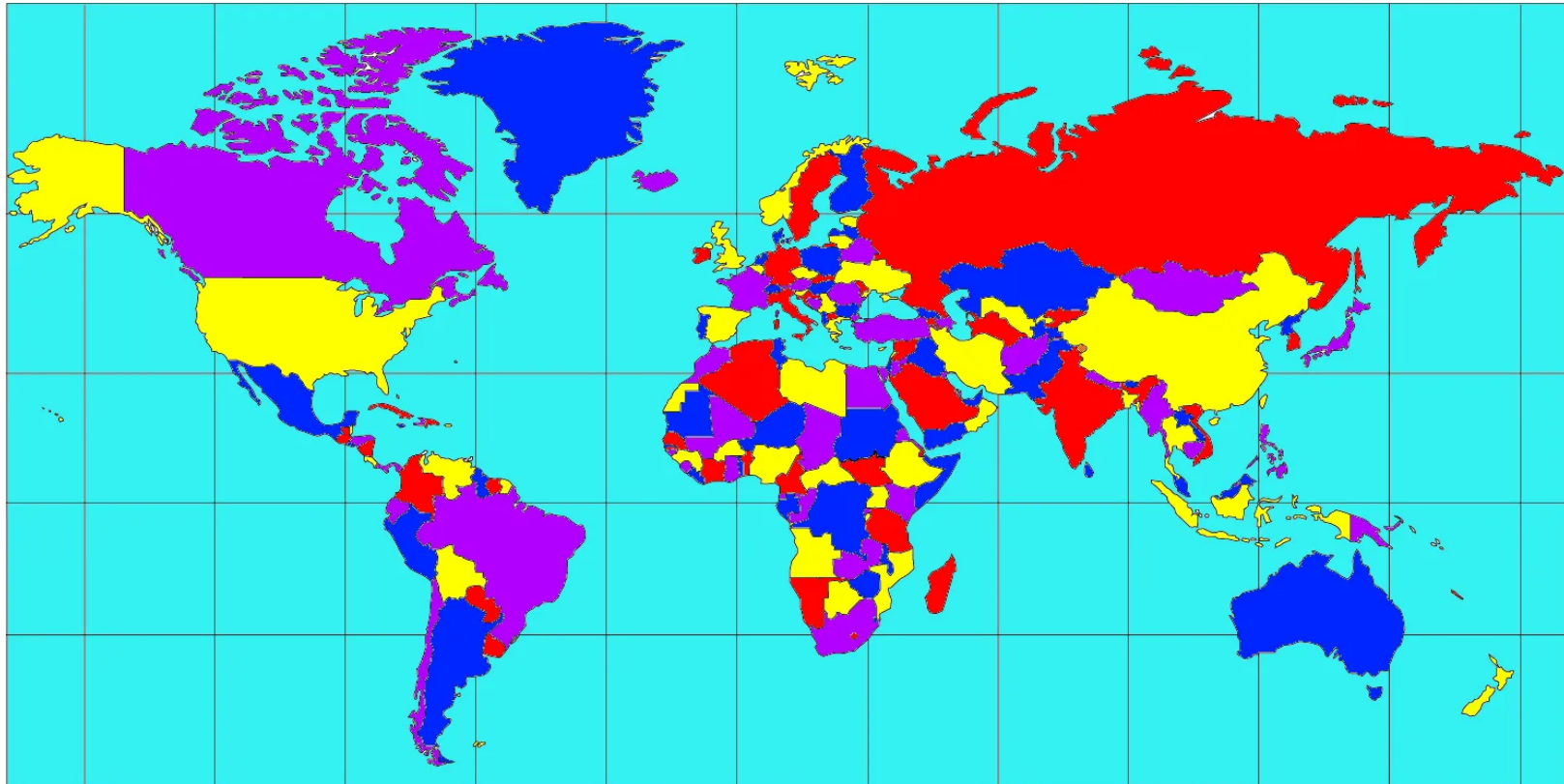
Proof III (*proof by contradiction*)

Assume to **the contrary** that there exists an integer n such that $3n + 2$ is odd and n is even. Since n is even, so is $3n$. Then 2 must be odd, leading to a **contradiction**.



Problem II

- **Four-color theorem** Given any separation of a plane into contiguous regions, producing a figure called a *map*, no more than four colors are required to color the regions of the map so that no two adjacent regions have the same color.



