No. Math.
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# UCL Reading Week Review.

A. Vector. Week 2. Vector.

1.2 $A+B=B+A$

$A+(B+C) = A+B+C$

1.4. $A = A_x \cdot \hat{x} + A_y \cdot \hat{y} + A_z \cdot \hat{z}$

1.5. $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\hat{a} = a/|a| = a(1/|a|)$ ← unit vector.

Example. 3.2 P10. $P = \vec{a} - \vec{b}$

$AP : PB = \nu : \mu$

$OP = \vec{P} = a + AP = 1/\mu+\nu\,(\mu a + \nu b)$.

1.6 $\vec{a}\cdot\vec{b} = |a||b|\cdot\cos\theta$ if $\theta = 0 + 2n\pi$ $\vec{a}\cdot\vec{b} = |a||b|$ Same Direction.

$W = F\cdot S$ $\quad \theta = \pi/2 + 2n\pi$ $\vec{a}\cdot\vec{b} = 0$ Orthonormals.

1.7 $\vec{a}\times\vec{b} = (|a||b|\cdot\sin\theta)\,\hat{n}$

Cross Product Rule.

$$\begin{vmatrix} i & j & k \\ & & \\ & & \end{vmatrix} = \sim$$

1.8. Triple Product.

$[a,b,c] = a\cdot(b\times c) = [b,c,a] = [c,a,b]$ ☆

$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}$

☆ $\varepsilon_{ijk}\cdot\varepsilon_{xyz} = \begin{bmatrix} \varepsilon_{ix} & \varepsilon_{iy} & \varepsilon_{iz} \\ \varepsilon_{jx} & \varepsilon_{jy} & \varepsilon_{jz} \\ \varepsilon_{kx} & \varepsilon_{ky} & \varepsilon_{kz} \end{bmatrix}$ and det of this matrix is the value of it.

☆ $\sum_k \varepsilon_{ik}\varepsilon_{jl}\varepsilon_{km} = \delta_{jl}(\sum_k \varepsilon_{ik}\cdot\varepsilon_{km}) = \delta_{jl}\cdot\delta_{im}$

$\delta_{ab}\delta_{bc} = \delta_{ac} \Rightarrow \varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$.

$A\times(B\times C) = b(a\cdot c) - c(a\cdot b)$

KOKUYO

## 2.1 Lines

For $r = a + \lambda b$ and $a+b+c=0$ $\quad r = a + \lambda(c-a)$

$\Rightarrow \lambda = (x - a_x)/b_x = (y - a_y)/b_y = (z - a_z)/b_z = \lambda$

Example: 7.1 P24.

$A(2,1,5)$ direction $b: (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.

$r = a + \lambda b$

$X = A_x + \lambda \cdot b_x = 2 + \lambda/\sqrt{3}$

$Y = 1 + \lambda/\sqrt{3}$ $\qquad \Rightarrow \lambda = \dfrac{X-2}{1/\sqrt{3}} = \dfrac{Y-1}{1/\sqrt{3}} = \dfrac{Z-5}{1/\sqrt{3}}$

$Z = 5 + \lambda/\sqrt{3}$ $\qquad \underline{X-2 = Y-1 = Z-5 = \lambda' = \lambda/\sqrt{3}}$.

Check Coordinates or sth...

## 2.2 Planes.

$r = a + \lambda u + \mu v$ Similar $= a + \lambda(b-a) + \mu(c-a)$.

define $w = \lambda(b-a) + \mu(c-a)$ distribute on the plane.

$r = a + w \rightarrow r - a = w$ define $\hat{n}$ Normal to $w$. ($\perp$)

$(r-a)\cdot\hat{n} = w\cdot\hat{n} = 0 \Rightarrow r\cdot\hat{n} = \underset{\sim}{a\cdot\hat{n}}$

distance from origion to plane.

$\hat{n} = a\hat{i} + b\hat{j} + c\hat{k}$

Example 7.2 $\quad x + 3y - z = 5 \quad 2x - 2y + 4z = 3$

Smallest Distance. ii) Direction of line for intersect.

i). for $ax + by + cz = d$ Proof $\hat{n} = (a, b, c)$ Normal to surface.

$P_1 = (x_1, y_1, z_1)$ $\quad$ define $w$ distribute on $p = (x_1 - x, y_1 - y, z_1 - z)$

$P_0 = (x, y, z)$ $\quad w \cdot \hat{n} = 0$ ~~th~~ $\hat{n} = (a, b, c)$

$(x_1 - x, y_1 - y, z_1 - z)(a, b, c) \Rightarrow P_1(a, b, c)$

Same as define. $= P_0(a, b, c)$

> $n_1 = 1i + 3j - 4k \quad n_2 = 2i - 2j + 4k$

direction of line $\rightarrow n_1 \times n_2 = W = 10i - 6j - 8k$.

