Calculus A2: Homework 12

May 17, 2022

p.870

1.

$f(x,y) = xe^y$. $f'_x = e^y, f''_{xx} = 0$, $f'_y = f''_{yy} = xe^y$, $f''_{xy} = e^y$.
At $(0,0)$, $f'_x = f''_{xy} = 1$, and all other derivatives at origin is zero.
Therefore, quadratic approximation is (for a small positive value of δ)

$$f(x,y) = x + xy, (0 \leq \sqrt{x^2+y^2} < \delta)$$

p.886

1.

$$\int_1^2 \int_0^4 2xydydx = 2\int_1^2 xdx \int_0^3 ydy = \frac{1}{2}(4-1)(16-0) = 24$$

5.

$$\int_0^3 \int_0^2 (4-y^2)dydx = \int_0^3 dx \int_0^2 (4-y^2)dy = 3\left(4y - \frac{1}{3}y^3\right)\Big|_0^2 = 16$$

25.

$$\iint\limits_{[-1,1]^2} (x^2+y^2)dydx = 2\int_{-1}^1 x^2dx \int_{-1}^1 dy = 2 \cdot \frac{2}{3} \cdot 2 = \frac{8}{3}$$

p.894

9.

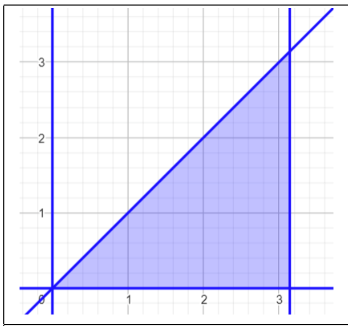
$$\text{(a) } R = \{(x,y) : 0 \leq x \leq 2 \wedge x^3 \leq y \leq 8\}, \text{ (b) } R = \{(x,y) : 0 \leq y \leq 8 \wedge 0 \leq x \leq \sqrt[3]{x}\}$$

15.

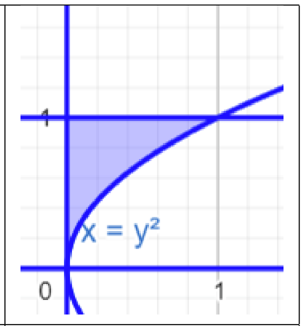
$$\text{(a) } R = \{(x,y) : 0 \leq x \leq \ln 3 \wedge e^{-x} \leq y \leq 1\}, \text{ (b) } R = \{(x,y) : 1/3 \leq y \leq 1 \wedge -\ln x \leq x \leq \ln 3\}$$

19.

$$\int_0^\pi \int_0^x x\sin ydydx = \int_0^\pi x \cdot (-\cos y)|_0^x dx = \int_0^\pi x(1-\cos x)dx$$
$$= \frac{\pi^2}{2} + \int_0^\pi \sin xdx - x\sin x|_0^\pi = 2 + \frac{\pi^2}{2}$$



Domain for 19



Domain for 23

23.

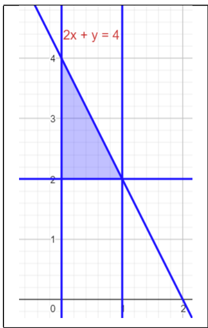
$$\int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy = \int_0^1 3y^2((e^y)^x)_0^{y^2} dy = \int_0^1 3y^2(e^{y^3} - 1)dy = (e^{y^3} - y^3)|_0^1 = e - 2$$

33.

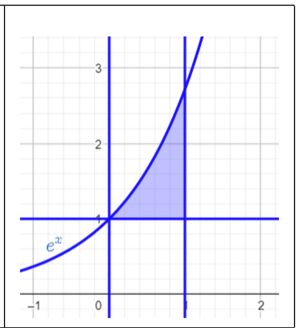
$$D = \{(x,y) : 0 \le y \le 4 - 2x \wedge 0 \le x \le 1\} = \{(x,y) : 0 \le x \le \frac{4-y}{2} \wedge 2 \le y \le 4\}$$

So

$$\int_0^1 \int_2^{4-2x} dy dx = \int_2^4 \int_0^{(4-y)/2} dx dy$$



Domain for 33



Domain for 37

37.

$$D = \{(x,y) : 1 \le y \le e^x \wedge 0 \le x \le 1\} = \{(x,y) : \ln y \le x \le 1 \wedge 1 \le y \le e\}$$

So

$$\int_0^1 \int_1^{e^x} dy dx = \int_1^e \int_{\ln y}^1 dx dy$$

57.

Let $D = \{(x,y) : x \ge 0 \wedge y \ge 0 \wedge x + y \le 2\}$.

$$\iint_D (x^2 + y^2) dy dx = \int_0^1 \int_x^{2-x} (x^2 + y^2) dy dx = \int_0^1 (x^2(2-x) + \frac{1}{3}(2-x)^3 - \frac{4}{3}x^3) dx$$

$$= \left(\frac{2}{3}x^3 - \frac{7}{12}x^4 - \frac{1}{12}(2-x)^4\right)_0^1 = \frac{4}{3}$$

69.

$$\int_1^{+\infty} \int_{e^{-x}}^1 \frac{1}{x^3 y} dy dx = \int_1^{+\infty} \frac{\ln y|_{e^{-x}}^1}{x^3} dx = \int_1^{+\infty} \frac{dx}{x^2}$$

converges, because

$$\lim_{\ell \to +\infty} \int_1^{\ell} \left| \frac{dx}{x^2} \right| < \lim_{\ell \to +\infty} \sum_{n=1}^{\ell} \frac{1}{n^2} < 3$$

Therefore,

$$\int_{1}^{+\infty}\int_{e^{-x}}^{1}\frac{1}{x^3y}dydx = \int_{1}^{+\infty}\frac{dx}{x^2} = -\frac{1}{x}\bigg|_{1}^{+\infty} = 1$$

p.899

1.

