离散数学(2). #6.

No.

Date.

1 (1-3) 选一.

(4-8)

(9, 10) 选一.

2. 皇.

*选一 | 证明.

选一 举反.

4. (1-5) 计 前束范式

(9, 10) 求 SNF.

1. (3).

$(\forall x)(P(x) \vee q) \rightarrow$

$(\exists x)(P(x) \wedge q)$

$= ((\exists x) \neg P(x) \wedge \neg q)$

$\vee ((\exists x) P(x) \wedge q)$.

证明:

$(\forall x)(P(x) \vee q) \rightarrow (\exists x)(P(x) \wedge q)$.

$\cancel{(\forall x)(P(x) \vee ((\exists x) P(x) \wedge q)}$

$= \neg (\forall x)(P(x) \vee q) \vee ((\exists x) P(x) \wedge q)$.

$= (\exists x) \neg (P(x) \vee q) \vee ((\exists x) P(x) \wedge q)$.

$= (\exists x)(\neg P(x) \wedge \neg q) \vee ((\exists x) P(x) \wedge q)$

(4). $(\forall y)(\exists x)((P(x) \rightarrow q) \vee S(y))$

$= ((\forall x) P(x) \rightarrow q) \vee (\forall y) S(y)$

证明:

$(\forall y)(\exists x)((P(x) \rightarrow q) \vee S(y))$

$= (\forall y)((\exists x)(P(x) \rightarrow q) \vee S(y))$

$\cancel{= (\forall y) \neg\neg((\exists x)(P(x) \rightarrow q) \vee \neg\neg S(y))}$

$\cancel{= \neg(\exists y) \neg((\exists x)(P(x) \rightarrow q) \wedge \neg S(y))}$

$(\exists x)(P(x) \rightarrow q) \vee (\forall y) S(y)$.

$= (\exists x)(\neg P(x) \vee q) \vee (\forall y) S(y)$.

$= ((\forall x) P(x) \rightarrow q) \vee (\forall y) S(y)$

(5). $(\forall x) P(x) \rightarrow q = (\exists x)(P(x) \rightarrow q)$

证明:

$(\forall x) P(x) \rightarrow q = \neg (\forall x) P(x) \vee q$

$= (\exists x) \neg P(x) \vee q$.

$= (\exists x)(\neg P(x) \vee q) = (\exists x)(P(x) \rightarrow q)$

(6). $(\exists x)(P(x) \rightarrow Q(x))$

$= (\forall x) P(x) \rightarrow (\exists x) Q(x)$.

证明:

$(\exists x)(P(x) \rightarrow Q(x))$.

$= (\exists x)(\neg P(x) \vee Q(x))$

$= (\exists x) \neg P(x) \vee (\exists x) Q(x)$.

$= \neg (\forall x) P(x) \vee (\exists x) Q(x)$.

$= (\forall x) P(x) \rightarrow (\exists x) Q(x)$.

No

Date:

(7). $(\exists x)P(x) \to (\forall x)Q(x)$

$\Rightarrow (\forall x)(P(x) \to Q(x))$

证明.

$(\exists x)P(x) \to (\forall x)Q(x)$.

$= \neg(\exists x)P(x) \vee (\forall x)Q(x)$.

$= (\forall x)\neg P(x) \vee (\forall x)Q(x)$

$\Rightarrow (\forall x)(\neg P(x) \vee Q(x))$.

$= (\forall x)(P(x) \to Q(x))$.

(8). $(\exists x)P(x) \wedge (\forall x)Q(x)$

$\Rightarrow (\exists x)(P(x) \wedge Q(x))$.

证明:

$(\exists x)P(x) \wedge (\forall x)Q(x)$.

$\cancel{= (\exists x)\neg\neg P(x) \wedge (\forall x)Q(x)}$.

$\cancel{\neq \neg(\forall x)\neg P(x) \wedge (\forall x)Q(x)}$.

$\cancel{\neq \neg(\forall x)\neg P(x) \wedge \neg\neg(\forall x)Q(x)}$.

$\cancel{= \neg((\forall x)\neg P(x) \vee}$

$\cancel{= (\exists x)P(x) \wedge \neg(\exists x)\neg Q(x)}$.

$\cancel{= \neg\neg(\exists x)P(x) \wedge \neg(\exists x)\neg Q(x)}$.

