760-Heuristics-Assignment

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1 Question 1

Neighbors of the current solution can be found by swapping one pot in any crucible with a pot in any other crucible. This can be formally defined as following:

$N(\mathbf{x}) = \{\mathbf{y}(\mathbf{x}, k, l, m, n), k = 1, 2, 3, ..., 16, l = k + 1, k + 2, k + 3, ...17, m = 1, 2, 3, n = 1, 2, 3\}$ where

$$\mathbf{y}(\mathbf{x}, k, l, m, n) = (y_{1,1}, y_{1,2}, y_{1,3}; y_{2,1}y_{2,2}y_{2,3}; ...y_{c,j}), y_{c,j} = \begin{cases} x_{l,n} & \text{if } c = k, j = m \\ x_{k,m} & \text{if } c = l, j = n \\ x_{c,j} & \text{otherwise} \end{cases}$$

2 Question 2

Let $h(x_c) = g(\overline{Al}[x_{c,avg}], \overline{Fe}[x_{c,avg}], \overline{Si}[x_{c,avg}])$ where

$$\overline{Al}[x_{c,avg}] = \frac{Al[x_{c,1}], Al[x_{c,2}], Al[x_{c,3}]}{3}$$

$$\overline{Fe}[x_{c,avg}] = \frac{Fe[x_{c,1}], Fe[x_{c,2}], Fe[x_{c,3}]}{3}$$

$$\overline{Si}[x_{c,avg}] = \frac{Si[x_{c,1}], Si[x_{c,2}], Si[x_{c,3}]}{3}$$

Algorithm 1 Sweep x

Let S contain all possible solutions of x
Let x, $x \in S$, be some initial configuration
Let x^* be some configuration that maximizes f(x)
Let I be the intermediate values
$I_i := h(x_i) \forall i \in c$
while not stopped **do**
  Compare each neighbor $y(x, k, l, m, n) \in N(x)$
  Let $d = h(y_k) + h(y_l) - I_k - I_l$
  if $d > 0$ for some $y \in N(x)$ **then**
    x := y
    $I_k := h(y_k)$
    $I_l := h(y_l)$
  else if $d \leq 0 \forall y \in N(x)$ **then**
    Stop
  end if
end while
$x^* := x$

3 Question 3

See Appendix A for all code and plots.

3.1 Question 3E

Best solution found using repeated next ascents with $n = 200$.

```
1 [27  4 45 ] 99.261 %Al, 0.274 %Fe, 0.110 %Si, $41.53, spread = 41
2 [ 8 13 37 ] 99.508 %Al, 0.154 %Fe, 0.153 %Si, $48.71, spread = 29
3 [50 40 30 ] 99.504 %Al, 0.143 %Fe, 0.238 %Si, $48.71, spread = 20
4 [17 51 15 ] 99.268 %Al, 0.162 %Fe, 0.386 %Si, $41.53, spread = 36
5 [24 23 46 ] 99.512 %Al, 0.053 %Fe, 0.180 %Si, $48.71, spread = 23
6 [28 12 32 ] 99.358 %Al, 0.096 %Fe, 0.348 %Si, $44.53, spread = 20
7 [18  3 42 ] 99.760 %Al, 0.040 %Fe, 0.139 %Si, $57.35, spread = 39
```

```
 8 [19 41 20 ] 99.262 %Al, 0.212 %Fe, 0.249 %Si, $41.53, spread = 22
 9 [11 25  5 ] 99.353 %Al, 0.185 %Fe, 0.325 %Si, $44.53, spread = 20
10 [ 9  6 21 ] 99.504 %Al, 0.139 %Fe, 0.253 %Si, $48.71, spread = 15
11 [ 7 36 26 ] 99.506 %Al, 0.098 %Fe, 0.268 %Si, $48.71, spread = 29
12 [47 39  2 ] 99.503 %Al, 0.114 %Fe, 0.270 %Si, $48.71, spread = 45
13 [22 33 10 ] 99.356 %Al, 0.187 %Fe, 0.219 %Si, $44.53, spread = 23
14 [34 29 38 ] 99.361 %Al, 0.209 %Fe, 0.241 %Si, $44.53, spread =  9
15 [14 43 49 ] 99.254 %Al, 0.224 %Fe, 0.307 %Si, $41.53, spread = 35
16 [ 1 35 31 ] 99.350 %Al, 0.075 %Fe, 0.336 %Si, $44.53, spread = 34
17 [48 44 16 ] 99.515 %Al, 0.143 %Fe, 0.241 %Si, $48.71, spread = 32
                                              Sum = $787.09, MxSprd = 45
```

