

760-Heuristics-Assignment

Leo Mooney

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1 Question 1

Neighbors of the current solution can be found by swapping one pot in any crucible with a pot in any other crucible. This can be formally defined as following:

$N(\mathbf{x}) = \{\mathbf{y}(\mathbf{x}, k, l, m, n), k = 1, 2, 3, \dots, 16, l = k + 1, k + 2, k + 3, \dots, 17, m = 1, 2, 3, n = 1, 2, 3\}$ where

$$\mathbf{y}(\mathbf{x}, k, l, m, n) = (y_{1,1}, y_{1,2}, y_{1,3}; y_{2,1}y_{2,2}y_{2,3}; \dots y_{c,j}), y_{c,j} = \begin{cases} x_{l,n} & \text{if } c = k, j = m \\ x_{k,m} & \text{if } c = l, j = n \\ x_{c,j} & \text{otherwise} \end{cases}$$

2 Question 2

Algorithm 1 Sweep x

```
Let  $x$  be the current configuration
Let  $I$  be the intermediate values
Let  $I_c = g(\overline{Al}[x_c], \overline{Fe}[x_c] \overline{Si}[x_c])$ 
for  $k=1 \dots 16$ ;  $m=1 \dots 3$ ;  $l=k \dots 17$ ;  $n=1 \dots 3$  do
     $y = y(x, k, l, m, n)$ 
     $d = g(\overline{Al}[y_k], \overline{Fe}[y_k] \overline{Si}[y_k]) + g(\overline{Al}[y_l], \overline{Fe}[y_l] \overline{Si}[y_l]) - I_k - I_l$ 
    if  $d > 0$  then
         $x := y$ 
         $I_k = g(\overline{Al}[y_k], \overline{Fe}[y_k] \overline{Si}[y_k])$ 
         $I_l = g(\overline{Al}[y_l], \overline{Fe}[y_l] \overline{Si}[y_l])$ 
    end if
end for
```

3 Question 3

3.1 Question 3A

See Appendix A for code.

3.2 Question 3B

See Appendix A for code.

3.3 Question 3C

See Appendix A for plot.

3.4 Question 3D

See Appendix A for plot.

3.5 Question 3E

See Appendix A for code and plots. Best solution found using repeated next ascents with $n = 200$

```
1 [27  4 45 ] 99.261 %Al, 0.274 %Fe, 0.110 %Si, $41.53, spread = 41
2 [ 8 13 37 ] 99.508 %Al, 0.154 %Fe, 0.153 %Si, $48.71, spread = 29
3 [50 40 30 ] 99.504 %Al, 0.143 %Fe, 0.238 %Si, $48.71, spread = 20
4 [17 51 15 ] 99.268 %Al, 0.162 %Fe, 0.386 %Si, $41.53, spread = 36
5 [24 23 46 ] 99.512 %Al, 0.053 %Fe, 0.180 %Si, $48.71, spread = 23
6 [28 12 32 ] 99.358 %Al, 0.096 %Fe, 0.348 %Si, $44.53, spread = 20
7 [18  3 42 ] 99.760 %Al, 0.040 %Fe, 0.139 %Si, $57.35, spread = 39
8 [19 41 20 ] 99.262 %Al, 0.212 %Fe, 0.249 %Si, $41.53, spread = 22
9 [11 25  5 ] 99.353 %Al, 0.185 %Fe, 0.325 %Si, $44.53, spread = 20
10 [ 9  6 21 ] 99.504 %Al, 0.139 %Fe, 0.253 %Si, $48.71, spread = 15
11 [ 7 36 26 ] 99.506 %Al, 0.098 %Fe, 0.268 %Si, $48.71, spread = 29
12 [47 39  2 ] 99.503 %Al, 0.114 %Fe, 0.270 %Si, $48.71, spread = 45
13 [22 33 10 ] 99.356 %Al, 0.187 %Fe, 0.219 %Si, $44.53, spread = 23
14 [34 29 38 ] 99.361 %Al, 0.209 %Fe, 0.241 %Si, $44.53, spread =  9
15 [14 43 49 ] 99.254 %Al, 0.224 %Fe, 0.307 %Si, $41.53, spread = 35
16 [ 1 35 31 ] 99.350 %Al, 0.075 %Fe, 0.336 %Si, $44.53, spread = 34
17 [48 44 16 ] 99.515 %Al, 0.143 %Fe, 0.241 %Si, $48.71, spread = 32
    Sum = $787.09, MxSprd = 45
```