Problem II

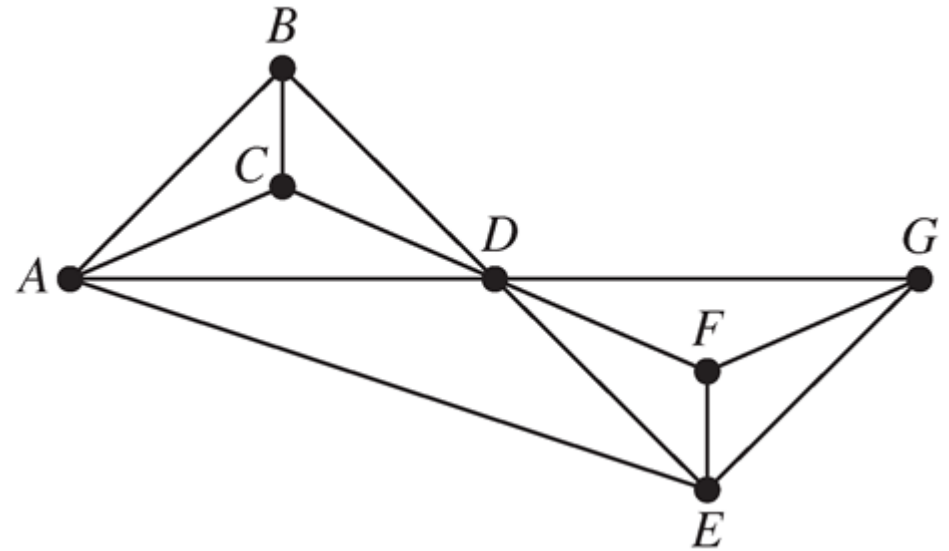
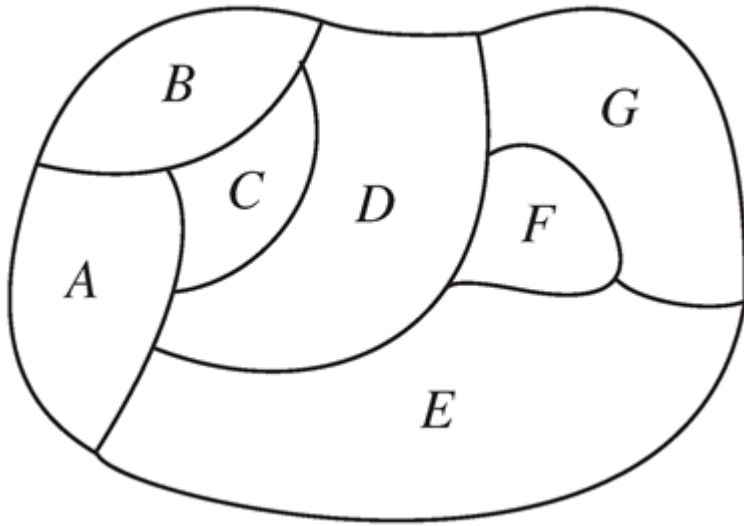
■ **Four-color theorem** (for planar graphs)

- ◇ first proposed by Francis Guthrie in 1852
- ◇ his brother Frederick Guthrie told Augustus De Morgan
- ◇ De Morgan wrote to William Hamilton
- ◇ Alfred Kempe proved it **incorrectly** in 1879
- ◇ Percy Heawood found an error in 1890 and proved the *five-color theorem*
- ◇ Finally, Kenneth Appel and Wolfgang Haken proved it with case by case analysis by computer in 1976 (*the first computer-aided proof*)
- ◇ Kempe's incorrect proof serves as a basis



Problem II

■ Four-color theorem



Problem II

- Applications of *graph colorings*

- ◇ Scheduling Final Exams

Vertices represent courses, and there is an edge between two vertices if there is a **common student** in the courses.

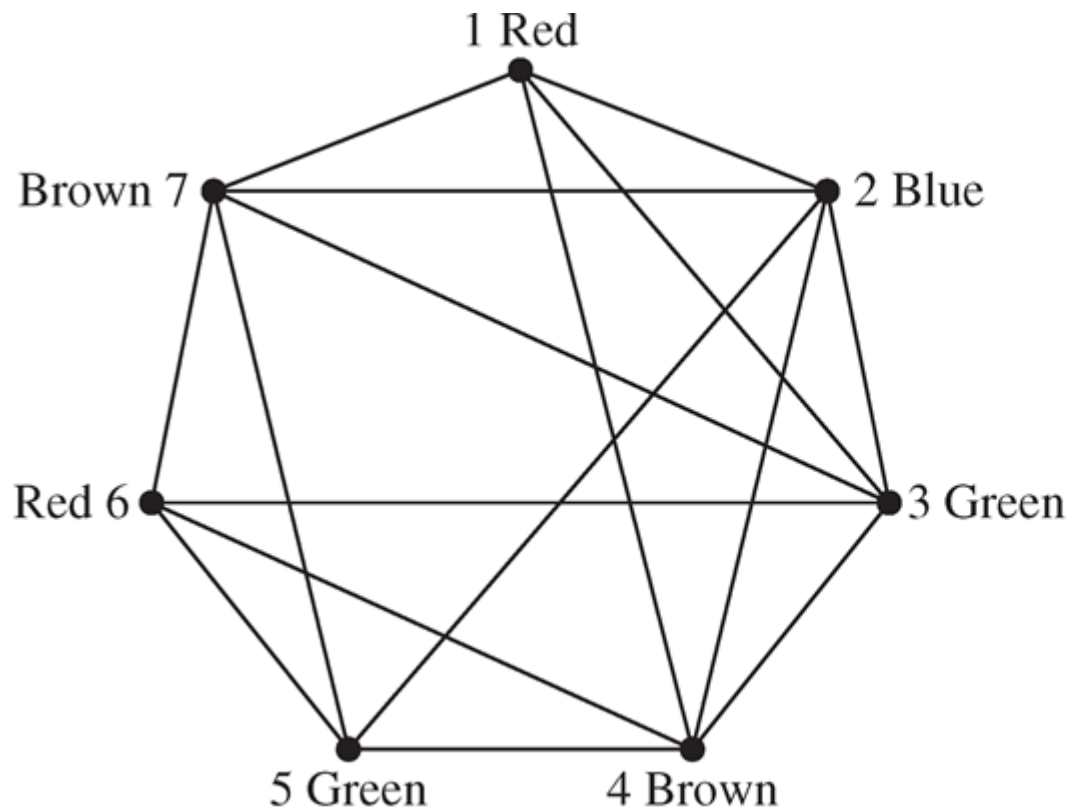


Problem II

■ Applications of *graph colorings*

◇ Scheduling Final Exams

Vertices represent courses, and there is an edge between two vertices if there is a **common student** in the courses.



