No.

Date 2021. 11. 18.

⑨.

(3 (3,4) 选 -.

15, 17, 18, 19, 26, 28

(30-33) 选.

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13 全集为 $\mathbb{N}$.

$A=\{1,2,7,8\}$.

$B=\{x \mid x^2<50\}$

$D=\{x \mid x=2^k \wedge k\in\mathbb{N} \wedge 0\le k\le 5\}$

(4). $(B-A)\cup D$

$=\{0,3,4,5,6\}\cup\{1,2,4,8,16,32\}$.

$=\{0,2,3,4,5,6,8,16,32\}$.

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15. $P(\emptyset)=\{\emptyset\}$

$PP(\emptyset)=\{\emptyset,\{\emptyset\}\}$.

$PPP(\emptyset)=\{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$.

(1). $\cup\{PPP(\emptyset), PP(\emptyset), P(\emptyset), \emptyset\}$.

$=PPP(\emptyset)$.

$=\{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$.

(2) $\cap\{PPP(\emptyset), PP(\emptyset), P(\emptyset)\}=P(\emptyset)$.

$=\{\emptyset\}$.

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17. (1). $(A-B)-C=A-(B\cup C)$

证:

$(A-B)-C$

$=(A\cap -B)\cap -C)$

$=A\cap(-B\cap -C)$

$=A\cap -(B\cup C)$

$=A-(B\cup C)$ ☒

(2). $(A-B)-C=(A-C)-(B-C)$.

证:

$(A-C)-(B-C)$

$=(A-C)\cap -(B\cap -C)$.

$=(A-C)\cap(-B\cup C)$.

$=((A-C)\cap -B)\cup((A-C)\cap C)$.

$=((A\cap -C)\cap -B)\cup((A\cap -C)\cap C)$

$=((A\cap -B)\cap -C)\cup\emptyset$

$=(A-B)-C$ ☒

(3). $A=B \Leftrightarrow A\oplus B=\emptyset$.

证:

$A\oplus B=\emptyset$.

$\Leftrightarrow \neg(\exists x)(x\in((A-B)\cup(B-A)))$.

$\Leftrightarrow (\forall x)(x\in -((A-B)\cup(B-A)))$

$\Leftrightarrow (\forall x)(x\in(-(A-B)\cap -(B-A)))$

$\Leftrightarrow (\forall x)(x\in(-(A\cap -B)\cap -(B\cap -A)))$

$\Leftrightarrow (\forall x)(x\in((-A\cup B)\cap(-B\cup A)))$

$\Leftrightarrow (\forall x)(x \in (-A \cup B)) \wedge (\forall x)(x \in (-B \cup A))$.

$\Leftrightarrow (\forall x)(x \in -A \vee x \in B) \wedge (\forall x)(x \in -B \vee x \in A)$.

$\Leftrightarrow (\forall x)(x \in A \to x \in B) \wedge (\forall x)(x \in B \to x \in A)$.

$\Leftrightarrow A \subseteq B \wedge B \subseteq A$

$\Leftrightarrow A = B$. ☒

(4). $A \subseteq C \wedge B \subseteq C \Leftrightarrow A \cup B \subseteq C$.

证:

$A \subseteq C \wedge B \subseteq C$

$\Leftrightarrow (\forall x)(x \in A \to x \in C) \wedge (\forall x)(x \in B \to x \in C)$.

$\Leftrightarrow (\forall x)[(x \in A \to x \in C) \wedge (x \in B \to x \in C)]$

$\Leftrightarrow (\forall x)[(\neg(x \in A) \vee x \in C) \wedge (\neg(x \in B) \vee x \in C)]$.

$\Leftrightarrow (\forall x)[(\neg(x \in A) \wedge \neg(x \in B)) \vee x \in C]$

$\Leftrightarrow (\forall x)[\neg(x \in A \vee x \in B) \vee x \in C]$.

$\Leftrightarrow (\forall x)[(x \in A \vee x \in B) \to x \in C]$.

$\Leftrightarrow (\forall x)[x \in (A \cup B) \to x \in C]$

$\Leftrightarrow A \cup B \subseteq C$. ☒

(5). $C \subseteq A \wedge C \subseteq B \Leftrightarrow C \subseteq A \cap B$.

证:

$C \subseteq A \cap B$

$\Leftrightarrow (\forall x)(x \in C \to x \in (A \cap B))$

$\Leftrightarrow (\forall x)(x \in C \to (x \in A \wedge x \in B))$

$\Leftrightarrow (\forall x)[(x \in C \to x \in A) \wedge (x \in C \to x \in B)]$

$\Leftrightarrow [(\forall x)(x \in C \to x \in A)] \wedge [(\forall x)(x \in C \to x \in B)]$

$\Leftrightarrow C \subseteq A \wedge C \subseteq B$. ☒.

