A Low Complexity Iterative Receiving Method Based on Message Passing in $\pi/4$-DQPSK Modulated Frequency-Hopping Communications

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***Abstract*—A low complexity iterative receiving scheme is proposed for $\pi/4$-DQPSK modulated frequency-hopping communication systems relying on message passing algorithm (MPA), where the receiving process is converted to the joint phase estimation and decoding process. Due to short duration of each hop pulse, coherent demodulation which costs much pilot overhead will reduce the system transmission efficiency. Besides, differential demodulation will lose bit error performance. In order to approximate the theoretically optimal performance with low pilot overhead and low complexity, this paper proposes a low complexity joint phase estimation and decoding iterative algorithm. We simplify the process of message passing and approximate the calculation of some probability messages, which omit lots of computation. Simulation results demonstrate that the proposed algorithm can reduce computational complexity by more than 60% compared with conventional MPA, and can also improve bit error performance by more than 1dB with only 4 pilot symbols per hop compared with differential demodulation.**

***Index Terms*—frequency-hopping, $\pi/4$-DQPSK, phase estimation, factor graph, message passing, Tikhonov.**

I. Introduction

FREQUENCY hopping (FH) communication is a common mode of spread spectrum communication, which is widely used in military and civil wireless communication area due to its anti-interference, anti-jamming and anti-fading ability [1]. In FH communications, a message is divided into multiple short pulses with different carrier frequencies, and it is difficult to guarantee the continuity of carrier phase between two adjacent hops. Since the initial phase of carrier is unknown, it is much easier to adopt the modulation mode which supports differential demodulation in FH communications.

In addition to the characteristic of constant envelope and high spectrum utilization, the $\pi/4$-DQPSK modulation supports differential demodulation, which is suitable to overcome the random phase in FH communications. However, differential demodulation has poor bit error performance, while coherent demodulation needs much pilot symbols to accurately estimate the phase, which will reduce the system transmission efficiency. Therefore, how to balance performance and pilot overhead in $\pi/4$-DQPSK modulated FH communication systems becomes an important issue.

In [2], a basic squaring technique was described to estimate the phase of each hop using few pilot symbols in BPSK and DPSK modulated systems. In [3], the squaring technique was further extended in QPSK and DQPSK modulated systems, where the square processing was replaced by the quartic processing. In [4], a class of nonlinear estimation algorithms was described to estimate the phase in fully m-ary PSK modulated systems. This nonlinear transformation involved multiplying the phase by m and applying a nonlinear function to the magnitude of signal samples, and a m-fold ambiguity was introduced in the phase estimation. Therefore, the performance of above algorithms is unsatisfactory in $\pi/4$-DQPSK modulated FH communication systems due to phase ambiguity. In [5]–[8], a class of joint phase estimation and decoding algorithms based on message passing was described to exchange probability information between decoder and phase estimator and achieve better performance through iterations. However, these algorithms bear a heavy computational load, which can not be directly applied in $\pi/4$-DQPSK modulated FH communication systems.

To deal with above problems, this paper proposes a low complexity iterative receiving algorithm based on message passing in $\pi/4$-DQPSK modulated FH communication systems. On the one hand, we iteratively pass the probability message between phase estimator and decoder, which will

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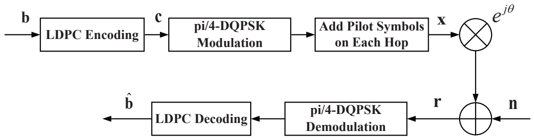

Fig. 1. FH communication baseband system block diagram.