Time Period

I

II

III

IV

Courses

1, 6

2

3, 5

4, 7



Problem II

■ Applications of *graph colorings*

◇ Scheduling Final Exams

Vertices represent courses, and there is an edge between two vertices if there is a **common student** in the courses.

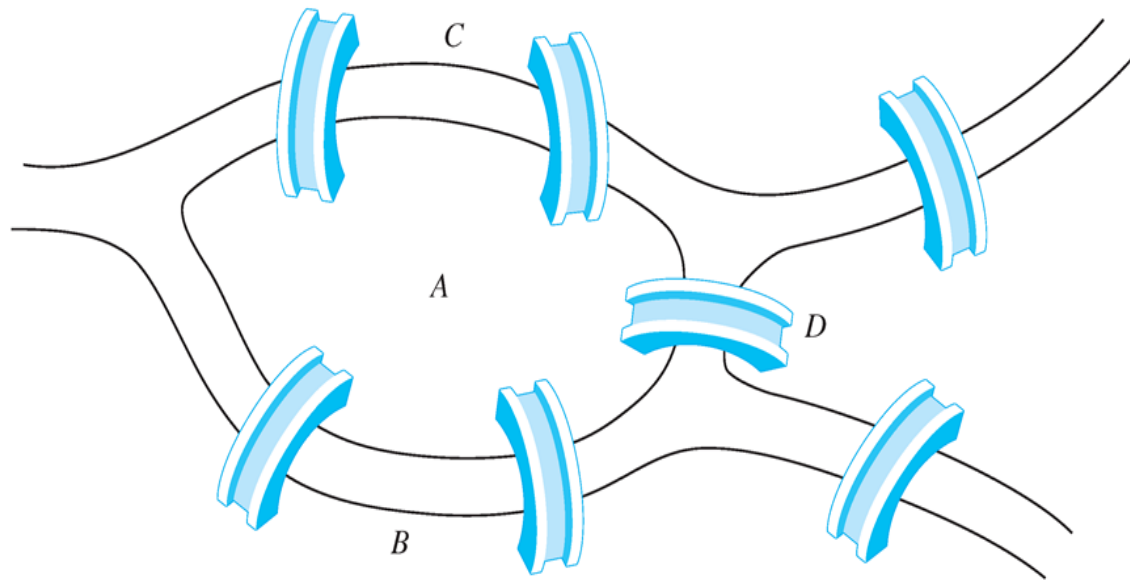
◇ Graph coloring is *computationally hard*! (?)



Problem III

■ Königsberg seven-bridge problem

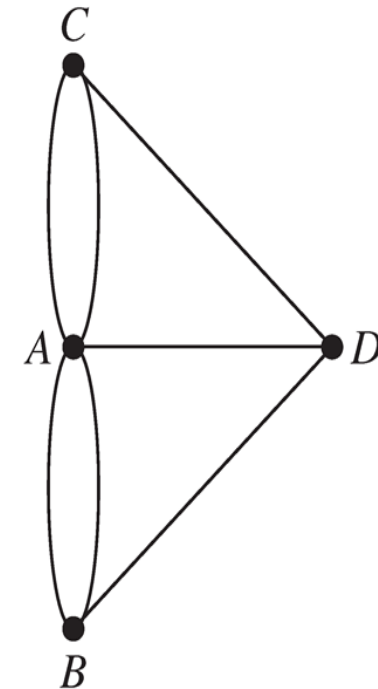
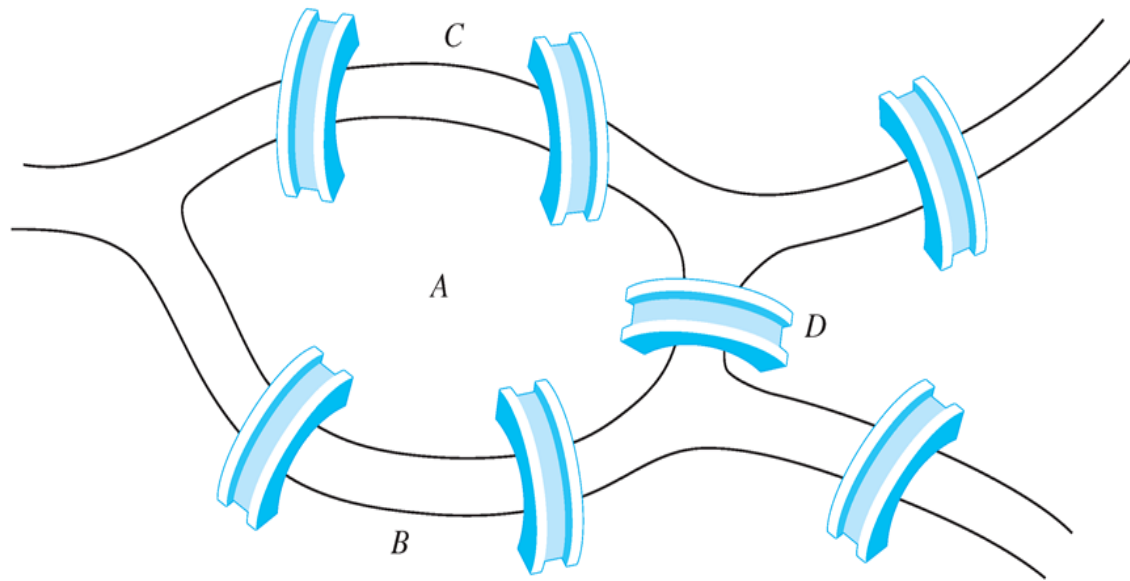
People wondered whether it was possible to start at some location in the town, travel across **all the bridges once** without crossing any bridge twice, and **return to the starting point**.



