PHAS 0006 - Thermal

Cheng Xu
Lin
Leo

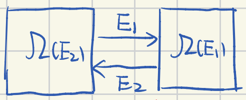
Week 1 Zero-Law Tempreture Heat capacity Microview of ideal gas

— State Ω in thermal concepts

- Quantum State : $\psi(x,t)$ — Stick on one special particle — ψ^2 = Propability of find quantum at (x,t)
- Thermal State : ψ — Statistic Information "packge" — ψ include P, V, T ... all informations

— Tempreture [Similar defination method with E in Quantum]

i). Two heat resovior



Two System in Same Steady State $\Downarrow$ Same Tempreture

Least Action Theory $d\,\Omega(E_1)\,\Omega(E_2)/dE_1$ or E_2 $=0$

$\Omega_2 \cdot d\Omega_1/dE_1 + \Omega_1 \, d\Omega_2/dE_2 \cdot dE_2/dE_1 = 0$ For steady state — $E_1+E_2=0$

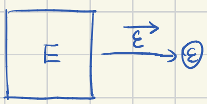
Similar on $dE_1 = -dE_2$

$\Downarrow$

$d\Omega_1/dE_1 \cdot 1/\Omega_1 = d\Omega_2/dE_2 \cdot 1/\Omega_2$

$d\ln\Omega_1/dE_1 = d\ln\Omega_2/dE_2$ = Define As Constant $1/k_BT$ ⟵ ☆ Here's define of T

ii). Heat Bath → Boltzman distribution



Propotion of small system have energy $\varepsilon \propto$ State number of exsist $\Omega(E-\varepsilon)\cdot\Omega(\varepsilon) = \Omega(E-\varepsilon)$

Assume ε small enough $\Omega(\varepsilon)$ = 1

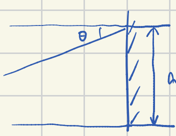
Focusing on $\Omega(E-\varepsilon)$: ☆ Method — Taylor expansion on $\ln\Omega(E-\varepsilon)$

$\ln(E-\varepsilon) = \ln(E) - \varepsilon\, \partial\ln(E)/dE$

$\ln(E-\varepsilon) = \ln(E) - \varepsilon/k_BT \Rightarrow \Omega(E-\varepsilon) = \Omega(E)\exp(-\varepsilon/k_BT)$

$\hookrightarrow P(\varepsilon) \propto \exp(-\varepsilon/k_BT)$

— Pressure [$F = dP/dt$]



Num. on possible particle $a\cdot v\,dt\cos\theta \cdot P(\varepsilon)$

$P(\tfrac{1}{2}mv^2) = \dfrac{\exp(-mv^2/2k_BT)}{\int \exp(-mv^2/2k_BT)\,dv} = \dfrac{\exp(-mv^2/2k_BT)}{\sqrt{-\pi\cdot\frac{2k_BT}{m}}}$

For fully speed distribution $f(v)$ = Probable · Velocity Space

$f(v) = 4\pi v^2 dv \cdot \exp(-mv^2/2k_BT)$

Similar normalize ($\alpha = -m/2k_BT$)

$\int f(v)\,dv = \int v\exp(-\alpha v^2)\,dv = -\frac{1}{2\alpha}\exp(-\alpha v^2)$

$= (2\pi k_BT/m)^{3/2}$

$f(v) = 4\pi\,(m/2\pi k_BT)^{3/2}\cdot v^2\exp(-mv^2/2k_BT)$

Distribute of particle on speed v

→ Distribute of particle on speed per unit volumn

solid angel $d\Omega = 2\pi\sin\theta\,d\theta$

$4\pi\,(m/2\pi k_BT)^{3/2}\cdot v^2\exp(-mv^2/2k_BT)\cdot \frac{1}{2}\sin\theta\,d\theta$

$d\Omega/4\pi$ (per unit angle)

$P = \int_0^\infty\int_0^{\pi/2} f(v)\cdot 2mv\cos\theta\cdot\frac{1}{2}\sin\theta\,d\theta\; v\cos\theta\; dv\,d\theta\cdot n$

$= mn\int_0^\infty f(v)\cdot v^2\,dv\int_0^{\pi/2}\cos^2\theta\sin\theta\,d\theta$

$= \frac{1}{3}mn\langle v^2\rangle$ $= nk_BT \Rightarrow P\cdot V = \underbrace{n\cdot V}_{N}k_BT$

with $\langle v^2\rangle = \int_0^\infty f(v)\cdot v^2\,dv = 3k_BT/m$

☆ Gauss Integration $I = \int_-^+ \exp(-ax^2)\,dx$

Try on $I^2 = \int_-^+\exp(-ax^2)\cdot dx\int_-^+\exp(-ay^2)\,dy \Rightarrow I = \sqrt{\pi/a}$

$= \int_-^+\exp(-a(x^2+y^2))\,dx\,dy$ $x^2+y^2=r^2$ ← Try polar coodinates

$= \int_-^+\exp(-ar^2)\cdot dr\,d\theta\,\boxed{\frac{dx\,dy}{dr\,d\theta}}$

☆ $\frac{dx\,dy}{dr\,d\theta} \neq \frac{d(x,y)}{d(r,\theta)}$ → Must Use Jacobian Matrix

$dx\,dy/dr\,d\theta = \begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{vmatrix} = r(\cos^2\theta+\sin^2\theta) = r$

$= \int_-^+\exp(-ar^2)\cdot r\,dr\,d\theta = \exp(-ar^2)\,1/a\;\pi\,\Big|_0^\infty$

$= \pi/a$

— Energy [Equipartition Theorm]

Consider Movement of one DOF. $H = E_k + E_p = \frac{1}{2}m\underset{\smile}{\dot{x}^2} + \frac{1}{2}k\underset{\smile}{x^2}$ ---

related with x^2 or $\dot{x}^2$

Assume $E = \alpha x^2$, $P(E) \propto \exp(-\beta \cdot \alpha x^2)$ $\square$

$P(E) = \exp(-\beta\alpha x^2) / \int_-^+ \exp(-\beta\alpha x^2) \cdot dx$

Avg. $\langle E \rangle = \int_-^+ E P(x) = \int_-^+ \alpha x^2 \exp(-\beta\alpha x^2) dx \Big/ \int_-^+ \exp(-\beta\alpha x^2) \cdot dx$

$= \frac{1}{2\beta}$ $\square$

★ $\langle E \rangle$ isn't affected by α $\leftarrow$ $= \frac{1}{2}k_B T$

$\Rightarrow \langle E \rangle$ only depends on tempreture and DOF.

Eg. Ideal Gas — 3 DOF $\langle E \rangle = \frac{3}{2}k_B T$