Drag - phase 1

$$D = \frac{1}{n+1} \sum_j c_j v_j^{n+1}$$

Drag has quadratic relationship to velocity $\left(F = \frac{1}{2}\rho c A v^2\right)$

so $n=2$ $\quad v = \dot{y}$ $\quad c = \frac{1}{2}\rho c A$

$$D = \frac{1}{2+1} c\dot{y}^3$$

$$D = \frac{1}{6}\rho c A \dot{y}^3$$

<u>free - fall</u>

$$\frac{\partial L}{\partial \dot{y}} = \dot{y}\left(m\left(\frac{L-y}{2L}\right) + M\right)$$

$$\frac{\partial L}{\partial y} = \frac{-m\dot{y}^2 - 2mgy + 2Lg(m+2M)}{4L}$$

$$\frac{\partial D}{\partial \dot{y}} = \frac{1}{2}\rho c A \dot{y}^2$$

$$\frac{-m\dot{y}^2 - 2mgy + 2Lg(m+2M)}{4L} - \ddot{y}\left(M + m\frac{L-y}{2L}\right) + \frac{m\dot{y}^2}{2L} - \frac{1}{2}\rho c A \dot{y}^2 = 0$$

$$\ddot{y}\left(M + \frac{m(L-y)}{2L}\right) = \frac{m\dot{y}^2}{2L} - \frac{\rho c A \dot{y}^2}{2} + \frac{-m\dot{y}^2 - 2mgy + 2Lg(m+2M)}{4L}$$

$$\ddot{y}\left(M + \frac{m(L-y)}{2L}\right) = \frac{m\dot{y}^2 - 2mgy + 2gLm + 4gLM}{4L} - \frac{\rho c A \dot{y}^2}{2}$$

$$\ddot{y}\left(\frac{2LM + m(L-y)}{2L}\right) = \frac{m\dot{y}^2 - 2mgy + 2gLm + 4gLM - 2L\rho c A \dot{y}^2}{4L}$$

$$\ddot{y} = \frac{m\dot{y}^2 - 2mgy + 2mgL + 4gLM - 2L\rho cA\dot{y}^2}{\cancel{4L}\;2}\left(\frac{2\cancel{L}}{2LM + m(L-y)}\right)$$

$$= \frac{m\dot{y}^2 - 2mgy + 2mgL + 4gLM - 2L\rho cA\dot{y}^2}{4LM + 2m(L-y)}$$

$$\ddot{y} = \frac{\frac{1}{2}m\dot{y}^2 - mgy + mgL + 2gLM - L\rho cA\dot{y}^2}{2LM + m(L-y)}$$

$$\ddot{y} = g\left(\frac{2ML + mL - my}{2ML + mL - my}\right) + \frac{\frac{1}{2}m\dot{y}^2 - L\rho cA\dot{y}^2}{2LM + m(L-y)}$$

$$\boxed{\ddot{y} = g + \frac{\left(\frac{1}{2}m - L\rho cA\right)\dot{y}^2}{2LM + m(L-y)}}$$