(6). $A \cap B = \emptyset \Leftrightarrow A \subseteq -B \Leftrightarrow B \subseteq -A$

证:

$A \cap B = \emptyset$

$\Leftrightarrow \neg(\exists x)(x \in A \wedge x \in B)$

$\Leftrightarrow (\forall x)[\neg(x \in A \wedge x \in B)]$ ①.

$\Leftrightarrow (\forall x)[\neg(x \in A) \vee x \in -B]$

$\Leftrightarrow (\forall x)(x \in A \to x \in -B)$.

$\Leftrightarrow A \subseteq -B$.

与此同时.

① $\Leftrightarrow (\forall x)[x \in -A \vee \neg(x \in B)]$

$\Leftrightarrow (\forall x)[x \in B \to x \in -A)$

$\Leftrightarrow B \subseteq -A$ ☒.

18 (1). $A-B=B$.

$\Rightarrow A-B\subseteq B$

$B\subseteq A-B$.

$\Rightarrow B\subseteq\emptyset$

$\Rightarrow B=\emptyset$.

$\Rightarrow A=\emptyset$.

故 $A=B=\emptyset$.

(2) $A-B=B-A$.

$\Rightarrow (A-B)-(B-A)=\emptyset$.

$\Rightarrow$.

$\Leftrightarrow (\forall x)(\exists N\ x\in A$

$x\notin A \vee (x\in A \wedge x\notin (B\cap C))$

$\Rightarrow (\forall x)$

$((x\notin A \vee x\in A)\wedge(x\notin A \vee$

$A\cap B\subseteq A$

$A\cap -B\subseteq A$.

(#) $\Rightarrow A\cap -(B\cap C)\cap(B\cap C)$ *

$= A\cap(B\cap C)$. *

$\Leftrightarrow A\cap B\cap C = A\cap\emptyset=\emptyset$. *

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* 18(1). $A-B=B$

* $\Rightarrow A=B=\emptyset$.

* (2) $A-B=B-A$.

* $\Rightarrow A=B$.

* (3). $A\cap B=A\cup B$

* $\Rightarrow A=B$.

* (4). $A\oplus B=A\Rightarrow B=\emptyset$.

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* 19. $(A-B)\cup(A-C)=A$.

* $\Leftrightarrow A\cap(-B\cup -C)=A$.

* $\Leftrightarrow A\cap -(B\cap C)=A$.

* $\Leftrightarrow A-B\cap C=A$.

$\Rightarrow A\subseteq A-B\cap C$.

$\Leftrightarrow (\forall x)(x\in A\rightarrow$

$x\in(A\wedge x\notin B\cap C)$.

反之

$(A-B)\cup(A-C)=A\Rightarrow A\cap B\cap C=\emptyset$ * ①

另一方面,

$A\cap B\cap C=\emptyset$, *

$(\forall x)(x\in A\rightarrow x\notin(B\cap C))$

$\Leftrightarrow (\forall x)(x\in A\rightarrow x\in(-B\vee -C))$.

$\Leftrightarrow (\forall x)[x\in -A\vee(x\in -B\vee x\in -C)]$

$(\forall x)(x\in A)$

$\Rightarrow (\forall x)(x\in -(B\cap C))$

$\Leftrightarrow (\forall x)(x\in -B\vee x\in -C)$.

$\Rightarrow (\forall x)(x\in A-B\vee x\in A-C)$.

$\Rightarrow (\forall x)(x\in((A-B)\cup(A-C)))$ ②

故 $A\subseteq(A-B)\cup(A-C)$.

No

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亦-有

$x \in B \cap C$

$(\forall x)(x \in ((A-B) \cup (A-C)))$

$\Rightarrow (\forall x)(x \in A-B \vee x \in A-C)$

$\Rightarrow (\forall x)(x \in A)$.

故 $(A-B) \cup (A-C) \subseteq A$. ③

由 ②, ③ 知.

$A \cap B \cap C = \emptyset$

$\Rightarrow (A-B) \cup (A-C) = A$ ④

由 ① ④ 知.

$(A-B) \cup (A-C) = A$

$\Leftrightarrow A \cap B \cap C = \emptyset$.

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(2). $(A-B) \cup (A-C) = \emptyset$.

$\Leftrightarrow A-B = \emptyset \wedge A-C = \emptyset$

$\Leftrightarrow A \subseteq B \wedge A \subseteq C$.

$\Leftrightarrow A \subseteq (B \cap C)$.

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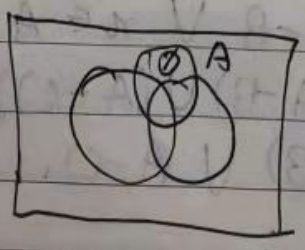
(3). $(A-B) \cap (A-C) = \emptyset$.