$=$

$(\exists x)P(x) \wedge (\forall y)Q(y)$.

$= (\exists x)(P(x) \wedge (\forall y)Q(y))$.

$\Rightarrow (\exists x)(P(x) \wedge Q(x))$

(9).

$((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)R(x))$

$\vee ((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)S(x))$

$= (\forall x)(P(x) \wedge Q(x))$

$\wedge (\exists x)(R(x) \vee S(x))$

证明.

$((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)R(x))$

$\vee ((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)R(x))$

$= ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)R(x))$

$\vee ((\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)S(x))$.

$\cancel{= (\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)R(x)}$

$= (\forall x)(P(x) \wedge Q(x)) \wedge ((\exists x)R(x) \vee (\exists x)S(x))$

$= (\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)(R(x) \vee S(x))$

2(1).

$\cancel{(\exists x)(P(x) \leftrightarrow Q(x))}$.

$\cancel{= (\exists x)((P(x) \wedge Q(x)) \vee (\neg P(x) \wedge \neg Q(x)))}$

$\cancel{= (\exists x)(P(x) \wedge Q(x)) \vee (\exists x)(\neg P(x) \wedge \neg Q(x))}$

$\cancel{\Rightarrow ((\exists x)P(x) \wedge (\exists x)Q(x))}$

$\cancel{\vee ((\exists x)\neg P(x) \wedge (\exists x)\neg Q(x))}$.

~~2. (1) 设个体域为{0,1}. 令 P(0) = Q(0) = P(1) = F, Q(1) = T. P(0) ↔ Q(0) 为真, 值!~~

证明(3), 举反(6).

(3) 的证明:

$((\exists x)P(x) \to (\forall x)Q(x))$
$\to (\forall x)(P(x) \to Q(x))$.

证明:

$((\exists x)P(x) \to (\forall x)Q(x))$.

$= \neg(\exists x)P(x) \vee (\forall x)Q(x)$.

$\neq \cancel{\neg(\exists x)P(x) \vee \neg(\exists x)\neg Q(x)}$

$= \cancel{\neg(\exists x)P(x) \vee \neg(\exists x)\neg Q(x)}$

$= \cancel{\neg((\exists x)P(x) \wedge (\exists x)\neg Q(x))}$

$= (\forall x)\neg P(x) \vee (\forall x)Q(x)$

$\Rightarrow (\forall x)(\neg P(x) \vee Q(x))$.

$= (\forall x)(P(x) \to Q(x))$

故 $(\exists x)P(x) \to (\forall x)Q(x)$.
"重言蕴涵",
$(\forall x)(P(x) \to Q(x))$,

原式普遍有效.

(6). $(\exists x)(P(x) \to Q(x))$
$\cancel{\to (\forall x)P(x) \to \exists x}$

$(\forall x)(P(x) \vee Q(x))$
$\to ((\forall x)P(x) \vee (\forall x)Q(x))$

解: 设个体域为{0,1}.
令 $P(0) = Q(1) = T$,
$P(1) = Q(0) = F$.

则

$(\forall x)(P(x) \vee Q(x))$
$= (P(0) \vee Q(0)) \wedge (P(1) \vee Q(1))$
$= T \wedge T = T$,

$(\forall x)P(x) \vee (\forall x)Q(x)$
$= (P(0) \wedge P(1)) \vee (Q(0) \wedge Q(1))$
$= F \vee F = F$, 即有 $T \to F$, 原式不是普遍有效.

2 (1). 不是普遍有效.
(2) 不是普遍有效.
(4) ~~普遍有效~~ 不是普遍有效.
(5) 普遍有效.
(7) 不是普遍有效.
(8). 不是普遍有效.

