760-Heuristics-Assignment

Leo Mooney

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1 Best Solution

```
 1 [13  4 26 ] 99.253 %Al, 0.210 %Fe, 0.373 %Si, $41.53, spread = 22
 2 [31 46 17 ] 99.500 %Al, 0.135 %Fe, 0.144 %Si, $48.71, spread = 29
 3 [49 14 29 ] 99.251 %Al, 0.207 %Fe, 0.314 %Si, $41.53, spread = 35
 4 [23 27 30 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread =  7
 5 [20 50 10 ] 99.351 %Al, 0.220 %Fe, 0.327 %Si, $44.53, spread = 40
 6 [ 8 34 19 ] 99.353 %Al, 0.244 %Fe, 0.155 %Si, $44.53, spread = 26
 7 [ 1 48 24 ] 99.355 %Al, 0.069 %Fe, 0.342 %Si, $44.53, spread = 47
 8 [32  6  2 ] 99.500 %Al, 0.146 %Fe, 0.227 %Si, $48.71, spread = 30
 9 [40 36  7 ] 99.501 %Al, 0.169 %Fe, 0.121 %Si, $48.71, spread = 33
10 [28 25 43 ] 99.252 %Al, 0.218 %Fe, 0.369 %Si, $41.53, spread = 18
11 [21 45 16 ] 99.357 %Al, 0.199 %Fe, 0.232 %Si, $44.53, spread = 29
12 [ 9 47 39 ] 99.501 %Al, 0.095 %Fe, 0.228 %Si, $48.71, spread = 38
13 [37 35 11 ] 99.501 %Al, 0.067 %Fe, 0.277 %Si, $48.71, spread = 26
14 [22 15  3 ] 99.500 %Al, 0.089 %Fe, 0.165 %Si, $48.71, spread = 19
15 [18 42 44 ] 99.750 %Al, 0.044 %Fe, 0.136 %Si, $57.35, spread = 26
16 [51  5 38 ] 99.351 %Al, 0.192 %Fe, 0.349 %Si, $44.53, spread = 46
17 [41 33 12 ] 99.353 %Al, 0.099 %Fe, 0.305 %Si, $44.53, spread = 29
                                            Sum = $790.09, MxSprd = 47
```

This was found by using simulated annealing in Python. I also tried simulated annealing in C but could not improve upon this solution. The parameters I used were $c_1 = 100$, $\alpha = 0.99999999$. I updated c_k every iteration.

2 Question 1

Neighbors of the current solution can be found by swapping one pot in any crucible with a pot in any other crucible. This can be formally defined as following:

$N(\mathbf{x}) = \{\mathbf{y}(\mathbf{x}, k, l, m, n), k = 1, 2, 3, ..., 16, l = k + 1, k + 2, k + 3, ...17, m = 1, 2, 3, n = 1, 2, 3\}$ where

$$\mathbf{y}(\mathbf{x},k,l,m,n) = (y_{1,1},y_{1,2},y_{1,3};y_{2,1}y_{2,2}y_{2,3};...y_{c,j}), y_{c,j} = \begin{cases} x_{l,n} & \text{if } c=k, j=m \\ x_{k,m} & \text{if } c=l, j=n \\ x_{c,j} & \text{otherwise} \end{cases}$$

3 Question 2

Algorithm 1 Sweep x

Let x be the current configuration
Let I be the intermediate values
Let $I_c = g(\overline{Al}[x_c], \overline{Fe}[x_c]\overline{Si}[x_c])$
for k=1...16; m=1...3; l=k...17; n=1..3 **do**
  $y = y(x,k,l,m,n)$
  $d = g(\overline{Al}[y_k], \overline{Fe}[y_k]\overline{Si}[y_k]) + g(\overline{Al}[y_l], \overline{Fe}[y_l]\overline{Si}[y_l]) - I_k - I_l$
  if $d > 0$ **then**
    $x := y$
    $I_k = g(\overline{Al}[y_k], \overline{Fe}[y_k]\overline{Si}[y_k])$
    $I_l = g(a(\overline{Al}[y_l], \overline{Fe}[y_l]\overline{Si}[y_l]))$
  end if
end for

4 Question 3

4.1 Question 3A

See Appendix A for code.

4.2 Question 3B

See Appendix A for code.

4.3 Question 3C

See Appendix A for plot.

4.4 Question 3D

See Appendix A for plot.

