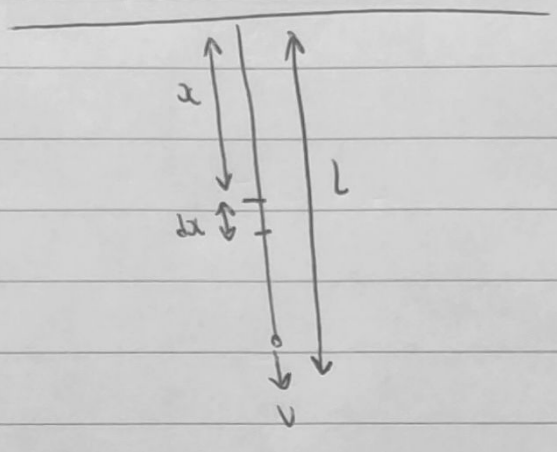
Stretching rope Kinetic Energy

Let x be some position along a rope of natural length L and constant density λ. Let dx be a small segment of rope of mass dm. Let the free end of the rope travel downwards with velocity V.



Assume the velocity of any segment of rope is $\propto$ to its position.

$$\Rightarrow v = x\frac{V}{L}$$

$$dk = \tfrac{1}{2}dmv^2 \qquad \text{(its kinetic energy)}$$

$$= \tfrac{1}{2}(\lambda dx)\left(\frac{xV}{L}\right)^2$$

$$= \tfrac{1}{2}\left(\frac{M}{L}dx\right)\left(\frac{x^2V^2}{L^2}\right)$$

$$= \tfrac{1}{2}\left(\frac{M}{L}\right)\left(\frac{V^2}{L^2}\right)x^2\,dx$$

$$k = \tfrac{1}{2}\frac{MV^2}{L^3}\int_0^L x^2\,dx = \tfrac{1}{2}\frac{MV^2}{L^3}\left[\frac{x^3}{3}\right]_0^L$$

$$= \tfrac{1}{2}\frac{MV^2}{L^3}\left[\frac{L^3}{3}\right]$$

$$\boxed{k = \tfrac{1}{6}MV^2}$$