# 760-Heuristics-Assignment

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## 1 Question 1

Neighbors of the current solution can be found by swapping one pot in any crucible with a pot in any other crucible. This can be formally defined as following:

$N(\mathbf{x}) = \{\mathbf{y}(\mathbf{x}, k, l, m, n), k = 1, 2, 3, ..., 16, l = k + 1, k + 2, k + 3, ...17, m = 1, 2, 3, n = 1, 2, 3\}$ where

$$\mathbf{y}(\mathbf{x}, k, l, m, n) = (y_{1,1}, y_{1,2}, y_{1,3}; y_{2,1}y_{2,2}y_{2,3}; ...y_{c,j}), y_{c,j} = \begin{cases} x_{l,n} & \text{if } c = k, j = m \\ x_{k,m} & \text{if } c = l, j = n \\ x_{c,j} & \text{otherwise} \end{cases}$$

# 2 Question 2

Let $h(x_c) = g(\overline{Al}[x_{c,avg}], \overline{Fe}[x_{c,avg}], \overline{Si}[x_{c,avg}])$ where

$$\overline{Al}[x_{c,avg}] = \frac{Al[x_{c,1}], Al[x_{c,2}], Al[x_{c,3}]}{3}$$
$$\overline{Fe}[x_{c,avg}] = \frac{Fe[x_{c,1}], Fe[x_{c,2}], Fe[x_{c,3}]}{3}$$
$$\overline{Si}[x_{c,avg}] = \frac{Si[x_{c,1}], Si[x_{c,2}], Si[x_{c,3}]}{3}$$

**Algorithm 1** Sweep x

Let $S$ contain all possible solutions of $x$
Let $x$, $x \in S$, be some initial configuration
Let $x^*$ be some configuration that maximizes f(x)
Let $I$ be the intermediate values
$I_i := h(x_i) \forall i \in c$
**while** not stopped **do**
    Compare each neighbor $y(x, k, l, m, n) \in N(x)$
    Let $d = h(y_k) + h(y_l) - I_k - I_l$
    **if** $d > 0$ for some $y \in N(x)$ **then**
        x := y
        $I_k := h(y_k)$
        $I_l := h(y_l)$
    **else if** $d \leq 0 \forall y \in N(x)$ **then**
        Stop
    **end if**
**end while**
$x^* := x$

# 3 Question 3

See Appendix A for all code and plots.

## 3.1 Question 3E

Best solution found using repeated next ascents with $n = 200$.

```
1 [27  4 45 ] 99.261 %Al, 0.274 %Fe, 0.110 %Si, $41.53, spread = 41
2 [ 8 13 37 ] 99.508 %Al, 0.154 %Fe, 0.153 %Si, $48.71, spread = 29
3 [50 40 30 ] 99.504 %Al, 0.143 %Fe, 0.238 %Si, $48.71, spread = 20
4 [17 51 15 ] 99.268 %Al, 0.162 %Fe, 0.386 %Si, $41.53, spread = 36
5 [24 23 46 ] 99.512 %Al, 0.053 %Fe, 0.180 %Si, $48.71, spread = 23
6 [28 12 32 ] 99.358 %Al, 0.096 %Fe, 0.348 %Si, $44.53, spread = 20
7 [18  3 42 ] 99.760 %Al, 0.040 %Fe, 0.139 %Si, $57.35, spread = 39
```

```
 8 [19 41 20 ] 99.262 %Al, 0.212 %Fe, 0.249 %Si, $41.53, spread = 22
 9 [11 25  5 ] 99.353 %Al, 0.185 %Fe, 0.325 %Si, $44.53, spread = 20
10 [ 9  6 21 ] 99.504 %Al, 0.139 %Fe, 0.253 %Si, $48.71, spread = 15
11 [ 7 36 26 ] 99.506 %Al, 0.098 %Fe, 0.268 %Si, $48.71, spread = 29
12 [47 39  2 ] 99.503 %Al, 0.114 %Fe, 0.270 %Si, $48.71, spread = 45
13 [22 33 10 ] 99.356 %Al, 0.187 %Fe, 0.219 %Si, $44.53, spread = 23
14 [34 29 38 ] 99.361 %Al, 0.209 %Fe, 0.241 %Si, $44.53, spread =  9
15 [14 43 49 ] 99.254 %Al, 0.224 %Fe, 0.307 %Si, $41.53, spread = 35
16 [ 1 35 31 ] 99.350 %Al, 0.075 %Fe, 0.336 %Si, $44.53, spread = 34
17 [48 44 16 ] 99.515 %Al, 0.143 %Fe, 0.241 %Si, $48.71, spread = 32
                                         Sum = $787.09, MxSprd = 45
```

