

# 760-Heuristics-Assignment

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## 1 Question 1

Neighbors of the current solution can be found by swapping one pot in any crucible with a pot in any other crucible. This can be formally defined as following:

$N(\mathbf{x}) = \{\mathbf{y}(\mathbf{x}, k, l, m, n), k = 1, 2, 3, \dots, 16, l = k + 1, k + 2, k + 3, \dots, 17, m = 1, 2, 3, n = 1, 2, 3\}$  where

$$\mathbf{y}(\mathbf{x}, k, l, m, n) = (y_{1,1}, y_{1,2}, y_{1,3}; y_{2,1}y_{2,2}y_{2,3}; \dots y_{c,j}), y_{c,j} = \begin{cases} x_{l,n} & \text{if } c = k, j = m \\ x_{k,m} & \text{if } c = l, j = n \\ x_{c,j} & \text{otherwise} \end{cases}$$

## 2 Question 2

Let  $h(x_c) = g(\overline{Al}[x_{c,avg}], \overline{Fe}[x_{c,avg}], \overline{Si}[x_{c,avg}])$  where

$$\overline{Al}[x_{c,avg}] = \frac{Al[x_{c,1}], Al[x_{c,2}], Al[x_{c,3}]}{3}$$

$$\overline{Fe}[x_{c,avg}] = \frac{Fe[x_{c,1}], Fe[x_{c,2}], Fe[x_{c,3}]}{3}$$

$$\overline{Si}[x_{c,avg}] = \frac{Si[x_{c,1}], Si[x_{c,2}], Si[x_{c,3}]}{3}$$

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### Algorithm 1 Sweep x

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Let  $S$  contain all possible solutions of  $x$ 
Let  $x, x \in S$ , be some initial configuration
Let  $x^*$  be some configuration that maximizes  $f(x)$ 
Let  $I$  be the intermediate values
 $I_i := h(x_i) \forall i \in c$ 
while not stopped do
    Compare each neighbor  $y(x, k, l, m, n) \in N(x)$ 
    Let  $d = h(y_k) + h(y_l) - I_k - I_l$ 
    if  $d > 0$  for some  $y \in N(x)$  then
         $x := y$ 
         $I_k := h(y_k)$ 
         $I_l := h(y_l)$ 
    else if  $d \leq 0 \forall y \in N(x)$  then
        Stop
    end if
end while
 $x^* := x$ 

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## 3 Question 3

See Appendix A for all code and plots.

### 3.1 Question 3E

Best solution found using repeated next ascents with  $n = 200$ .

```

1 [27  4 45 ] 99.261 %Al, 0.274 %Fe, 0.110 %Si, $41.53, spread = 41
2 [ 8 13 37 ] 99.508 %Al, 0.154 %Fe, 0.153 %Si, $48.71, spread = 29
3 [50 40 30 ] 99.504 %Al, 0.143 %Fe, 0.238 %Si, $48.71, spread = 20
4 [17 51 15 ] 99.268 %Al, 0.162 %Fe, 0.386 %Si, $41.53, spread = 36
5 [24 23 46 ] 99.512 %Al, 0.053 %Fe, 0.180 %Si, $48.71, spread = 23
6 [28 12 32 ] 99.358 %Al, 0.096 %Fe, 0.348 %Si, $44.53, spread = 20
7 [18  3 42 ] 99.760 %Al, 0.040 %Fe, 0.139 %Si, $57.35, spread = 39

```

```

8 [19 41 20 ] 99.262 %Al, 0.212 %Fe, 0.249 %Si, $41.53, spread = 22
9 [11 25 5 ] 99.353 %Al, 0.185 %Fe, 0.325 %Si, $44.53, spread = 20
10 [ 9 6 21 ] 99.504 %Al, 0.139 %Fe, 0.253 %Si, $48.71, spread = 15
11 [ 7 36 26 ] 99.506 %Al, 0.098 %Fe, 0.268 %Si, $48.71, spread = 29
12 [47 39 2 ] 99.503 %Al, 0.114 %Fe, 0.270 %Si, $48.71, spread = 45
13 [22 33 10 ] 99.356 %Al, 0.187 %Fe, 0.219 %Si, $44.53, spread = 23
14 [34 29 38 ] 99.361 %Al, 0.209 %Fe, 0.241 %Si, $44.53, spread = 9
15 [14 43 49 ] 99.254 %Al, 0.224 %Fe, 0.307 %Si, $41.53, spread = 35
16 [ 1 35 31 ] 99.350 %Al, 0.075 %Fe, 0.336 %Si, $44.53, spread = 34
17 [48 44 16 ] 99.515 %Al, 0.143 %Fe, 0.241 %Si, $48.71, spread = 32
Sum = $787.09, MxSprd = 45