Problem III

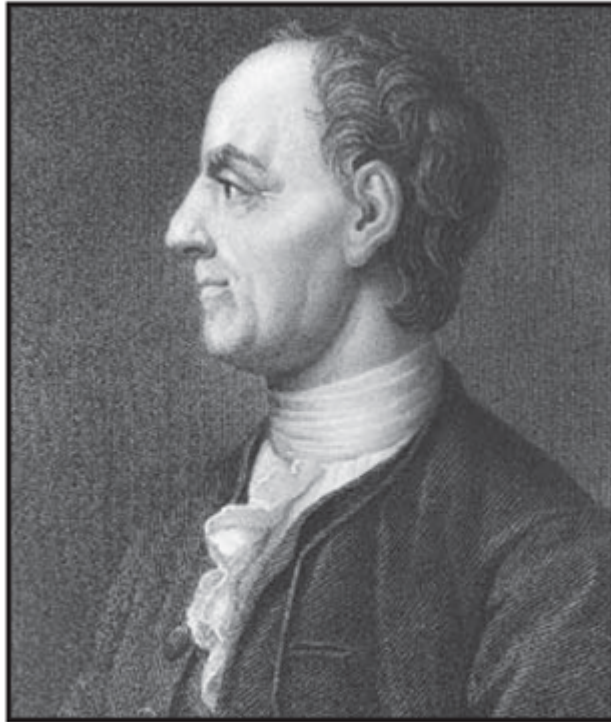
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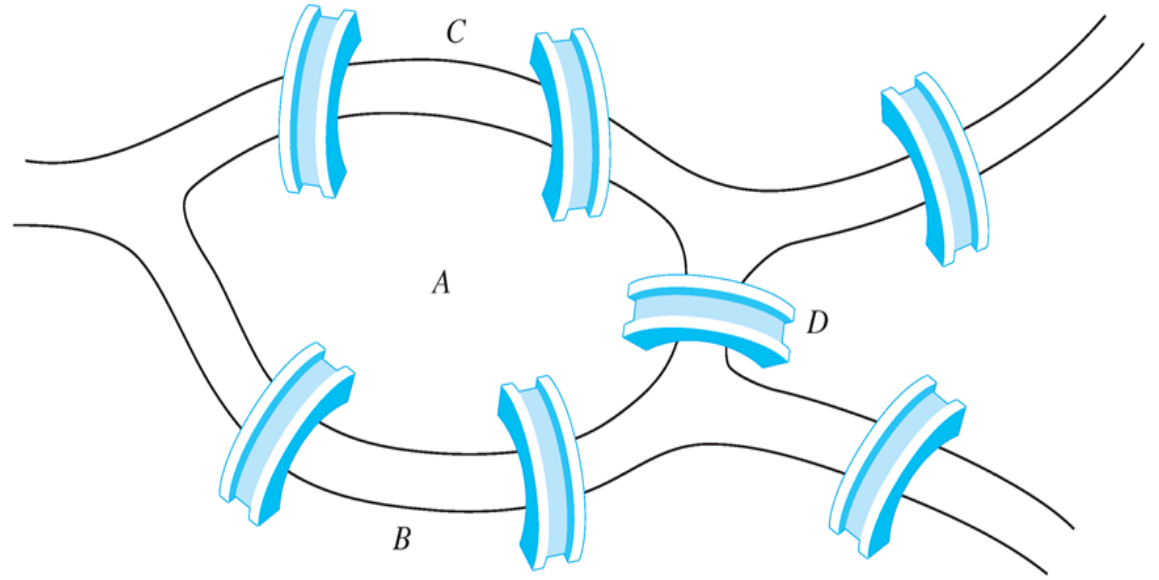


