

## 第六章部分习题解答

9. 此题有误。应加上条件 $a, b, c, d \in R$ , 否则结论不对。例如 $w = z = \frac{iz}{i}$ , 则 $w$ 是恒同映射, 但 $ad - bc = i^2 - 0 = -1 < 0$ .

10. 此题也有误。应改 $|w| < 1$ 为 $|w| = 1$ . 当 $z \rightarrow \infty$ 时, 有 $|w| \rightarrow 1$ , 故有 $|a| = |c|$ ,  $ad - bc \neq 0$ .

12. 令 $w^1 = w - 1$ , 则有 $|z| < 1 \rightarrow |w^1| < 1$ , 因而有

$$w^1 = w - 1 = e^{i\theta} \frac{z - z_0}{1 - \overline{z_0}z},$$

即

$$w = 1 + e^{i\theta} \frac{z - z_0}{1 - \overline{z_0}z}, \theta \in R, |z_0| < 1.$$

17. 令 $w = \frac{z-i}{cz+d}$ , 分别将 $z_1 = 1, z_2 = -i$ 代入, 分别得 $w_1 = 1, w_2 = -1$ , 从而得到 $c, d$ 的值。再将 $z = 0$ 代入, 如果 $Im(w(0)) = \frac{-i}{d} > 0$ , 则映为上复平面, 否则为下复平面。

19(3).

$$z_1 = z^4, \quad z_2 = \frac{z_1 + 2^4}{z_1 - 2^4}, \quad w = z_2^2 = \left( \frac{z^4 + 2^4}{z^4 - 2^4} \right)^2.$$

19(8).

$$z_1 = \frac{z+2}{2-z}, (ad-bc > 0), \quad z_2 = z_1\pi, \quad z_3 = z_2i, \quad w = e^{z_3} = e^{\pi i \left( \frac{z+2}{2-z} \right)}.$$

19(9).

$$z_1 = \frac{z-a}{b-a}\pi, \quad z_2 = z_1i, \quad w = e^{z_2} = e^{\frac{z-a}{b-a}\pi i}.$$