Week 7 – PHAS0004 ASU – Note

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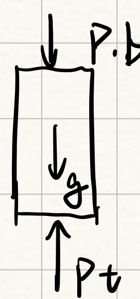
Reading Section

Chap 10. Hydrostatic Equation.

$F = ma = dm \cdot d^2r/dt^2 = F_g + F_{p,t} + F_{p,b}$



$F_{p.b} = -(F_{p,t} + dF_p)$

$dF_p = A \cdot dP$

$F_g = G M \rho A dr / r^2$

$F = -G \frac{M \rho A dr}{r^2} - A \cdot dp$ For Static, $F = 0$

$-G \frac{M \rho A dr}{r^2} = A \cdot dp$

$dM = \rho(4\pi r^2 dr)$

$dM_r/dr = 4\pi r^2 \rho$

$-GM\rho \cdot dr/r^2 = dp$

Simply Solve dP/dr

$dP/dr = -\rho g$

[Fundation of astro] Chap 14. Hydro Equlibrium.

$dP/dr = -GM(r)/r^2 \rho$ For $p = nk_BT = \rho kT/\mu m_p$

For only H, He, metal Composition: $X, Y, Z = (\rho_H, \rho_{He}, \rho_{metal})/\rho$

Particial Density

$= M_{H/He/metal}/V$

14.1 Particle Number Density (num. of particle per volumn)

num = Mass / Mass of one particle. $\rho Y/4m_p$.

i). $H = 2(\rho X/m_p) \longrightarrow \frac{\rho X = \rho_H}{m_p}$ = num of pairs.

1 electron
1 core

ii). $He = 3\cdot(\rho Y/4m_p) = \frac{3}{4}\cdot \rho Y/m_p$

iii). ☆ Set particle number = $A/2$ For full Ionized.

$$Metal = \frac{A}{2}(\rho Z/Am_p) = \frac{1}{2}\rho Z/m_p$$

$$n_{total} = \rho/m_p(2X + \tfrac{3}{4}Y + \tfrac{1}{2}Z)$$

↓

$$\mu_{averge\,+Ionized} = M/nm_p = (2X + \tfrac{3}{4}Y + \tfrac{1}{2}Z)^{-1}$$

$$\mu_{avg.\,+Neutron} = (X + Y/4 + Z/A)^{-1}$$ $[\mu . m_p]$

Week 7. Slides.

Hydrostatic equilibrium : $dP/dr = -\rho g$ ☐

Mass Conservation : $dM/dr = 4\pi r^2 \rho(r)$ ☐

Equation of state : $pv = nRT \rightarrow p = N/V\, k_B T = \rho/m\, k_B T$

$P(r) = \rho(r) k T(r) / \mu m_p$ $\square$

$[\mu m_p]$

Energy Transport

i). Radiative Transport.

Force : $F = (P_{rad}(r) - P_{rad}(r+dr)) 4\pi r^2$ ☆ $P_{rad} = a/3\, T^4$

$$= -a \cdot 4/3\, T^3 \cdot 4\pi r^2 \cdot dr = -16\pi/3\, a r^2 T^3 \cdot dT$$

Tempreture : ☆ relate momentum in forth with T.

$$F = a \cdot b \cdot \mathcal{L}/c \cdot dr = -16\pi/3\, a r^2 T^3 \cdot dT$$

$$\Rightarrow dT/dr = \frac{3ab\mathcal{L}}{16\pi a r^2 T^3 c} = \frac{3k\rho\mathcal{L}}{16\pi a r^2 T^3 c} \quad \square$$

[$a, b = k, \rho$ (k is capture prob. ρ is densty)]

ii). Convective Transport : $dT/dr = (1 - 1/\gamma) \cdot T(r)/P(r) \cdot dP/dr$

· Idiabatic During Upwards..