Problem III

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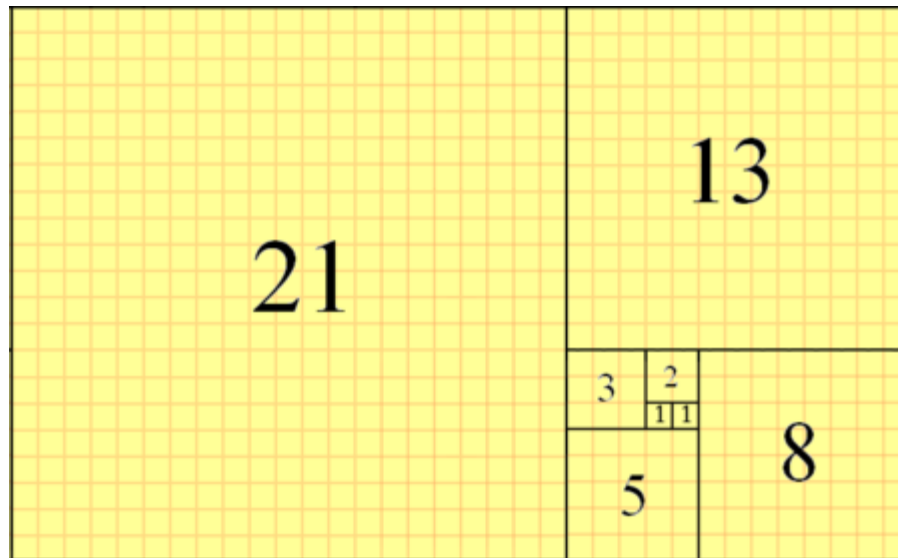
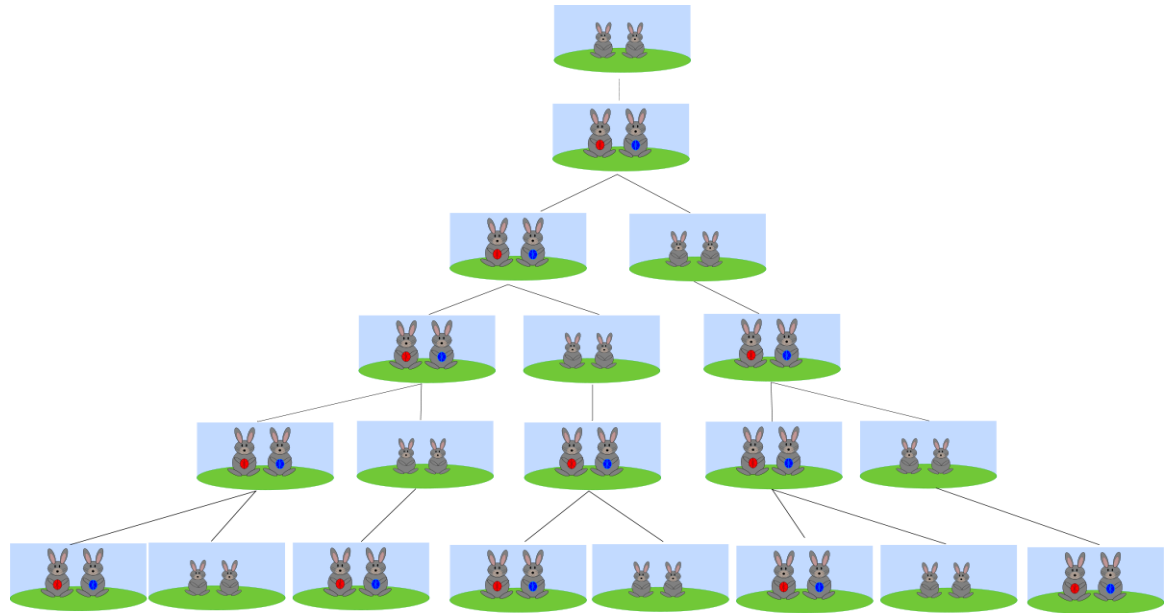
Leonhard Euler



Theorem A connected multigraph with at least two vertices has an *Euler circuit* if and only if each of its vertices has **even** degree.

Problem IV

■ Fibonacci number



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$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$



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◇ What is the closed-form expression of F_n ?



Problem IV

■ Fibonacci number

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$

◇ What is the closed-form expression of F_n ?

Consider $x^n = x^{n-1} + x^{n-2}$, with $x \neq 0$. There are two different roots

$$\phi = \frac{1 + \sqrt{5}}{2}, \quad \psi = \frac{1 - \sqrt{5}}{2}$$

Then F_n can be the form of $a\phi^n + b\psi^n$. By $F_0 = 0$ and $F_1 = 1$, we have $a + b = 0$ and $\phi a + \psi b = 1$, leading to $a = \frac{1}{\sqrt{5}}$, $b = -a$. Therefore,

