Calculus A(1): Homework 3

October 23, 2021

1.2

Problem 93.

For what value of k is the line $2x + ky = 3$ perpendicular to the line $4x + y = 1$? For what value of k are the lines parallel?

Solution 93.

For any line $y = ax + b$, a is the slope and b is the y-intercept. $2x + ky = 3 \Leftrightarrow y = -\frac{2}{k}x + \frac{3}{k}$, $4x + y = 1 \Leftrightarrow y = -4x + 1$.
If the lines are perpendicular, then $-\frac{2}{k} \cdot (-4) = -1 \Rightarrow k = -8$.
If the lines are parallel, then $-\frac{2}{k} = -4 \Rightarrow k = 1/2$.

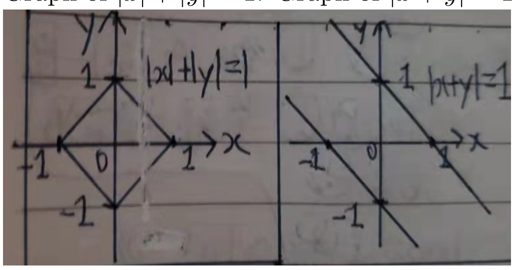
1.3

Problem 22.

Graph $|x| + |y| = 1$ and $|x + y| = 1$ and explain why they are not graphs of functions of x.

Solution 22.

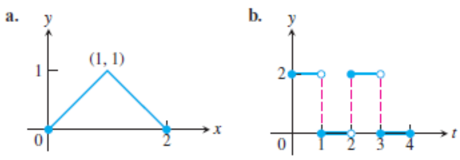
Graph of $|x| + |y| = 1$: Graph of $|x + y| = 1$:



Since $\exists a$ s.t. $x = a$ and the each of the graphs have two intersections, neither are functions.

Problem 27.

Find a formula for each function graphed.



Solution 27.

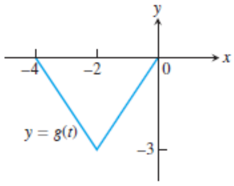
(a) $y = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

(b) $y = \begin{cases} 2, & 0 \le x < 1 \vee 2 \le x < 3 \\ 0, & 1 \le x < 2 \vee 3 \le x \le 4 \end{cases}$

1.5

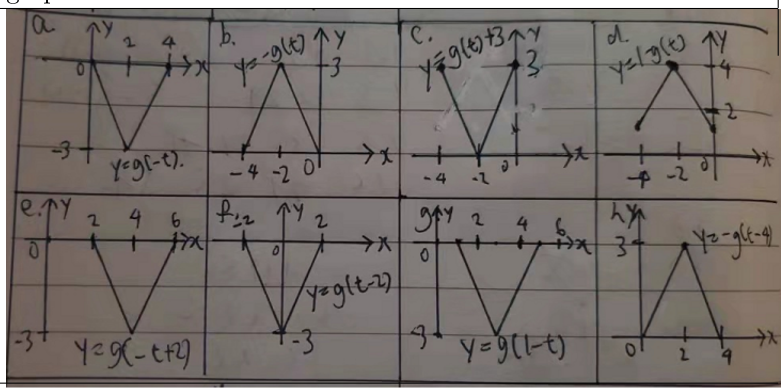
Problem 50.

The accompanying figure shows the graph of a function $g(t)$ with domain $[-4, 0]$ and range $[-3, 0]$. Find the domains and ranges of the following functions, and sketch their graphs.



a. $g(-t)$ **b.** $-g(t)$ **c.** $g(t) + 3$ **d.** $1 - g(t)$ **e.** $g(-t + 2)$ **f.** $g(t - 2)$ **g.** $g(1 - t)$ **h.** $-g(t - 4)$

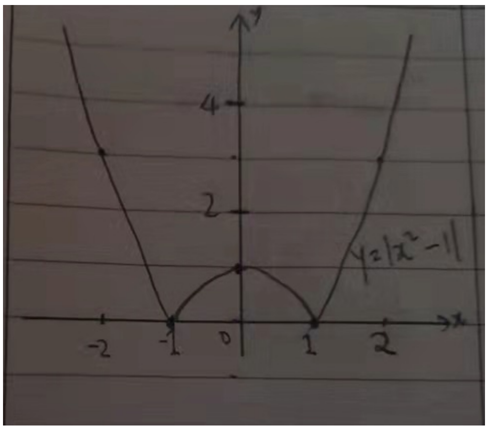
Solution 50.