以下证 $A \subseteq (B \cup C)$ 为其充要条件.

$A \cap (A-B) \cap (A-C) = \emptyset$

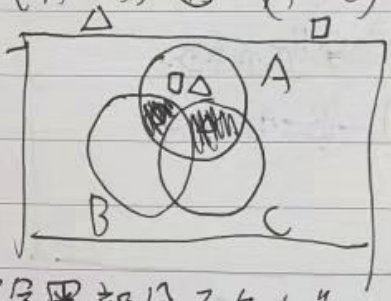
$\Leftrightarrow (A \cap -B) \cap (A \cap -C) = \emptyset$

$\Leftrightarrow A \cap -B \cap -C = \emptyset$

$\Leftrightarrow A \cap -(B \cup C) = \emptyset$.

$\Leftrightarrow A \subseteq B \cup C$.

(4). $(A-B) \oplus (A-C) = \emptyset$



"涂黑部分不存在".

$\Leftrightarrow ((A-B) \cup (A-C)) - ((A-B) \cap (A-C)) = \emptyset$.

$\Leftrightarrow (A \cap (-B \cup -C)) - (A \cap -B \cap -C) = \emptyset$

$\Leftrightarrow (A - (B \cap C)) - (A - (B \cup C)) = \emptyset$.

$\Leftrightarrow A - (B \cap C) \subseteq A - (B \cup C)$.

$\Leftrightarrow (A-B)-(A-C) \cup (A-C)-(A-B) = \emptyset$

$\Leftrightarrow (A-B)-(A-C) = \emptyset$ ①

$\wedge (A-C)-(A-B) = \emptyset$ ②

~~$A \cap \sim B \cap \sim(A \cap C)$~~

$= A \cap \sim(B \cup (A \cap \sim C))$

$\Leftrightarrow A-B \subseteq A-C \wedge A-C \subseteq A-B$

$\Leftrightarrow A-B = A-C.$

26. (1). $A \times B = \emptyset$.

$\Rightarrow |A \times B| = |\emptyset| = 0$

$\Rightarrow |A||B| = 0.$

$\Rightarrow A = \emptyset \vee B = \emptyset.$

(2). $A = A \times A.$

$\Rightarrow |A| = |A|^2$

$\Leftrightarrow |A| = 0 \vee |A| = 1.$

设 $A = \{a\}$

$A \times A = \{\langle a, a \rangle\}.$

$= \{\{a, \{a, a\}\}\}.$

$\{a, \{a, a\}\} \neq a,$

因此 $|A| = 1$ 与 $A = A \times A$ 矛盾,

故 $A = A \times A \Rightarrow A = \emptyset.$

另一方面 $A = \emptyset \Rightarrow A \times A = \emptyset \times \emptyset$

显然有 $= \emptyset = A.$

故 $A = A \times A \Leftrightarrow A = \emptyset.$

28. 全集为 $\mathbb{Z}_+$.

$A = \{x : 2 | x\}.$

$B = \{x : 3 | x\}$

$C = \{x : 5 | x\}.$

$|A| = \frac{250-2}{2} + 1 = 127$

$|B| = \frac{249-3}{3} + 1 = 83$

$|C| = \frac{250-5}{5} + 1 = 50.$

$|A \cap B| = |\{x : 6 | x\}|$

$= \frac{246-6}{6} + 1 = 41$

$|B \cap C| = \frac{240-15}{15} + 1 = 16$

$|C \cap A| = \frac{250-10}{10} + 1 = 25.$

$|A \cap B \cap C| = \frac{240-30}{30} + 1 = 8.$

原题等价求 $|A \cup B \cup C|$

$|A \cup B \cup C| = |A| + |B| + |C|$

$- |A \cap B| - |B \cap C| - |C \cap A|$

$+ |A \cap B \cap C|$

$= 127 + 83 + 50 - 41 - 16 - 25 + 8$

$= 184.$ #

32. 设 $P(x)$: x 为素数.

由子集公理.

$(\exists A)(\forall n)(n\in A \leftrightarrow n\in \mathbb{N}^* \wedge P(n))$.

故 $\exists A$,

$A=\{n\in\mathbb{N} \mid n$ 为素数$\}$

为素数之全体.

33. A^+ 为 A 的后继.

$A^+ = A\cup\{A\}$.

A 为传递集, 即

$(\forall x)(x\in A \to x\subseteq A)$.

而

$(\forall x)(x\in A^+)$

$\Rightarrow (\forall x)(x\in A \vee x\in\{A\})$

$\Rightarrow (\forall x)(x\subseteq A \vee x = A)$.

$\Rightarrow (\forall x)(x\subseteq A^+)$

故 $x\in A^+ \Rightarrow x\subseteq A^+$

A^+ 为传递集合. 证毕.