Calculus A Homework 1

30 Sept 2021

Assigned exercises:

1. Prove that is a Dedekind cut.

Definition:

A Dedekind cut T is a non-empty proper subset of such that:

Proof:

Let

Let

By definition, p<q.

1. Let be a map such that we have f(f(n))=f(n). Show that f is injective if and only if f is surjective.

Proof:

1. Assume f is injective, but f is not surjective.

Let

It is given that f(f(n))=f(n) so f(m)=m.

Means that

Thus m does not exist and , arising a contradiction to that f is not surjective.

1. Assume f is surjective, but f is not injective.

That contradicts the assumption of f that f is not a one-to-one function. Q.E.D.

1. Let be two Cauchy sequences of rationals. Show that is also a Cauchy sequence.

Proof:

are both Cauchy,

Also, are both bounded, so let M be the sum of the upper bounds of

Thus is also Cauchy.

Q.E.D.

4. Show that if is a Cauchy sequence of rationals, then is also a Cauchy sequence. Is the converse true (justify)?

Proof:

is Cauchy, so

For , , so is also Cauchy.

Q.E.D.

Assume is not. So the converse is false.

Bonus exercises:

1. Prove that is irrational.

Proof:

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Thus, 2|m and 2|n contradicts gcd (m,n)=1, hence is irrational. (Q.E.D.)