$PV^\gamma = C \qquad V = nk_BT/P$

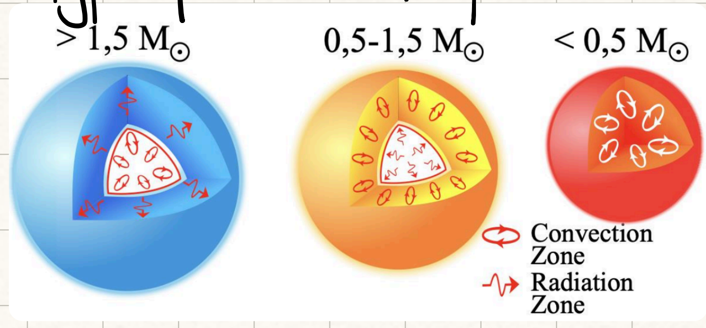
$P^{1-\gamma} T^\gamma = C$

$\downarrow$ ii). Take $\ln$,

$(1-\gamma)\ln P + \gamma \cdot \ln T = \ln C$

iii). Take d/dr ☆

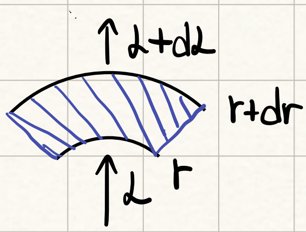
Type of heat transfer process.



$$(1-\gamma)\cdot 1/P \cdot dP/dr - \gamma \cdot 1/T \cdot dT/dr = 0$$

$$\Rightarrow dT/dr = (1-1/\gamma)\cdot T(r)/P(r) \cdot dP/dr \quad \square$$

Generation of energy



For energy produce at rate ε per mass.

$$dL = (4\pi r^2 dr)\cdot \varepsilon \cdot \rho \longrightarrow dL/dr = 4\pi r^2 \rho(r)\varepsilon(r) \quad \square$$

Mean Free Path: $l = 1/n\sigma$

I. Maxwell-Boltzman Distribution. 1D

$$g(\varepsilon) \propto \exp(\varepsilon/k_BT)$$

For $g(E_k) \propto \exp(mv^2/2k_BT)$

To nomalize $$\underbrace{A\cdot\int_{-\infty}^{+\infty}\exp(-mv^2/2k_BT)\cdot dv}_{\sqrt{\frac{2k_BT\pi}{m}}} = 1$$

$$\Rightarrow A = \sqrt{m/2k_BT\pi}$$

$$\Rightarrow f(v_x) = \sqrt{\frac{m}{2\pi k_BT}}\exp(-mv^2/2k_BT)$$

II. Take to 3D.

$$P(\text{density}) = P(x) \times P(y) \times P(z)$$
$$= (m/2\pi k_B T)^{3/2} \cdot \exp(-mv^2/2k_B T)$$

$$F(v)dv = P(\text{density}) \times \text{volumn} = (m/2\pi k_B T)^{3/2} \cdot \exp(-mv^2/2k_B T) \cdot 4\pi v^2 \cdot dv$$

III. Mean free time.

For P of no collision $P(t) = P(t+dt) + dP(t)/dt \cdot dt$

☆ Assume $dP(t)$ = cross section area · speed · sth. constant · dt · $P(t)$

$$P(t+dt) - P(t) = -n\sigma v\, dt \cdot P$$
$$dP/dt = P(t+dt) - P(t) / dt$$
$$1/P \cdot dP/dt = -n\sigma v \qquad \text{For } P(0) = 1$$

$P(t) = \exp(-n\sigma v t)$. // $P(t+dt) = \exp(-n\sigma v t) \cdot n\sigma v \cdot dt$

☆ why normaliz on $P(t+dt)$

$P(t+dt)$ → collision in $t \sim t+dt$ probability

$$f(t) = P(t) \cdot n\sigma v$$

Integration from $0 \sim \infty$ on $f(t)$ must $= 1$

Mean Free time $\tau = \langle t \rangle = \int_0^\infty t \cdot f(t) \cdot dt = 1/n\sigma v$

IV. Cross Section Area σ

$\sigma = \pi(a_1+a_2)^2$ For $a_1 = a_2$ $\sigma = 4\pi r^2$

V. Mean Free Path. $\lambda = \langle v \rangle \cdot \tau$

☆ $\langle v \rangle \neq v$ Move is relatively $\rightarrow$ $V = \langle V_r \rangle = \langle V_2 - V_1 \rangle$

$\langle V_r^2 \rangle = \langle V_1^2 \rangle + \langle V_2 \rangle^2 - \langle 2 V_1 \cdot V_2 \rangle$

$\uparrow$

$\langle 2 V_1 V_2 \cos\theta \rangle$

$\langle \cos\theta \rangle$ must $= 0$

$\Rightarrow \langle V_r^2 \rangle = 2\langle V^2 \rangle$ $\langle V_r \rangle \approx \sqrt{2}\langle V \rangle$

