No.

Date. 2021. 11.26　　离散数学(1).
week 11.

(10). 13前3.

14, 15, 17, 18 $\begin{cases}\text{一证} \\ \text{一举反}\end{cases}$, 19, 20, 22前2. 27(1).

14. ~~R满足~~

解:

R满足

~~$(\forall x)\wedge\langle x,y\rangle\in R$~~

$(\forall x)(\forall y)(x\in A\wedge y\in A\wedge\langle x,y\rangle\in R \rightarrow \langle y,x\rangle\in R)$ ①

和

$(\forall x)(\forall y)(\forall z)(x\in A\wedge y\in A\wedge z\in A \wedge\langle x,y\rangle\in R\wedge\langle y,z\rangle\in R\rightarrow\langle x,z\rangle\in R)$. ②

对②令 $z=x$, 同时使用① 有

$(\forall x)(\forall y)(x\in A\wedge y\in A\wedge\langle x,y\rangle\in R \rightarrow \langle x,x\rangle\in R)$. ③

但若R自反, 就必须满足

$(\forall x)(x\in A\rightarrow\langle x,x\rangle\in R)$,

其与③不同。

设R是在{1,2,3}上的空关系,

即 $R=\emptyset$.

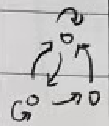
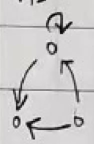
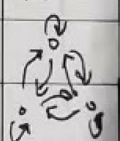
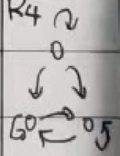
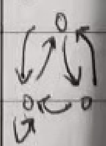
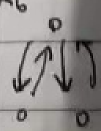
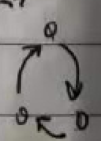
显然 ①② 前提皆为假. 即

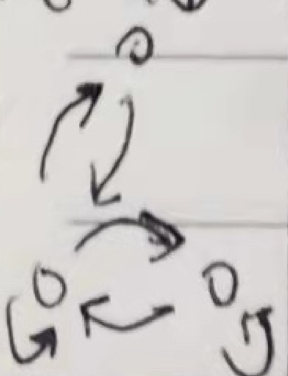
R为对称、传递关系.

但 $R=\emptyset$ 且 $\{1,2,3\}\neq\emptyset$, 故

R不是自反关系。 ☒.

15. 解:

	自反	非自反	对称	反对称	传递
R_1: 	X	X	X	~~X~~ X	X
R_2: 	X	X	X	✓	✓
R_3: 	✓	X	✓	X	✓
R_4 	✓	X	X	X	✓
R_5 	X	X	X	X	X
R_6 	X	✓	✓	X	X
R_7 	X	✓	X	✓	X

R8. 	√	×	√	×	×

⇒

(1) <

~~17. 关系R在A上.~~

Date.

17(1). 证明: ($\Rightarrow$).

~~假~~ R 自反.

即对任意 $\langle x,y\rangle \in A^2$.

$\langle x,y\rangle \in I_A \Leftrightarrow x=y \Rightarrow xRy$.

故 $I_A \subseteq R$.

($\Leftarrow$):

I_A ~~是~~ $\subseteq R$, ~~所以~~ 因此.

~~即~~ $(\forall x)(x \in A \rightarrow xRx)$.

~~故~~ 故 R 自反.

综上, R 自反 $\Leftrightarrow I_A \subseteq R$.

(2). 证明:

($\Rightarrow$)

R 非自反.

即对任意 $\langle x,y\rangle \in A^2$

$\langle x,y\rangle \in I_A \Leftrightarrow x=y \Rightarrow x\cancel{R}y$.

故 $I_A \cap R = \emptyset$.

($\Leftarrow$).

$I_A \cap R = \emptyset$, 因此

$(\forall x)(x \in A \rightarrow x \notin R)$, 故

R 非自反.

综上, R 非自反 $\Leftrightarrow I_A \cap R = \emptyset$.

(3). 证明:

($\Rightarrow$)

R 传递, 即对任意 $\langle x,y\rangle \in A^2$

$\langle x,y\rangle \in R \circ R$.

$\Leftrightarrow (\exists z)($~~xRy~~$\langle x,z\rangle \in R \wedge \langle z,y\rangle \in R)$.

$\Rightarrow \langle x,y\rangle \in R$, 故 $R \circ R \subseteq R$.

($\Leftarrow$).

设 $R \circ R \subseteq R$. 即

~~$(\forall x)(\forall y)$~~

~~$(x \in A \wedge y \in A \wedge \langle x,y\rangle \in R \circ R \rightarrow \langle x,y\rangle \in R)$.~~

~~即~~

~~$(\forall x)(\forall y)(\exists z)$~~

~~$(x \in A \wedge y \in A \wedge z \in A \wedge \langle x,z\rangle \in R \wedge \langle z,y\rangle \in R \rightarrow \langle x,y\rangle \in R)$.~~

~~即~~

$(\forall x)(\forall y)(\forall z)$

$($~~xRy~~ $x \in A \wedge y \in A \wedge z \in A \wedge xRz \wedge zRy \Rightarrow xRy$

$\langle x,y\rangle \in R \circ R \Rightarrow \langle x,y\rangle \in R)$,

故 R 传递.

$\circ R_2 = \{\langle 3,1\rangle\}$.

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18(1). 真.

证明:

R_1, R_2 在A上自反.