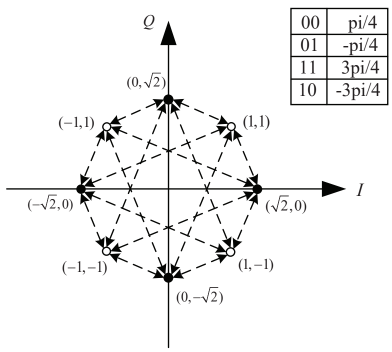

Fig. 2. $\pi/4$-DQPSK constellation diagram.

not only reduce the pilot overhead but also improve the bit error performance; on the other hand, we reduce the computational complexity by simplifying the message passing process and approximating the calculation of some probability messages. Simulation results show that the proposed algorithm can achieve a desired performance with few pilot symbols and low complexity. The main contributions and innovations of this work are summarized as follows,

- In this paper, message passing algorithm is firstly applied in $\pi/4$-DQPSK modulated FH communication systems, and we propose an algorithm with low complexity and high performance.
- We combine phase estimation and decoding and pass the probability message between phase estimator and decoder. Through iterations, the bit error performance of proposed algorithm is improved with low pilot overhead.
- In order to reduce the computational load, we simplify the message passing process from phase variable node to symbol variable node and approximate the calculation of some probability messages by replacing the accumulation of probability products with the maximum of probability products.

The rest of this paper is organized as follows. Section II describes the system model and formulates the maximum a $posteriori$ probability (MAP) criterion. In Section III, the factor graph is given to represent the probabilistic structure, and a low complexity joint phase estimation and decoding algorithm is proposed. Simulation results and performance analysis are provided in Section IV. Section V concludes this paper.

II. System Model

In this paper, we consider a $\pi/4$-DQPSK modulated FH communication system protected by low-density parity check

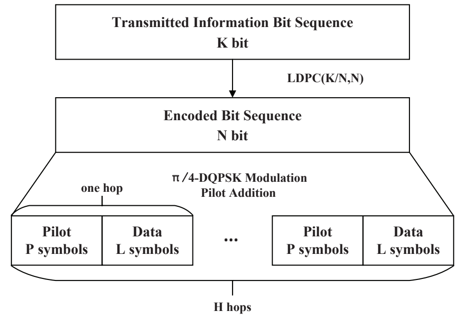

Fig. 3. The illustration of frame format diagram.

(LDPC) code, which is shown in Fig. 1. The transmitted information bit sequence is represented by $\mathbf{b}$, which is firstly converted into sequence $\mathbf{c}$ through LDPC encoding and then modulated to the transmitted symbol sequence $\mathbf{x}$ after $\pi/4$-DQPSK modulation and pilot symbol addition. The constellation diagram of $\pi/4$-DQPSK is shown in Fig. 2, where two encoded bits correspond to the phase difference of adjacent transmitted symbols. As is shown in Fig. 3, we suppose that the length of LDPC code block is N and the code rate is K/N. One LDPC code block is divided into H hops, and each hop contains $L = N/(H * \log_2 M)$ data symbols and P pilot symbols, where M is the modulation order. The received signal after ideal filtering and sampling can be written as,

$$r_k^i = x_k^i e^{j\theta^i} + n_k^i, 1 \leq i \leq H, 1 \leq k \leq L+P, \qquad (1)$$

where r_k^i and x_k^i denote the k-th received and transmitted symbol of the i-th hop respectively. θ^i denotes the unknown phase of the i-th hop, which is subject to the uniform distribution, i.e. $\theta^i \sim U(0, 2\pi)$. n_k^i denotes the k-th noise sample of the i-th hop, which is a zero-mean complex white Gaussian random variable with variance of σ^2. In order to minimize the bit error rate (BER) , according to MAP criterion, we get

$$\hat{b}_i = \underset{b_i}{\operatorname{argmax}} \; p(b_i \,|\mathbf{r}\,). \qquad (2)$$

The posterior probability in (2) can be obtained by the marginalization of joint posterior probability, which has the form as,

$$p(b_i \,|\mathbf{r}\,) = \sum_{\mathbf{b} \backslash b_i, \theta} p(\mathbf{b}, \theta \,|\mathbf{r}\,). \qquad (3)$$