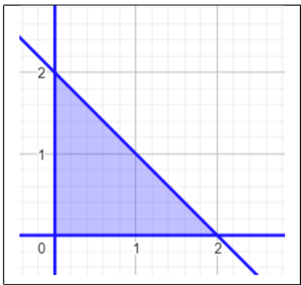
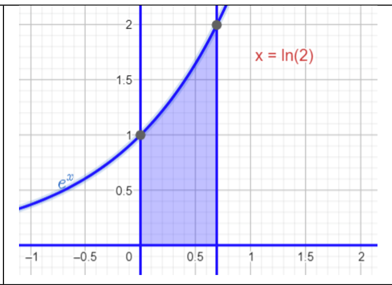
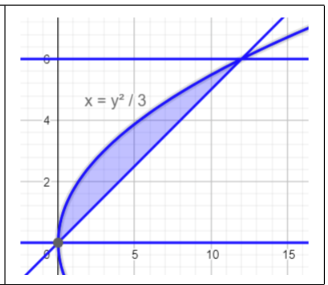
$$D = \{(x,y) : x \geq 0 \wedge y \geq 0 \wedge x + y \leq 2\}$$

$$\iint\limits_{D} dS = \int_{0}^{2}\int_{0}^{2-x} dydx = \int_{0}^{2}(2-x)dx = 2(2) - \frac{1}{2}(4) = 2$$

5.

$$D = \{(x,y) : 0 \leq y \leq e^x \wedge 0 \leq x \leq \ln 2\}$$

$$\iint\limits_{D} dS = \int_{0}^{\ln 2}\int_{0}^{e^x} dydx = \int_{0}^{\ln 2} e^x dx = 2 - 1 = 1$$

		
Domain for 1:	Domain for 5:	Domain for 13

13.

The vertices of the region are $(0,0)$ and $(12,6)$.

$$\int_{0}^{6}\int_{y^2/3}^{2y} dxdy = \int_{0}^{6}(2y - y^2/3)dy = 6^2 - 6^3/9 = 12$$

19.

(a)

$$\text{Average} = \frac{1}{\pi^2}\iint\limits_{[0,\pi]^2}\sin(x+y)dS = \frac{1}{\pi^2}\int_{0}^{\pi}(\cos x - \cos(x+\pi))dx = \frac{1}{\pi^2}\int_{0}^{\pi}2\cos x dx = 0$$

(b)

$$\text{Average} = \frac{2}{\pi^2}\iint\limits_{[0,\pi]\times[0,\pi/2]}\sin(x+y)dS = \frac{2}{\pi^2}\int_{0}^{\pi}(\cos x - \cos(x+\pi/2))dx = \frac{2}{\pi^2}\int_{0}^{\pi}(\cos x + \sin x)dx = \frac{4}{\pi^2}$$

p.904

1.

$$D = \{(r,\varphi) : 0 \leq r \leq 9 \wedge \pi/2 \leq \varphi \leq 2\pi\}$$

3.

$$D = \{(r,\varphi) : 0 \leq r \leq \csc\varphi \wedge \pi/4 \leq \varphi \leq 3\pi/4\}$$

11.

Analysis: By given boundaries, it can be easily seen that ∂D is the circle $\rho = 2$ on the first quadrant and the two axes, so

$$\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)dxdy = \int_0^{\frac{\pi}{2}} \int_0^2 \rho^2 \cdot \rho d\rho d\varphi = 2\pi$$

19.

Similar to 11(here is $\rho = \ln 2$),

$$\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{s^2+y^2}} dxdy = \int_0^{\frac{\pi}{2}} \int_0^{\ln 2} e^{\rho} \rho d\rho d\varphi = \frac{\pi}{2} \int_0^{\ln 2} \rho e^{\rho} d\rho = \frac{\pi}{2} \left(2\ln 2 - \int_0^{\ln 2} e^{\rho} d\rho \right) = \pi \ln 2 - \frac{\pi}{2}$$

27.

$$\text{Area} = \frac{1}{2} \int_0^{\frac{\pi}{2}} (2\sqrt{(2-\sin 2\theta)})^2 d\theta = 2\left(2\theta + \frac{1}{2}\cos(2\theta)\right)\bigg|_0^{\frac{\pi}{2}} = 2(\pi - 2)$$

33.

Let D be a disk with radius a, then average height

$$= \frac{1}{\pi a^2} \iint\limits_D \sqrt{a^2-x^2-y^2} dydx = \frac{1}{\pi a^2} \int_0^{2\pi} d\theta \int_0^a \sqrt{a^2-r^2} r dr = \frac{2}{a^2} \frac{1}{3} (a^2)^{(3/2)} = \frac{2a}{3}$$