Best solution found using repeated steepest ascents with $n = 200$.

```
 1 [ 1 48 15 ] 99.256 %Al, 0.121 %Fe, 0.300 %Si, $41.53, spread = 47
 2 [29 36 10 ] 99.369 %Al, 0.216 %Fe, 0.272 %Si, $44.53, spread = 26
 3 [33  6 34 ] 99.268 %Al, 0.253 %Fe, 0.272 %Si, $41.53, spread = 28
 4 [42 16 49 ] 99.502 %Al, 0.161 %Fe, 0.163 %Si, $48.71, spread = 33
 5 [43 24 39 ] 99.508 %Al, 0.067 %Fe, 0.243 %Si, $48.71, spread = 19
 6 [47  8  5 ] 99.366 %Al, 0.188 %Fe, 0.317 %Si, $44.53, spread = 42
 7 [ 4 26 19 ] 99.253 %Al, 0.243 %Fe, 0.283 %Si, $41.53, spread = 22
 8 [31 28 46 ] 99.365 %Al, 0.139 %Fe, 0.196 %Si, $44.53, spread = 18
 9 [ 3 11 23 ] 99.502 %Al, 0.058 %Fe, 0.246 %Si, $48.71, spread = 20
10 [32 27 35 ] 99.501 %Al, 0.087 %Fe, 0.224 %Si, $48.71, spread =  8
11 [22 40 51 ] 99.353 %Al, 0.122 %Fe, 0.349 %Si, $44.53, spread = 29
12 [13 38  2 ] 99.504 %Al, 0.177 %Fe, 0.160 %Si, $48.71, spread = 36
13 [50 25 37 ] 99.500 %Al, 0.188 %Fe, 0.231 %Si, $48.71, spread = 25
14 [20 30 17 ] 99.522 %Al, 0.111 %Fe, 0.277 %Si, $48.71, spread = 13
15 [ 9 18 44 ] 99.750 %Al, 0.037 %Fe, 0.099 %Si, $57.35, spread = 35
16 [45 41 12 ] 99.258 %Al, 0.171 %Fe, 0.284 %Si, $41.53, spread = 33
17 [21  7 14 ] 99.360 %Al, 0.168 %Fe, 0.347 %Si, $44.53, spread = 14
                                         Sum = $787.09, MxSprd = 47
```

# 4 Quesion 4

There does not seem to be a significant difference in the objective value for next and steepest ascent. Steepest ascent also took far longer to converge on each local optimum. This means that fastest ascent is better than steepest ascent; however, this conclusion is from limited information so it not conclusive evidence.

# 5 Question 5

(a) You would expect the problem's objective function to have lots of plateus because the objective function is not continuous. This means there will be lots of cases where two pots are swapped and the quality, and thus value, will remain constant.

(b) The best outcome is when the crucible is just above the min/max requirements to enter the grade boundary. This is because when sitting too far above the grade boundary there are wasted resources (i.e. the quality doesn't need to be that good). The proposed function will add a non-linear gradient to the existing value function to encourage sitting on the grade boundary. This non-linear gradient will take the form of a quadratic with a local minimum at the current grade boundary. The maximum value this quadratic will take will be 1 at the next grade boundary and the minimum 0. This can be formally defined as follows

$$g'(\overline{Al}, \overline{Fe}, \overline{Si}) = g(\overline{Al}, \overline{Fe}, \overline{Si}) + \frac{(p-r)^2}{(q-r)^2}$$

Where
$p$ is which of $\overline{Al}, \overline{Fe}, \overline{Si}$ is closest to the current grade min/max requirement.
$r$ is the current grade min/max requirement for $p$.
$q$ is the next highest grade min/max requirement for $p$.

The effect this will have is that there will be a gradient encouraging neighbors to be closer to the next grade boundary. Furthermore, neighbors that are already closer to the next grade boundary will be further encouraged than those who are far away from the next grade boundary.

Let us consider a simplified example where the crucible only has one property, $Al$. If $Al > 95$ then the value is \$10. If $Al > 97$ then the value is \$20. Now let us consider solution x with two crucibles $x_k, x_l$, and a neighbor y with two crucibles $y_k, y_l$.
$Al[x_{k,1}] = 95.3, Al[x_{k,2}] = 95.2, Al[x_{k,3}] = 96.5, \overline{Al}[x_k] = 95.67$
$Al[x_{l,1}] = 96.8, Al[x_{l,2}] = 96.9, Al[x_{l,3}] = 95.4, \overline{Al}[x_l] = 96.37$
$Al[y_{k,1}] = 95.3, Al[y_{k,2}] = 95.2, Al[y_{k,3}] = 95.4, \overline{Al}[y_k] = 95.30$
$Al[y_{l,1}] = 96.8, Al[y_{l,2}] = 96.9, Al[y_{l,3}] = 96.5, \overline{Al}[y_l] = 96.73$

In this example using the original grading function there would be no difference between the objective value for $x$ and $y$ so the neighbor would not be accepted in standard next/steepest ascent. However, accepting the new solution would likely allow a better solution to be found, as crucible $l$ is closer to the \$97 boundary while $k$ is closer to the \$95 boundary, which it will not drop below without good reason due to the sharp change in grading function $g()$ at this boundary.

Using the modified $g'()$ The crucible values are as follows
$x_k = \$10.112225$ $x_l = \$10.469225$ $y_k = \$10.022500$ $y_l = \$10.748225$

This results in a change of objective value of 0.189275, meaning the better solution will be accepted. The greatest magnitude this gradient value can take is 1, as it is stil desirable to have a steep gradient change when changing grades.