Best solution found using repeated steepest ascents with $n = 200$.

```
 1 [ 1 48 15 ] 99.256 %Al, 0.121 %Fe, 0.300 %Si, $41.53, spread = 47
 2 [29 36 10 ] 99.369 %Al, 0.216 %Fe, 0.272 %Si, $44.53, spread = 26
 3 [33  6 34 ] 99.268 %Al, 0.253 %Fe, 0.272 %Si, $41.53, spread = 28
 4 [42 16 49 ] 99.502 %Al, 0.161 %Fe, 0.163 %Si, $48.71, spread = 33
 5 [43 24 39 ] 99.508 %Al, 0.067 %Fe, 0.243 %Si, $48.71, spread = 19
 6 [47  8  5 ] 99.366 %Al, 0.188 %Fe, 0.317 %Si, $44.53, spread = 42
 7 [ 4 26 19 ] 99.253 %Al, 0.243 %Fe, 0.283 %Si, $41.53, spread = 22
 8 [31 28 46 ] 99.365 %Al, 0.139 %Fe, 0.196 %Si, $44.53, spread = 18
 9 [ 3 11 23 ] 99.502 %Al, 0.058 %Fe, 0.246 %Si, $48.71, spread = 20
10 [32 27 35 ] 99.501 %Al, 0.087 %Fe, 0.224 %Si, $48.71, spread =  8
11 [22 40 51 ] 99.353 %Al, 0.122 %Fe, 0.349 %Si, $44.53, spread = 29
12 [13 38  2 ] 99.504 %Al, 0.177 %Fe, 0.160 %Si, $48.71, spread = 36
13 [50 25 37 ] 99.500 %Al, 0.188 %Fe, 0.231 %Si, $48.71, spread = 25
14 [20 30 17 ] 99.522 %Al, 0.111 %Fe, 0.277 %Si, $48.71, spread = 13
15 [ 9 18 44 ] 99.750 %Al, 0.037 %Fe, 0.099 %Si, $57.35, spread = 35
16 [45 41 12 ] 99.258 %Al, 0.171 %Fe, 0.284 %Si, $41.53, spread = 33
17 [21  7 14 ] 99.360 %Al, 0.168 %Fe, 0.347 %Si, $44.53, spread = 14
                                              Sum = $787.09, MxSprd = 47
```

4 Quesion 4

There does not seem to be a significant difference in the objective value for next and steepest ascent. Steepest ascent also took far longer to converge on each local optimum. This means that fastest ascent is better than steepest ascent; however, this conclusion is from limited information so it not conclusive evidence.

5 Question 5

(a) You would expect the problem's objective function to have lots of plateus because the objective function is not continuous. This means there will be lots

of cases where two pots are swapped and the quality, and thus value, will remain constant.

(b) The best outcome is when the crucible is just above the min/max requirements to enter the grade boundary. This is because when sitting too far above the grade boundary there are wasted resources (i.e. the quality doesn't need to be that good). The proposed function will add a non-linear gradient to the existing value function to encourage sitting on the grade boundary. This non-linear gradient will take the form of a quadratic with a local minimum in the center of the grade boundary. The maximum value this quadratic will take will be 1 and the minimum 0. This can be formally defined as follows

$$g'(\overline{Al}, \overline{Fe}, \overline{Si}) = g(\overline{Al}, \overline{Fe}, \overline{Si}) + \frac{(p - (r + \frac{q-r}{2}))^2}{(\frac{q-r}{2})^2}$$

6 Question 6

$$g''(\overline{Al}, \overline{Fe}, \overline{Si}, x_{c1}, x_{c2}, x_{c3}, s) = \begin{cases} g(\overline{Al}, \overline{Fe}, \overline{Si}) - 20 * (s_c - s) & \text{if } s_c > s \\ g(\overline{Al}, \overline{Fe}, \overline{Si}) & \text{otherwise} \end{cases}$$

where $s_c = \max(x_{c1}, x_{c2}, x_{c3}) - \min(x_{c1}, x_{c2}, x_{c3})$

6.1 Task 6

See Appendix A for code.

6.1.1 Max Spread = 6

```
 1 [38 34 40 ] 99.371 %Al, 0.241 %Fe, 0.163 %Si, $44.53, spread =  6
 2 [17 13 19 ] 99.270 %Al, 0.297 %Fe, 0.234 %Si, $41.53, spread =  6
 3 [35 36 39 ] 99.615 %Al, 0.043 %Fe, 0.196 %Si, $48.71, spread =  4
 4 [ 6  8 10 ] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread =  4
 5 [30 32 27 ] 99.558 %Al, 0.093 %Fe, 0.170 %Si, $48.71, spread =  5
 6 [37 42 41 ] 99.501 %Al, 0.115 %Fe, 0.198 %Si, $48.71, spread =  5
 7 [24 22 21 ] 99.532 %Al, 0.037 %Fe, 0.276 %Si, $48.71, spread =  3
 8 [33 29 28 ] 99.256 %Al, 0.151 %Fe, 0.303 %Si, $41.53, spread =  5
 9 [11 14  9 ] 99.381 %Al, 0.080 %Fe, 0.297 %Si, $44.53, spread =  5
10 [ 5  7  3 ] 99.512 %Al, 0.188 %Fe, 0.212 %Si, $48.71, spread =  4
11 [46 47 43 ] 99.359 %Al, 0.172 %Fe, 0.315 %Si, $44.53, spread =  4
12 [26 25 31 ] 99.393 %Al, 0.138 %Fe, 0.325 %Si, $44.53, spread =  6
13 [49 48 44 ] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread =  5
14 [20 18 23 ] 99.544 %Al, 0.057 %Fe, 0.220 %Si, $48.71, spread =  5
15 [15 16 12 ] 99.369 %Al, 0.127 %Fe, 0.302 %Si, $44.53, spread =  4
16 [ 2  4  1 ] 99.356 %Al, 0.144 %Fe, 0.280 %Si, $44.53, spread =  3
17 [50 51 45 ] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread =  6
```

```
                                                  Sum = $777.27, MxSprd =  6
```