$$F_n = \frac{\phi^n - \psi^n}{\sqrt{5}}$$



Problem IV

■ Fibonacci number

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \text{ for } n \geq 2$$

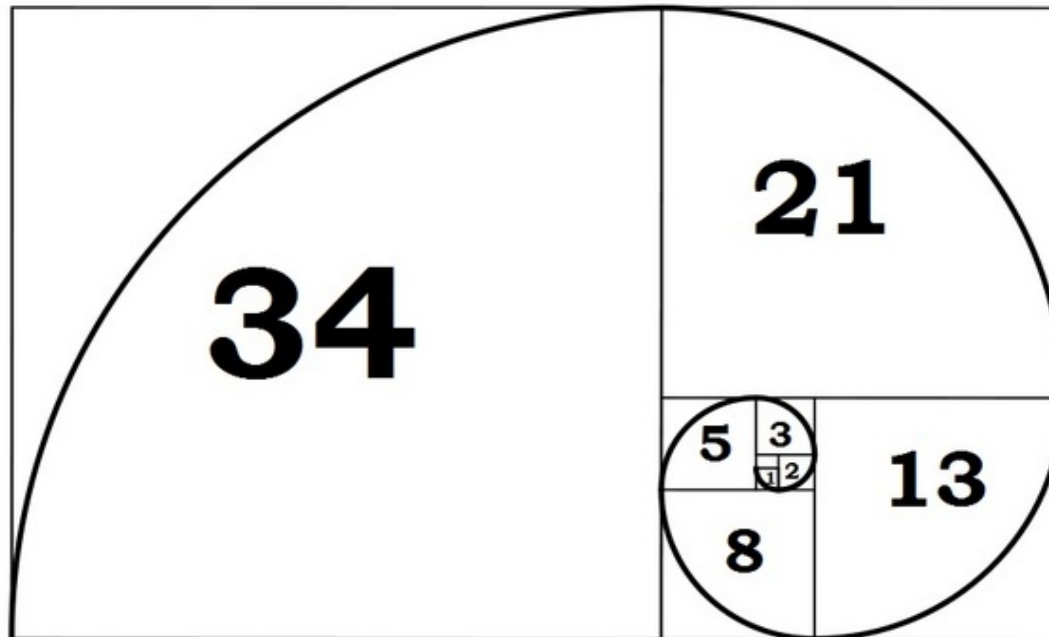
◇ What is t

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roots

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we have $a +$

$b = -a$. Therefore,

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two different

and $F_1 = 1$,
 $\frac{1}{\sqrt{5}}$,

Let's start!

“All you need is a cool head, a large sheet of paper, and a fairly decent handwriting.”



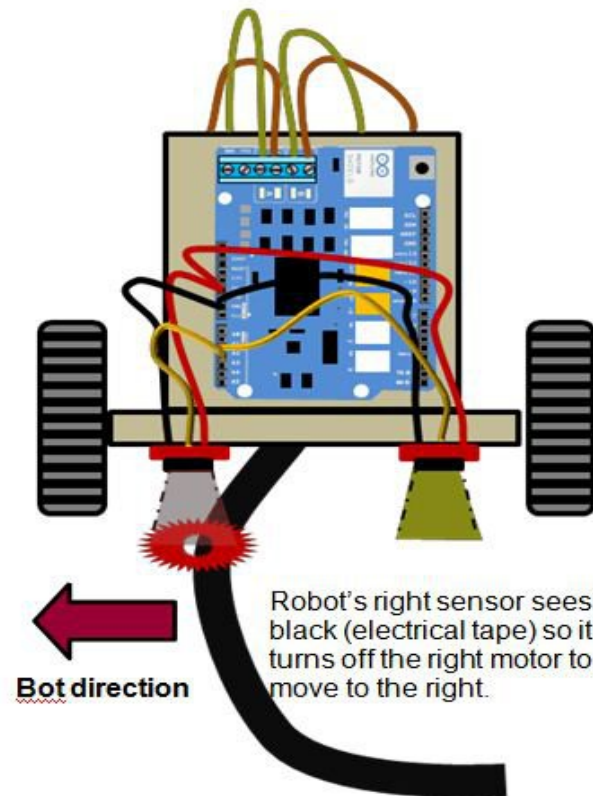
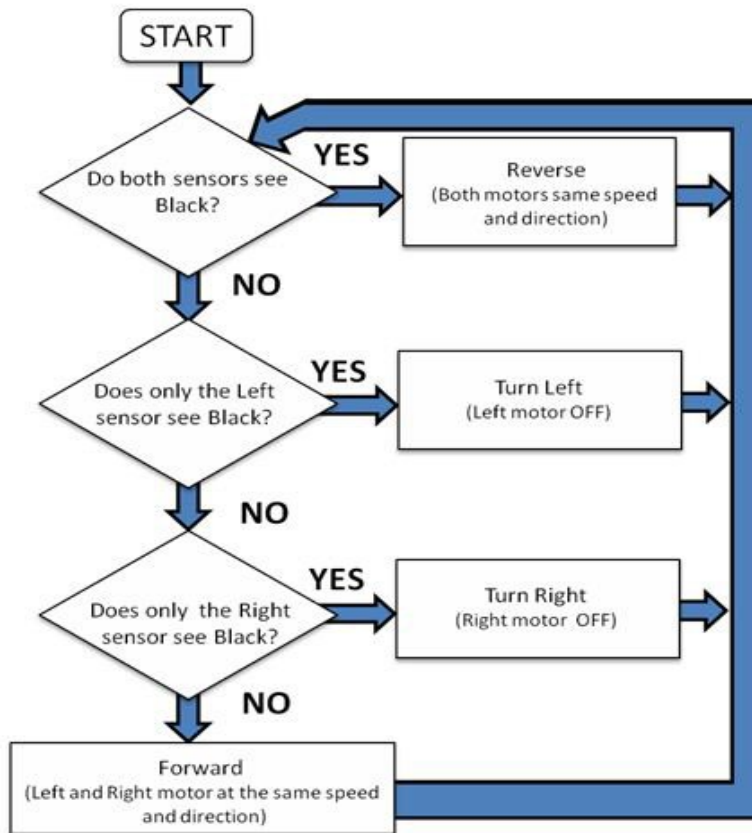
Logic

- **Logic** is the basis of all mathematical reasoning.
 - ◇ *syntax* of statements
 - ◇ the *meaning* of statements
 - ◇ the rules of logical *inference*



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Propositional Logic

- A *proposition* is a declarative statement that is either true or false.



Propositional Logic

- A *proposition* is a *declarative* statement that is *either true or false*.

Examples:

- ◇ SUSTech is located in Shenzhen. (T)
- ◇ $1 + 1 = 2$ (T)
- ◇ $2 + 2 = 3$ (F)



Propositional Logic

- A *proposition* is a *declarative* statement that is *either true or false*.

