Lab4 Solution

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Lab4.A:Postman

- Lida Pu is a postman from Twilight Company. There are N houses along a long straight street, and it takes d_i seconds for Lida Pu to walk from the i^{th} to the $(i+1)^{th}$ house. Lida Pu begins his work at the first house at time 0 everyday, singing songs while walking down the street, and finishes his work at the last house.
- Lida Pu is in charge of M letters: the i^{th} letter which should be sent to house b_i would appear at house a_i at time t_i punctually ($a_i < b_i$). As a responsible postman, Lida Pu would not miss any letters, so he would not leave certain house until all letters from it appears. Also, as an experienced postman, Lida Pu collects and sends letters within no time.
- However, one day Lida Pu is accused of noise making. Thus he is required to run instead of walking and singing. Yet he could run K times at most. When Lida Pu runs from the i^{th} house, he could reduce the original d_i by 1. Note that Lida Pu could run from the same house multiple times, but $0 \leq d_i$ should be satisfied at all times.
- Suppose Lida Pu arrives at the i^{th} house at time x_i, then the waiting time of the j^{th} letter is $w_j = x_{b_i} - t_j$. Now Lida Pu wants to minimize $f = \sum_{j=1}^{M} w_j$. Please calculate it for him.

Sample Input 1

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5 4 2
3 2 1 6
1 3 2
4 5 6
2 5 10
2 3 6
```

Sample Output 1