4.5 Question 3E

See Appendix A for code and plots. Best solution found using repeated next ascents with $n = 200$

```
 1 [27  4 45 ] 99.261 %Al, 0.274 %Fe, 0.110 %Si, $41.53, spread = 41
 2 [ 8 13 37 ] 99.508 %Al, 0.154 %Fe, 0.153 %Si, $48.71, spread = 29
 3 [50 40 30 ] 99.504 %Al, 0.143 %Fe, 0.238 %Si, $48.71, spread = 20
 4 [17 51 15 ] 99.268 %Al, 0.162 %Fe, 0.386 %Si, $41.53, spread = 36
 5 [24 23 46 ] 99.512 %Al, 0.053 %Fe, 0.180 %Si, $48.71, spread = 23
 6 [28 12 32 ] 99.358 %Al, 0.096 %Fe, 0.348 %Si, $44.53, spread = 20
 7 [18  3 42 ] 99.760 %Al, 0.040 %Fe, 0.139 %Si, $57.35, spread = 39
 8 [19 41 20 ] 99.262 %Al, 0.212 %Fe, 0.249 %Si, $41.53, spread = 22
 9 [11 25  5 ] 99.353 %Al, 0.185 %Fe, 0.325 %Si, $44.53, spread = 20
10 [ 9  6 21 ] 99.504 %Al, 0.139 %Fe, 0.253 %Si, $48.71, spread = 15
11 [ 7 36 26 ] 99.506 %Al, 0.098 %Fe, 0.268 %Si, $48.71, spread = 29
12 [47 39  2 ] 99.503 %Al, 0.114 %Fe, 0.270 %Si, $48.71, spread = 45
13 [22 33 10 ] 99.356 %Al, 0.187 %Fe, 0.219 %Si, $44.53, spread = 23
14 [34 29 38 ] 99.361 %Al, 0.209 %Fe, 0.241 %Si, $44.53, spread =  9
15 [14 43 49 ] 99.254 %Al, 0.224 %Fe, 0.307 %Si, $41.53, spread = 35
16 [ 1 35 31 ] 99.350 %Al, 0.075 %Fe, 0.336 %Si, $44.53, spread = 34
17 [48 44 16 ] 99.515 %Al, 0.143 %Fe, 0.241 %Si, $48.71, spread = 32
                                            Sum = $787.09, MxSprd = 45
```

Best solution found using repeated steepest ascents with $n = 200$

```
 1 [ 1 48 15 ] 99.256 %Al, 0.121 %Fe, 0.300 %Si, $41.53, spread = 47
 2 [29 36 10 ] 99.369 %Al, 0.216 %Fe, 0.272 %Si, $44.53, spread = 26
 3 [33  6 34 ] 99.268 %Al, 0.253 %Fe, 0.272 %Si, $41.53, spread = 28
 4 [42 16 49 ] 99.502 %Al, 0.161 %Fe, 0.163 %Si, $48.71, spread = 33
 5 [43 24 39 ] 99.508 %Al, 0.067 %Fe, 0.243 %Si, $48.71, spread = 19
 6 [47  8  5 ] 99.366 %Al, 0.188 %Fe, 0.317 %Si, $44.53, spread = 42
 7 [ 4 26 19 ] 99.253 %Al, 0.243 %Fe, 0.283 %Si, $41.53, spread = 22
 8 [31 28 46 ] 99.365 %Al, 0.139 %Fe, 0.196 %Si, $44.53, spread = 18
 9 [ 3 11 23 ] 99.502 %Al, 0.058 %Fe, 0.246 %Si, $48.71, spread = 20
10 [32 27 35 ] 99.501 %Al, 0.087 %Fe, 0.224 %Si, $48.71, spread =  8
11 [22 40 51 ] 99.353 %Al, 0.122 %Fe, 0.349 %Si, $44.53, spread = 29
12 [13 38  2 ] 99.504 %Al, 0.177 %Fe, 0.160 %Si, $48.71, spread = 36
13 [50 25 37 ] 99.500 %Al, 0.188 %Fe, 0.231 %Si, $48.71, spread = 25
14 [20 30 17 ] 99.522 %Al, 0.111 %Fe, 0.277 %Si, $48.71, spread = 13
15 [ 9 18 44 ] 99.750 %Al, 0.037 %Fe, 0.099 %Si, $57.35, spread = 35
16 [45 41 12 ] 99.258 %Al, 0.171 %Fe, 0.284 %Si, $41.53, spread = 33
17 [21  7 14 ] 99.360 %Al, 0.168 %Fe, 0.347 %Si, $44.53, spread = 14
                                            Sum = $787.09, MxSprd = 47
```

5 Quesion 4

6 Question 5

(a) You would expect the problem's objective function to have lots of plateus because the objective function is not continuous. This means there will be lots of cases where two pots are swapped and the quality, and thus value, will remain constant.

(b)

7 Question 6

$$g''(\overline{Al}, \overline{Fe}, \overline{Si}, x_{c1}, x_{c2}, x_{c3}, s) = \begin{cases} g(\overline{Al}, \overline{Fe}, \overline{Si}) - 10000 * (s_c - s) & \text{if } s_c > s \\ g(\overline{Al}, \overline{Fe}, \overline{Si}) \text{otherwise} \end{cases}$$

where $s_c = \max(x_{c1}, x_{c2}, x_{c3}) - \min(x_{c1}, x_{c2}, x_{c3})$