Best solution found using repeated steepest ascents with $n = 200$

```
1 [ 1 48 15 ] 99.256 %Al, 0.121 %Fe, 0.300 %Si, $41.53, spread = 47
2 [29 36 10 ] 99.369 %Al, 0.216 %Fe, 0.272 %Si, $44.53, spread = 26
3 [33  6 34 ] 99.268 %Al, 0.253 %Fe, 0.272 %Si, $41.53, spread = 28
4 [42 16 49 ] 99.502 %Al, 0.161 %Fe, 0.163 %Si, $48.71, spread = 33
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5 [43 24 39 ] 99.508 %Al, 0.067 %Fe, 0.243 %Si, $48.71, spread = 19
6 [47 8 5 ] 99.366 %Al, 0.188 %Fe, 0.317 %Si, $44.53, spread = 42
7 [ 4 26 19 ] 99.253 %Al, 0.243 %Fe, 0.283 %Si, $41.53, spread = 22
8 [31 28 46 ] 99.365 %Al, 0.139 %Fe, 0.196 %Si, $44.53, spread = 18
9 [ 3 11 23 ] 99.502 %Al, 0.058 %Fe, 0.246 %Si, $48.71, spread = 20
10 [32 27 35 ] 99.501 %Al, 0.087 %Fe, 0.224 %Si, $48.71, spread = 8
11 [22 40 51 ] 99.353 %Al, 0.122 %Fe, 0.349 %Si, $44.53, spread = 29
12 [13 38 2 ] 99.504 %Al, 0.177 %Fe, 0.160 %Si, $48.71, spread = 36
13 [50 25 37 ] 99.500 %Al, 0.188 %Fe, 0.231 %Si, $48.71, spread = 25
14 [20 30 17 ] 99.522 %Al, 0.111 %Fe, 0.277 %Si, $48.71, spread = 13
15 [ 9 18 44 ] 99.750 %Al, 0.037 %Fe, 0.099 %Si, $57.35, spread = 35
16 [45 41 12 ] 99.258 %Al, 0.171 %Fe, 0.284 %Si, $41.53, spread = 33
17 [21 7 14 ] 99.360 %Al, 0.168 %Fe, 0.347 %Si, $44.53, spread = 14
Sum = $787.09, MxSprd = 47

```

4 Question 4

There does not seem to be a significant difference in the objective value for next and steepest ascent. Steepest ascent also took far longer to converge on each local optimum. This means that fastest ascent is better than steepest ascent; however, this conclusion is from limited information so it not conclusive evidence.

5 Question 5

(a) You would expect the problem's objective function to have lots of plateaus because the objective function is not continuous. This means there will be lots of cases where two pots are swapped and the quality, and thus value, will remain constant.

(b)

6 Question 6

$$g''(\overline{Al}, \overline{Fe}, \overline{Si}, x_{c1}, x_{c2}, x_{c3}, s) = \begin{cases} g(\overline{Al}, \overline{Fe}, \overline{Si}) - 10000 * (s_c - s) & \text{if } s_c > s \\ g(\overline{Al}, \overline{Fe}, \overline{Si}) & \text{otherwise} \end{cases}$$

where $s_c = \max(x_{c1}, x_{c2}, x_{c3}) - \min(x_{c1}, x_{c2}, x_{c3})$

6.1 Task 6

See Appendix A for code.