$\Leftrightarrow (\forall x)(x\in A \rightarrow xR_1x) \land (\forall x)(x\in A \rightarrow xR_2x)$

$\Leftrightarrow (\forall x)((x\notin A \lor xR_1x) \land (x\notin A \lor xR_2x))$

$\Leftrightarrow (\forall x)(x\notin A \lor (xR_1x \land xR_2x))$

$\Leftrightarrow (\forall x)(x\in A \rightarrow (xR_1x \land xR_2x))$.

$\Rightarrow (\forall x)(x\in A \rightarrow x(R_1\circ R_2)x)$

(2) 假.

证明:

~~设 x, y, z ∈ A.~~

~~$R_1 = \{\langle x,y\rangle, \langle z,y\rangle\}$~~

$R_2 = \{\langle y,x\rangle, \langle y,z\rangle\}$.

$R_2 = \{\langle x,y\rangle, \langle z,y\rangle\}$.

$R_1\circ R_2 = \{\langle x,x\rangle, \langle x,z\rangle, \langle z,x\rangle, \langle z,z\rangle\}$.

~~此时 R_1, R_2 非自反 但 $R_1\circ R_2$ 自反~~

设 x, y ∈ A, |A| = 2

~~设~~ $R_1 = \{\langle x,y\rangle, \langle y,x\rangle\}$ 故为非自反, 但

$R_1\circ R_1 = \{\langle x,x\rangle, \langle y,y\rangle\}$ 自反.

(3). 假

(4). 假.

19(1).

解: 令 $R=\emptyset$ 即可.

(2).

解: ~~$R=\{\langle 1,2\rangle, \langle 2,1\rangle,$ [illegible]~~, $\langle 2,3\rangle, \langle 3,4\rangle\}$.

$R=\{\langle 1,1\rangle, \langle 2,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle, \langle 2,1\rangle, \langle 1,2\rangle\}$.

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解:

$A=\{1,2,3,4\}$.

$R=\{\langle 1,2\rangle, \langle 4,3\rangle, \langle 2,2\rangle, \langle 2,1\rangle, \langle 3,1\rangle\}$

R不是传递的, 因为

~~[illegible] $\in R \land \langle 3,1\rangle \in R$~~

因 $\langle 4,3\rangle \in R \land \langle 3,1\rangle \in R$

~~$\langle 3,1\rangle \notin R$~~. $\langle 4,1\rangle \notin R$.

$R_1=\{\langle 1,2\rangle, \langle 4,3\rangle, \langle 2,2\rangle, \langle 2,1\rangle, \langle 3,1\rangle, \langle 1,1\rangle, \langle 3,2\rangle, \langle 4,1\rangle, \langle 4,2\rangle\}$.

22. 解:

$R_1\circ R_2 = \{\langle c,d\rangle\}$.

$R_2\circ R_1 = \{\langle a,d\rangle, \langle a,c\rangle\}$.

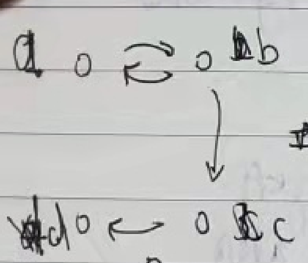
27. $A = \{a, b, c, d\}$.

$R = \{ \langle a,b\rangle, \langle b,a\rangle, \langle b,c\rangle, \langle c,d\rangle \}$.

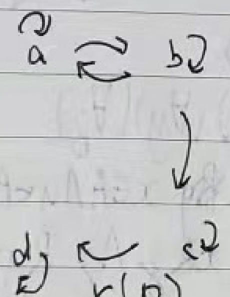
$M(r(R))$ $M(R) + I_4$.

$$= \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} + \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix}$$

$$= \begin{bmatrix} 1&1&0&0\\ 1&1&1&0\\ 0&0&1&1\\ 0&0&0&1 \end{bmatrix}$$



R



r(R).

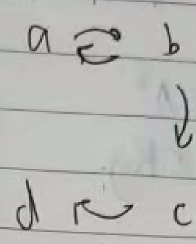
$r(R) = \{ \langle a,a\rangle, \langle a,b\rangle, \langle b,a\rangle, \langle b,b\rangle, \langle b,c\rangle, \langle c,c\rangle, \langle c,d\rangle, \langle d,d\rangle \}$.

$M(s(R)) = M(R) + M(R^{-1})$

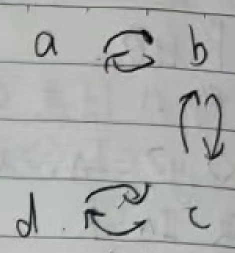
$= M(R) + M(R)^T$

$$= \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0 \end{bmatrix}$$

$$= \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&1&0&1\\ 0&0&1&0 \end{bmatrix}$$



R.



S(R)

$S(R) = \{ \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 3,4\rangle, \langle 4,3\rangle \}$

$M(R \circ R)$

$= M(R^2) = M(R)^2$

$$= \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

$$M(R^3) = \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 0&1&0&1\\ 1&0&1&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

$$M(R^4) = \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} = \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

$M(t(R)) = M(R) + M(R^2) + M(R^3)$

$$= \begin{bmatrix} 0&1&0&0\\ 1&0&1&0\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix} + \begin{bmatrix} 1&0&1&0\\ 0&1&0&1\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&1&1&1\\ 1&0&1&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1&1&1&1\\ 1&1&1&1\\ 0&0&0&1\\ 0&0&0&0 \end{bmatrix}$$

t(R):