Drag - Phase 2

$$\frac{\partial L}{\partial y} = Mg + \frac{mg}{2} - k(y-l)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = M\ddot{y} + \frac{1}{3}m\ddot{y}$$

$$\frac{\partial D}{\partial \dot{y}} = \frac{1}{2}\rho C A \dot{y}^2 \qquad \text{(from Drag - phase 1)}$$

Sub into $\frac{\partial L}{\partial y} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial D}{\partial \dot{y}} = 0$

$$Mg + \frac{mg}{2} - k(y-l) - M\ddot{y} - \frac{1}{3}m\ddot{y} - \frac{1}{2}\rho C A \dot{y}^2 = 0$$

$$\ddot{y}\left(M + \frac{m}{3}\right) = Mg + \frac{mg}{2} - k(y-l) - \frac{1}{2}\rho C A \dot{y}^2$$

$$\ddot{y} = \left(\frac{3}{3M+m}\right)\left(\frac{2Mg + mg - 2k(y-l) - \rho C A \dot{y}^2}{2}\right)$$

$$\boxed{\ddot{y} = \frac{6Mg + 3mg - 6k(y-l) - 3\rho C A \dot{y}^2}{6M + 2m}}$$

Lets add a Linear term to account for energy lost in the rope itself (Due to heating)

$$D = \frac{1}{2} f \dot{y}^2$$
$$\frac{\partial D}{\partial \dot{y}} = f\dot{y}$$

going through the same steps as above.

$$\ddot{y}\left(M + \frac{m}{3}\right) = Mg + \frac{mg}{2} - k(y-l) - \frac{1}{2}\rho C A \dot{y}^2 - f\dot{y}$$

$$\ddot{y}\left(\frac{3M+m}{3}\right) = \frac{2Mg + mg - 2k(y-l) - \rho C A \dot{y}^2 - 2f\dot{y}}{2}$$

$$mg = k\blacksquare(2L)$$

$$k = \frac{mg}{2L}$$

$$\ddot{y} = \frac{6Mg + 3mg - 6k(\blacksquare y - L) - 3\rho C A \dot{y}^2 - 6f\dot{y}}{6M + 2m}$$