PHAS0006 - Thermal

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Week 2.

- Maxwell - Boltzman Distribution $f(v)$ ☐

$\langle v \rangle = \int_0^{\infty} v \cdot f(v) \cdot dv = \int_0^{+} 4\pi \cdot (m/2\pi k_B T)^{3/2} \cdot v^2 \exp(-mv^2/2k_B T) \cdot v \cdot dv$

☆ $\int_0^{+} v^n \exp(-\alpha v^2) \, dv$ Try on $\Gamma(z) = \int_0^{+} t^{z-1} e^{-t} dt = (z-1)!$ for $z \in n^+$ → $\Gamma(1/2) = \sqrt{\pi}$

$= c \cdot \int_0^{+} v^n \exp(\alpha v^2) \cdot dv$ with $\alpha = -m/2k_B T$ $\quad C = 4\pi (m/2\pi k_B T)^{3/2}$

$t = \alpha v^2$

$= c \cdot \int_0^{+} (t/\alpha)^{n/2} \exp(t) \, dt \cdot 1/2\alpha \cdot (t/\alpha)^{-1/2}$

$v = (t/\alpha)^{1/2}$.

$= c \int_0^{+} (t/\alpha)^{1/2(n-1)} \exp(t) \, dt \cdot 1/2\alpha$

$= C \cdot \alpha^{-1/2(n+1)} \cdot \int_0^{+} t^{1/2(n-1)} \cdot \exp(t) \cdot dt = C\alpha^{-1/2(n+1)} \cdot \Gamma(1/2(n+1))$

Back to $\langle v \rangle \; [n=3] = C\alpha^{-2} \cdot \Gamma(2) = 4\pi (m/2\pi k_B T)^{3/2} \cdot (-m/2k_B T)^{-2} \cdot \Gamma(2)$

$= 4 \, (m/2k_B T)^{-1/2} \cdot \pi^{-1/2} = (8k_B T/\pi m)^{1/2}$

Similar On Root Mean Square of distribution. $\sqrt{\langle v^2 \rangle} = (C\alpha^{-5/2} \cdot \Gamma(5/2))^{1/2}$ → $\Gamma(z+1) = z \cdot \Gamma(z)$

$= [C\alpha^{-5/2} \cdot \Gamma(1/2) \cdot 3/2 \cdot 1/2]^{1/2}$

$= [\sqrt{\pi} \cdot 4\pi \cdot (m/2\pi k_B T)^{3/2} \cdot (-m/2k_B T)^{-5/2} \cdot 3/4]^{1/2}$

$= [(m/2k_B T)^{-1} \, 3/4]^{1/2} = (3k_B T/m)^{1/2}$

$\langle E \rangle = 1/2 m \langle v^2 \rangle = 3/2 k_B T$ ☐

Equipartition ☐

Degree of Freedom ☐