function	domain	range	graph
$g(-t)$	$[0, 4]$	$[-3, 0]$	
$-g(t)$	$[-4, 0]$	$[0, 3]$	
$g(t) + 3$	$[-4, 0]$	$[0, 3]$	
$1 - g(t)$	$[-4, 0]$	$[1, 4]$	
$g(-t + 2)$	$[2, 6]$	$[-3, 0]$	
$g(t - 2)$	$[-2, 2]$	$[-3, 0]$	
$g(1 - t)$	$[1, 5]$	$[-3, 0]$	
$-g(t - 4)$	$[0, 4]$	$[0, 3]$	

Problem 69.

Graph the function $y = |x^2 - 1|$

Solution 69.



Problem 79.

Assume that is an even function, g is an odd function, and both and g are defined on the entire real line $\mathbb{R}$. Which of the following (where defined) are even? odd?
a.fg **b.**f/g **c.**g/f **d.**$f^2 = ff$ **e.**$g^2 = gg$ **f.**$f \circ g$ **g.**$g \circ f$ **h.**$f \circ f$ **i.**$g \circ g$

Solution 79.

$f(-x) = f(x), g(-x) = -g(x)$

fg	f/g	g/f	$f^2 = ff$	$g^2 = gg$	$f \circ g$	$g \circ f$	$f \circ f$	$g \circ g$
odd	odd	odd	even	even	even	even	even	odd

1.6

Problem 49.

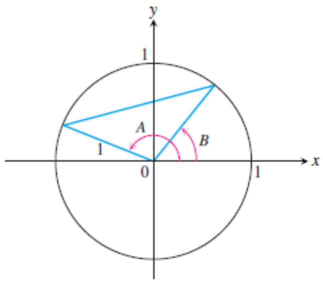
Find the value of $\sin^2 \frac{\pi}{12}$

Solution 49.

$$\sin^2 \frac{\pi}{12} = \frac{1 - \cos \frac{\pi}{6}}{2} = \frac{2 - \sqrt{3}}{4}$$

Problem 53.

Apply the law of cosines to the triangle in the accompanying figure to derive the formula for $\cos(A - B)$.



Solution 53.

Let $P_A = (\cos A, \sin A), P_B = (\cos B, \sin B)$.
On one hand,

$$|P_A P_B|^2 = 1^2 + 1^2 - 2(1)(1)\cos(A - B) = 2 - 2\cos(A - B)$$

On the other hand,

$$|P_A P_B|^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$$
$$= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B$$
$$= 2 - 2\cos A \cos B - 2\sin A \sin B$$

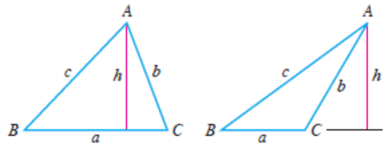
Thus,

$$2 - 2\cos A \cos B - 2\sin A \sin B = 2 - 2\cos(A - B) \Rightarrow \cos(A - B) = \cos A \cos B + \sin A \sin B$$

Problem 57.

The law of sines The *law of sines* says that if a,b and c are the sides of opposite the angles A, B and C in a triangle, then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Use the accompanying figures and the identity $\sin(\pi - \theta) = \sin\theta$ if required, to derive the law.

Solution 57.

Consider the area of that triangle, and denote $S_{\triangle ABC}$ as the area of $\triangle ABC$.
Case $\triangle ABC$ is an acute triangle, $S_{\triangle ABC} = \frac{1}{2}ah = \frac{1}{2}ab\sin C$
Case $\triangle ABC$ is an obtuse triangle, $S_{\triangle ABC} = \frac{1}{2}ah = \frac{1}{2}ab\sin(\pi - C) = \frac{1}{2}ab\sin C$
Similarly, $S_{\triangle ABC} = \frac{1}{2}bc\sin A$, $S_{\triangle ABC} = \frac{1}{2}ca\sin B$.
Hence, $\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.