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“declarative” + “either true or false”



Propositional Logic

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Examples:

- ◇ SUSTech is located in Shenzhen. (T)
- ◇ $1 + 1 = 2$ (T)
- ◇ $2 + 2 = 3$ (F)

“*declarative*” + “*either true or false*”

- ◇ No parking.
- ◇ How old are you?
- ◇ $x + 2 = 5$
- ◇ She is very talented.



Propositions

- What about the following?
 - ◇ There are infinitely many twin prime numbers.
 - ◇ This is an unsettled conjecture (a.k.a. twin-prime conjecture).
 - ◇ There are other life forms on other planets in the universe.



Propositions

- What about the following?
 - ◇ There are infinitely many twin prime numbers.
 - ◇ This is an unsettled conjecture (a.k.a. twin-prime conjecture).
 - ◇ There are other life forms on other planets in the universe.
- Usually, a proposition is **condition-based**.
 - ◇ If you would donate 1 billion, you will become the president.



Compound Propositions

- More complex propositions can be built from elementary statements using **logical connectives**.



Compound Propositions

- More complex propositions can be built from elementary statements using **logical connectives**.

Example:

- ◇ Proposition *A*: **It rains outside**.
- ◇ Proposition *B*: **We will watch a movie**.
- ◇ A new proposition: If **it rains outside** then **we will watch a movie**.



Compound Propositions

- More complex propositions can be built from elementary statements using **logical connectives**.
- **Logical connectives:**
 - ◇ *Negation*
 - ◇ *Conjunction*
 - ◇ *Disjunction*
 - ◇ *Exclusive or*
 - ◇ *Implication*
 - ◇ *Biconditional*



Negation

- Let p be a proposition. The statement “It is not the case that p .” is called the *negation of p* , denoted by $\neg p$, and read as “not p ”.



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Examples:

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- ◇ $\neg p$ – SUSTech is not located in Shenzhen.
It is not the case that SUSTech is located in Shenzhen.



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Examples:

- ◇ p – SUSTech is located in Shenzhen. (T)
- ◇ $\neg p$ – SUSTech is not located in Shenzhen.
It is not the case that SUSTech is located in Shenzhen.
- ◇ $5 + 2 \neq 8$
- ◇ 10 is not a prime number.
- ◇ It is not the case that classes begin at 8:00am.



Negation

- A *truth table* displays the relationships between truth values (T or F) of different propositions.

p	$\neg p$
T	F
F	T



Negation

- A *truth table* displays the relationships between truth values (T or F) of different propositions.

p	$\neg p$
T	F
F	T

Rows: contains all possible values of elementary propositions.



Conjunction (and)

- Let p and q be propositions. The *conjunction* of p and q , denoted by $p \wedge q$, is **true** when both p and q are true and is **false** otherwise.



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- ◇ q – $5 + 2 = 8$
- ◇ $p \wedge q$ – SUSTech is located in Shenzhen **and** $5 + 2 = 8$



Conjunction (and)

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Examples:

- ◇ p – SUSTech is located in Shenzhen.
- ◇ q – $5 + 2 = 8$
- ◇ $p \wedge q$ – SUSTech is located in Shenzhen **and** $5 + 2 = 8$
- ◇ There are infinitely many twin prime numbers **and** $2 + 2 = 3$
- ◇ 2 is a prime number **and** 9 is a prime power.



Disjunction (or)

- Let p and q be propositions. The *disjunction* of p and q , denoted by $p \vee q$, is **false** when both p and q are false and is **true** otherwise.



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Examples:

- ◇ p – SUSTech is located in Shenzhen.
- ◇ q – $5 + 2 = 8$
- ◇ $p \vee q$ – SUSTech is located in Shenzhen **or** $5 + 2 = 8$



Disjunction (or)

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Examples:

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- ◇ q – $5 + 2 = 8$
- ◇ $p \vee q$ – SUSTech is located in Shenzhen **or** $5 + 2 = 8$
- ◇ There are infinitely many twin prime numbers **or** $2 + 2 = 3$
- ◇ 2 is a prime number **or** 9 is a prime power.



Truth Tables

■ Conjunction and disjunction

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

Rows: all possible values of elementary propositions (2^n).



Exclusive or

- Let p and q be propositions. The *exclusive or* of p and q , denoted by $p \oplus q$, is *true* when *exactly one* of p and q is true and is *false* otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F



Conditional Statement (implication)

- Let p and q be propositions. The *conditional statment* (a.k.a. *implication*) $p \rightarrow q$, is the proposition “if p , then q ”, is **false** when p is true and q is false, and **true** otherwise. In $p \rightarrow q$, p is called the *hypothesis* and q is called the *conclusion*.



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p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T



Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
 - ◇ if p then q
 - ◇ p implies q
 - ◇ p is sufficient for q
 - ◇ q is necessary for p
 - ◇ q follows from p
 - ◇ q unless $\neg p$
 - ◇ p only if q



Implication

- $p \rightarrow q$ is read in a variety of equivalent ways:
 - ◇ if p then q
 - ◇ p implies q
 - ◇ p is **sufficient** for q
 - ◇ q is **necessary** for p
 - ◇ q follows from p
 - ◇ **q unless $\neg p$**
 - ◇ **p only if q**

Example:

- ◇ If you get 100 on the final, then you will get an A.

p

q



Implication

- The *converse* of $p \rightarrow q$ is $q \rightarrow p$.
The *contrapositive* of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.
The *inverse* of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.



