Eom for non-Stretched Cord.

$$L = T - V$$

$$= \frac{1}{2} m \frac{(L-y)}{2L} \dot{y}^2 + \frac{1}{2} M \dot{y}^2 + M g y + m \frac{(L-y)}{2L} g\left(y + \frac{L-y}{4}\right)$$

$$+ m \frac{L+y}{2L} g \frac{L+y}{4}$$

$$\therefore \frac{\partial L}{\partial y} = \frac{-m\dot{y}^2 - 2mgy + 2gL(m+2M)}{4L}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = \ddot{y}\left(M + m\frac{L-y}{2L}\right) - \frac{m\dot{y}^2}{2L}$$

plug into Euler-Lagrange equation: $\frac{\partial L}{\partial y} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = 0$

$$\frac{-m\dot{y}^2 - 2mgy + 2gL(m+M)}{4L} - \ddot{y}\left(M + m\frac{(L-y)}{2L}\right) + \frac{m\dot{y}^2}{2L} = 0$$

$$\ddot{y}\left(M + m\frac{(L-y)}{2L}\right) = \frac{m\dot{y}^2}{2L} + \frac{-m\dot{y}^2 - 2mgy + 2gL(m+2M)}{4L}$$

$$\ddot{y}\left(\frac{2LM + m(L-y)}{2L}\right) = \frac{m\dot{y}^2 - 2mgy + 2gLm + 4gLM}{4L}$$

$$\ddot{y} = \left(\frac{m\dot{y}^2 - 2mgy + 2gLm + 4gLM}{4L}\right)\left(\frac{2L}{2LM + m(L-y)}\right)$$

$$\ddot{y} = \left(\frac{m\dot{y}^2 - 2mgy + 2gLm + 4gLM}{4LM + 2m(L-y)}\right)$$

$$\ddot{y} = g\left(\frac{2ML + mL - my}{2ML + mL - my}\right) + \frac{\frac{1}{2}m\dot{y}^2}{2ML + mL - my}$$

$$\boxed{\ddot{y} = g + \frac{\frac{1}{2}m\dot{y}^2}{m(L-y) + 2ML}}$$