Calculus A2: Homework 15

June 2, 2022

p.1001

17.

$$f(x,y) = 1 - y/2$$
$$R_{xy} = \{(x,y) : x^2 + y^2 \le 1\}$$
$$S = \iint\limits_{R_{xy}} \sqrt{(f'_x)^2 + (f'_y)^2 + 1}dS = \iint\limits_{R} \frac{\sqrt{5}}{2}dS = \frac{\sqrt{5}\pi}{2}$$

19.

$$f(x,y) = 2\sqrt{x^2 + y^2}$$
$$R_{xy} = \{(x,y) : 1 \le x^2 + y^2 \le 9\}$$
$$S = \iint\limits_{R_{xy}} \sqrt{(f'_x)^2 + (f'_y)^2 + 1}dS = \iint\limits_{R_{xy}} \sqrt{\frac{4x^2 + 4y^2}{x^2 + y^2} + 1}dS = \iint\limits_{R_{xy}} \sqrt{5}dS = 8\sqrt{5}\pi$$

37.

$$F(x,y,z) = x^2 + y^2 - z, \nabla F = 2x\mathbf{i} + 2y\mathbf{j} - \mathbf{k}$$
$$R = \{(x,y) : 0 \le x^2 + y^2 \le 2\}$$
$$S = \iint\limits_{R} \left|\frac{\nabla F}{\nabla F \cdot \mathbf{k}}\right| dS = \iint\limits_{R} \sqrt{4x^2 + 4y^2 + 1}dS = \int_0^{2\pi} d\alpha \int_0^{\sqrt{2}} r\sqrt{4r^2 + 1}dr = \frac{2\pi}{12}((8+1)^{(3/2)} - 1) = \frac{13\pi}{3}$$

39.

$$F(x,y,z) = x + 2y + 2z - 5, \nabla F = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$
$$R = \{(x,y) : |y| \le 1 \land y^2 \le x \le 2 - y^2\}$$
$$S = \iint\limits_{R} \left|\frac{\nabla F}{\nabla F \cdot \mathbf{k}}\right| dS = \iint\limits_{R} \frac{3}{2}dS = \frac{3}{2} \cdot 4(1 - \int_0^1 y^2 dy) = 6(1 - 1/3) = 4$$

p.1012

1.

$$S : \mathbf{r}(x,z) = x\mathbf{i} + x^2\mathbf{j} + z\mathbf{k}, x \in [0,2], z \in [0,3]$$
$$d\sigma = |\mathbf{r}'_x \times \mathbf{r}'_z|dxdz = |(\mathbf{i} + 2x\mathbf{j}) \times \mathbf{k}|dxdz = \sqrt{4x^2 + 1}dxdz$$

$$\iint\limits_{S} Gd\sigma = \int_0^3 dz \int_0^2 x\sqrt{4x^2 + 1}dx = 3 \cdot \frac{1}{12}((17)^{3/2} - 1) = \frac{17^{3/2} - 1}{4}$$

3.

$$S : \mathbf{r}(\phi,\theta) = \sin\phi\cos\theta\mathbf{i} + \sin\phi\sin\theta\mathbf{j} + \cos\phi\mathbf{k}, \phi \in [0,\pi], \theta \in [0,2\pi]$$

$$d\sigma = |\mathbf{r}'_\phi \times \mathbf{r}'_\theta| d\phi d\theta = |(\cos\phi\cos\theta\mathbf{i} + \cos\phi\sin\theta\mathbf{j} - \sin\phi\mathbf{k}) \times (-\sin\phi\sin\theta\mathbf{i} + \sin\phi\cos\theta\mathbf{j})| d\phi d\theta$$

$$= \sin\phi|\sin\phi\cos\theta\mathbf{i} + \sin\phi\sin\theta\mathbf{j} + \cos\phi\mathbf{k}| d\phi d\theta = \sin\phi d\phi d\theta$$

$$\iint_S G d\sigma = \int_0^\pi \sin^3\phi d\phi \int_0^{2\pi} \cos^2\theta d\theta = \int_0^\pi \sin\phi(1-\cos^2\phi) d\phi \int_0^{2\pi} \frac{1+\cos 2\theta}{2} d\theta$$

$$= \left(-\cos\phi + (1/3)\cos^3\phi\right)_0^\pi \left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)_0^{2\pi} = \frac{4\pi}{3}$$

19.

$$S : \mathbf{r}(x,y) = x\mathbf{i} + y\mathbf{j} + (4-y^2)\mathbf{k}, x \in [0,1], y \in [-2,2]$$

$$\mathbf{n} d\sigma = (\mathbf{r}_x \times \mathbf{r}_y) dx dy = (\mathbf{i} \times (\mathbf{j} - 2y\mathbf{k})) dx dy = (2y\mathbf{j} + \mathbf{k}) dx dy$$

$$\iint_S \mathbf{F}\cdot\mathbf{n} d\sigma = \int_{-2}^{2}\int_0^1 (2xy - 3(4-y^2)) dx dy = \int_{-2}^{2} (y + 3y^2 - 12) dy = 2((2)^3 - 12(2)) = -32$$

21.

$$S : \mathbf{r}(\phi,\theta) = a\sin\phi\cos\theta\mathbf{i} + a\sin\phi\sin\theta\mathbf{j} + a\cos\phi\mathbf{k}, \phi \in [0,\pi/2], \theta \in [0,\pi/2]$$

$$\mathbf{n} d\sigma = (\mathbf{r}'_\phi \times \mathbf{r}'_\theta) d\phi d\theta = a^2\sin\phi(\sin\phi\cos\theta\mathbf{i} + \sin\phi\sin\theta\mathbf{j} + \cos\phi\mathbf{k}) d\phi d\theta$$

$$\iint_S \mathbf{F}\cdot\mathbf{n} d\sigma = \int_0^{\pi/2} d\theta \int_0^{\pi/2} (a\cos\phi \cdot a^2\sin\phi\cos\phi) d\phi = \frac{\pi}{2}\cdot\left(-\frac{a^3}{3}\cos^3\phi\right)_0^{\pi/2} = \frac{\pi a^3}{6}$$