```

Best solution found using repeated steepest ascents with  $n = 200$ .

```

1 [ 1 48 15 ] 99.256 %Al, 0.121 %Fe, 0.300 %Si, $41.53, spread = 47
2 [29 36 10 ] 99.369 %Al, 0.216 %Fe, 0.272 %Si, $44.53, spread = 26
3 [33 6 34 ] 99.268 %Al, 0.253 %Fe, 0.272 %Si, $41.53, spread = 28
4 [42 16 49 ] 99.502 %Al, 0.161 %Fe, 0.163 %Si, $48.71, spread = 33
5 [43 24 39 ] 99.508 %Al, 0.067 %Fe, 0.243 %Si, $48.71, spread = 19
6 [47 8 5 ] 99.366 %Al, 0.188 %Fe, 0.317 %Si, $44.53, spread = 42
7 [ 4 26 19 ] 99.253 %Al, 0.243 %Fe, 0.283 %Si, $41.53, spread = 22
8 [31 28 46 ] 99.365 %Al, 0.139 %Fe, 0.196 %Si, $44.53, spread = 18
9 [ 3 11 23 ] 99.502 %Al, 0.058 %Fe, 0.246 %Si, $48.71, spread = 20
10 [32 27 35 ] 99.501 %Al, 0.087 %Fe, 0.224 %Si, $48.71, spread = 8
11 [22 40 51 ] 99.353 %Al, 0.122 %Fe, 0.349 %Si, $44.53, spread = 29
12 [13 38 2 ] 99.504 %Al, 0.177 %Fe, 0.160 %Si, $48.71, spread = 36
13 [50 25 37 ] 99.500 %Al, 0.188 %Fe, 0.231 %Si, $48.71, spread = 25
14 [20 30 17 ] 99.522 %Al, 0.111 %Fe, 0.277 %Si, $48.71, spread = 13
15 [ 9 18 44 ] 99.750 %Al, 0.037 %Fe, 0.099 %Si, $57.35, spread = 35
16 [45 41 12 ] 99.258 %Al, 0.171 %Fe, 0.284 %Si, $41.53, spread = 33
17 [21 7 14 ] 99.360 %Al, 0.168 %Fe, 0.347 %Si, $44.53, spread = 14
Sum = $787.09, MxSprd = 47

```

## 4 Question 4

There does not seem to be a significant difference in the objective value for next and steepest ascent. Steepest ascent also took far longer to converge on each local optimum. This means that fastest ascent is better than steepest ascent; however, this conclusion is from limited information so it not conclusive evidence.

## 5 Question 5

(a) You would expect the problem's objective function to have lots of plateaus because the objective function is not continuous. This means there will be lots

Pot Number	Al	Fe	Si
1	99.23	0.063	0.054
2	99.23	0.063	0.054
3	99.23	0.063	0.054
Average Quality	3	3	3

of cases where two pots are swapped and the quality, and thus value, will remain constant.

(b) One function that could add a slope to plateaus is to give a slightly higher objective value when the means of the elements in the pot are further away from their worst quality. The maximum distance from the worst quality should be the best quality, as there should be no incentive to improve on a quality that results in the highest grade. The gradient at which a higher objective value is given should be lower for aluminium than iron or silicon as the distance between the worst mean quality for aluminium is greater than that of the other two. This can be described by the following definition.

$$g'(\overline{Al}, \overline{Fe}, \overline{Si}) = g(\overline{Al}, \overline{Fe}, \overline{Si}) + a(\overline{Al}) + b(\overline{Fe}) + c(\overline{Si})$$

Where

$$a(\overline{Al}) = \begin{cases} 0.1 * (\overline{Al} - 99.1) & \text{if } \overline{Al} \leq 99.9 \\ 0 & \text{otherwise} \end{cases}$$

$$b(\overline{Fe}) = \begin{cases} 0.09 - \overline{Fe} & \text{if } \overline{Fe} \geq 0.03 \\ 0 & \text{otherwise} \end{cases}$$

$$c(\overline{Si}) = \begin{cases} 0.09 - \overline{Si} & \text{if } \overline{Si} \geq 0.01 \\ 0 & \text{otherwise} \end{cases}$$

Consider a crucible with the properties. Note the specific properties of these pots are not consistent with those provided as it is just an example of a possible configuration.