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## 2.3 Sphere.

$|r-c|^2 = a^2$ r is any point on sphere, c is center

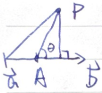
$(x-c_x)^2+(y-c_y)^2+(z-c_z)^2 = a^2$ a is radius.

## 3.1 Distance.

i). Point to Line.

☆ For line with unit vector $\hat{b}$ and Point P, b cross point A with $\vec{a}$ as position.

Smallest distance = $|(p-a)\times\hat{b}|$ Proof:




$d = |p-a|\cdot\sin\theta = |(p-a)\times\hat{b}|$ whereas $\hat{b} = b/|b|$

☆ To get minmum distance, require a point's position and corespoding angel.

Example 8.1 Minimal $P(1,2,1)$ to $r = a+\lambda b$

when $\lambda=0$ $r=a=(1,1,1)$ $a=i+j+k$ $b=2i-j+3k$

$d=|(p-a)\times\hat{b}| = (0,1,0)\times\hat{b} \Rightarrow d=\sqrt{13/14}$

$\hat{b} = 1/\sqrt{14}\,(2i-j+3k)$

ii). Point to plane.

Similar $d=|(a-p)\cdot\hat{n}|$

E. 8.2 $A(0,1,0)$ $B(2,3,1)$ $C(5,7,2)$; $P(1,2,3)$.

ABC Is on a plane, find distance from P to plane.

ABC构成三角形中任两条边 cross product 必垂直于 plane.

$(b-c)\times(a-b) = -2i+j+2k \rightarrow \hat{n} = 1/3\,(-2i+j+2k)$

$d=|(-1,-1,-3)\cdot(-2,1,2)\cdot 1/3| = 5/3$.

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B. Complex    Week 3. Complex Numbers.

1.1 Useful Skill.

$(z + 1/z)^n = 2^n \cos^n\theta$    $(z - 1/z)^n = (2i)^n \sin^n\theta$

$z^n + 1/z^n = 2\cos(n\theta)$    $z^n - 1/z^n = 2i\sin(n\theta)$.

$\arg(z) = \tan^{-1}(\mathrm{Im}[\,]/\mathrm{Re}[\,])$.

1.2 Complex Root.

Define as n for $z^n = 1 \Rightarrow z = 1^{1/n}$

i). Useless solution $1 = z \Rightarrow \cos(2\pi k) + i\sin(2\pi k)$. $k$ = any.

ii). $z_k = 1^{1/n} = (\exp(2\pi k i))^{1/n} = \cos(2\pi k/n) + i\sin(2\pi k/n)$

Example $(-8)^{1/3}$ Find roots. $\Rightarrow z^3 = -8$

$z = (-8)^{1/3} = 8^{1/3}(\exp(i(\pi + 2k\pi)))^{1/3} = 2\exp(i\pi(2k+1)/3)$.

$z_0 = 2\exp(i\pi/3) = 1 + i\sqrt{3}$

$z_1 = -2$

$z_2 = 2\exp(i5\pi/3) = 1 - i\sqrt{3}$

1.3 Hyperbolic Functions.

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

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C. Differentiation                Week 4. Differentiation

1.1 Implicit differentiate.

$x^2+y^2=1 \quad dy/dx$ ?

$2x + 2y \cdot dy/dx = 0 \Rightarrow dy/dx = -x/y$

1.2 Vector dif.

$d(a \cdot b)/dt = a \cdot db/dt + b \cdot da/dt$

$d(a \times b)/dt = a \times db/dt + da/dt \times b$

D. Integration                Week 5. Integration.

1.1 Special Skill

$\int \tan x \cdot dx = -\ln|\cos x| + c \qquad \int \cos x \sin^n x \cdot dx = \frac{\sin^{n+1} x}{n+1} + c$

$\int \sinh x \cdot dx \rightleftarrows \int \cosh x \cdot dx \qquad \int \sin x \cos^n x \cdot dx = -\cos^{n+1} x/n+1 + c$

$\int dx/\sqrt{a^2-x^2} = \arcsin(x/a) + c = -\arccos(x/a) + c'$

$\int a\,dx/a^2+x^2 = \arctan(x/a)+c$

$\int dx/\sqrt{a^2+x^2} = \text{arcsinh}(x/a)+c \qquad \int a\,dx/a^2-x^2 = \text{arctanh}(x/a)+c$