6.1.2 Max Spread = 8

```
 1 [15 13 11 ] 99.255 %Al, 0.157 %Fe, 0.277 %Si, $41.53, spread =  4
 2 [ 2  1  4 ] 99.356 %Al, 0.144 %Fe, 0.280 %Si, $44.53, spread =  3
 3 [46 49 51 ] 99.399 %Al, 0.152 %Fe, 0.259 %Si, $44.53, spread =  5
 4 [26 19 24 ] 99.355 %Al, 0.140 %Fe, 0.305 %Si, $44.53, spread =  7
 5 [32 29 36 ] 99.509 %Al, 0.087 %Fe, 0.197 %Si, $48.71, spread =  7
 6 [34 35 33 ] 99.358 %Al, 0.156 %Fe, 0.278 %Si, $44.53, spread =  2
 7 [ 8  3 10 ] 99.527 %Al, 0.176 %Fe, 0.194 %Si, $48.71, spread =  7
 8 [48 50 44 ] 99.544 %Al, 0.129 %Fe, 0.241 %Si, $48.71, spread =  6
 9 [14  6  7 ] 99.253 %Al, 0.249 %Fe, 0.360 %Si, $41.53, spread =  8
10 [16 18 17 ] 99.542 %Al, 0.132 %Fe, 0.258 %Si, $48.71, spread =  2
11 [30 27 23 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread =  7
12 [20 22 21 ] 99.516 %Al, 0.060 %Fe, 0.277 %Si, $48.71, spread =  2
13 [43 45 38 ] 99.350 %Al, 0.226 %Fe, 0.125 %Si, $44.53, spread =  7
14 [ 9  5 12 ] 99.510 %Al, 0.106 %Fe, 0.258 %Si, $48.71, spread =  7
15 [40 39 47 ] 99.378 %Al, 0.169 %Fe, 0.269 %Si, $44.53, spread =  8
16 [28 25 31 ] 99.278 %Al, 0.204 %Fe, 0.285 %Si, $41.53, spread =  6
17 [42 41 37 ] 99.501 %Al, 0.115 %Fe, 0.198 %Si, $48.71, spread =  5
                                                  Sum = $781.45, MxSprd =  8
```

6.1.3 Max Spread = 11

```
 1 [40 49 51 ] 99.301 %Al, 0.205 %Fe, 0.295 %Si, $41.53, spread = 11
 2 [35 33 34 ] 99.358 %Al, 0.156 %Fe, 0.278 %Si, $44.53, spread =  2
 3 [ 5  3  7 ] 99.512 %Al, 0.188 %Fe, 0.212 %Si, $48.71, spread =  4
 4 [29 31 28 ] 99.257 %Al, 0.160 %Fe, 0.310 %Si, $41.53, spread =  3
 5 [ 4  6  9 ] 99.386 %Al, 0.217 %Fe, 0.204 %Si, $44.53, spread =  5
 6 [36 32 26 ] 99.524 %Al, 0.048 %Fe, 0.266 %Si, $48.71, spread = 10
 7 [39 44 47 ] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread =  8
 8 [25 18 16 ] 99.527 %Al, 0.153 %Fe, 0.256 %Si, $48.71, spread =  9
 9 [41 50 48 ] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread =  9
10 [14 22 24 ] 99.362 %Al, 0.083 %Fe, 0.335 %Si, $44.53, spread = 10
11 [13 23 19 ] 99.251 %Al, 0.258 %Fe, 0.213 %Si, $41.53, spread = 10
12 [30 27 20 ] 99.530 %Al, 0.104 %Fe, 0.212 %Si, $48.71, spread = 10
13 [46 42 45 ] 99.502 %Al, 0.140 %Fe, 0.056 %Si, $48.71, spread =  4
14 [21 10 17 ] 99.369 %Al, 0.239 %Fe, 0.344 %Si, $44.53, spread = 11
15 [37 43 38 ] 99.519 %Al, 0.169 %Fe, 0.143 %Si, $48.71, spread =  6
16 [11 12 15 ] 99.361 %Al, 0.068 %Fe, 0.309 %Si, $44.53, spread =  4
17 [ 8  1  2 ] 99.526 %Al, 0.042 %Fe, 0.230 %Si, $48.71, spread =  7
                                                  Sum = $781.45, MxSprd = 11
```

7 My Best Solutions

These were found by using standard simulated annealing in C.