## 6 Question 6

$$g''(\overline{Al}, \overline{Fe}, \overline{Si}, x_{c1}, x_{c2}, x_{c3}, s) = \begin{cases} g(\overline{Al}, \overline{Fe}, \overline{Si}) - 20 * (s_c - s) & \text{if } s_c > s \\ g(\overline{Al}, \overline{Fe}, \overline{Si}) & \text{otherwise} \end{cases}$$

where  $s_c = \max(x_{c1}, x_{c2}, x_{c3}) - \min(x_{c1}, x_{c2}, x_{c3})$

### 6.1 Task 6

See Appendix A for code.

### 6.1.1 Max Spread = 6

1	[38 34 40 ]	99.371	%Al,	0.241	%Fe,	0.163	%Si,	\$44.53,	spread = 6
2	[17 13 19 ]	99.270	%Al,	0.297	%Fe,	0.234	%Si,	\$41.53,	spread = 6
3	[35 36 39 ]	99.615	%Al,	0.043	%Fe,	0.196	%Si,	\$48.71,	spread = 4
4	[ 6 8 10 ]	99.352	%Al,	0.257	%Fe,	0.275	%Si,	\$44.53,	spread = 4
5	[30 32 27 ]	99.558	%Al,	0.093	%Fe,	0.170	%Si,	\$48.71,	spread = 5
6	[37 42 41 ]	99.501	%Al,	0.115	%Fe,	0.198	%Si,	\$48.71,	spread = 5
7	[24 22 21 ]	99.532	%Al,	0.037	%Fe,	0.276	%Si,	\$48.71,	spread = 3
8	[33 29 28 ]	99.256	%Al,	0.151	%Fe,	0.303	%Si,	\$41.53,	spread = 5
9	[11 14 9 ]	99.381	%Al,	0.080	%Fe,	0.297	%Si,	\$44.53,	spread = 5
10	[ 5 7 3 ]	99.512	%Al,	0.188	%Fe,	0.212	%Si,	\$48.71,	spread = 4
11	[46 47 43 ]	99.359	%Al,	0.172	%Fe,	0.315	%Si,	\$44.53,	spread = 4
12	[26 25 31 ]	99.393	%Al,	0.138	%Fe,	0.325	%Si,	\$44.53,	spread = 6
13	[49 48 44 ]	99.511	%Al,	0.169	%Fe,	0.145	%Si,	\$48.71,	spread = 5
14	[20 18 23 ]	99.544	%Al,	0.057	%Fe,	0.220	%Si,	\$48.71,	spread = 5
15	[15 16 12 ]	99.369	%Al,	0.127	%Fe,	0.302	%Si,	\$44.53,	spread = 4
16	[ 2 4 1 ]	99.356	%Al,	0.144	%Fe,	0.280	%Si,	\$44.53,	spread = 3
17	[50 51 45 ]	99.256	%Al,	0.198	%Fe,	0.351	%Si,	\$41.53,	spread = 6
									Sum = \$777.27, MxSprd = 6

### 6.1.2 Max Spread = 8

1	[15 13 11 ]	99.255	%Al,	0.157	%Fe,	0.277	%Si,	\$41.53,	spread = 4
2	[ 2 1 4 ]	99.356	%Al,	0.144	%Fe,	0.280	%Si,	\$44.53,	spread = 3
3	[46 49 51 ]	99.399	%Al,	0.152	%Fe,	0.259	%Si,	\$44.53,	spread = 5
4	[26 19 24 ]	99.355	%Al,	0.140	%Fe,	0.305	%Si,	\$44.53,	spread = 7
5	[32 29 36 ]	99.509	%Al,	0.087	%Fe,	0.197	%Si,	\$48.71,	spread = 7
6	[34 35 33 ]	99.358	%Al,	0.156	%Fe,	0.278	%Si,	\$44.53,	spread = 2
7	[ 8 3 10 ]	99.527	%Al,	0.176	%Fe,	0.194	%Si,	\$48.71,	spread = 7
8	[48 50 44 ]	99.544	%Al,	0.129	%Fe,	0.241	%Si,	\$48.71,	spread = 6
9	[14 6 7 ]	99.253	%Al,	0.249	%Fe,	0.360	%Si,	\$41.53,	spread = 8
10	[16 18 17 ]	99.542	%Al,	0.132	%Fe,	0.258	%Si,	\$48.71,	spread = 2
11	[30 27 23 ]	99.505	%Al,	0.105	%Fe,	0.199	%Si,	\$48.71,	spread = 7
12	[20 22 21 ]	99.516	%Al,	0.060	%Fe,	0.277	%Si,	\$48.71,	spread = 2
13	[43 45 38 ]	99.350	%Al,	0.226	%Fe,	0.125	%Si,	\$44.53,	spread = 7
14	[ 9 5 12 ]	99.510	%Al,	0.106	%Fe,	0.258	%Si,	\$48.71,	spread = 7
15	[40 39 47 ]	99.378	%Al,	0.169	%Fe,	0.269	%Si,	\$44.53,	spread = 8
16	[28 25 31 ]	99.278	%Al,	0.204	%Fe,	0.285	%Si,	\$41.53,	spread = 6
17	[42 41 37 ]	99.501	%Al,	0.115	%Fe,	0.198	%Si,	\$48.71,	spread = 5
									Sum = \$781.45, MxSprd = 8