The joint posterior probability in (3) can be factored as follows,

$$\begin{aligned} p(\mathbf{b}, \theta \,|\mathbf{r}\,) &\propto p(\mathbf{r} \,|\mathbf{b}, \theta\,) p(\mathbf{b}, \theta) \\ &= p(\mathbf{r} \,|\mathbf{b}, \theta\,) p(\mathbf{b}) p(\theta) \\ &= I\left[\mathbf{c} = q(\mathbf{b})\right] p(\mathbf{r} \,|\mathbf{c}, \theta\,) p(\mathbf{b}) p(\theta) \\ &= I\left[\mathbf{x} = m(\mathbf{c})\right] I\left[\mathbf{c} = q(\mathbf{b})\right] p(\mathbf{r} \,|\mathbf{x}, \theta\,) p(\mathbf{b}) p(\theta), \end{aligned} \qquad (4)$$

where $I[\cdot]$ denotes Dirac function; $m[\cdot]$ denotes $\pi/4$-DQPSK mapping rule; $q[\cdot]$ denotes LDPC encoding constraints. Since the channel observations are independent,

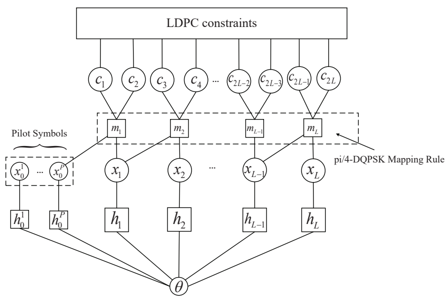

Fig. 4. Factor graph for joint phase estimation and LDPC decoding.

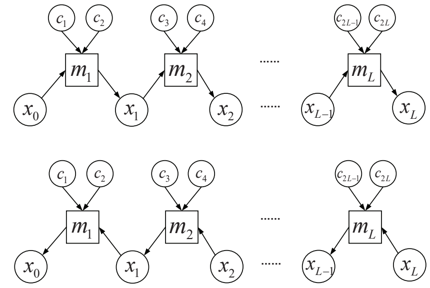

Fig. 5. Message passing process from function node m to variable node x.

the channel transfer function $p(\mathbf{r}\,|\mathbf{x},\theta)$ in (4) can be factored into $p(\mathbf{r}\,|\mathbf{x},\theta) = \prod_{i=1}^{H}\prod_{k=1}^{L+P} p(r_k^i\,|x_k^i,\theta^i)$, where $p(r_k^i\,|x_k^i,\theta^i) = \mathcal{N}(r_k^i;x_k^i e^{j\theta^i},\sigma^2)$. Similarly, $I\left[\mathbf{x}=m(\mathbf{c})\right] = \prod_{i=1}^{H}\prod_{k=P+1}^{L+P} I\left[x_k^i = m(c_{2k-1}^i, c_{2k,}^i x_{k-1}^i)\right]$. Since $p(\mathbf{b})$ and $p(\theta)$ are constants, the joint posterior probability can be expressed as,

$$\begin{aligned} p(\mathbf{b},\theta\,|\mathbf{r}) \propto I\left[\mathbf{c}=q(\mathbf{b})\right]\prod_{i=1}^{H}\prod_{k=P+1}^{L+P} I\left[x_k^i = m(c_{2k-1}^i, c_{2k,}^i x_{k-1}^i)\right] \\ *\prod_{i=1}^{H}\prod_{k=1}^{L+P} p(r_k^i\,|x_k^i,\theta^i). \end{aligned} \tag{5}$$

III. Proposed Approach

A. Factor Graph Representation

The carrier phase of different hops is estimated independently, so we take one hop as an example. According to above analysis, the probabilistic structure of (5) can be represented by a factor graph as shown in Fig. 4, where the function node m_k denotes the $\pi/4$-DQPSK mapping rule and the function node h_k denotes channel transfer function $p(r_k\,|x_k,\theta)$. Since there are cycles in this graph, we choose to pass the probability message from bottom to top first and then update the probability message from top to bottom. This whole process is defined as one iteration.