1.2. Still Special Skill — Trigonometric Substitution.

i). $\int dx/a+b\cos x \;//\; \int dx/a+b\sin x$

try $t = \tan(x/2) \;//\; t = \tan(x)$

$dt/dx = \frac{1}{2}(1+\tan^2(x/2)) = \frac{1}{2}(1+t^2)$

Since $\sin x = 2\tan(x/2)/1+\tan^2(x/2) = 2t/1+t^2$

$\cos x = 1-\tan^2(x/2)/1+\tan^2(x/2) = 1-t^2/1+t^2$

或结合和差角/三角换元解决

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ii) 先求导解决 other varience.

Example: $I=\int_0^1 x^b-1/\ln x \cdot dx \Rightarrow I'(b)=d/db[\int_0^1 x^b-1/\ln x \cdot dx]$

$=\int_0^1 d/db(x^b-1/\ln x)\cdot dx$

$=\int_0^1 [1/\ln x \cdot (x^b \cdot \ln x)]\cdot dx$

$=\int_0^1 x^b \cdot dx = 1/b+1$

$\rightarrow I(b) = \int 1/b+1 \cdot db = \ln(b+1)+C$

1.3 Differentiate Respect to peremeter

☆ Example: $I=\int xe^{-ax}\cdot dx$

$= -d/da(e^{-ax}) \Rightarrow I = -\int d/da \cdot \exp(-ax)\cdot dx$

$= d/da(\exp(-ax)/+a + C(a))$

$= -\exp(-ax)(1+ax)/a^2+C.$

1.4. Reduction Formule. 循环积分

$I_n=\int_0^1 (1-x^3)^n \cdot dx$

> Seperate $I_n = \int_0^1 (1-x^3)(1-x^3)^{n-1}\cdot dx$

$=[\int_0^1 (1-x^3)^{n-1} - \int x^3(1-x^3)^{n-1}]dx$

$= I_{n-1} - \int_0^1 x \cdot x^2(1-x^3)^{n-1}\cdot dx$

$= I_{n-1} - [x/3n(1-x^3)^n]_0^1 - 1/3n\int_0^1 (1-x^3)^n \cdot dx$

$= I_{n-1} - 1/3n\, I_n$

$\Rightarrow I_n = 3n/3n+1 \cdot I_{n-1}$

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# Math Review uck    Week 7+8 Partial Differentiation.

## 1. Stationary Point in $f(x,y)$

$A = f_{xx}$ $B = f_{xy}$ $C = f_{yy}$ Set $\Delta = AC - B^2$

| | | |
|---|---|---|
| $\Delta > 0$ | $A > 0$ | local min |
| $\Delta > 0$ | $A < 0$ | local max |
| $\Delta < 0$ | ~ | Saddlepoint. |
| $\Delta = 0$ | ~ | N/A |

Proof: $f(x+h, y+m) = f(x,y) + (h f_x + m f_y) + \frac{1}{2}(h^2 f_{xx} + 2 f_{xy} hm + m^2 f_{yy})$

☆ $\Delta f = f(x+h, y+m) - f(x,y)$ determine point's state.

$\Delta f = \frac{1}{2}(h^2 f_{xx} + 2hm f_{xy} + m^2 f_{yy})$

$= \frac{1}{2} f_{xx}(h^2 + 2hm\, f_{xy}/f_{xx}) + \frac{1}{2} m^2 f_{yy}$

$= f_{xx} \underbrace{(h + f_{xy}/f_{xx} \cdot m)^2}_{>0} + (f_{xx} f_{yy} - f_{xy}^2)/f_{xx} \cdot \underbrace{m^2}_{>0}$

$\Rightarrow$ Determine $A$ and $AC - B^2 = \Delta$.

## 2. Curve Integration

For $V =$ ~~[illegible]~~ $a\hat{i} + b\hat{j}$ and curve $(a,b)$

$\int V \cdot dl \rightarrow \int V_a\, da + V_b\, da\, db/da = \sim$ Convert to $(a,b)$ domain

## 3. Several Inequality

☆ $\langle n \rangle \geq \sqrt[n]{a_1 a_2 \cdots a_n} \longrightarrow \sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$ ☆

☆ $\|u\|\|v\| \geq |u \cdot v|$    $a^2 + b^2 \geq 2ab$

## Week 7. Pareial Derivitave.

25.2 $\nabla f(x,y) = \partial f/\partial x\, i + \partial f/\partial y\, j = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$