### 6.1.3 Max Spread = 11

1	[40 49 51 ]	99.301	%Al,	0.205	%Fe,	0.295	%Si,	\$41.53,	spread = 11
2	[35 33 34 ]	99.358	%Al,	0.156	%Fe,	0.278	%Si,	\$44.53,	spread = 2
3	[ 5 3 7 ]	99.512	%Al,	0.188	%Fe,	0.212	%Si,	\$48.71,	spread = 4

```

4 [29 31 28 ] 99.257 %Al, 0.160 %Fe, 0.310 %Si, $41.53, spread = 3
5 [ 4  6  9 ] 99.386 %Al, 0.217 %Fe, 0.204 %Si, $44.53, spread = 5
6 [36 32 26 ] 99.524 %Al, 0.048 %Fe, 0.266 %Si, $48.71, spread = 10
7 [39 44 47 ] 99.500 %Al, 0.124 %Fe, 0.270 %Si, $48.71, spread = 8
8 [25 18 16 ] 99.527 %Al, 0.153 %Fe, 0.256 %Si, $48.71, spread = 9
9 [41 50 48 ] 99.351 %Al, 0.151 %Fe, 0.329 %Si, $44.53, spread = 9
10 [14 22 24 ] 99.362 %Al, 0.083 %Fe, 0.335 %Si, $44.53, spread = 10
11 [13 23 19 ] 99.251 %Al, 0.258 %Fe, 0.213 %Si, $41.53, spread = 10
12 [30 27 20 ] 99.530 %Al, 0.104 %Fe, 0.212 %Si, $48.71, spread = 10
13 [46 42 45 ] 99.502 %Al, 0.140 %Fe, 0.056 %Si, $48.71, spread = 4
14 [21 10 17 ] 99.369 %Al, 0.239 %Fe, 0.344 %Si, $44.53, spread = 11
15 [37 43 38 ] 99.519 %Al, 0.169 %Fe, 0.143 %Si, $48.71, spread = 6
16 [11 12 15 ] 99.361 %Al, 0.068 %Fe, 0.309 %Si, $44.53, spread = 4
17 [ 8  1  2 ] 99.526 %Al, 0.042 %Fe, 0.230 %Si, $48.71, spread = 7
Sum = $781.45, MxSprd = 11

```

## 7 My Best Solutions

These were found by using standard simulated annealing in C.

### 7.1 No Max Spread

```

1 [13  4 26 ] 99.253 %Al, 0.210 %Fe, 0.373 %Si, $41.53, spread = 22
2 [31 46 17 ] 99.500 %Al, 0.135 %Fe, 0.144 %Si, $48.71, spread = 29
3 [49 14 29 ] 99.251 %Al, 0.207 %Fe, 0.314 %Si, $41.53, spread = 35
4 [23 27 30 ] 99.505 %Al, 0.105 %Fe, 0.199 %Si, $48.71, spread = 7
5 [20 50 10 ] 99.351 %Al, 0.220 %Fe, 0.327 %Si, $44.53, spread = 40
6 [ 8 34 19 ] 99.353 %Al, 0.244 %Fe, 0.155 %Si, $44.53, spread = 26
7 [ 1 48 24 ] 99.355 %Al, 0.069 %Fe, 0.342 %Si, $44.53, spread = 47
8 [32  6  2 ] 99.500 %Al, 0.146 %Fe, 0.227 %Si, $48.71, spread = 30
9 [40 36  7 ] 99.501 %Al, 0.169 %Fe, 0.121 %Si, $48.71, spread = 33
10 [28 25 43 ] 99.252 %Al, 0.218 %Fe, 0.369 %Si, $41.53, spread = 18
11 [21 45 16 ] 99.357 %Al, 0.199 %Fe, 0.232 %Si, $44.53, spread = 29
12 [ 9 47 39 ] 99.501 %Al, 0.095 %Fe, 0.228 %Si, $48.71, spread = 38
13 [37 35 11 ] 99.501 %Al, 0.067 %Fe, 0.277 %Si, $48.71, spread = 26
14 [22 15  3 ] 99.500 %Al, 0.089 %Fe, 0.165 %Si, $48.71, spread = 19
15 [18 42 44 ] 99.750 %Al, 0.044 %Fe, 0.136 %Si, $57.35, spread = 26
16 [51  5 38 ] 99.351 %Al, 0.192 %Fe, 0.349 %Si, $44.53, spread = 46
17 [41 33 12 ] 99.353 %Al, 0.099 %Fe, 0.305 %Si, $44.53, spread = 29
Sum = $790.09, MxSprd = 47

