南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

# CS215 DISCRETE MATHEMATICS FOR COMPUTER SCIENCE

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# Review

01. Propositional Logic
02. Predicate Logic
03. Mathematical Proofs
04. Sets
05. Functions
06. Complexity of Algorithms
07. Number Theory
    Groups, Rings and Fields
08. Cryptography
09. Mathematical Induction
10. Recursion
11. Counting
12. Relation
13. Graphs
14. Tree

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02. Predicate Logic
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06. Complexity of Algorithms
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    Groups, Rings and Fields

    Discrete Probability

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# Logic

- Logical connectives

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  $\neg p,\ p \vee q,\ p \wedge q,\ p \oplus q,\ p \rightarrow q,\ p \leftrightarrow q$

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  $\neg p$, $p \vee q$, $p \wedge q$, $p \oplus q$, $p \rightarrow q$, $p \leftrightarrow q$

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  De Morgan's laws, communtative laws, distributive laws, ...

- Predicate logic

  contains variables

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  De Morgan's laws, communtative laws, distributive laws, ...

- Predicate logic

  contains variables

- Quantified statements

  universal, existential, equivalence

# Methods of Proving Theorems

- Basic methods to prove theorems:
  - *direct proof*
    - $p \to q$ is proved by showing that if $p$ is true then $q$ follows
  - *proof by contrapositive*
    - show the contrapositive $\neg q \to \neg p$
  - *proof by contradiction*
    - show that $(p \land \neg q)$ contradicts the assumptions
  - *proof by cases*
    - give proofs for all possible cases
  - *proof of equivalence*
    - $p \leftrightarrow q$ is replaced with $(p \to q) \land (q \to p)$

- function?

# Set, Function

- function?

  one-to-one (injective) function?

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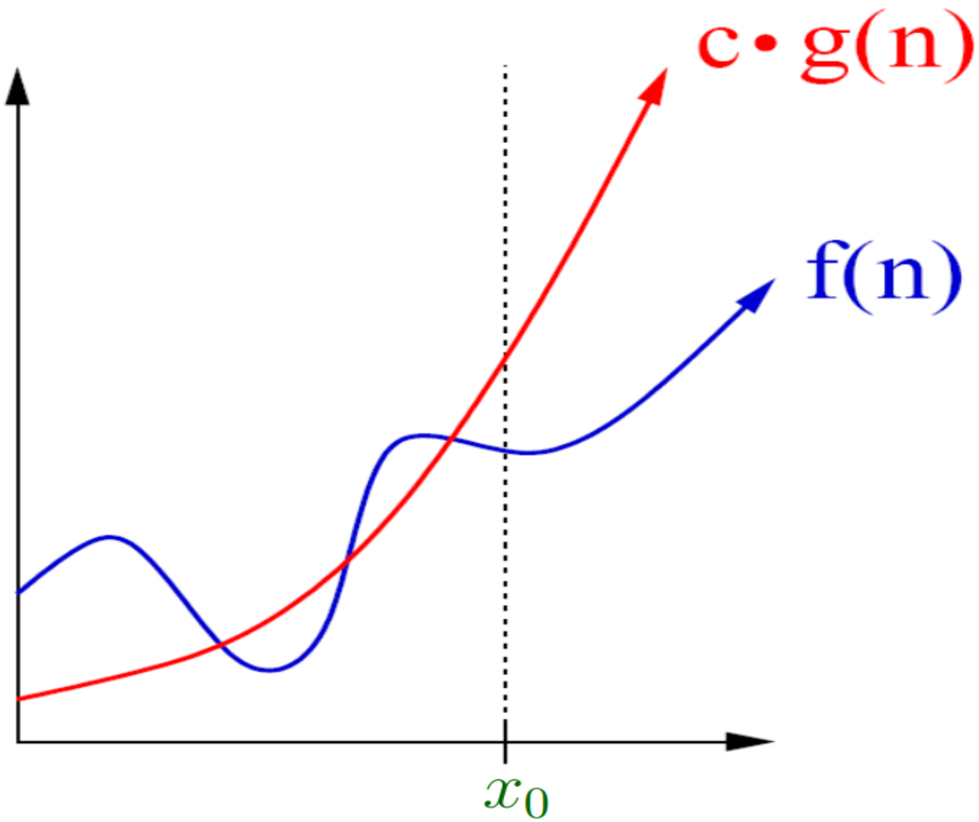
- counting the number of such functions?

# Big-$O$ Notation

- Let $f$ and $g$ be functions from the set of integers or the set of real numbers to the set of real numbers. We say that $f(n) = O(g(n))$ (reads: $f(n)$ is $O$ of $g(n)$), if there exist some positive constants $C$ and $k$ such that $|f(n)| \leq C|g(n)|$, whenever $n > k$.



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  Congruence relation

# Number Theory

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    How to find inverses?

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  GCD and Euclidean Algorithm

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  Back substitution

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 2 \pmod{7} \end{aligned}$$

- Fermat's Little Theorem

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  Euler's Theorem

  Primitive roots, multiplicative order

# Cryptography

- Fermat's Little Theorem

  Euler's Theorem

  Primitive roots, multiplicative order

  RSA cryptosystem

  DLP, Diffie-Hellman protocol

# Mathematical Induction

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3. We conclude on the basis of the principle of mathematical induction that $P(n)$ is true for all $n \geq b$.

# Recurrence

- Iterating a recurrence

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  bottom up or top down

  prove by induction, complexity, ...

# Counting

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  **Theorem** If $N$ is a positive integer and $k$ is an integer with $1 \leq k \leq n$, then there are
  $P(n, k) = n(n-1)(n-2)\cdots(n-k+1)$
  $k$-element permutations with $n$ distinct elements.

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  The Binomial Theorem, Trinomial

- **Definition** An *$r$-combination* with repetition allowed, or a *multiset of size $r$*, chosen from a set of $n$ elements, is an unordered selection of elements with repetition allowed.

  **Example** Find # multisets of size 17 from the set $\{1, 2, 3\}$.

  This is equivalent to finding the # nonnegative solutions to $x_1 + x_2 + x_3 = 17$.

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- Combinatorial proof

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  **Definition** A relation $R$ on a set $A$ is called a *partial ordering* if it is reflexive, antisymmetric, and transitive.

# Graphs & Trees

- Basic concepts

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  connected graph, simple graph, isomophism, chromatic number, planar graph, Euler circuit, Hamilton circuit, shortest path, bipartite graph, complete graph, special graphs ($K_n$, $K_{m,n}$, $C_n$, $W_n$, $Q_n$), m-ary tree, tree traversal, spanning tree ...

# Good Luck!