4. (1). $(\forall x)(P(x) \to (\exists y)Q(x,y))$

$= (\forall x)(\neg P(x) \vee (\exists y)Q(x,y))$

$= (\forall x)(\exists y)(\neg P(x) \vee Q(x,y))$

$=$

(2). $(\forall x)(\forall y)(\forall z)$

$(P(x,y,z) \wedge ((\exists u)Q(x,u) \to (\exists w)Q(y,w))).$

$= (\forall x)(\forall y)(\forall z)$

$(\neg(P(x,y,z) \wedge (\exists u)Q(x,u)) \vee (\exists w)Q(y,w))$

$=$

$(\forall x)(\forall y)(\forall z)$

$((\neg P(x,y,z) \vee \neg(\exists u)Q(x,u)) \vee (\exists w)Q(y,w)).$

$= (\forall x)(\forall y)(\forall z)((\neg P(x,y,z) \vee (\forall u)\neg Q(x,u)) \vee (\exists w)Q(y,w)).$

$= (\forall x)(\forall y)(\forall z)$

$((\forall u)(\neg P(x,y,z) \vee \neg Q(x,u)) \vee (\exists w)Q(y,w))$

$= (\forall x)(\forall y)(\forall z)(\forall u)(\exists w).$

$(\neg P(x,y,z) \vee \neg Q(x,u) \vee Q(y,w))$

$= (\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge$

$(\neg(\exists w)Q(x,u) \vee (\exists w)Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge$

$((\forall w)\neg Q(x,u) \vee (\exists w)Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(P(x,y,z) \wedge$

$(\forall u)(\exists w)(\neg Q(x,u) \vee Q(y,w)))$

$= (\forall x)(\forall y)(\forall z)(\forall u)(\exists w)$

$(P(x,y,z) \wedge (\neg Q(x,u) \vee Q(y,w)))$

(3). $(\exists x)P(x,y) \leftrightarrow (\forall z)(Q(z))$

$= ((\exists x)P(x,y) \wedge (\forall z)Q(z))$

$\vee(\neg(\exists x)P(x,y) \wedge \neg(\forall z)Q(z))$

$= (\exists x)(\forall z)$

$=$

$=$

$\neg((\exists x)P(x,y) \wedge (\forall z)Q(z))$

$\wedge((\exists x)P(x,y) \vee (\forall z)Q(z)).$

(3). $(\exists x) P(x,y) \leftrightarrow (\forall z) Q(z)$

$= \{\neg(\exists x) P(x,y) \vee (\forall z) Q(z)\}$
$\wedge \{(\exists x) P(x,y) \vee \neg(\forall z) Q(z)\}.$

$= \{(\forall x) \neg P(x,y) \vee (\forall z) Q(z)\}$
$\wedge \{(\exists u) P(u,y) \vee (\exists v) \neg Q(v)\}$

$= (\forall x)(\forall z)(\exists u)(\exists v)$
$\{(\neg P(x,y) \vee Q(z)) \wedge (P(u,y) \vee \neg Q(v))\}$

(4). $(\neg(\exists x) P(x) \vee (\forall y) Q(y)) \to (\forall z) R(z).$

$= \neg(\neg(\exists x) P(x) \vee (\forall y) Q(y)) \vee (\forall z) R(z).$

$= ((\exists x) P(x) \wedge (\exists y) \neg Q(y)) \vee (\forall z) R(z)$

$= (\exists x)(\exists y)(\forall z)((P(x) \wedge \neg Q(y)) \vee R(z))$

(5). $(\forall x)\{P(x) \to (\forall y)[[P(y) \to (Q(x) \to Q(y))]] \vee (\forall z) P(z)\}$

$= (\forall x)\{\neg P(x) \vee [(\forall y)(\neg P(y) \vee [\neg Q(x) \vee Q(y)])] \vee (\forall z) P(z)\}$

$= \cancel{(\forall x)(\forall y)(\forall z)(\neg P(x) \vee \neg Q(x) \vee Q(y) \vee R(z))}$

$= (\forall x)(\forall y)(\forall z)$
$(\neg P(x) \vee \neg P(y) \vee \neg Q(x) \vee Q(y) \vee P(z))$

~~const. pointer~~

(9). $(\forall x)\{P(x) \to (\exists y) Q(x,y)\} \vee (\forall z) R(z).$

$= (\forall x)(\neg P(x) \vee (\exists y) Q(x,y)) \vee (\forall z) R(z)$

$= (\forall x)(\exists y)(\forall z)(\neg P(x) \vee Q(x,y) \vee R(z))$

Skolem:

$(\forall x)(\forall z)(\neg P(x) \vee Q(x,f(x)) \vee R(z))$

(10):

$(\exists y)(\forall x)(\forall z)(\exists u)(\forall v)$
$P(x,y,z,u,v).$

$=$

$(\forall x)(\forall z)(\forall v) P(x,a,z,f(x,z),v).$