7.1 No Max Spread

```
 1 [13  4 26 ] 99.253 %Al, 0.210 %Fe, 0.373 %Si, $41.53, spread = 22
 2 [31 46 17 ] 99.500 %Al, 0.135 %Fe, 0.144 %Si, $48.71, spread = 29
 3 [49 14 29 ] 99.251 %Al, 0.207 %Fe, 0.314 %Si, $41.53, spread = 35
 4 [23 27 30 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread =  7
 5 [20 50 10 ] 99.351 %Al, 0.220 %Fe, 0.327 %Si, $44.53, spread = 40
 6 [ 8 34 19 ] 99.353 %Al, 0.244 %Fe, 0.155 %Si, $44.53, spread = 26
 7 [ 1 48 24 ] 99.355 %Al, 0.069 %Fe, 0.342 %Si, $44.53, spread = 47
 8 [32  6  2 ] 99.500 %Al, 0.146 %Fe, 0.227 %Si, $48.71, spread = 30
 9 [40 36  7 ] 99.501 %Al, 0.169 %Fe, 0.121 %Si, $48.71, spread = 33
10 [28 25 43 ] 99.252 %Al, 0.218 %Fe, 0.369 %Si, $41.53, spread = 18
11 [21 45 16 ] 99.357 %Al, 0.199 %Fe, 0.232 %Si, $44.53, spread = 29
12 [ 9 47 39 ] 99.501 %Al, 0.095 %Fe, 0.228 %Si, $48.71, spread = 38
13 [37 35 11 ] 99.501 %Al, 0.067 %Fe, 0.277 %Si, $48.71, spread = 26
14 [22 15  3 ] 99.500 %Al, 0.089 %Fe, 0.165 %Si, $48.71, spread = 19
15 [18 42 44 ] 99.750 %Al, 0.044 %Fe, 0.136 %Si, $57.35, spread = 26
16 [51  5 38 ] 99.351 %Al, 0.192 %Fe, 0.349 %Si, $44.53, spread = 46
17 [41 33 12 ] 99.353 %Al, 0.099 %Fe, 0.305 %Si, $44.53, spread = 29
                                          Sum = $790.09, MxSprd = 47
```

7.2 Max Spread = 6

```
 1 [44,49,48,] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread=5
 2 [18,19,21,] 99.512 %Al, 0.163 %Fe, 0.187 %Si, $48.71, spread=3
 3 [43,47,46,] 99.359 %Al, 0.172 %Fe, 0.315 %Si, $44.53, spread=4
 4 [9,11,12,] 99.535 %Al, 0.015 %Fe, 0.265 %Si, $48.71, spread=3
 5 [27,24,22,] 99.502 %Al, 0.068 %Fe, 0.194 %Si, $48.71, spread=5
 6 [36,39,34,] 99.502 %Al, 0.139 %Fe, 0.173 %Si, $48.71, spread=5
 7 [25,23,28,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, $41.53, spread=5
 8 [38,37,42,] 99.650 %Al, 0.118 %Fe, 0.067 %Si, $52.44, spread=5
 9 [51,50,45,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread=6
 10 [2,7,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
 11 [29,26,31,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, $44.53, spread=5
 12 [16,14,20,] 99.298 %Al, 0.164 %Fe, 0.379 %Si, $41.53, spread=6
 13 [33,32,30,] 99.541 %Al, 0.070 %Fe, 0.160 %Si, $48.71, spread=3
 14 [6,8,10,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread=4
 15 [3,1,4,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, $44.53, spread=3
 16 [13,15,17,] 99.276 %Al, 0.220 %Fe, 0.261 %Si, $41.53, spread=4
 17 [35,40,41,] 99.335 %Al, 0.141 %Fe, 0.317 %Si, $41.53, spread=6
Sum = $779.18, MxSprd = 6
```