Implication

- The *converse* of $p \rightarrow q$ is $q \rightarrow p$.
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Examples:

- ◇ If you get 100 on the final, then you will get an A . ($p \rightarrow q$)
- ◇ If you get an A , then you get 100 on the final. ($q \rightarrow p$)
- ◇ If you don't get an A , then you don't get 100 on the final.
($\neg q \rightarrow \neg p$)
- ◇ If you don't get 100 on the final, then you don't get an A .
($\neg p \rightarrow \neg q$)



Implication

- The *converse* of $p \rightarrow q$ is $q \rightarrow p$.
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Examples:

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- ◇ If you get an A, then you get 100 on the final. ($q \rightarrow p$)
- ◇ If you don't get an A, then you don't get 100 on the final.
($\neg q \rightarrow \neg p$)
- ◇ If you don't get 100 on the final, then you don't get an A.
($\neg p \rightarrow \neg q$)

$\neg q \rightarrow \neg p$ is *equivalent* to $p \rightarrow q$



Biconditional

- Let p and q be propositions. The *biconditional statement* (a.k.a. *bi-implications*) $p \leftrightarrow q$, is the proposition “ p if and only if q ”, is **true** when p and q have the same truth values, and **false** otherwise.



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p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T



Compound Propositions

- A *proposition* is a *declarative* statement that is *either true or false*. More complex propositions can be built from elementary statements using *logical connectives*.
- Logical connectives:
 - ◇ $\neg p$ (*Negation*)
 - ◇ $p \wedge q$ (*Conjunction*)
 - ◇ $p \vee q$ (*Disjunction*)
 - ◇ $p \oplus q$ (*Exclusive or*)
 - ◇ $p \rightarrow q$ (*Implication*)
 - ◇ $p \leftrightarrow q$ (*Biconditional*)



Compound Propositions

- p : 2 is a prime (T)
 q : 6 is a prime (F)



Compound Propositions

- p : 2 is a prime (T)
 q : 6 is a prime (F)
- Determine the truth value of the following.

$$\neg p$$

$$p \wedge q$$

$$p \wedge \neg q$$

$$p \vee q$$

$$p \oplus q$$

$$p \rightarrow q$$

$$q \rightarrow p$$



Compound Propositions

- p : 2 is a prime (T)
 q : 6 is a prime (F)
- Determine the truth value of the following.

$$\neg p \quad F$$

$$p \wedge q \quad F$$

$$p \wedge \neg q \quad T$$

$$p \vee q \quad T$$

$$p \oplus q \quad T$$

$$p \rightarrow q \quad F$$

$$q \rightarrow p \quad T$$



Constructing the Truth Table

- Construct a truth table for $p \vee q \rightarrow \neg r$



Constructing the Truth Table

- Construct a truth table for $p \vee q \rightarrow \neg r$

p	q	r	$\neg r$	$p \vee q$	$p \vee q \rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T



Computer Representation of True and False

- A *bit* is sufficient to represent two possible values: 0 (*false*) or 1 (*true*)
- A variable that takes on values 0 and 1 is called a *Boolean variable*.
- A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.



Computer Representation of True and False

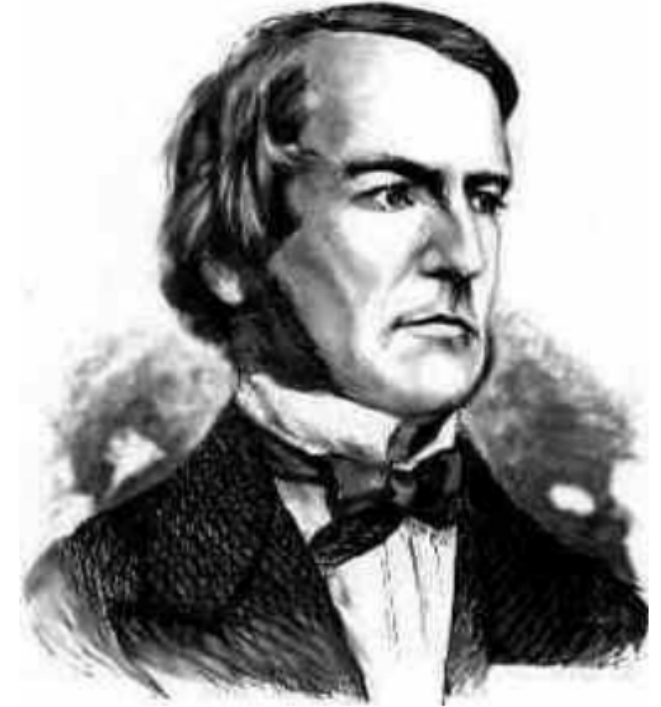
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- A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.
- *bitwise operations*: replace “T” and “F” with 1 and 0.

$$\begin{array}{r} 1011\ 0011 \\ \vee\ 0110\ 1010 \\ \hline 1111\ 1011 \end{array}$$



George Boole

- British mathematician (b. 1815, d. 1864)
 - ◇ The inventor of Boolean algebra
 - ◇ Truth tables are an example of B.A.



- ◇ Although Boole's work was not originally perceived as particularly interesting, even by other mathematicians, he is now seen as one of the founders of the field of Computer Science.



Next Lecture

- applications of propositional logic, predicate logic, ...