*Note: This new objective function with next ascent ended up working better than simulated annealing and was used for find my best solutions.*

# 6 Question 6

$$g''(\overline{Al}, \overline{Fe}, \overline{Si}, x_{c1}, x_{c2}, x_{c3}, s) = \begin{cases} g(\overline{Al}, \overline{Fe}, \overline{Si}) - 20 * (s_c - s) & \text{if } s_c > s \\ g(\overline{Al}, \overline{Fe}, \overline{Si}) & \text{otherwise} \end{cases}$$

where $s_c = \max(x_{c1}, x_{c2}, x_{c3}) - \min(x_{c1}, x_{c2}, x_{c3})$

## 6.1 Task 6

See Appendix A for code.

### 6.1.1 Max Spread = 6

```
 1 [38 34 40 ] 99.371 %Al, 0.241 %Fe, 0.163 %Si, $44.53, spread =  6
 2 [17 13 19 ] 99.270 %Al, 0.297 %Fe, 0.234 %Si, $41.53, spread =  6
 3 [35 36 39 ] 99.615 %Al, 0.043 %Fe, 0.196 %Si, $48.71, spread =  4
 4 [ 6  8 10 ] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread =  4
 5 [30 32 27 ] 99.558 %Al, 0.093 %Fe, 0.170 %Si, $48.71, spread =  5
 6 [37 42 41 ] 99.501 %Al, 0.115 %Fe, 0.198 %Si, $48.71, spread =  5
 7 [24 22 21 ] 99.532 %Al, 0.037 %Fe, 0.276 %Si, $48.71, spread =  3
 8 [33 29 28 ] 99.256 %Al, 0.151 %Fe, 0.303 %Si, $41.53, spread =  5
 9 [11 14  9 ] 99.381 %Al, 0.080 %Fe, 0.297 %Si, $44.53, spread =  5
10 [ 5  7  3 ] 99.512 %Al, 0.188 %Fe, 0.212 %Si, $48.71, spread =  4
11 [46 47 43 ] 99.359 %Al, 0.172 %Fe, 0.315 %Si, $44.53, spread =  4
12 [26 25 31 ] 99.393 %Al, 0.138 %Fe, 0.325 %Si, $44.53, spread =  6
13 [49 48 44 ] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread =  5
14 [20 18 23 ] 99.544 %Al, 0.057 %Fe, 0.220 %Si, $48.71, spread =  5
15 [15 16 12 ] 99.369 %Al, 0.127 %Fe, 0.302 %Si, $44.53, spread =  4
16 [ 2  4  1 ] 99.356 %Al, 0.144 %Fe, 0.280 %Si, $44.53, spread =  3
17 [50 51 45 ] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread =  6
                                         Sum = $777.27, MxSprd =  6
```

### 6.1.2 Max Spread = 8

```
 1 [15 13 11 ] 99.255 %Al, 0.157 %Fe, 0.277 %Si, $41.53, spread =  4
 2 [ 2  1  4 ] 99.356 %Al, 0.144 %Fe, 0.280 %Si, $44.53, spread =  3
 3 [46 49 51 ] 99.399 %Al, 0.152 %Fe, 0.259 %Si, $44.53, spread =  5
 4 [26 19 24 ] 99.355 %Al, 0.140 %Fe, 0.305 %Si, $44.53, spread =  7
 5 [32 29 36 ] 99.509 %Al, 0.087 %Fe, 0.197 %Si, $48.71, spread =  7
```

```
 6 [34 35 33 ] 99.358 %Al, 0.156 %Fe, 0.278 %Si, $44.53, spread =  2
 7 [ 8  3 10 ] 99.527 %Al, 0.176 %Fe, 0.194 %Si, $48.71, spread =  7
 8 [48 50 44 ] 99.544 %Al, 0.129 %Fe, 0.241 %Si, $48.71, spread =  6
 9 [14  6  7 ] 99.253 %Al, 0.249 %Fe, 0.360 %Si, $41.53, spread =  8
10 [16 18 17 ] 99.542 %Al, 0.132 %Fe, 0.258 %Si, $48.71, spread =  2
11 [30 27 23 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread =  7
12 [20 22 21 ] 99.516 %Al, 0.060 %Fe, 0.277 %Si, $48.71, spread =  2
13 [43 45 38 ] 99.350 %Al, 0.226 %Fe, 0.125 %Si, $44.53, spread =  7
14 [ 9  5 12 ] 99.510 %Al, 0.106 %Fe, 0.258 %Si, $48.71, spread =  7
15 [40 39 47 ] 99.378 %Al, 0.169 %Fe, 0.269 %Si, $44.53, spread =  8
16 [28 25 31 ] 99.278 %Al, 0.204 %Fe, 0.285 %Si, $41.53, spread =  6
17 [42 41 37 ] 99.501 %Al, 0.115 %Fe, 0.198 %Si, $48.71, spread =  5
                                           Sum = $781.45, MxSprd =  8
```