26.1 Total defrential: $df = (\partial f/\partial x) \cdot dx + (\partial f/\partial y) \cdot dy$

For $f(x,y)$ $df = df/dt \cdot dt/dx + df/dt \cdot dt/dy$

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26.3 Derivitive in a direction

For linear path $r(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 + n_x t \\ y_0 + n_y t \end{pmatrix}$

have $\hat{e} = n_x \hat{i} + n_y \hat{j}$ where $n_x^2 + n_y^2 = 1$

$df/dt|_{0,\hat{e}} = \partial f/\partial x + \partial f/\partial y = (\nabla f)_0 \cdot \hat{e}$ ☆

Example: $f(x,y) = 4x^2 y$ 沿 $v = 2\hat{i} + 4\hat{j}$ 的变化率

$\nabla f = (8xy, 4x^2)$ $\hat{e} = 2/2\sqrt{5}\,\hat{i} + 4/2\sqrt{5}\,\hat{j}$

$(\nabla f)_0 \hat{e} = \dots$

26.4 exact derivitive $df = A(x,y)\,dx + B(x,y)\,dy$

if $\partial A(x,y)/\partial y = \partial B(x,y)/\partial x$ derivative exact.

27.2 Stationary Points under constrain.

For $\phi(x,y) = c$ and $f(x,y)$ ☆ Lagrange Multiplier

$df = \partial f/\partial x \cdot dx + \partial f/\partial y \cdot dy$

$d\phi = \partial\phi/\partial x \cdot dx + \partial\phi/\partial y \cdot dy$ To solve $df = 0$

Since $\phi(x,y) = c\ d\phi = 0$

$\Rightarrow df + \lambda d\phi$ also $= 0$.

$(\partial f/\partial x + \lambda \cdot \partial\phi/\partial x)\,dx + (\partial f/\partial y + \lambda\,\partial\phi/\partial y)\cdot dy = 0$

Seprate to $dx$, $dy$ and $\phi(x,y) = c$.

Example: Solve $f(x,y) = xy^{3/2}$ and $x + 2y = 100$.

$\partial f/\partial x = y^{3/2} + \lambda = 0$ $\partial f/\partial y = 3/2\,x \cdot y^{1/2} + \lambda \cdot 2 = 0$

$\Rightarrow y = 3/4\,x \rightarrow x = 40\ y = 30$

28. Line/Space Integrals.

For Line path $C$ in Space $F(r)$ (vector, position dependent Feild)

$I = \int_C F \cdot dr = \int_{t_A}^{t_B} [F(r(t)) \cdot dr/dt]\,dt$ for $r = x(t)\hat{i} + y(t)\hat{j}$

Example: $F = xy\hat{i} - y^2\hat{j}$ from $r_A(0,0)$ to $r_B(2,1)$ ☆

Since define as straight line → $x = 2t$, $y = t$. Goal is to convert to all $(t)$

$r = 2t\hat{i} + t\hat{j}$ $t: 0 \sim 1$

$I = \int_0^1 (xy\hat{i} - y^2\hat{j}) \cdot (2\hat{i} + \hat{j}) \cdot dt = \int_0^1 4t^2 - t^2 \cdot dt = 1$ Finally.

Loop: $\oint_c (\nabla f) \, dr = 0$.

29. None Cartesien Coordinates

29.2 Polar Coordinates 2D.

define $e_r = \cos\theta\hat{i} + \sin\theta\hat{j}$ $e_\theta = -r\sin\theta\hat{i} + r\cos\theta\hat{j}$

where $r$ point out ward from origion. $\theta$ counterclockwise point

Example. Motion 2D in polar coordinate.

$r(t) = r(t) \cdot \hat{r}(t)$ $v = dr/dt = dr/dt \cdot \hat{r} + r \cdot d\hat{r}/dt$

★ $d\hat{r}/d\theta = -\sin\theta\hat{i} + \cos\theta\hat{j} = \hat{\theta}$ $= dr/dt \cdot \hat{r} + r \cdot d\hat{r}/d\theta \cdot d\theta/dt$

$d\hat{\theta}/d\theta = -\cos\theta\hat{i} - \sin\theta\hat{j} = -\hat{r}$ $= v \cdot \hat{r} + r \cdot d\theta/dt \cdot \hat{\theta}$

$$a = d(v \cdot \hat{r} + r \cdot d\theta/dt \cdot \hat{\theta})/dt = (d^2r/dt^2 + d\hat{r}/d\theta \cdot d\theta/dt)\hat{r}$$
$$+ (\dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r})\hat{\theta}$$
$$= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$