6.1.1 Max Spread = 6

1	[38 34 40]	99.371	%Al,	0.241	%Fe,	0.163	%Si,	\$44.53,	spread = 6
2	[17 13 19]	99.270	%Al,	0.297	%Fe,	0.234	%Si,	\$41.53,	spread = 6
3	[35 36 39]	99.615	%Al,	0.043	%Fe,	0.196	%Si,	\$48.71,	spread = 4
4	[6 8 10]	99.352	%Al,	0.257	%Fe,	0.275	%Si,	\$44.53,	spread = 4
5	[30 32 27]	99.558	%Al,	0.093	%Fe,	0.170	%Si,	\$48.71,	spread = 5
6	[37 42 41]	99.501	%Al,	0.115	%Fe,	0.198	%Si,	\$48.71,	spread = 5
7	[24 22 21]	99.532	%Al,	0.037	%Fe,	0.276	%Si,	\$48.71,	spread = 3
8	[33 29 28]	99.256	%Al,	0.151	%Fe,	0.303	%Si,	\$41.53,	spread = 5
9	[11 14 9]	99.381	%Al,	0.080	%Fe,	0.297	%Si,	\$44.53,	spread = 5
10	[5 7 3]	99.512	%Al,	0.188	%Fe,	0.212	%Si,	\$48.71,	spread = 4
11	[46 47 43]	99.359	%Al,	0.172	%Fe,	0.315	%Si,	\$44.53,	spread = 4
12	[26 25 31]	99.393	%Al,	0.138	%Fe,	0.325	%Si,	\$44.53,	spread = 6
13	[49 48 44]	99.511	%Al,	0.169	%Fe,	0.145	%Si,	\$48.71,	spread = 5
14	[20 18 23]	99.544	%Al,	0.057	%Fe,	0.220	%Si,	\$48.71,	spread = 5
15	[15 16 12]	99.369	%Al,	0.127	%Fe,	0.302	%Si,	\$44.53,	spread = 4
16	[2 4 1]	99.356	%Al,	0.144	%Fe,	0.280	%Si,	\$44.53,	spread = 3
17	[50 51 45]	99.256	%Al,	0.198	%Fe,	0.351	%Si,	\$41.53,	spread = 6
Sum = \$777.27, MxSprd = 6									

6.1.2 Max Spread = 8

1	[15 13 11]	99.255	%Al,	0.157	%Fe,	0.277	%Si,	\$41.53,	spread = 4
2	[2 1 4]	99.356	%Al,	0.144	%Fe,	0.280	%Si,	\$44.53,	spread = 3
3	[46 49 51]	99.399	%Al,	0.152	%Fe,	0.259	%Si,	\$44.53,	spread = 5
4	[26 19 24]	99.355	%Al,	0.140	%Fe,	0.305	%Si,	\$44.53,	spread = 7
5	[32 29 36]	99.509	%Al,	0.087	%Fe,	0.197	%Si,	\$48.71,	spread = 7
6	[34 35 33]	99.358	%Al,	0.156	%Fe,	0.278	%Si,	\$44.53,	spread = 2
7	[8 3 10]	99.527	%Al,	0.176	%Fe,	0.194	%Si,	\$48.71,	spread = 7
8	[48 50 44]	99.544	%Al,	0.129	%Fe,	0.241	%Si,	\$48.71,	spread = 6
9	[14 6 7]	99.253	%Al,	0.249	%Fe,	0.360	%Si,	\$41.53,	spread = 8
10	[16 18 17]	99.542	%Al,	0.132	%Fe,	0.258	%Si,	\$48.71,	spread = 2
11	[30 27 23]	99.505	%Al,	0.105	%Fe,	0.199	%Si,	\$48.71,	spread = 7
12	[20 22 21]	99.516	%Al,	0.060	%Fe,	0.277	%Si,	\$48.71,	spread = 2
13	[43 45 38]	99.350	%Al,	0.226	%Fe,	0.125	%Si,	\$44.53,	spread = 7
14	[9 5 12]	99.510	%Al,	0.106	%Fe,	0.258	%Si,	\$48.71,	spread = 7
15	[40 39 47]	99.378	%Al,	0.169	%Fe,	0.269	%Si,	\$44.53,	spread = 8
16	[28 25 31]	99.278	%Al,	0.204	%Fe,	0.285	%Si,	\$41.53,	spread = 6
17	[42 41 37]	99.501	%Al,	0.115	%Fe,	0.198	%Si,	\$48.71,	spread = 5
Sum = \$781.45, MxSprd = 8									

6.1.3 Max Spread = 11

1	[40 49 51]	99.301	%Al,	0.205	%Fe,	0.295	%Si,	\$41.53,	spread = 11
2	[35 33 34]	99.358	%Al,	0.156	%Fe,	0.278	%Si,	\$44.53,	spread = 2
3	[5 3 7]	99.512	%Al,	0.188	%Fe,	0.212	%Si,	\$48.71,	spread = 4