B. A Low Complexity Joint Phase Estimation and LDPC Decoding Algorithm

At the first iteration of the proposed algorithm, we use the pilot symbols to estimate the phase, and the received signal can be expressed as,

$$r_k = x_k e^{j\hat{\theta}} + n_k, \tag{6}$$

where $\hat{\theta}$ denotes the phase estimate. Due to $p(r_k\,\Big|x_k,\hat{\theta}) = \mathcal{N}(r_k;x_k e^{j\hat{\theta}},\sigma^2)$, the message from function node h_k to variable node x_k can be expressed as,

$$\mu_{h_k\to x_k}(x_k) \propto e^{-\frac{(r_k - x_k e^{j\hat{\theta}})^2}{2\sigma^2}}. \tag{7}$$

In the proposed algorithm, the above process is considered to approximate the complete message passing process from variable node θ up to variable node x_k.

According to the sum-product algorithm (SPA) [5], the message from variable node x_k to function node m_k can be expressed as $\mu_{x_k\to m_k}(x_k) = \mu_{h_k\to x_k}(x_k)\mu_{m_{k+1}\to x_k}(x_k)$, and we approximate it as $\mu_{x_k\to m_k}(x_k) \approx \mu_{h_k\to x_k}(x_k)$. Similarly, we get $\mu_{x_{k-1}\to m_k}(x_{k-1}) \approx \mu_{h_{k-1}\to x_{k-1}}(x_{k-1})$. Then the message from function node m_k to variable node c_{2k-1} and c_{2k} can be calculated by,

$$\begin{aligned} \mu_{m_k\to c_{2k-1}}(c_{2k-1}) &= \sum_{x_{k-1},x_k,c_{2k}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1})\mu_{c_{2k}\to m_k}(c_{2k}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right] \\ &\approx \sum_{x_{k-1},x_k,c_{2k}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right] \\ &\approx \max_{x_{k-1},x_k,c_{2k}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right], \end{aligned} \tag{8}$$

$$\begin{aligned} \mu_{m_k\to c_{2k}}(c_{2k}) &= \sum_{x_{k-1},x_k,c_{2k-1}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1})\mu_{c_{2k-1}\to m_k}(c_{2k-1}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right] \\ &\approx \sum_{x_{k-1},x_k,c_{2k-1}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right] \\ &\approx \max_{x_{k-1},x_k,c_{2k-1}} \mu_{x_k\to m_k}(x_k)\mu_{x_{k-1}\to m_k}(x_{k-1}) I\left[x_k = m(c_{2k-1},c_{2k},x_{k-1})\right], \end{aligned} \tag{9}$$

where we approximate the accumulation of probability products with the maximum of probability products. Next, the log-likelihood ratio (LLR) message passed to LDPC decoder can be expressed as,

$$LLR = \ln\frac{p(c_j=1)}{p(c_j=0)} = \ln\frac{\mu_{m_k\to c_j}(c_j=1)}{\mu_{m_k\to c_j}(c_j=0)}, \tag{10}$$

where c_j denotes c_{2k} or c_{2k-1}. According to (7) and (10), complex exponential and logarithmic operations can be replaced by subtractions. Finally, LDPC decoder outputs the decoded bit sequence.

Now the message is updated from top to bottom. Firstly, the extrinsic information output by LDPC decoder is passed from variable node c_{2k-1} and c_{2k} to function node m_k, and the message passing process from function node m_k to variable node x_k and x_{k-1} is shown in Fig. 5. Assuming that the message is passed from left to right first and then from right

Algorithm 1 A low complexity joint phase estimation and LDPC decoding algorithm

Input: $\boldsymbol{r} = [r_1, r_2, ..., r_{L+P}], \sigma^2, IterNum.$

Initialize: Using pilot symbols to estimate phase.