```

### 7.2 Max Spread = 6

```

1 [44,49,48,] 99.511 %Al, 0.169 %Fe, 0.145 %Si, $48.71, spread=5
2 [18,19,21,] 99.512 %Al, 0.163 %Fe, 0.187 %Si, $48.71, spread=3

```

3 [43,47,46,] 99.359 %Al, 0.172 %Fe, 0.315 %Si, \$44.53, spread=4  
 4 [9,11,12,] 99.535 %Al, 0.015 %Fe, 0.265 %Si, \$48.71, spread=3  
 5 [27,24,22,] 99.502 %Al, 0.068 %Fe, 0.194 %Si, \$48.71, spread=5  
 6 [36,39,34,] 99.502 %Al, 0.139 %Fe, 0.173 %Si, \$48.71, spread=5  
 7 [25,23,28,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, \$41.53, spread=5  
 8 [38,37,42,] 99.650 %Al, 0.118 %Fe, 0.067 %Si, \$52.44, spread=5  
 9 [51,50,45,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, \$41.53, spread=6  
 10 [2,7,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, \$48.71, spread=5  
 11 [29,26,31,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, \$44.53, spread=5  
 12 [16,14,20,] 99.298 %Al, 0.164 %Fe, 0.379 %Si, \$41.53, spread=6  
 13 [33,32,30,] 99.541 %Al, 0.070 %Fe, 0.160 %Si, \$48.71, spread=3  
 14 [6,8,10,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, \$44.53, spread=4  
 15 [3,1,4,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, \$44.53, spread=3  
 16 [13,15,17,] 99.276 %Al, 0.220 %Fe, 0.261 %Si, \$41.53, spread=4  
 17 [35,40,41,] 99.335 %Al, 0.141 %Fe, 0.317 %Si, \$41.53, spread=6  
 Sum = \$779.18, MxSprd = 6

### 7.3 Max Spread = 8

1 [8,1,6,] 99.358 %Al, 0.130 %Fe, 0.315 %Si, \$44.53, spread=7  
 2 [24,27,21,] 99.506 %Al, 0.091 %Fe, 0.245 %Si, \$48.71, spread=6  
 3 [26,31,29,] 99.372 %Al, 0.094 %Fe, 0.350 %Si, \$44.53, spread=5  
 4 [32,40,34,] 99.353 %Al, 0.199 %Fe, 0.210 %Si, \$44.53, spread=8  
 5 [46,44,41,] 99.507 %Al, 0.108 %Fe, 0.191 %Si, \$48.71, spread=5  
 6 [22,15,18,] 99.514 %Al, 0.064 %Fe, 0.164 %Si, \$48.71, spread=7  
 7 [17,13,20,] 99.359 %Al, 0.191 %Fe, 0.305 %Si, \$44.53, spread=7  
 8 [38,33,35,] 99.501 %Al, 0.105 %Fe, 0.167 %Si, \$48.71, spread=5  
 9 [47,39,43,] 99.370 %Al, 0.153 %Fe, 0.341 %Si, \$44.53, spread=8  
 10 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, \$48.71, spread=5  
 11 [51,45,50,] 99.256 %Al, 0.198 %Fe, 0.351 %Si, \$41.53, spread=6  
 12 [12,16,19,] 99.362 %Al, 0.204 %Fe, 0.275 %Si, \$44.53, spread=7  
 13 [11,9,14,] 99.381 %Al, 0.080 %Fe, 0.297 %Si, \$44.53, spread=5  
 14 [36,30,37,] 99.654 %Al, 0.098 %Fe, 0.129 %Si, \$52.44, spread=7  
 15 [10,3,4,] 99.357 %Al, 0.278 %Fe, 0.244 %Si, \$44.53, spread=7  
 16 [49,48,42,] 99.513 %Al, 0.147 %Fe, 0.139 %Si, \$48.71, spread=7  
 17 [28,25,23,] 99.267 %Al, 0.185 %Fe, 0.332 %Si, \$41.53, spread=5  
 Sum = \$784.00, MxSprd = 8