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4 [29 31 28 ] 99.257 %Al, 0.160 %Fe, 0.310 %Si, $41.53, spread = 3
5 [ 4  6  9 ] 99.386 %Al, 0.217 %Fe, 0.204 %Si, $44.53, spread = 5
6 [36 32 26 ] 99.524 %Al, 0.048 %Fe, 0.266 %Si, $48.71, spread = 10
7 [39 44 47 ] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread = 8
8 [25 18 16 ] 99.527 %Al, 0.153 %Fe, 0.256 %Si, $48.71, spread = 9
9 [41 50 48 ] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread = 9
10 [14 22 24 ] 99.362 %Al, 0.083 %Fe, 0.335 %Si, $44.53, spread = 10
11 [13 23 19 ] 99.251 %Al, 0.258 %Fe, 0.213 %Si, $41.53, spread = 10
12 [30 27 20 ] 99.530 %Al, 0.104 %Fe, 0.212 %Si, $48.71, spread = 10
13 [46 42 45 ] 99.502 %Al, 0.140 %Fe, 0.056 %Si, $48.71, spread = 4
14 [21 10 17 ] 99.369 %Al, 0.239 %Fe, 0.344 %Si, $44.53, spread = 11
15 [37 43 38 ] 99.519 %Al, 0.169 %Fe, 0.143 %Si, $48.71, spread = 6
16 [11 12 15 ] 99.361 %Al, 0.068 %Fe, 0.309 %Si, $44.53, spread = 4
17 [ 8  1  2 ] 99.526 %Al, 0.042 %Fe, 0.230 %Si, $48.71, spread = 7
Sum = $781.45, MxSprd = 11

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7 My Best Solutions

These were found by using simulated annealing in C.

7.1 No Max Spread

```

1 [13  4 26 ] 99.253 %Al, 0.210 %Fe, 0.373 %Si, $41.53, spread = 22
2 [31 46 17 ] 99.500 %Al, 0.135 %Fe, 0.144 %Si, $48.71, spread = 29
3 [49 14 29 ] 99.251 %Al, 0.207 %Fe, 0.314 %Si, $41.53, spread = 35
4 [23 27 30 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread = 7
5 [20 50 10 ] 99.351 %Al, 0.220 %Fe, 0.327 %Si, $44.53, spread = 40
6 [ 8 34 19 ] 99.353 %Al, 0.244 %Fe, 0.155 %Si, $44.53, spread = 26
7 [ 1 48 24 ] 99.355 %Al, 0.069 %Fe, 0.342 %Si, $44.53, spread = 47
8 [32  6  2 ] 99.500 %Al, 0.146 %Fe, 0.227 %Si, $48.71, spread = 30
9 [40 36  7 ] 99.501 %Al, 0.169 %Fe, 0.121 %Si, $48.71, spread = 33
10 [28 25 43 ] 99.252 %Al, 0.218 %Fe, 0.369 %Si, $41.53, spread = 18
11 [21 45 16 ] 99.357 %Al, 0.199 %Fe, 0.232 %Si, $44.53, spread = 29
12 [ 9 47 39 ] 99.501 %Al, 0.095 %Fe, 0.228 %Si, $48.71, spread = 38
13 [37 35 11 ] 99.501 %Al, 0.067 %Fe, 0.277 %Si, $48.71, spread = 26
14 [22 15  3 ] 99.500 %Al, 0.089 %Fe, 0.165 %Si, $48.71, spread = 19
15 [18 42 44 ] 99.750 %Al, 0.044 %Fe, 0.136 %Si, $57.35, spread = 26
16 [51  5 38 ] 99.351 %Al, 0.192 %Fe, 0.349 %Si, $44.53, spread = 46
17 [41 33 12 ] 99.353 %Al, 0.099 %Fe, 0.305 %Si, $44.53, spread = 29
Sum = $790.09, MxSprd = 47

```

7.2 Max Spread = 6

```

1 [44,49,48,] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread=5
2 [18,19,21,] 99.512 %Al, 0.163 %Fe, 0.187 %Si, $48.71, spread=3

```

3 [43,47,46,] 99.359 %Al, 0.172 %Fe, 0.315 %Si, \$44.53, spread=4
 4 [9,11,12,] 99.535 %Al, 0.015 %Fe, 0.265 %Si, \$48.71, spread=3
 5 [27,24,22,] 99.502 %Al, 0.068 %Fe, 0.194 %Si, \$48.71, spread=5
 6 [36,39,34,] 99.502 %Al, 0.139 %Fe, 0.173 %Si, \$48.71, spread=5
 7 [25,23,28,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, \$41.53, spread=5
 8 [38,37,42,] 99.650 %Al, 0.118 %Fe, 0.067 %Si, \$52.44, spread=5
 9 [51,50,45,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, \$41.53, spread=6
 10 [2,7,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, \$48.71, spread=5
 11 [29,26,31,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, \$44.53, spread=5
 12 [16,14,20,] 99.298 %Al, 0.164 %Fe, 0.379 %Si, \$41.53, spread=6
 13 [33,32,30,] 99.541 %Al, 0.070 %Fe, 0.160 %Si, \$48.71, spread=3
 14 [6,8,10,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, \$44.53, spread=4
 15 [3,1,4,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, \$44.53, spread=3
 16 [13,15,17,] 99.276 %Al, 0.220 %Fe, 0.261 %Si, \$41.53, spread=4
 17 [35,40,41,] 99.335 %Al, 0.141 %Fe, 0.317 %Si, \$41.53, spread=6
 Sum = \$779.18, MxSprd = 6