$$\hat{\theta} = angle(\sum_{k=1}^{P} r_k x_k^*)$$

for $l = 1$ to $IterNum$ **do**

- $\forall k : \mu_{h_k \to x_k}(x_k) \propto e^{-\frac{(r_k - x_k e^{j\hat{\theta}})^2}{2\sigma^2}}$;
- $\forall k : \mu_{x_k \to m_k}(x_k) \approx \mu_{h_k \to x_k}(x_k)$;
- $\forall k$:Updating $\mu_{m_k \to c_{2k}}(x_k)$ and $\mu_{m_k \to c_{2k+1}}(x_k)$ using $(8), (9)$;
- Updating LLR using (10) ;
- Updating decoded bit sequence and extrinsic information output by LDPC decoder ;
- $\forall k$:Updating $\mu_{m_k \to x_k}(x_k)$ using (11) ;
- $\forall k$:Updating $\mu_{m_{k+1} \to x_k}(x_k)$ using (12);
- $\forall k$:Updating $\mu_{x_k \to h_k}(x_k)$ using (15);
- $\forall k : \mu_{h_k \to \theta}(\theta) \propto \mathcal{T}(\theta; \frac{2r_k {\alpha_k}^*}{\sigma^2 + \beta_k - |\alpha_k|^2})$;
- $\forall k$:Updating phase estimate $\hat{\theta}$ using (21) .

end for

Output: Decoded bit sequence.

to left, then we get

$$\begin{aligned} \mu_{m_k \to x_k}(x_k) &= \sum_{x_{k-1}, c_{2k-1}, c_{2k}} \mu_{x_{k-1} \to m_k}(x_{k-1}) \mu_{c_{2k} \to m_k}(c_{2k}) \mu_{c_{2k-1} \to m_k}(c_{2k-1}) I\left[x_k = m(c_{2k-1}, c_{2k}, x_{k-1})\right] \\ &\approx \max_{x_{k-1}, c_{2k-1}, c_{2k}} \mu_{x_{k-1} \to m_k}(x_{k-1}) \mu_{c_{2k} \to m_k}(c_{2k}) \mu_{c_{2k-1} \to m_k}(c_{2k-1}) I\left[x_k = m(c_{2k-1}, c_{2k}, x_{k-1})\right], \end{aligned} \tag{11}$$

$$\begin{aligned} \mu_{m_{k+1} \to x_k}(x_k) &= \sum_{x_{k+1}, c_{2k+1}, c_{2k+2}} \mu_{x_{k+1} \to m_{k+1}}(x_{k+1}) \mu_{c_{2k+1} \to m_{k+1}}(c_{2k+1}) \mu_{c_{2k+2} \to m_{k+1}}(c_{2k+2}) I\left[x_{k+1} = m(c_{2k+1}, c_{2k+2}, x_k)\right] \\ &\approx \max_{x_{k+1}, c_{2k+1}, c_{2k+2}} \mu_{x_{k+1} \to m_{k+1}}(x_{k+1}) \mu_{c_{2k+1} \to m_{k+1}}(c_{2k+1}) \mu_{c_{2k+2} \to m_{k+1}}(c_{2k+2}) I\left[x_{k+1} = m(c_{2k+1}, c_{2k+2}, x_k)\right], \end{aligned} \tag{12}$$

where

$$\mu_{x_{k-1} \to m_k}(x_{k-1}) = \begin{cases} \mu_{h_{k-1} \to x_{k-1}}(x_{k-1}) & if\ k = 1, \\ \mu_{h_{k-1} \to x_{k-1}}(x_{k-1}) \mu_{m_{k-1} \to x_{k-1}}(x_{k-1}) & others. \end{cases} \tag{13}$$

$$\mu_{x_{k+1} \to m_{k+1}}(x_{k+1}) = \begin{cases} \mu_{h_{k+1} \to x_{k+1}}(x_{k+1}) & if\ k = L-1, \\ \mu_{h_{k+1} \to x_{k+1}}(x_{k+1}) \mu_{m_{k+2} \to x_{k+1}}(x_{k+1}) & others. \end{cases} \tag{14}$$

Next, the message from variable node x_k to function node h_k can be expressed as,

$$\mu_{x_k \to h_k}(x_k) = \mu_{m_k \to x_k}(x_k) \mu_{m_{k+1} \to x_k}(x_k), \tag{15}$$

and the message from function node h_k to variable node θ is expressed as,

$$\mu_{h_k \to \theta}(\theta) = \sum_{x_k \in \mathbf{x}} p(r_k | x_k, \theta) \mu_{x_k \to h_k}(x_k). \tag{16}$$