#### 6.1.3 Max Spread = 11

```
 1 [40 49 51 ] 99.301 %Al, 0.205 %Fe, 0.295 %Si, $41.53, spread = 11
 2 [35 33 34 ] 99.358 %Al, 0.156 %Fe, 0.278 %Si, $44.53, spread =  2
 3 [ 5  3  7 ] 99.512 %Al, 0.188 %Fe, 0.212 %Si, $48.71, spread =  4
 4 [29 31 28 ] 99.257 %Al, 0.160 %Fe, 0.310 %Si, $41.53, spread =  3
 5 [ 4  6  9 ] 99.386 %Al, 0.217 %Fe, 0.204 %Si, $44.53, spread =  5
 6 [36 32 26 ] 99.524 %Al, 0.048 %Fe, 0.266 %Si, $48.71, spread = 10
 7 [39 44 47 ] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread =  8
 8 [25 18 16 ] 99.527 %Al, 0.153 %Fe, 0.256 %Si, $48.71, spread =  9
 9 [41 50 48 ] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread =  9
10 [14 22 24 ] 99.362 %Al, 0.083 %Fe, 0.335 %Si, $44.53, spread = 10
11 [13 23 19 ] 99.251 %Al, 0.258 %Fe, 0.213 %Si, $41.53, spread = 10
12 [30 27 20 ] 99.530 %Al, 0.104 %Fe, 0.212 %Si, $48.71, spread = 10
13 [46 42 45 ] 99.502 %Al, 0.140 %Fe, 0.056 %Si, $48.71, spread =  4
14 [21 10 17 ] 99.369 %Al, 0.239 %Fe, 0.344 %Si, $44.53, spread = 11
15 [37 43 38 ] 99.519 %Al, 0.169 %Fe, 0.143 %Si, $48.71, spread =  6
16 [11 12 15 ] 99.361 %Al, 0.068 %Fe, 0.309 %Si, $44.53, spread =  4
17 [ 8  1  2 ] 99.526 %Al, 0.042 %Fe, 0.230 %Si, $48.71, spread =  7
                                           Sum = $781.45, MxSprd = 11
```

## 7 My Best Solutions

These were found by using my modified objective function from Q5 in C with next ascent. This gave slightly better results than simulated annealing in C. I am happy to provide the code if required for the competition, just email me.

### 7.1 No Max Spread

```
 1 [35 23 19 ] 99.356 %Al, 0.172 %Fe, 0.239 %Si, $44.53, spread = 16
 2 [ 5 49 26 ] 99.351 %Al, 0.188 %Fe, 0.316 %Si, $44.53, spread = 44
```

```
 3 [ 6 43  7 ] 99.350 %Al, 0.235 %Fe, 0.300 %Si, $44.53, spread = 37
 4 [24 33  1 ] 99.350 %Al, 0.056 %Fe, 0.283 %Si, $44.53, spread = 32
 5 [12 48 45 ] 99.352 %Al, 0.166 %Fe, 0.235 %Si, $44.53, spread = 36
 6 [13 39 37 ] 99.500 %Al, 0.154 %Fe, 0.170 %Si, $48.71, spread = 26
 7 [17 20 34 ] 99.351 %Al, 0.201 %Fe, 0.308 %Si, $44.53, spread = 17
 8 [29 38 41 ] 99.355 %Al, 0.154 %Fe, 0.261 %Si, $44.53, spread = 12
 9 [11  2 16 ] 99.500 %Al, 0.087 %Fe, 0.272 %Si, $48.71, spread = 14
10 [47  3 15 ] 99.351 %Al, 0.172 %Fe, 0.287 %Si, $44.53, spread = 44
11 [28 50 22 ] 99.350 %Al, 0.126 %Fe, 0.329 %Si, $44.53, spread = 28
12 [51  9 36 ] 99.517 %Al, 0.070 %Fe, 0.264 %Si, $48.71, spread = 42
13 [27 25  4 ] 99.350 %Al, 0.253 %Fe, 0.204 %Si, $44.53, spread = 23
14 [14 40 46 ] 99.351 %Al, 0.173 %Fe, 0.232 %Si, $44.53, spread = 32
15 [30  8 10 ] 99.500 %Al, 0.167 %Fe, 0.227 %Si, $48.71, spread = 22
16 [21 32 31 ] 99.501 %Al, 0.089 %Fe, 0.201 %Si, $48.71, spread = 11
17 [44 42 18 ] 99.750 %Al, 0.044 %Fe, 0.136 %Si, $57.35, spread = 26
                                           Sum = $790.73, MxSprd = 44
```

### 7.2 Max Spread = 6

```
 1 [44,49,48,] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread=5
 2 [18,19,21,] 99.512 %Al, 0.163 %Fe, 0.187 %Si, $48.71, spread=3
 3 [43,47,46,] 99.359 %Al, 0.172 %Fe, 0.315 %Si, $44.53, spread=4
 4 [9,11,12,] 99.535 %Al, 0.015 %Fe, 0.265 %Si, $48.71, spread=3
 5 [27,24,22,] 99.502 %Al, 0.068 %Fe, 0.194 %Si, $48.71, spread=5
 6 [36,39,34,] 99.502 %Al, 0.139 %Fe, 0.173 %Si, $48.71, spread=5
 7 [25,23,28,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, $41.53, spread=5
 8 [38,37,42,] 99.650 %Al, 0.118 %Fe, 0.067 %Si, $52.44, spread=5
 9 [51,50,45,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread=6
 10 [2,7,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
 11 [29,26,31,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, $44.53, spread=5
 12 [16,14,20,] 99.298 %Al, 0.164 %Fe, 0.379 %Si, $41.53, spread=6
 13 [33,32,30,] 99.541 %Al, 0.070 %Fe, 0.160 %Si, $48.71, spread=3
 14 [6,8,10,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread=4
 15 [3,1,4,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, $44.53, spread=3
 16 [13,15,17,] 99.276 %Al, 0.220 %Fe, 0.261 %Si, $41.53, spread=4
 17 [35,40,41,] 99.335 %Al, 0.141 %Fe, 0.317 %Si, $41.53, spread=6
Sum = $779.18, MxSprd = 6
```