7.3 Max Spread = 8

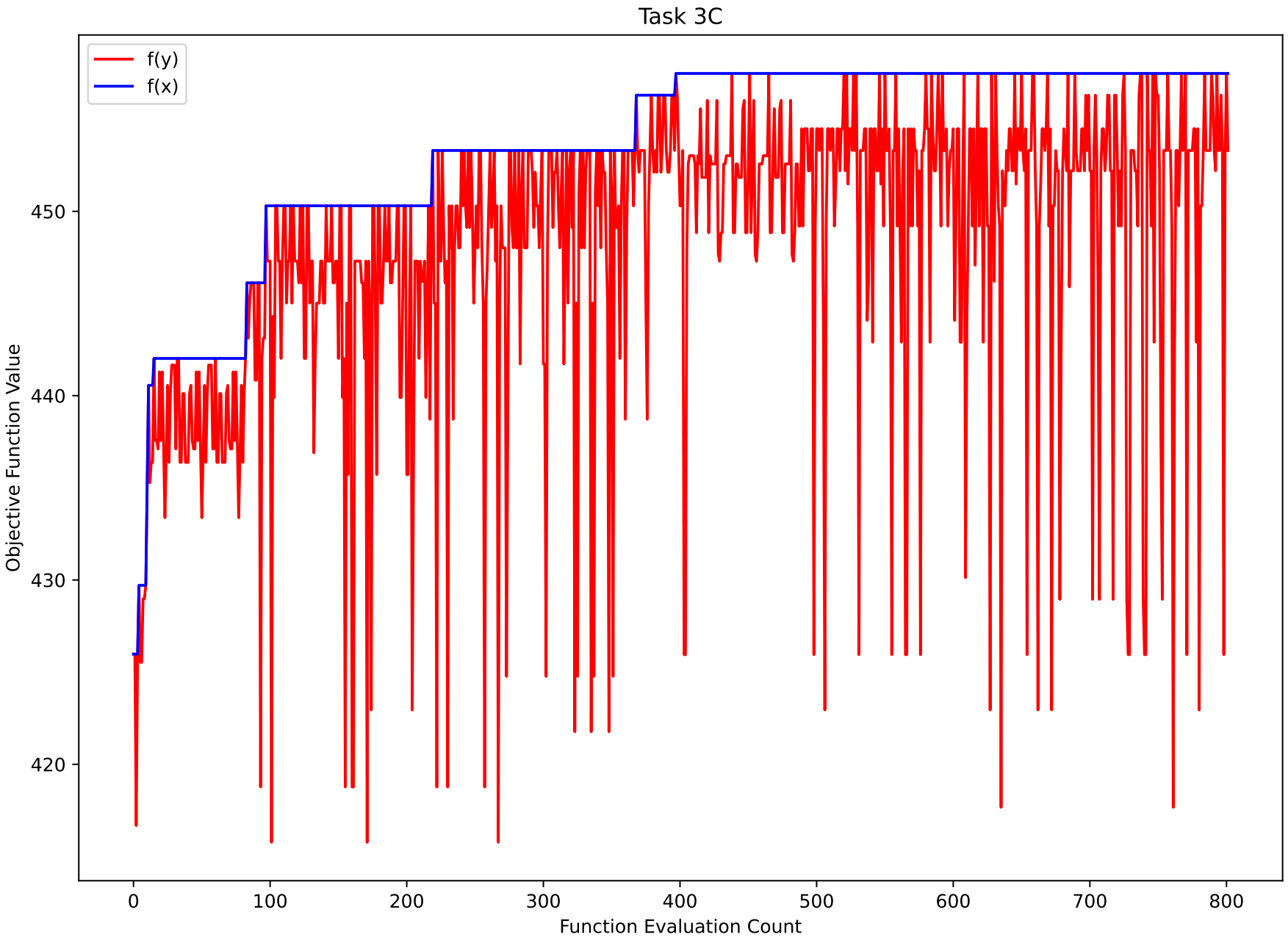
```
 1 [8,1,6,] 99.358 %Al, 0.130 %Fe, 0.315 %Si, $44.53, spread=7
 2 [24,27,21,] 99.506 %Al, 0.091 %Fe, 0.245 %Si, $48.71, spread=6
 3 [26,31,29,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, $44.53, spread=5
 4 [32,40,34,] 99.353 %Al, 0.199 %Fe, 0.210 %Si, $44.53, spread=8
 5 [46,44,41,] 99.507 %Al, 0.108 %Fe, 0.191 %Si, $48.71, spread=5
 6 [22,15,18,] 99.514 %Al, 0.064 %Fe, 0.164 %Si, $48.71, spread=7
 7 [17,13,20,] 99.359 %Al, 0.191 %Fe, 0.305 %Si, $44.53, spread=7
 8 [38,33,35,] 99.501 %Al, 0.105 %Fe, 0.167 %Si, $48.71, spread=5
 9 [47,39,43,] 99.370 %Al, 0.153 %Fe, 0.341 %Si, $44.53, spread=8
 10 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
 11 [51,45,50,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, $41.53, spread=6
 12 [12,16,19,] 99.362 %Al, 0.204 %Fe, 0.275 %Si, $44.53, spread=7
 13 [11,9,14,] 99.381 %Al, 0.080 %Fe, 0.297 %Si, $44.53, spread=5
 14 [36,30,37,] 99.654 %Al, 0.098 %Fe, 0.129 %Si, $52.44, spread=7
 15 [10,3,4,] 99.357 %Al, 0.278 %Fe, 0.244 %Si, $44.53, spread=7
 16 [49,48,42,] 99.513 %Al, 0.147 %Fe, 0.139 %Si, $48.71, spread=7
 17 [28,25,23,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, $41.53, spread=5
Sum = $784.00, MxSprd = 8
```