```
30
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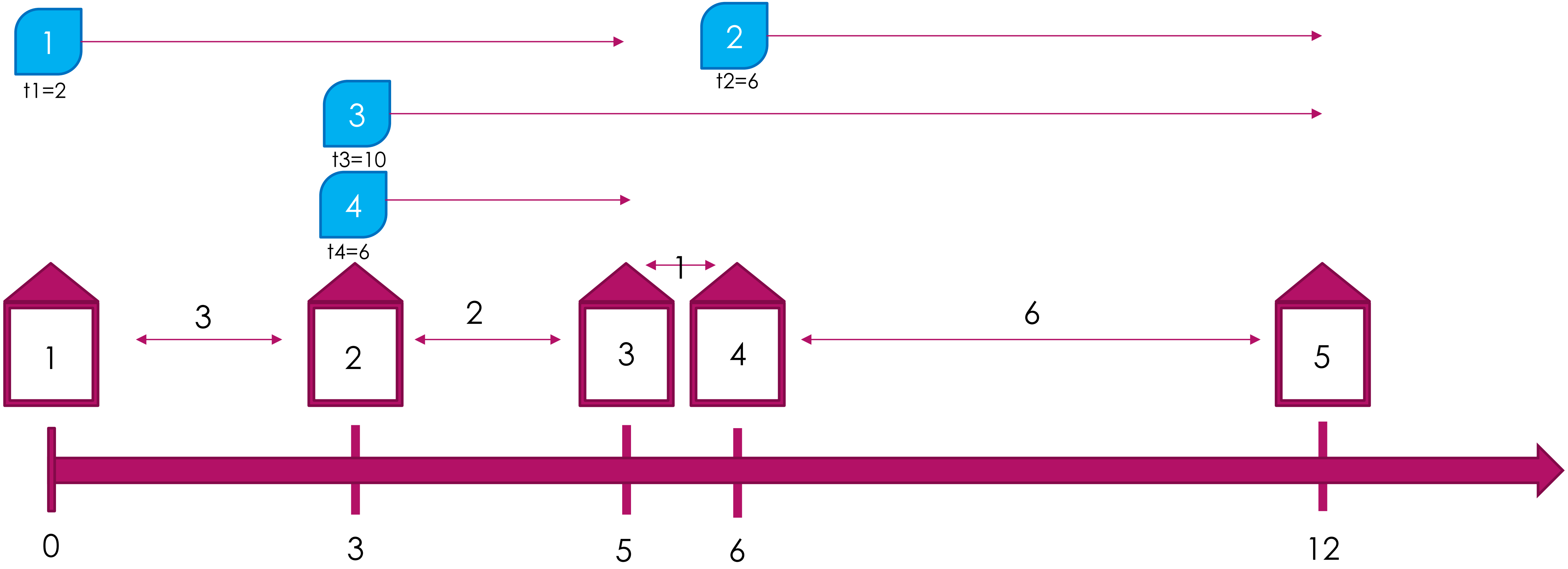