Due to $p(r_k | x_k, \theta) = \mathcal{N}(r_k; x_k e^{j\theta}, \sigma^2)$, $\mu_{h_k \to \theta}(\theta)$ equals to the sum of multiple Gaussian distributions. Then, we approximate $\mu_{h_k \to \theta}(\theta)$ to a single Gaussian distribution via minimizing Kullback-Leibler (KL) divergence. Assuming that $\mu_{x_k \to h_k}(x_k)$ is the probability density function of x_k, and we define

$$\alpha_k = \sum_{x_k \in \mathbf{x}} x_k \mu_{x_k \to h_k}(x_k), \tag{17}$$

TABLE I
SIMULATION PARAMETERS

PARAMETERS	VALUE
LDPC Parameters	(1/2,5120,2560)
Symbol Rate	128 ksps
Hop Rate	1000 hops/s
E_b/N_0	(4.1:0.2:6.5) dB
PilotNum	1,2,3,4,5,6
IterNum	5

$$\beta_k = \sum_{x_k \in \mathbf{x}} |x_k|^2 \mu_{x_k \to h_k}(x_k), \tag{18}$$

where α_k denotes the mean of x_k, and β_k denotes the second moment of x_k. Thus, $\mu_{h_k \to \theta}(\theta)$ in (16) can be indicated as,

$$\begin{aligned} \mu_{h_k \to \theta}(\theta) &\approx \mathcal{N}(r_k; \alpha_k e^{j\theta}, \sigma^2 + \beta_k - |\alpha_k|^2) \\ &\propto \mathcal{T}(\theta; \frac{2r_k {\alpha_k}^*}{\sigma^2 + \beta_k - |\alpha_k|^2}), \end{aligned} \tag{19}$$

where $\mathcal{T}(\theta; z)$ denotes the Tikhonov distribution of variable θ with parameter z,

$$\mathcal{T}(\theta; z) = \frac{e^{real(ze^{-j\theta})}}{2\pi I_0(|z|)}. \tag{20}$$

According to the property of Tikhonov distribution, phase estimate $\hat{\theta}$ can be calculated by

$$\hat{\theta} = angle(\sum_{k=1}^{L} \frac{2r_k {\alpha_k}^*}{\sigma^2 + \beta_k - |\alpha_k|^2}). \tag{21}$$

At this point, phase estimate is updated and the next iteration can be started from (6). A detailed algorithm description is shown in Algorithm 1.

IV. Simulation Results and Performance Analysis

The performance of proposed algorithm is evaluated in $\pi/4$-DQPSK modulated FH communication system protected by LDPC code. The symbol-to-noise ratio (SNR) is set as E_b/N_0 from 4.1 dB to 6.5 dB, where E_b denotes the average received signal energy per bit and N_0 denotes the one-sided power spectral density of white-Gaussian noise. The symbol rate is set to 128 ksps and the hop rate is set to 1000 hops per second. Each frame contains one fixed rate 1/2 LDPC code of length 5120, which is divided into 21 hops. $PilotNum$ is set from 1 to 6, which denotes the number of pilot symbols in one hop. A summary of the simulation parameters is given in Table I.

Firstly, the BER performance of proposed algorithm with different $PilotNum$ is simulated. According to the simulation result, we choose to insert 4 pilot symbols in each hop and compare the BER performance of proposed algorithm with five different algorithm as follows,

1) Differential Algorithm:

Differential algorithm leverages the carrier phase difference of the received symbol to demodulate, which does not need the pilot symbol. However, differential algorithm has the very poor

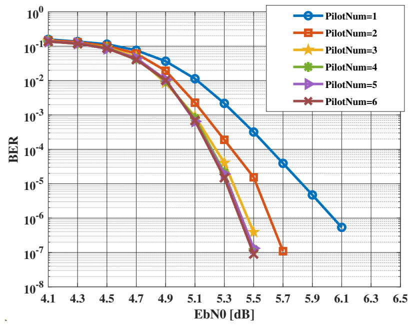

Fig. 6. Comparison of the proposed algorithm on BER with different $PilotNum$.