### 7.3 Max Spread = 8

```
 1 [8,1,6,] 99.358 %Al, 0.130 %Fe, 0.315 %Si, $44.53, spread=7
 2 [24,27,21,] 99.506 %Al, 0.091 %Fe, 0.245 %Si, $48.71, spread=6
 3 [26,31,29,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, $44.53, spread=5
 4 [32,40,34,] 99.353 %Al, 0.199 %Fe, 0.210 %Si, $44.53, spread=8
 5 [46,44,41,] 99.507 %Al, 0.108 %Fe, 0.191 %Si, $48.71, spread=5
 6 [22,15,18,] 99.514 %Al, 0.064 %Fe, 0.164 %Si, $48.71, spread=7
```

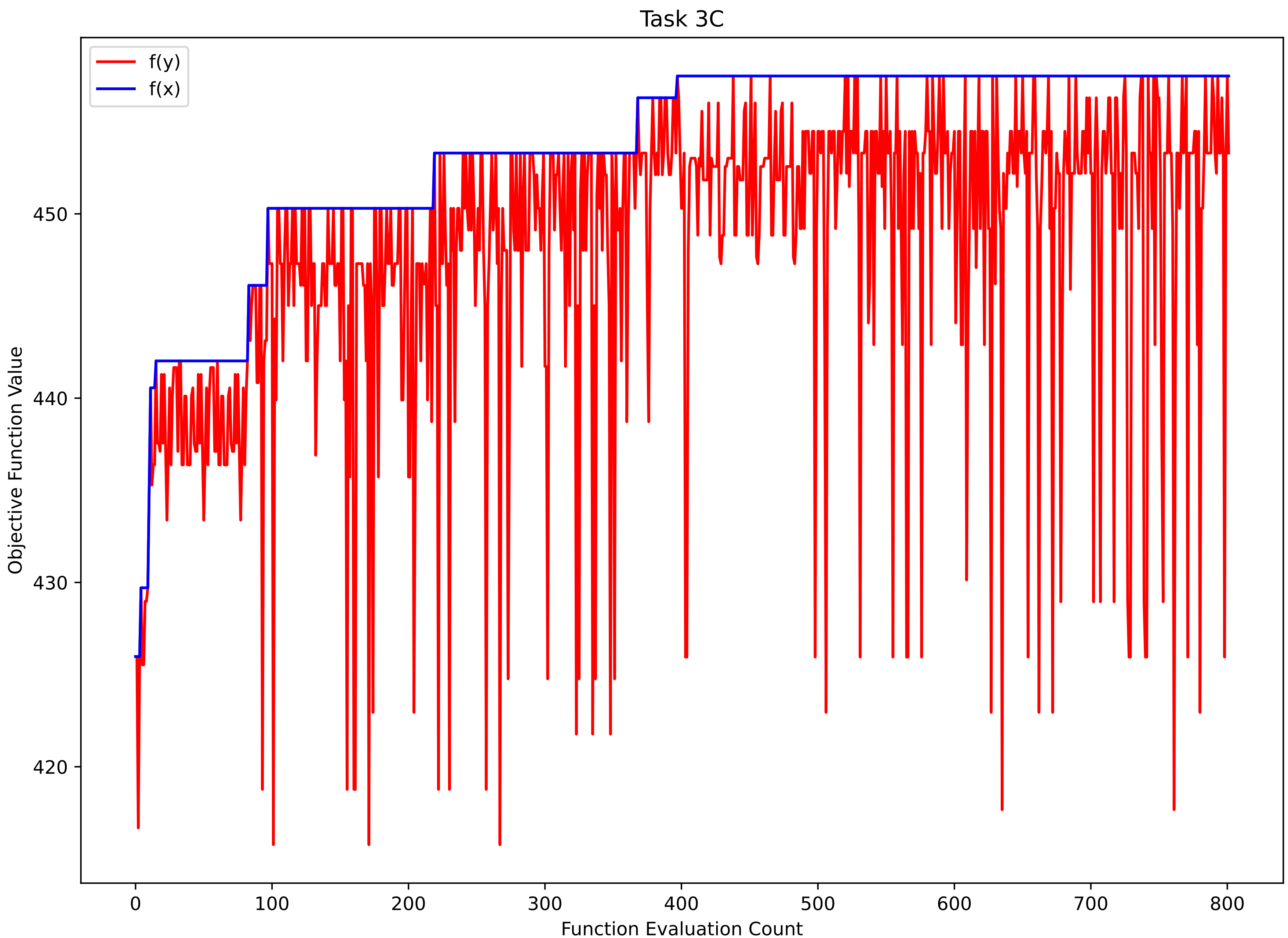
```
 7 [17,13,20,] 99.359 %Al, 0.191 %Fe, 0.305 %Si, $44.53, spread=7
 8 [38,33,35,] 99.501 %Al, 0.105 %Fe, 0.167 %Si, $48.71, spread=5
 9 [47,39,43,] 99.370 %Al, 0.153 %Fe, 0.341 %Si, $44.53, spread=8
 10 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
 11 [51,45,50,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread=6
 12 [12,16,19,] 99.362 %Al, 0.204 %Fe, 0.275 %Si, $44.53, spread=7
 13 [11,9,14,] 99.381 %Al, 0.080 %Fe, 0.297 %Si, $44.53, spread=5
 14 [36,30,37,] 99.654 %Al, 0.098 %Fe, 0.129 %Si, $52.44, spread=7
 15 [10,3,4,] 99.357 %Al, 0.278 %Fe, 0.244 %Si, $44.53, spread=7
 16 [49,48,42,] 99.513 %Al, 0.147 %Fe, 0.139 %Si, $48.71, spread=7
 17 [28,25,23,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, $41.53, spread=5
Sum = $784.00, MxSprd = 8
```