$\lambda = \sqrt{2}\langle v \rangle \cdot 1/n\sigma v = \sqrt{2} \cdot 1/n\sigma$ $\square$

Logic: ①. Maxwell by $g(\varepsilon) \propto \exp(\varepsilon/k_B T)$

②. 1D Maxwell to 3D (cube normalize function · Area unit)

③. Guess $dP = n\sigma v \cdot dt \cdot P(t)$

④. < Relative Speed > approach by $\sqrt{\langle v^2 \rangle} = \sqrt{2} v$

Energy Produce in Stars :

Ex. Sun

Nuclear happend when 10^{-15}m

$$V = e^2/4\pi\varepsilon_0 \cdot 1/r \approx 1.4\,\text{Mev}$$

No nuclear happen

Avg. kenetic $E = 3/2 k_B T \sim 2\,\text{kev}$

☆ Quantum Tunelling Transmission Rate. $P \approx \exp(-2G)$

Proof : Begin with Origional form Schd. Equation.

$$-\hbar^2/2\mu \cdot d^2\psi(r)/dr^2 + V(r)\psi(r) = E\psi(r)$$

$$d^2\psi/dr^2 + 2\mu/\hbar^2 (E - V(r))\psi = 0$$

☆ define $K(r) = \sqrt{2m/\hbar^2 (V(x) - E)}$ As decay constant.

☆ why $K(r)$ is decay coeffecient ?

$$d^2\psi/dr^2 + 2\mu/\hbar^2 (E - V(r))\psi = 0$$

$E > V$ $\quad \psi'' + k^2\psi = 0$

$k = \sqrt{\frac{2m}{\hbar^2}(E - V(r))}$ with $\psi = \pm \exp(ikx)$

$E < V$

$K = \sqrt{\frac{2m}{\hbar^2}(V(r) - E)}$ with ψ same.

$$d^2\psi/dr^2 + K^2\psi = 0 \rightarrow \psi(r) \text{ normally} = A(r)\exp(ikx)$$

☆ ↓ (Wentzel - K - B Method)

$$☆\ \psi(r) = A(r)\exp\left(\pm\int K(r)\cdot dr\right)$$

$\Rightarrow$ Inside barrier : $\psi(r) \sim 1/\sqrt{K(r)} \cdot \exp\left(-\int_{r_{in}}^{r} K(r')\cdot dr'\right)$

$$P \text{ of transmission} = \left|\frac{\psi(R_c)}{\psi(0)}\right|^2 = \exp\left(-2\int_0^{R_c} K(r)\cdot dr\right)$$

$$\Rightarrow P = e^{-2G} \quad \text{where } G = \int_0^{R_c} k(r)\cdot dr \quad \square$$

☆ Core is wkb method.

Number of proton fusions per unit volume, time.

Mean free path $l_{pp} = \sqrt{2}\cdot 1/n\sigma$

$\sim$ time $t_{pp} = l/v$

Nuclear reactor $1/t$ per second . total N particle

calculate $\frac{1}{2}$ for statistic $\Rightarrow N_{pp} = \frac{1}{2}\frac{n_p}{t_{pp}}$

$$= \frac{1}{2} n_p^2 \sigma_{pp} v$$

$$= \frac{1}{2} n_p^2 \sigma_{pp} \underbrace{\sqrt{2E/m_p}}_{\text{velocity}}$$

$$☆\ \sigma_{pp} = \underbrace{\pi (h/p)^2}_{\substack{\text{use wave vector} \\ \text{as possible Area Throw}}} \cdot \underbrace{P_{\text{throw barrier}}}_{P \sim \exp(-2G)} \cdot \underbrace{P_{\text{fussion happen}}}_{\text{Sth. Constant.}}$$

$$\langle N_{pp} \rangle = \int_0^\infty N_{pp}(E) \cdot F(E) \cdot dE \;\propto\; \int_0^\infty \exp(-2G)\, \exp(-E/k_BT) \cdot dE$$

Maxwell Boltzman factor

Change Form

$\exp(-\sqrt{E_G/E})$

E ↑↑ increase

prob. of cross ↑

E ↑↑ decrease

High energy particle ↓↓↓

$\square$