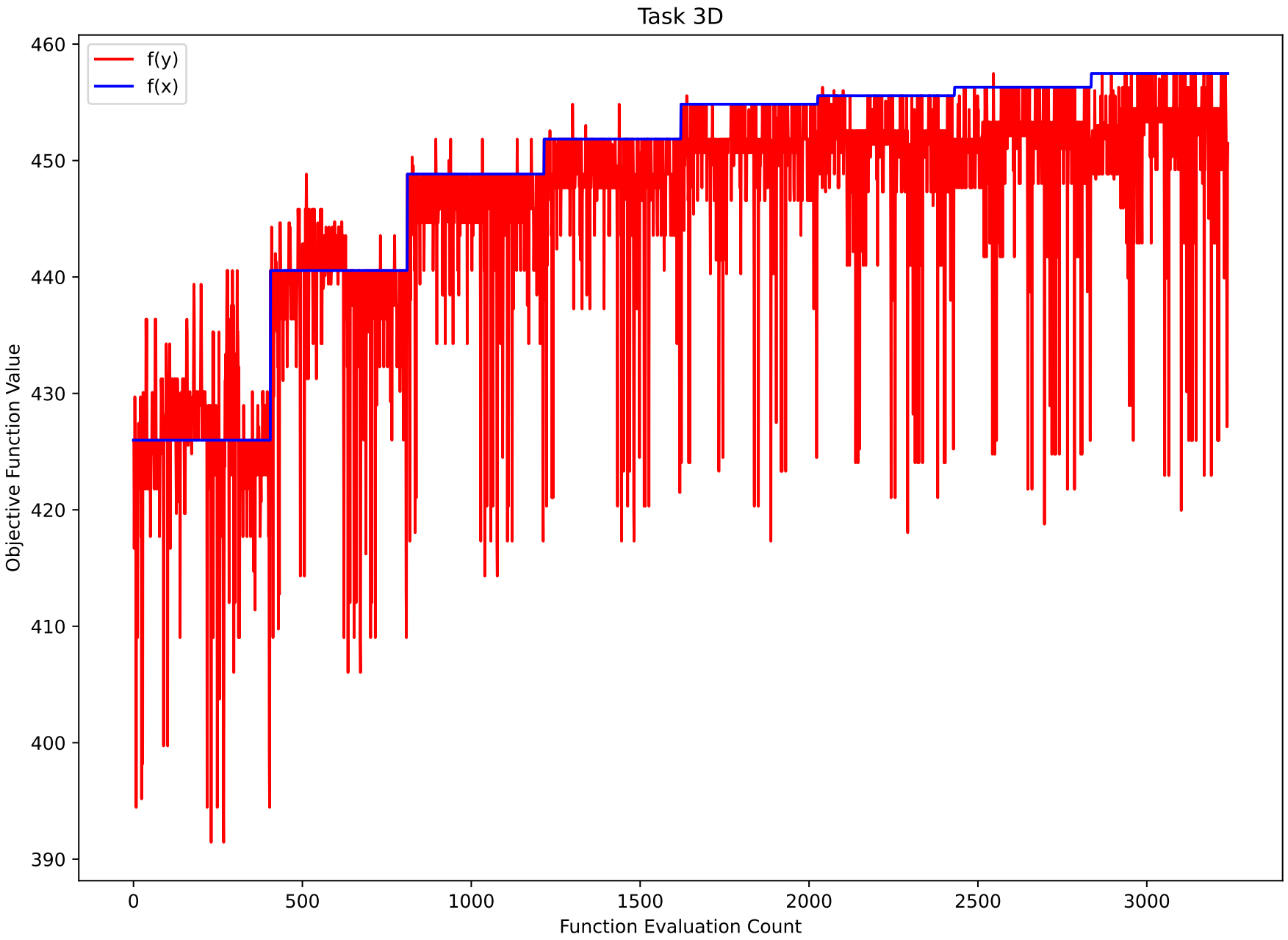
7.4 Max Spread = 11

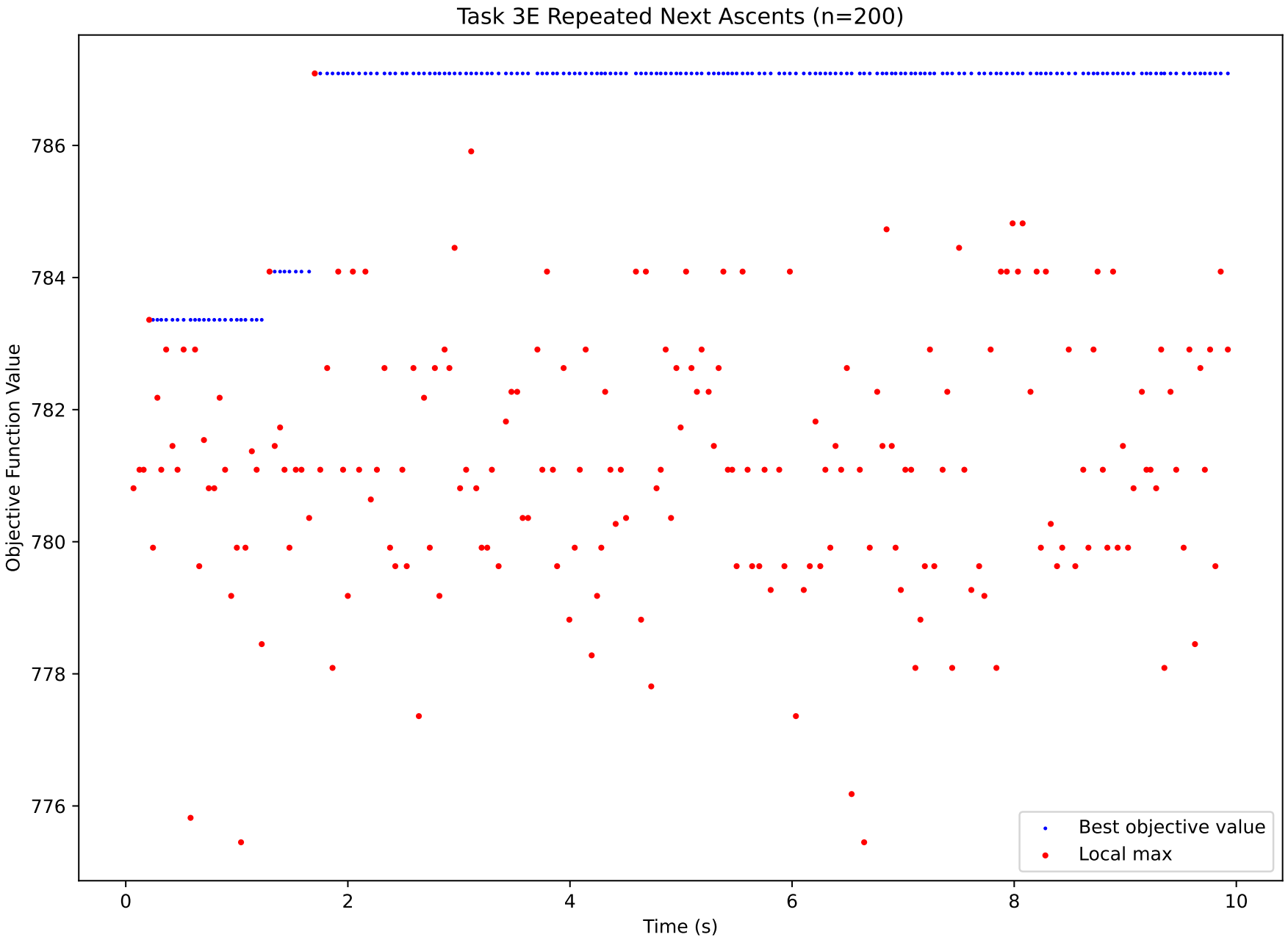
```
 1 [24,19,26,] 99.355 %Al, 0.140 %Fe, 0.305 %Si, $44.53, spread=7
 2 [27,33,37,] 99.500 %Al, 0.148 %Fe, 0.086 %Si, $48.71, spread=10
 3 [38,28,34,] 99.262 %Al, 0.236 %Fe, 0.270 %Si, $41.53, spread=10
 4 [48,50,41,] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread=9
 5 [49,43,51,] 99.294 %Al, 0.189 %Fe, 0.367 %Si, $41.53, spread=8
 6 [44,47,39,] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread=8
 7 [42,36,45,] 99.504 %Al, 0.145 %Fe, 0.069 %Si, $48.71, spread=9
 8 [35,46,40,] 99.504 %Al, 0.110 %Fe, 0.193 %Si, $48.71, spread=11
 9 [4,3,1,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, $44.53, spread=3
 10 [10,6,8,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread=4
 11 [32,31,21,] 99.501 %Al, 0.089 %Fe, 0.201 %Si, $48.71, spread=11
 12 [14,15,12,] 99.251 %Al, 0.139 %Fe, 0.370 %Si, $41.53, spread=3
 13 [9,11,17,] 99.512 %Al, 0.072 %Fe, 0.221 %Si, $48.71, spread=8
 14 [30,20,23,] 99.503 %Al, 0.072 %Fe, 0.255 %Si, $48.71, spread=10
 15 [29,25,18,] 99.505 %Al, 0.134 %Fe, 0.271 %Si, $48.71, spread=11
 16 [16,13,22,] 99.374 %Al, 0.167 %Fe, 0.288 %Si, $44.53, spread=9
 17 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
Sum = $785.63, MxSprd = 11
```