### 7.4 Max Spread = 11

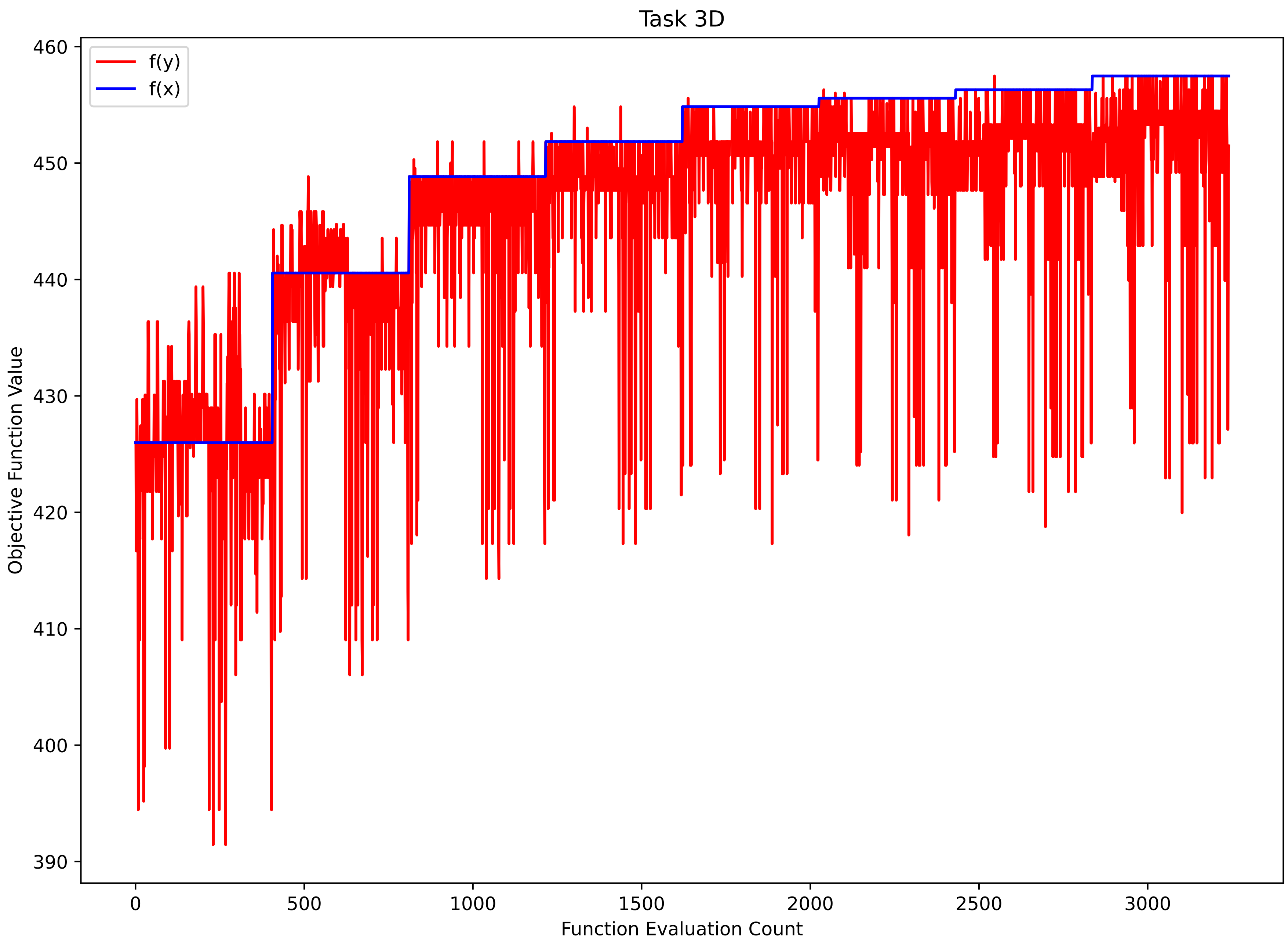
```
 1 [49,46,51,] 99.399 %Al, 0.152 %Fe, 0.259 %Si, $44.53, spread=5
 2 [26,37,32,] 99.514 %Al, 0.072 %Fe, 0.266 %Si, $48.71, spread=11
 3 [19,21,29,] 99.356 %Al, 0.204 %Fe, 0.263 %Si, $44.53, spread=10
 4 [20,30,23,] 99.503 %Al, 0.072 %Fe, 0.255 %Si, $48.71, spread=10
 5 [25,28,36,] 99.360 %Al, 0.186 %Fe, 0.275 %Si, $44.53, spread=11
 6 [2,1,4,] 99.356 %Al, 0.144 %Fe, 0.280 %Si, $44.53, spread=3
 7 [17,16,6,] 99.354 %Al, 0.238 %Fe, 0.341 %Si, $44.53, spread=11
 8 [34,35,33,] 99.358 %Al, 0.156 %Fe, 0.278 %Si, $44.53, spread=2
 9 [45,43,38,] 99.350 %Al, 0.226 %Fe, 0.125 %Si, $44.53, spread=7
 10 [10,3,13,] 99.353 %Al, 0.271 %Fe, 0.287 %Si, $44.53, spread=10
 11 [39,44,47,] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread=8
 12 [7,9,5,] 99.504 %Al, 0.163 %Fe, 0.166 %Si, $48.71, spread=4
 13 [42,40,31,] 99.500 %Al, 0.131 %Fe, 0.120 %Si, $48.71, spread=11
 14 [11,8,14,] 99.375 %Al, 0.082 %Fe, 0.312 %Si, $44.53, spread=6
 15 [22,27,24,] 99.502 %Al, 0.068 %Fe, 0.194 %Si, $48.71, spread=5
 16 [15,12,18,] 99.502 %Al, 0.068 %Fe, 0.241 %Si, $48.71, spread=6
 17 [48,41,50,] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread=9
Sum = $786.27, MxSprd = 11
```