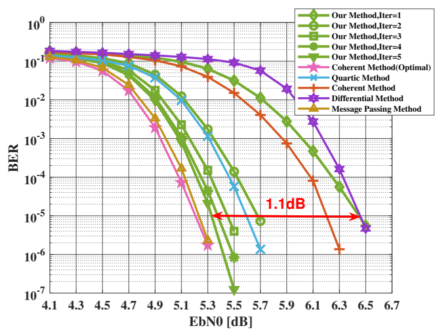

Fig. 7. Comparison of different receiving algorithms on BER with different E_b/N_0.

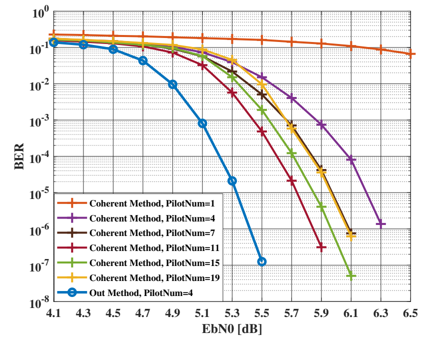

Fig. 8. Comparison of coherent algorithm on BER with different $PilotNum$

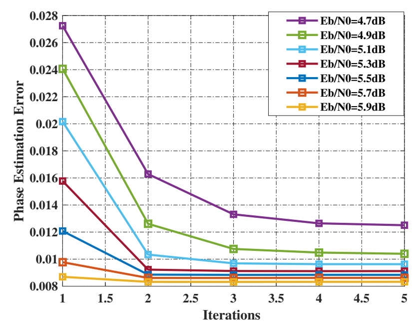

Fig. 9. Phase Estimation Error of the proposed algorithm versus Iterations with different E_b/N_0.

performance due to amplifying the noise after the difference of two adjacent received symbols.

2) Quartic Algorithm:

Quartic algorithm firstly performs the quartic operation on the received symbol, and then applies the double angle formula twice to transform $e^{j4\theta}$ to $e^{j\theta}$ for phase estimation. Finally, the received symbol is coherently demodulated after phase estimation. Quartic algorithm modifies all symbols of one hop so that pilot and data symbols both are used to estimate the phase. However, quartic algorithm exists phase ambiguity of π or $\pi/2$, which will lead to the reduction of performance.

3) Coherent Algorithm (Optimal):

Based on the assumption that the carrier has been precisely synchronized, the coherent algorithm (optimal) demodulates the received symbol directly. In theory, this algorithm has the optimal demodulation performance under the white Gaussian noise.

4) Coherent Algorithm:

Coherent algorithm firstly leverages pilot symbols to estimate phase, then demodulates the received symbol after phase estimation. Due to the limited number of pilot symbols per hop, the carrier phase can not be accurately estimated, thus there will be a large loss compared with coherent algorithm (optimal).

5) Message Passing Algorithm:

Message Passing algorithm [5] passes probability message between decoder and phase estimator, and achieves better performance with only few pilot symbols through iterations. However, message passing algorithm can not be directly applied in the actual system due to its high complexity.

Next, in order to further illustrate the superiority of proposed algorithm in reducing pilot overhead, the BER performance of coherent algorithm with different $PilotNum$ is simulated and compared with that of the proposed algorithm.

Finally, in order to observe phase estimation in each iteration of the proposed algorithm, phase estimation error is also simulated and analyzed.

A. BER of proposed algorithm with different $PilotNum$

Fig. 6 shows BER versus E_b/N_0 using the proposed algorithm with different $PilotNum = 1, 2, 3, 4, 5, 6$. Each hop is inserted with $PilotNum$ pilot symbols. It can be observed that the BER performance is apparently improved with increase of $PilotNum$ when it is less than 4, which demonstrates that the phase estimation gets more accurate with more pilot symbols. However, the BER performance is extremely approximate when $PilotNum$ is greater than or equal to 4. Besides, too many pilot symbols will lead to the reduction of system transmission efficiency, thus we choose to insert 4 pilot symbols in each hop for subsequent simulations.

