考试课程 Calculus (A2) 2022年4月 （**A** 卷）

系_______ 班_______ 姓名______________ 学号______________

Note: The $\mathbf{T}, \mathbf{N}, \mathbf{B}$ represent the Frenet frame, and 's' represents the arc length parameter in the related questions.

1. Fill in the following blanks (每空4分，共40分，直接填在试卷的横线上)

(1) Find a unit vector that has the same direction as the vector $< -4, 2, 4 >$: ____________

(2) Vectors $\mathbf{a} =< 4, 1, \frac{1}{4} >, \mathbf{b} =< 6, -3, -8 >$, then $\mathbf{a} \cdot \mathbf{b} =$ ____________.

(3) Vectors $\mathbf{a} =< -1, 2, 5 >, \mathbf{b} =< 3, 4, -1 >$, the angle between them is ____________.

(4) Vectors $\mathbf{a} =< 1, 3, -2 >, \mathbf{b} =< -1, 0, 5 >$. then $\mathbf{a} \times \mathbf{b} =$____________.

(5) Find the equation of the plane through the origin and perpendicular to the vector $< 1, -2, 5 >$, the equation of the plane is __________________.

(6) Given a line $x = 3 - t, y = 2 + t, z = 5t$; and a plane $x - y + 2z = 9$, find the point at which the line intersects the plane, the coordinate of the point is ____________.

(7) The vector function $\mathbf{r}(t) =< t, 1, 2\sqrt{t} >$, then its derivative $\mathbf{r}'(t) =$ ____________.

(8) The distance between $P_1(-1, 1, 5)$ and $P_2(2, 5, 0)$ is ____________.

(9) Given two points $P_1(1, 4, 5), P_2(4, -2, 7)$, the midpoint of line segment P_1P_2 is ____________.

(10) Vector function $\mathbf{r}(t) =< t, -t^3, 3t^5 >$, then the integral $\int_0^2 \mathbf{r}(t)dt =$ ____________.

(continued at next page)

2. Answer the following questions in detail（共60分）

(11) (18分) Ideal projectile motion equation is

$$\mathbf{r}(t) = (v_0 \cos\alpha)t\ \mathbf{i} + \left((v_0 \sin\alpha)t - \frac{1}{2}gt^2\right)\mathbf{j},$$

with initial speed v_0 and launch angle α, g is the acceleration due to gravity. please derive (i) the maximum height $y_{\max}$, (ii) the total flight time $t_{\max}$, (iii) the range R (the distance from the origin to the point of impact).

(12) (24分) For the space curve $\mathbf{r}(t) = (6\sin 2t)\mathbf{i} + (6\cos 2t)\mathbf{j} + 5t\mathbf{k}$, please find $\{\mathbf{T}, \mathbf{N}, \mathbf{B}$, and κ (curvature)$\}$.

(13) (12分) (i) For a function $f(x,y)$ and a real number L, please give the definition of

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

in ϵ and δ description.

(ii) Then find

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2+y^4},$$

and prove your result.

(14) (6分) Suppose that a function $f(x,y)$ is continuous on xy-plane ($\mathbb{R}^2$), $f(x,y) > 0$ when $x^2+y^2 \neq 0$, and

$$f(cx,cy) = c^2 f(x,y), \quad \forall c > 0, \forall (x,y) \in \mathbb{R}^2,$$

prove that $\exists a$ and b ($0 < a \leq b$), such that

$$a(x^2+y^2) \leq f(x,y) \leq b(x^2+y^2), \quad \forall (x,y) \in \mathbb{R}^2.$$