### 7.4 Max Spread = 11

1 [24,19,26,] 99.355 %Al, 0.140 %Fe, 0.305 %Si, \$44.53, spread=7  
 2 [27,33,37,] 99.500 %Al, 0.148 %Fe, 0.086 %Si, \$48.71, spread=10  
 3 [38,28,34,] 99.262 %Al, 0.236 %Fe, 0.270 %Si, \$41.53, spread=10  
 4 [48,50,41,] 99.351 %Al, 0.151 %Fe, 0.329 %Si, \$44.53, spread=9  
 5 [49,43,51,] 99.294 %Al, 0.189 %Fe, 0.367 %Si, \$41.53, spread=8  
 6 [44,47,39,] 99.500 %Al, 0.124 %Fe, 0.270 %Si, \$48.71, spread=8

```

7 [42,36,45,] 99.504 %Al, 0.145 %Fe, 0.069 %Si, $48.71, spread=9
8 [35,46,40,] 99.504 %Al, 0.110 %Fe, 0.193 %Si, $48.71, spread=11
9 [4,3,1,] 99.363 %Al, 0.151 %Fe, 0.284 %Si, $44.53, spread=3
10 [10,6,8,] 99.352 %Al, 0.257 %Fe, 0.275 %Si, $44.53, spread=4
11 [32,31,21,] 99.501 %Al, 0.089 %Fe, 0.201 %Si, $48.71, spread=11
12 [14,15,12,] 99.251 %Al, 0.139 %Fe, 0.370 %Si, $41.53, spread=3
13 [9,11,17,] 99.512 %Al, 0.072 %Fe, 0.221 %Si, $48.71, spread=8
14 [30,20,23,] 99.503 %Al, 0.072 %Fe, 0.255 %Si, $48.71, spread=10
15 [29,25,18,] 99.505 %Al, 0.134 %Fe, 0.271 %Si, $48.71, spread=11
16 [16,13,22,] 99.374 %Al, 0.167 %Fe, 0.288 %Si, $44.53, spread=9
17 [7,2,5,] 99.505 %Al, 0.182 %Fe, 0.208 %Si, $48.71, spread=5
Sum = $785.63, MxSprd = 11

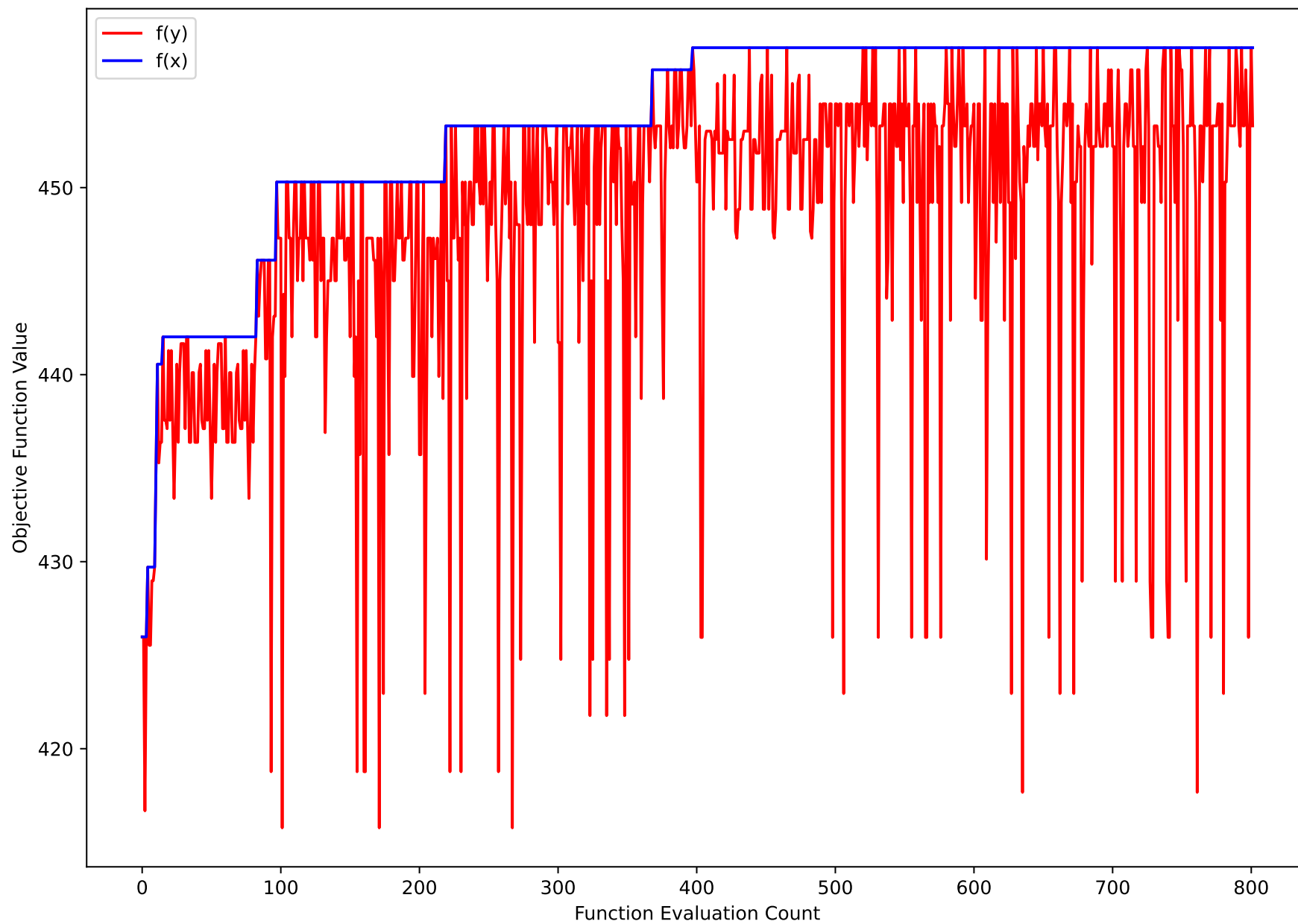
```