B. BER of different algorithms

Fig. 7 shows BER versus E_b/N_0 using different receiving algorithms. Each hop is inserted with 4 pilot symbols. It can be observed that coherent algorithm has the optimal

BER performance when carrier is accurately synchronized, but poor BER performance when using 4 pilot symbols to estimate phase, which demonstrates that coherent algorithm is not suitable for FH systems. Compared with the theoretically optimal BER performance, the BER performance of quartic algorithm is reduced by about 0.4 dB due to phase ambiguity, and the BER performance of differential algorithm is reduced by more than 1.2 dB due to amplifying the noise. As for the proposed algorithm, it can be seen that the BER performance is improved with the increase of iterations since the soft information becomes more accurate in the iterative process. Compared with differential algorithm, the BER performance of proposed algorithm is improved by about 1.1 dB. Meanwhile, the BER performance of proposed algorithm is reduced by about 0.05 dB compared with conventional MPA and less than 0.1 dB compared with coherent algorithm (optimal).

C. BER of coherent algorithm with different $PilotNum$

Fig. 8 shows BER versus E_b/N_0 using coherent algorithm with different $PilotNum = 1, 4, 7, 11, 15, 19$ and the proposed algorithm with $PilotNum = 4$. Each hop is inserted with $PilotNum$ pilot symbols. It can be observed that the BER performance of coherent algorithm is optimal when $PilotNum$ equals to 11, rather than being improved continuously with increase of $PilotNum$, which demonstrates that excessive pilot symbols will not be able to improve the BER performance. This is because the real SNR of system decreases with the increase of pilot symbols when E_b/N_0 is given. Moreover, compared with coherent algorithm inserted 11 pilot symbols , the BER performance of proposed algorithm inserted 4 pilot symbols is increased by about 0.4 dB, and the pilot overhead is reduced by more than 60%, which proves the superiority of proposed algorithm in reducing pilot overhead and achieving better BER performance.

D. Phase Estimation Error of proposed algorithm

Fig. 9 shows phase estimation error versus iterations with different E_b/N_0, and the phase estimation error is defined as,

$$Err = E\left[\left|\theta - \hat{\theta}\right|\right]. \tag{22}$$

It can be observed that the phase estimation error decreases with the increase of iterations, which demonstrates that phase estimation gets more precise according to the extrinsic information output by the decoder in the iterative process.

Furthermore, it can also be observed that, with the increase of E_b/N_0, the phase estimation error gets smaller and the convergence rate gets faster. However, since the phase estimation error can not be completely eliminated by iterations, the proposed algorithm will not be able to achieve the theoretically optimal BER performance with the increase of iterations.

E. Computational Complexity Analysis

Due to high complexity of conventional MPA, the algorithm is proposed and applied in $\pi/4$-DQPSK modulated FH communication systems. Thus, the computational complexity is compared only between the proposed algorithm and MPA. The computational complexity is summarized in Table II, where M denotes the modulation order and N denotes the number of symbols per hop. Besides, the complicated function includes exponential function, logarithmic function and Bessel function.

TABLE II
COMPUTATION COMPLEXITY

	MPA	Our Algorithm
Addition	$O(M^2 N \log_2 M + 2M^2 N + 5MN)$	$O(2MN)$
Multiplication	$O(M^2 N \log_2 M + 2M^2 N + 6MN)$	$O(11MN)$
Complicated Function	$O(3MN + 2N \log_2 M)$	$O(MN + N \log_2 M)$

As is shown in Table II, compared with conventional MPA, the computational complexity of proposed algorithm is reduced by more than 60% when $M = 4$ and $N = 124$, and we mainly reduce the computational complexity by simplifying message passing process and probability calculation.

V. CONCLUSION

This paper proposes a low complexity iterative receiving algorithm based on message passing in $\pi/4$-DQPSK modulated FH systems. We combine phase estimation and decoding and pass the probability message between phase estimator and decoder, where the Gaussian distribution is approximated by the Tikhonov distribution. Meanwhile, we simplify the message passing process and continuously improve the phase estimation accuracy and bit error performance through iterations. Simulation results and analysis demonstrate that the proposed receiving algorithm can achieve a much better bit error performance with only few pilot symbols and low complexity.

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