NAME :

STUDENT ID:

NOTE: There are 3 Parts to this MIDTERM EXAM (total of 7 pages). For Part 1 (multiple choice) be sure to indicate your answer clearly as no partial credit will be awarded. Each question has a unique right answer.

Part 3 consists of a bonus exercise, which is not compulsory, and can give you some extra points. It is advised to finish all the compulsory questions before attempting the bonus question. If your total is > 100 points, your assigned grade will be 100.

In order to receive full credit for Parts 2 and 3, you must show work to explain your reasoning. If you require additional sheets for Parts 2 or 3, be sure to put your name and ID to each additional page that you turn in with this exam. Use of calculators will NOT be permitted. You have 90 minutes complete this test.

Part 1 (25pts)

1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that for all $x \in \mathbb{R}$, we have $g(x) = f(x+2) - 3$. The graph of g is obtained from the graph of f by

   (A) shifting from 2 units on the right and 3 units down.
   (B) shifting from 2 units on the right and 3 units up.
   (C) shifting from 2 units on the left and 3 units up.
   (D) shifting from 2 units on the left and 3 units down.
   (E) None of the above

2. What is the value of the following limit:

$$\lim_{x \to 0^+} \sqrt{x^3+2} - x^{3/2} \ ?$$

   (A) $\sqrt{2}$
   (B) 1
   (C) 0
   (D) $-\sqrt{2}$
   (E) None of the above.

3. Which of the assertions below is equivalent to "The function $f : \mathbb{R} \to \mathbb{R}$ does not have a limit at $x = c$."?

   (A) $\exists \ell \in \mathbb{R}$ such that $\forall \epsilon > 0$ $\exists \delta > 0$ such that for all $x \in \mathbb{R}$ with $0 <\mid x - c \mid< \delta$, we have $\mid f(x) - \ell \mid> \epsilon$.
   (B) $\forall \ell \in \mathbb{R}$ $\exists \epsilon > 0$ such that $\forall \delta > 0$ $\exists x \in \mathbb{R}$ such that $0 <\mid x - c \mid< \delta$ and $\mid f(x) - \ell \mid> \epsilon$.
   (C) $\forall \ell \in \mathbb{R}$ $\forall \epsilon > 0$ $\exists \delta > 0$ and $\exists x \in \mathbb{R}$ such that $0 <\mid x - c \mid< \delta$ and $\mid f(x) - \ell \mid> \epsilon$.
   (D) $\exists \ell \in \mathbb{R}$ $\exists \epsilon > 0$ such that $\forall \delta > 0$ and $\exists x \in \mathbb{R}$ such that $0 <\mid x - c \mid< \delta$ and $\mid f(x) - \ell \mid< \epsilon$.
   (E) None of the above.

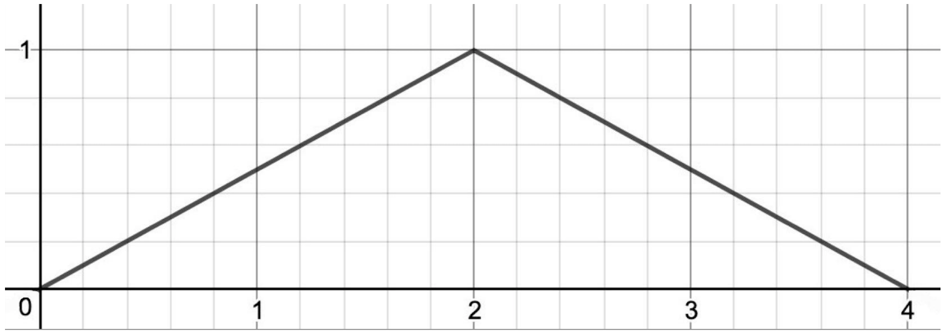
4. How many asymptotes (vertical and horizontal) does the function $f(x) = \frac{x^3+x+2}{x^3-4x^2+3x}$ have?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) None of the above.

5. What is the (smallest) period of the function $f(x) = (\sin(x))^2$?

   (A) $\frac{\pi}{2}$
   (B) π
   (C) 2π
   (D) $\frac{3\pi}{2}$
   (E) None of the above.

Part 2a. (20 pts) Consider the function $f : [0, 4] \to \mathbb{R}$ whose graph is represented below.



Give a formula for $f(x)$ in terms of the absolue value. Draw the graph of each function:

1. $y = f(-x)$.

2. $y = -f(x + 1)$.

3. $y = 1 + f(2x)$.

Part 2b. (10pts) Show that the equation $x^2+y^2+2x-6y+8=0$ is the equation of a circle, and determine its center and radius.

Part 2c. (15pts) Does the following limit exist? If yes, what is its value? (You need to justify your answer).

$$\lim_{x\to 0}\frac{\sqrt{1+\sin(x)}-1}{x}$$

Part 2d. (15pts)
Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Show that there exists functions $u : \mathbb{R} \to \mathbb{R}$ and $v : \mathbb{R} \to \mathbb{R}$ such that:

1. u is even and v is odd.
2. $f = u + v$, i.e. for any $x \in \mathbb{R}$ we have $f(x) = u(x) + v(x)$.

Part 2e. (15pts) Let $f : (0, +\infty) \to \mathbb{R}$ be defined by $f(x) = \cos(\frac{1}{\sqrt{x}})$ for $x > 0$. Does

$$\lim_{x \to 0^+} f(x)$$

exists? Justify your answer carefully.

Part 3 (BONUS QUESTION).

Let $(a_n)_{n\geq 1}$ be a sequence of real numbers such that $\lim_{n\to+\infty}(a_{n+1} - a_n) = 0$. Prove that $\lim_{n\to+\infty} \frac{a_n}{n} = 0$.