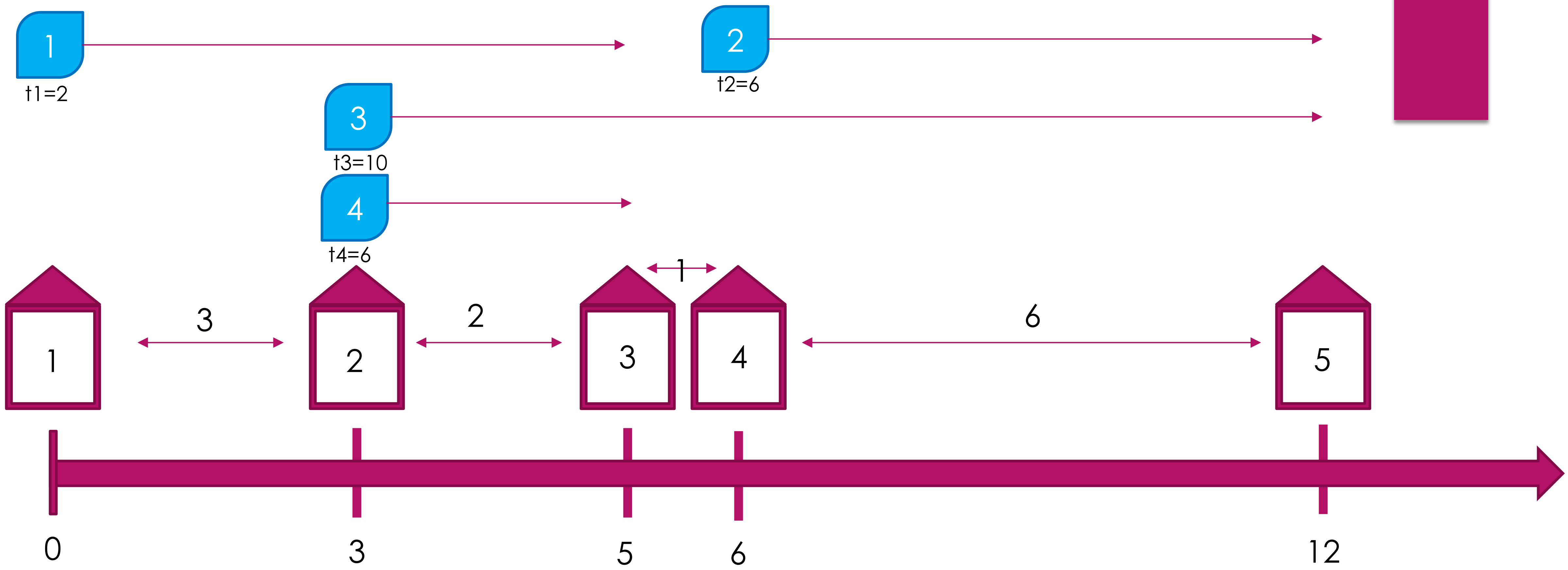
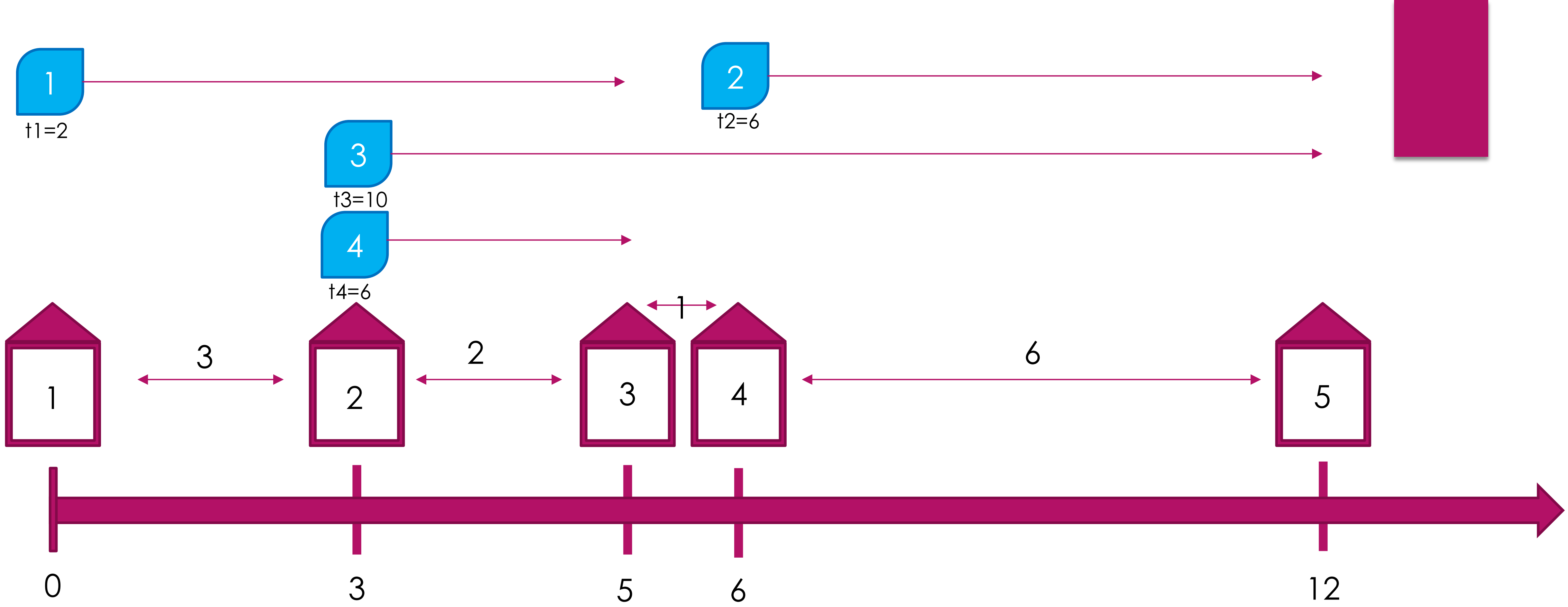