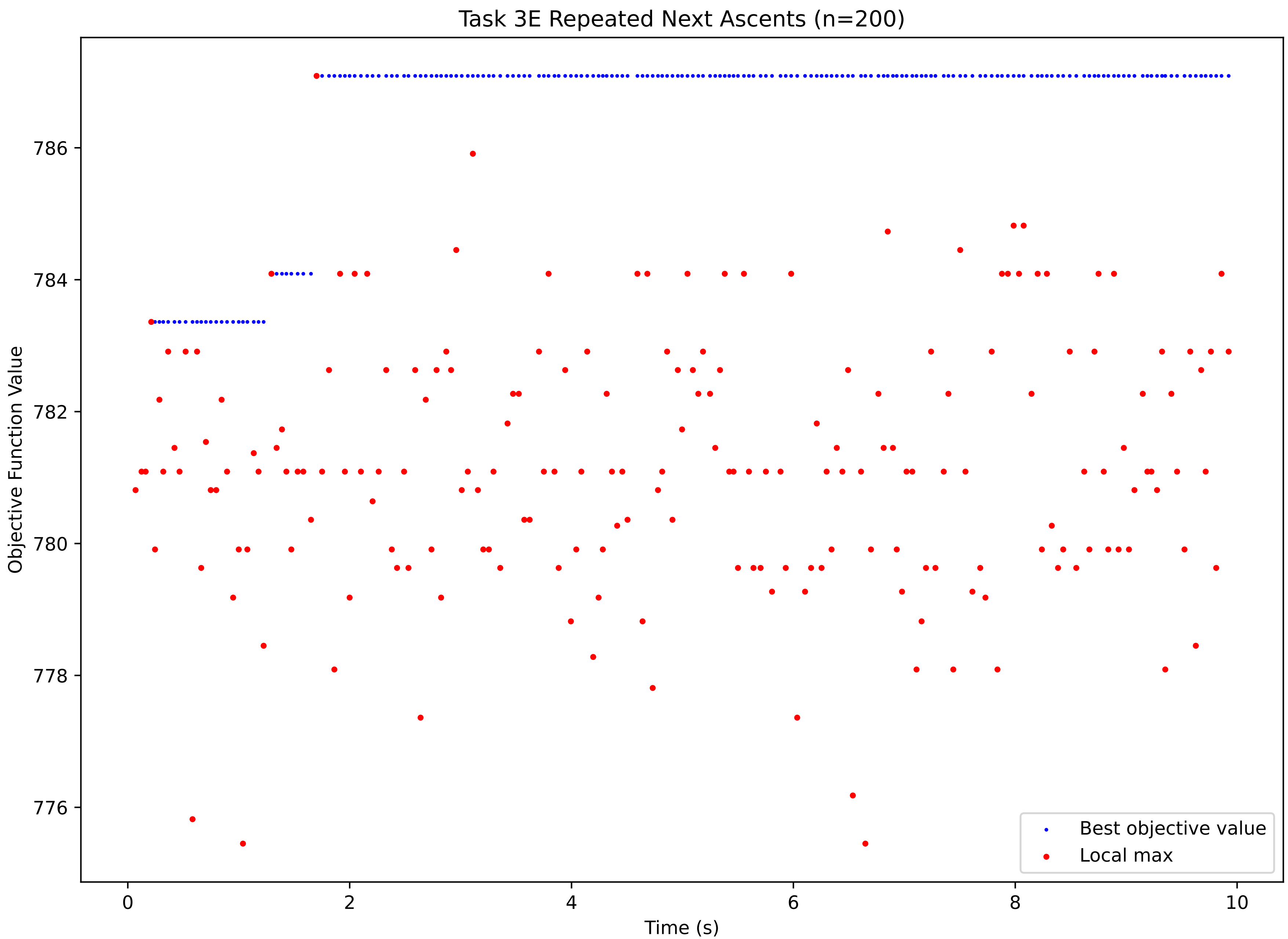
## 8 Appendix A

See overleaf

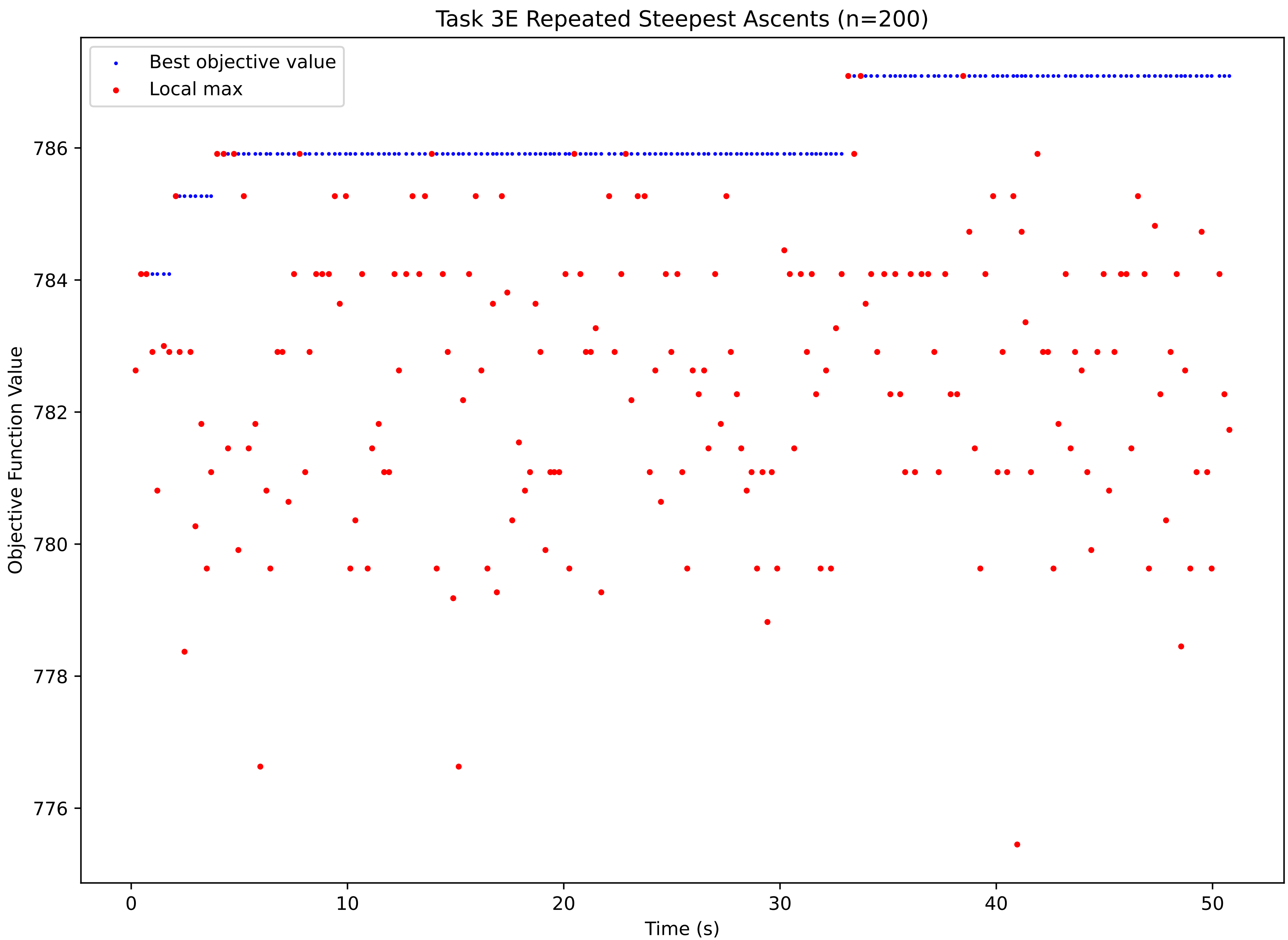


Task 3C
f(y)
f(x)
Objective Function Value
420
430
440
450
0
100
200
300
400
500
600
700
800
Function Evaluation Count


Task 3D
f(y)
f(x)
Objective Function Value
Function Evaluation Count
460
450
440
430
420
410
400
390
0
500
1000
1500
2000
2500
3000



Task 3E Repeated Next Ascents (n=200)
Objective Function Value
Time (s)
786
784
782
780
778
776
0
2
4
6
8
10
Best objective value
Local max


Task 3E Repeated Steepest Ascents (n=200)
Best objective value
Local max
Objective Function Value
Time (s)
776
778
780
782
784
786
0
10
20
30
40
50