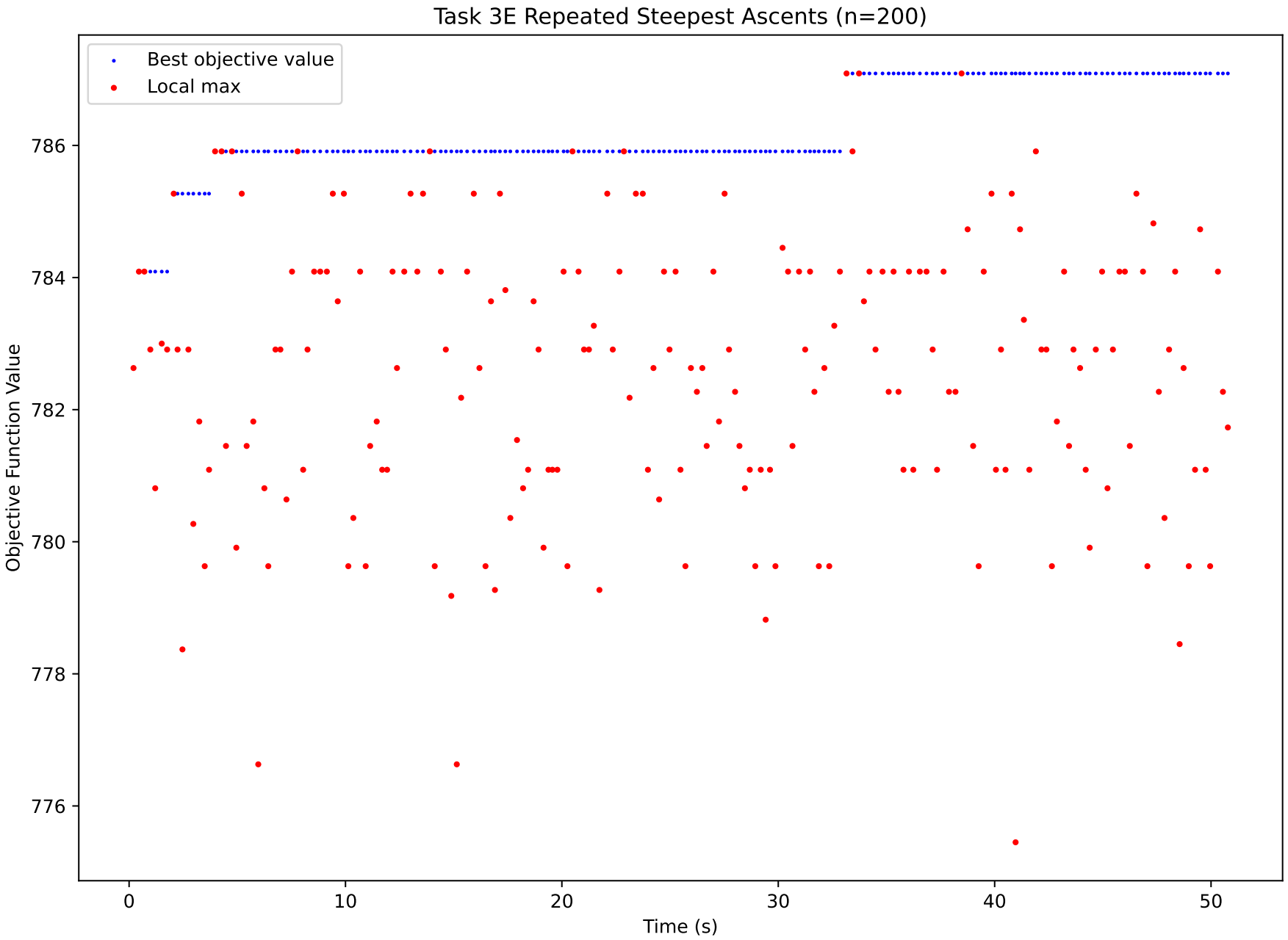
8 Appendix A

See overleaf


Task 3C
f(y)
f(x)
Objective Function Value
450
440
430
420
0
100
200
300
400
500
600
700
800
Function Evaluation Count


Task 3D
f(y)
f(x)
460
450
440
430
420
410
400
390
Objective Function Value
0
500
1000
1500
2000
2500
3000
Function Evaluation Count


Task 3E Repeated Next Ascents (n=200)
Objective Function Value
786
784
782
780
778
776
0
2
4
6
8
10
Time (s)
Best objective value
Local max


Task 3E Repeated Steepest Ascents (n=200)
Best objective value
Local max
Objective Function Value
786
784
782
780
778
776
0
10
20
30
40
50
Time (s)