No running:

Time = 2, take letter 1 from house 1→2;
Time = 5, arrive house 2;
Time = 10, take letter 1,3,4 from house 2→3;
Time = 12, arrive house 3; letter 1 wait 10, letter 4 wait 6.
Time = 12, take letter 3,4 from house 3→4;
Time = 13, take letter 2,3 from house 4→5;
Time = 19, arrive house 5. letter 2 wait 13, letter 3 wait 9.

Total: **38**



Running K times, k=2:

Time = 2, take letter 1 from house 1→2;
Time = 5, arrive house 2;
Time = 10, take letter 1,3,4 from house 2→3;
Time = 10, running 2 times, arrive house 3;
Time = **10**, take letter 3,4 from house 3→4;
Time = **11**, take letter 2,3 from house 4→5;
Time = **17**, arrive house 5.

letter 1 wait **8**, letter 4 wait **4**.
letter 2 wait **11**, letter 3 wait **7**.

Total: **30**

分析：

- 依据贪心算法，显然每次跑步应该在受影响的信件数最多的那一段使用，这也是大部分同学很容易想到的
- 本题的难点在于怎么统计出受影响信件数最多的一段。
- 邮递员到房子i之后，可能需要等信，也可能不需要等信。
  - 如果邮递员需要等信或者到房子i的时间与最晚到达房子的信的时间相等，那么从i-1段到i段使用跑步加速后只能影响到在房子i送达的信，目的地不是房子i和在房子i出现的信是不受影响的。
  - 如果邮差到达房子i的时间晚于最晚出现在房子i的信的时间，那么除了影响目的地为房子i的信，还影响目的地在房子i+1的信，如果邮差到达房子i+1还晚于最晚出现在i+1的信，那么还将持续影响目的地在房子i+2的信，依次类推，一直到在某房子j需要等信出现或者两个时间相等为止。

输入分析：输入处理时就应该统计要用到的变量

- 邮差在每一段路程的时间
- 统计每个房子作为目的地的信件数
- 每个房子最晚出现信件的时刻
- 正常步行到达和离开每一个房子的时刻
- 每个房子早点到时影响的总信件数

5 4 2
3 2 1 6
1 3 2
4 5 6
2 5 10
2 3 6

正常步行到达和离开各个房子的时间：
LeaveTime[i]=Max(Arrivetime[i], Latest[i])
ArriveTime[i]=LeaveTime[i-1]+FlightTime[i-1]

House interval	1	2	3	4
Walking Time	3	2	1	6

- 如果邮递员需要等信或者到房子i的时间与最晚到达房子的信的时间相等，那么从i-1段到i段使用跑步加速后只能影响到在房子i送达的信，目的地不是房子i和在房子i出现的信是不受影响的。
- 如果邮差到达房子i的时间晚于最晚出现在房子i的信的时间，那么除了影响目的地为房子i的信，还影响目的地在房子i+1的信，如果邮差到达房子i+1还晚于最晚出现在i+1的信，那么还将持续影响目的地在房子i+2的信，依次类推，一直到在某房子j需要等信出现或者两个时间相等为止。

在每个房子最晚出现的信件时刻：Max

House	1	2	3	4	5
Latest Time	2	10	0	6	0

House	1	2	3	4	5
Arrive Time	0	5	12	13	19
Leave Time	2	10	12	13	19

每个房子作为目的地的信件数

House	1	2	3	4	5
Letter Number	0	0	2	0	2

2表示在1到2段跑步加速，值为影响的信件数依次类推

House interval	1	2	3	4	5
InfluNumber	0	0	4	2	2

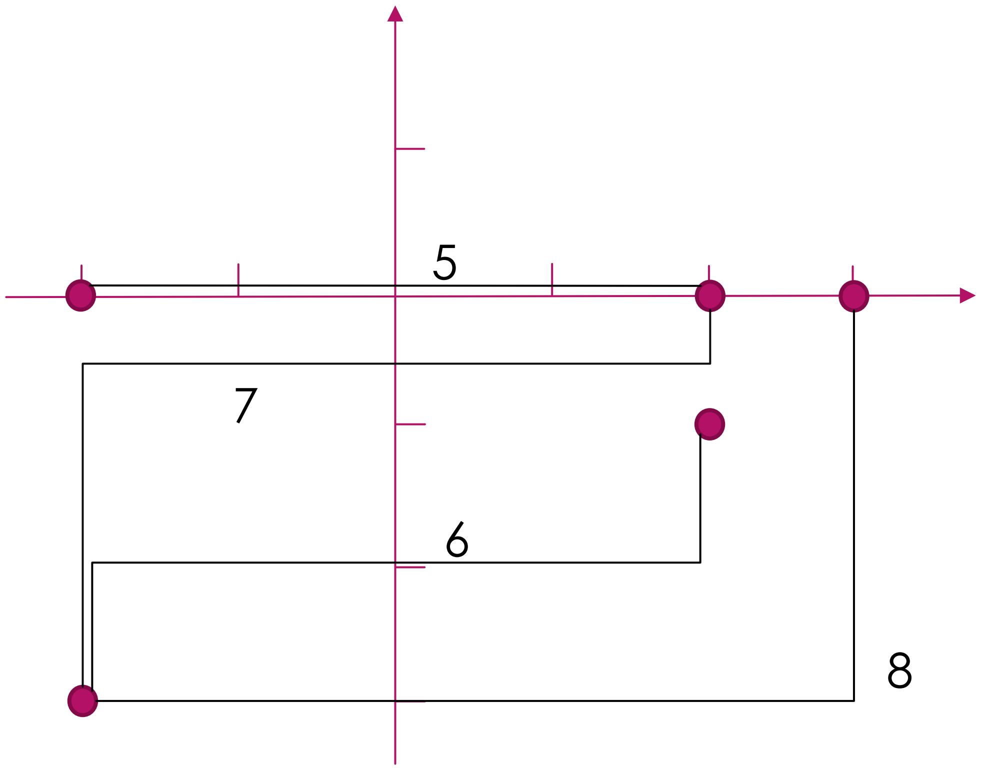
可从后往前计算Influ Number
然后更新Walking Time, Arrive Time, Leave Time
重复