## 8 Appendix A

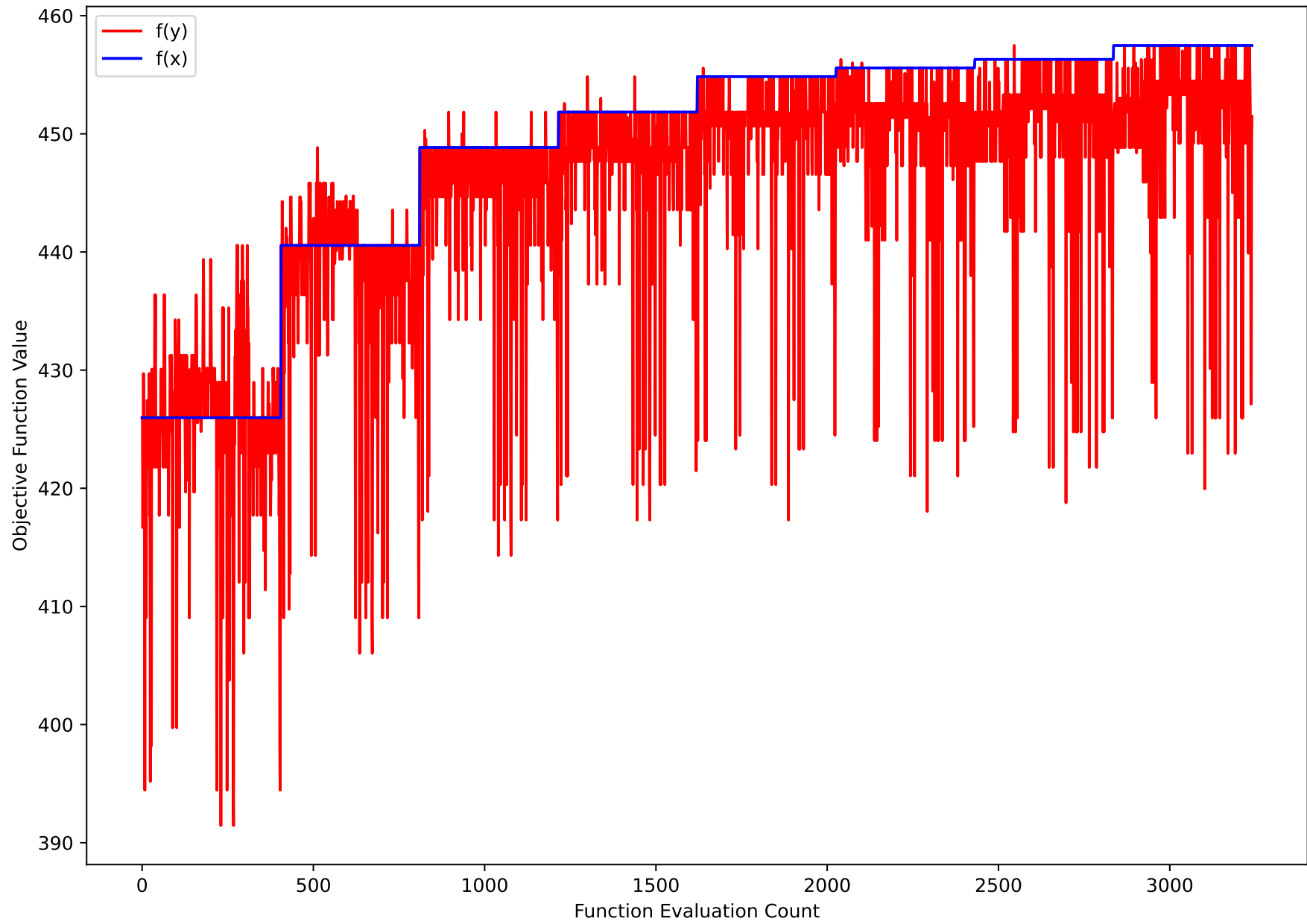
See overleaf



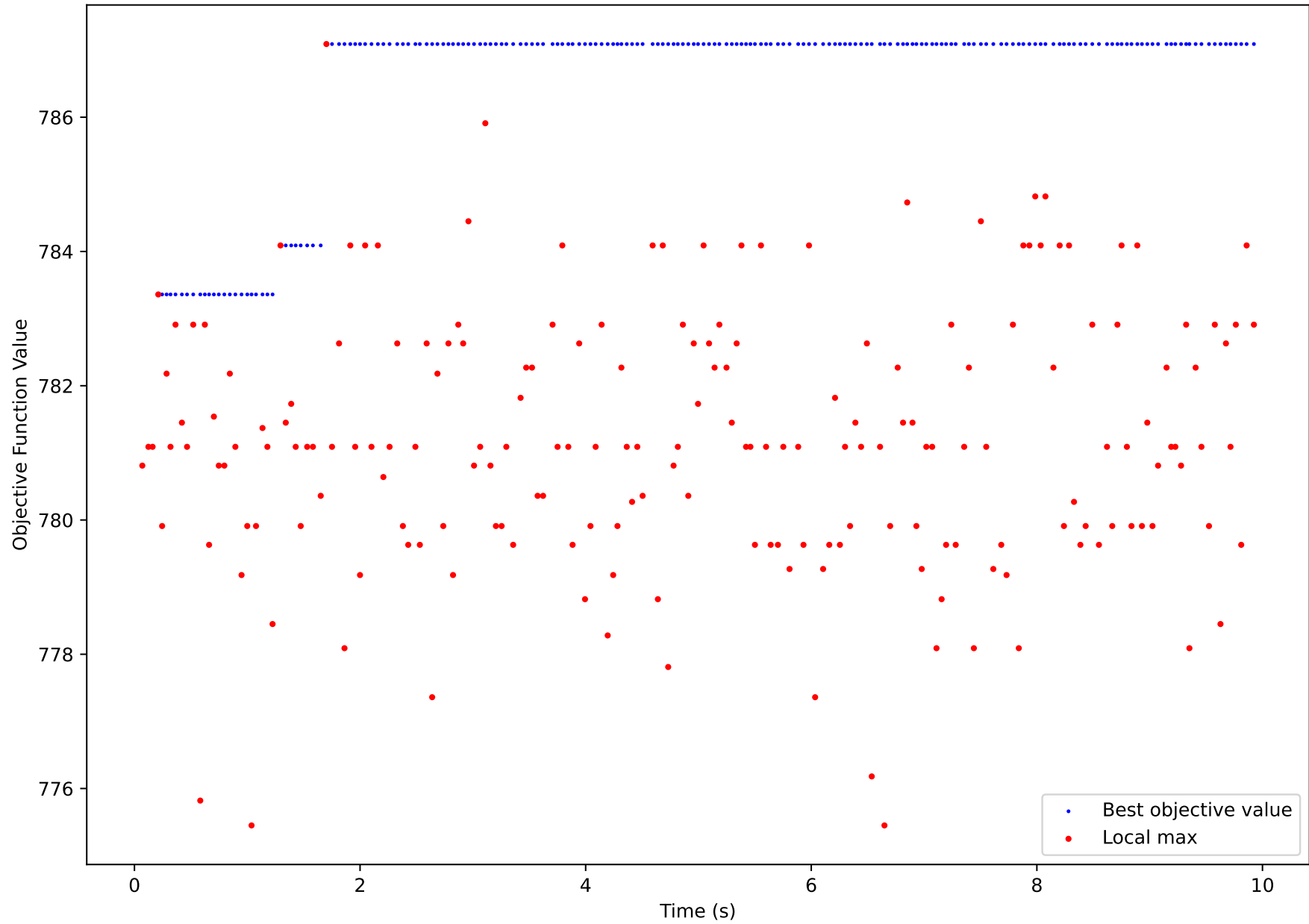
Task 3C



Task 3D



Task 3E Repeated Next Ascents (n=200)



Task 3E Repeated Steepest Ascents (n=200)