7.3 Max Spread = 8

1 [8,1,6,] 99.358 %Al, 0.130 %Fe, 0.315 %Si, \$44.53, spread=7
 2 [24,27,21,] 99.506 %Al, 0.091 %Fe, 0.245 %Si, \$48.71, spread=6
 3 [26,31,29,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, \$44.53, spread=5
 4 [32,40,34,] 99.353 %Al, 0.199 %Fe, 0.210 %Si, \$44.53, spread=8
 5 [46,44,41,] 99.507 %Al, 0.108 %Fe, 0.191 %Si, \$48.71, spread=5
 6 [22,15,18,] 99.514 %Al, 0.064 %Fe, 0.164 %Si, \$48.71, spread=7
 7 [17,13,20,] 99.359 %Al, 0.191 %Fe, 0.305 %Si, \$44.53, spread=7
 8 [38,33,35,] 99.501 %Al, 0.105 %Fe, 0.167 %Si, \$48.71, spread=5
 9 [47,39,43,] 99.370 %Al, 0.153 %Fe, 0.341 %Si, \$44.53, spread=8
 10 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, \$48.71, spread=5
 11 [51,45,50,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, \$41.53, spread=6
 12 [12,16,19,] 99.362 %Al, 0.204 %Fe, 0.275 %Si, \$44.53, spread=7
 13 [11,9,14,] 99.381 %Al, 0.080 %Fe, 0.297 %Si, \$44.53, spread=5
 14 [36,30,37,] 99.654 %Al, 0.098 %Fe, 0.129 %Si, \$52.44, spread=7
 15 [10,3,4,] 99.357 %Al, 0.278 %Fe, 0.244 %Si, \$44.53, spread=7
 16 [49,48,42,] 99.513 %Al, 0.147 %Fe, 0.139 %Si, \$48.71, spread=7
 17 [28,25,23,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, \$41.53, spread=5
 Sum = \$784.00, MxSprd = 8

7.4 Max Spread = 11

1 [24,19,26,] 99.355 %Al, 0.140 %Fe, 0.305 %Si, \$44.53, spread=7
 2 [27,33,37,] 99.500 %Al, 0.148 %Fe, 0.086 %Si, \$48.71, spread=10
 3 [38,28,34,] 99.262 %Al, 0.236 %Fe, 0.270 %Si, \$41.53, spread=10
 4 [48,50,41,] 99.351 %Al, 0.151 %Fe, 0.329 %Si, \$44.53, spread=9
 5 [49,43,51,] 99.294 %Al, 0.189 %Fe, 0.367 %Si, \$41.53, spread=8
 6 [44,47,39,] 99.500 %Al, 0.124 %Fe, 0.270 %Si, \$48.71, spread=8

7 [42,36,45,] 99.504 %Al, 0.145 %Fe, 0.069 %Si, \$48.71, spread=9
 8 [35,46,40,] 99.504 %Al, 0.110 %Fe, 0.193 %Si, \$48.71, spread=11
 9 [4,3,1,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, \$44.53, spread=3
 10 [10,6,8,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, \$44.53, spread=4
 11 [32,31,21,] 99.501 %Al, 0.089 %Fe, 0.201 %Si, \$48.71, spread=11
 12 [14,15,12,] 99.251 %Al, 0.139 %Fe, 0.370 %Si, \$41.53, spread=3
 13 [9,11,17,] 99.512 %Al, 0.072 %Fe, 0.221 %Si, \$48.71, spread=8
 14 [30,20,23,] 99.503 %Al, 0.072 %Fe, 0.255 %Si, \$48.71, spread=10
 15 [29,25,18,] 99.505 %Al, 0.134 %Fe, 0.271 %Si, \$48.71, spread=11
 16 [16,13,22,] 99.374 %Al, 0.167 %Fe, 0.288 %Si, \$44.53, spread=9
 17 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, \$48.71, spread=5
 Sum = \$785.63, MxSprd = 11