Lab4.B: Servers

- N servers are scattered on a 2D plane. The coordinate of the i^{th} server is (x_i, y_i). No two servers share the same coordinate.
- The famous engineer, **RHC**, wants all N servers to be connected. Initially, there are no link among these servers. **RHC** can create as many undirected links as he wants between any pair of servers. If any two servers can reach to each other by these links, the N servers are said to be connected.
- However, if two servers connected by a link are too close to each other, interference will occur. Therefore, if **the Manhattan Distance** of two servers is less than some constant D, **RHC** will refuse to link them.
- With greater D comes greater performance of these servers. However, he still needs to guarantee their connectivity. Help **RHC** find the maximal D.

Sample Input

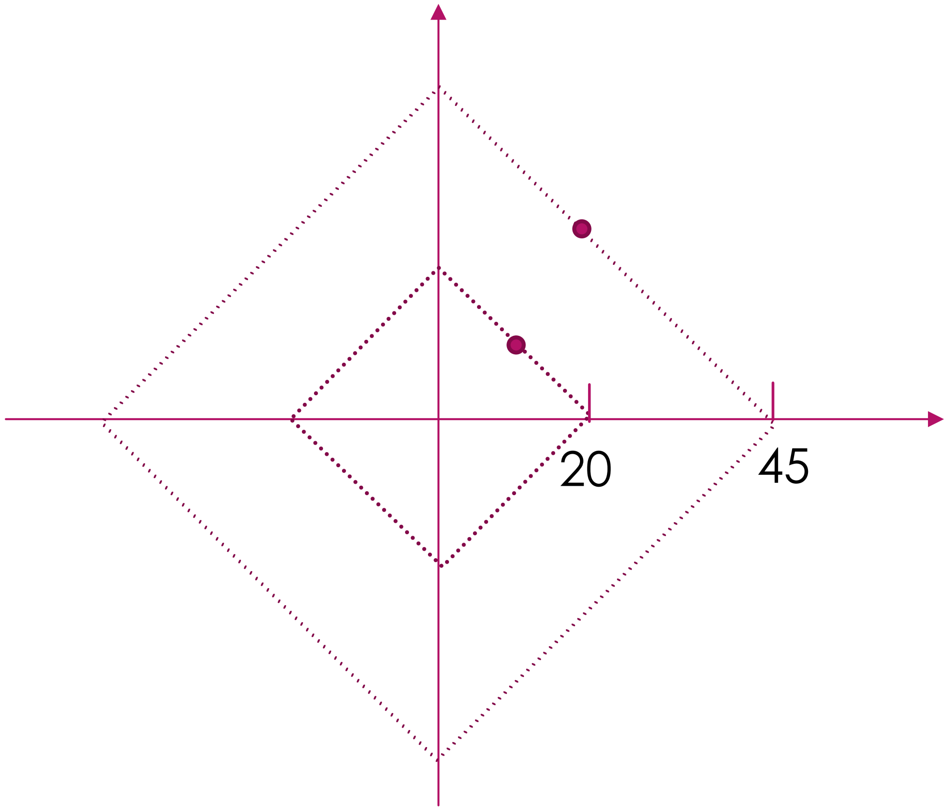
5
-2 -3
-2 0
2 -1
2 0
3 0

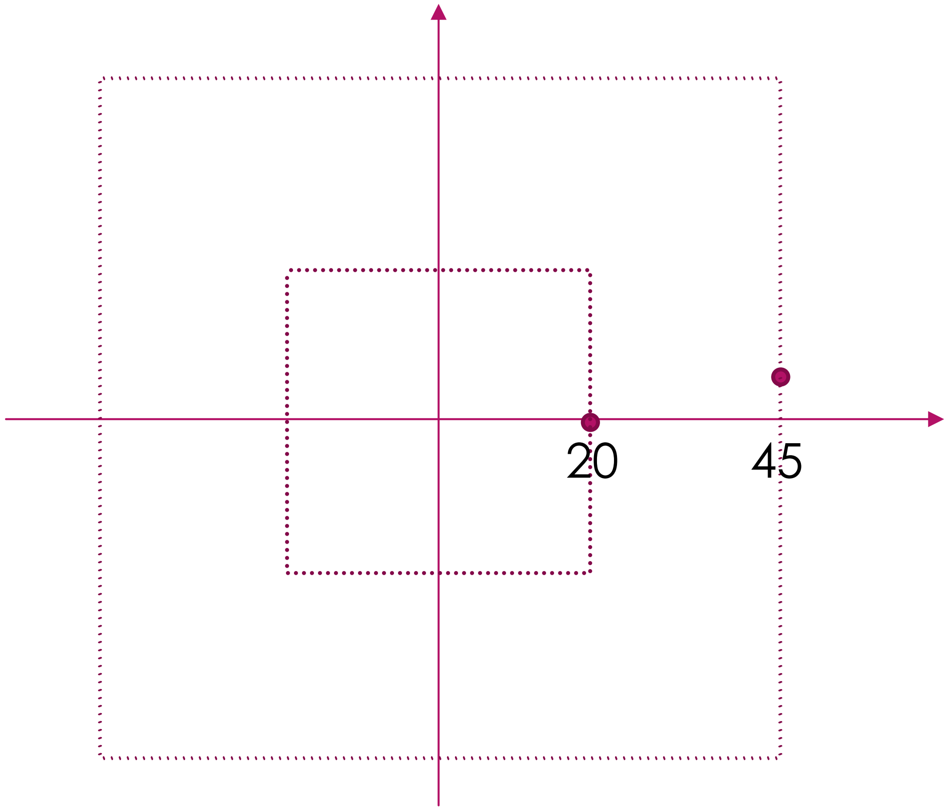


Sample Output
5

Hint

1.Manhattan Distance of (x_1, y_1) and (x_2, y_2) : $d_m(p1, p2) = |x_1 - x_2| + |y_1 - y_2|$.
2.Chebyshev Distance of (x_1, y_1) and (x_2, y_2): $d_c(p1, p2) = \max(|x_1 - x_2|, |y_1 - y_2|)$.
3.Let $x' = x + y, y' = x - y$, $then\ d_m(p1, p2) = d_c(p1', p2')$.





p1 (25,20)
p2 (10,10)

p1' (45,5)
p2' (20,0)

Cave Cows 3

- Question link: https://www.luogu.com.cn/problem/P5098
- To do: seeking the largest Manhattan distance between any two coordinates.

$$(x, y) \xrightarrow{(x+y,\, x-y)} (x', y')$$

$$\Downarrow$$

$$\max(\max\_x' - \min\_x', \max\_y' - \min\_y')$$