EFDC Technical Memorandum
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John Hamrick

Bed Thermal Model

Accurate simulation of temperature in deep lakes and reservoirs can be enhanced by properly accounting for the thermal mass of the underlying earth. This section describes the new EFDC bed thermal model. The model is based on the assumption that temperature is seasonally invariant at a specified elevation below the ground surface. This elevation of constant temperature and the corresponding temperature are user specified and can be viewed as calibration parameters.

The bed thermal model is based on solving the one-dimensional thermal diffusion or heat conduction equation

$$\partial_t \left(\rho_b c_{pb} T \right) - \partial_Z \left(k_b \partial_Z T \right) = 0 \tag{1}$$

in the bed, where ρ_b, c_{pb}, and k_b are the density, specific heat and thermal conductivity of the bed material and Z is a physical Cartesian vertical coordinate. Equation (1) is solved by discretization over vertical layers in the bed. The boundary conditions for (1) are

$$-k_b \partial_Z T = \rho_b c_{pb} J_{bw} \quad ; \quad Z = Z_b \tag{2}$$

$$T = T_{CT} \quad ; \quad Z = Z_{CT} \tag{3}$$

where Z_b is the elevation at the bed-water column interface, and Z_{CT} is the elevation at which the bed temperature can be assumed constant or seasonally invariant. The temperature flux from the bed to the water column is

$$J_{bw} = \omega \left(T_b - T_w \right) \tag{4}$$

where ω is a transfer coefficient, having units of velocity, to be subsequently defined and T_b is the temperature in the top layer of the bed and T_w is the temperature in the bottom layer of the water column.

Equation (1) is solved by introducing a dimensionless bed thermal coordinate

$$z = \frac{Z - Z_{CT}}{Z_b - Z_{CT}} = \frac{Z - Z_{CT}}{H_{CT}} \tag{5}$$

where H_{CT} is the bed thermal thickness or local distance for the water column-bed interface to the elevation at which the bed temperature is assumed to be locally constant. Introducing (5), equation (1) becomes

$$\partial_t T - \partial_z \left(\frac{k_b}{\rho_b c_{pb} H_{CT}^2} \partial_z T \right) = 0 \tag{6}$$

and the boundary conditions become

$$-\frac{k_b}{\rho_b c_{pb} H_{CT}} \partial_z T = J_{bw} \quad ; \quad z = 1 \tag{7}$$

$$T = T_{CT} \quad ; \quad z = 0 \tag{8}$$

The bed thermal diffusivity is defined by

$$D_b = \frac{k_b}{\rho_b c_{pb}} \tag{9}$$

with (6) and (7) becoming

$$\partial_t T - \partial_z \left(\frac{D_b}{H_{CT}^2} \partial_z T \right) = 0 \tag{10}$$

$$-\frac{D_b}{H_{CT}} \partial_z T = J_{bw} = \omega \left(T_b - T_w \right) \quad ; \quad z = 1 \tag{11}$$

The transfer velocity in (11) is determined by requiring that the heat flux be continuous at the bed-water interface

$$-\rho_b c_{pb} \frac{D_b}{H_{CT}} \partial_z T = -\rho_w c_{pw} \frac{D_w}{H} \partial_\sigma T \tag{12}$$

where D_w is the sum of the turbulent diffusion coefficient and the thermal diffusivity in the water column, H is the water column depth and σ is a dimensionless vertical coordinate in the water column. Discretizing (12) over the top half layer of the bed and the bottom half layer of the water column gives

$$J_{bw} = -\frac{2D_b}{\Delta_b H_{CT}} \left(T_{bw} - T_b \right) = -\left(\frac{\rho_w c_{pw}}{\rho_b c_{pb}} \right) \frac{2D_w}{\Delta_W H_W} \left(T_w - T_{bw} \right) \tag{13}$$

where Δ is the dimensionless thickness of the bed and water column layers. Solving (13) for the interface temperature gives

$$T_{bw} = \frac{\frac{rD_w}{\Delta_W H_W} T_w + \frac{D_b}{\Delta_b H_{CT}} T_b}{\frac{rD_w}{\Delta_W H_W} + \frac{D_b}{\Delta_b H_{CT}}} \tag{14}$$

where

$$r = \left(\frac{\rho_w c_{pw}}{\rho_b c_{pb}} \right) \tag{15}$$

Solving for the flux gives

$$J_{bw} = 2 \frac{\frac{rD_w}{\Delta_W H_W} \frac{D_b}{\Delta_b H_{CT}}}{\frac{rD_w}{\Delta_W H_W} + \frac{D_b}{\Delta_b H_{CT}}} \left(T_b - T_w \right) = \omega \left(T_b - T_w \right) \tag{16}$$

The water column diffusivity near the bed is given by

$$\frac{D_w}{\Delta_W H_W} = \frac{\kappa}{4} \sqrt{\tau_b} + \frac{D_{wm}}{\Delta_W H_W} = \frac{\kappa}{4} \sqrt{c_b} q + \frac{D_{wm}}{\Delta_W H_W} \tag{18}$$

where D_{wm} molecular thermal diffusivity of water, c_b is the bottom (log law based) resistance coefficient, κ the von Karman constant, and q is the velocity magnitude in the bottom layer of the water column. The transfer velocity is then

$$\omega = 2 \frac{r \left(\frac{\kappa}{4} \sqrt{c_b} q + \frac{D_w}{\Delta_W H_W} \right) \frac{D_b}{\Delta_b H_{CT}}}{r \left(\frac{\kappa}{4} \sqrt{c_b} q + \frac{D_w}{\Delta_W H_W} \right) + \frac{D_b}{\Delta_b H_{CT}}} \tag{19}$$

which reverts to the diffusive resistance in series form under quiescent conditions. For highly turbulent conditions in the water column, (19) simplifies to.

$$\omega = \frac{2D_b}{\Delta_b H_{CT}} \tag{20}$$

Indicating that heat transfer is controlled by the bed thermal transfer rate.

For application in EFDC, the bed thermal diffusivity is an input parameter. Estimates of the bed thermal diffusivity range from 5E-7 m^2/s for saturated sand to 15E-7 for bed rock. A lower bound for the bed thermal diffusivity is the water value of 1.4E-7, the value for water. The coefficient, $0.25\kappa\sqrt{c_b}$ is also input parameter. Although this coefficient could be internally computed, the EFDC model's turbulent closure scheme does not resolve convective turbulent processes at small scale near the bed, thus external specification provides more user control. Using a bottom resistance coefficient on the order of 0.0025 gives a value of 0.005.

Equation (10) is solved in EFDC by discretizing over *K* bed layers of uniform dimensionless thickness, using a fully implicit in time scheme

$$T^{new} - \frac{\Delta_t D_b}{\Delta_b H_{CT}^2}\left(\partial_z T^{new}\right)_{top} + \frac{\Delta_t D_b}{\Delta_b H_{CT}^2}\left(\partial_z T^{new}\right)_{bot} = T^{old} \tag{21}$$

The spatially discrete forms of (21) for the bottom layer, an intermediate layer, and the top layer are

$$\left(1 + 2\frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}\right)T_1^{new} - \frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}T_2^{new} = T_1^{old} + \frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}T_{TC} \tag{22}$$

$$-\frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}T_{k-1}^{new} + \left(1 + 2\frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}\right)T_k^{new} - \frac{\Delta_t D_b}{\Delta_b^2 H_{CT}^2}T_{k+1}^{new} = T_k^{old} \tag{23}$$

$$-\frac{\Delta_t D_b}{\Delta_b H_{CT}^2}T_{K-1}^{new} + \left(1 + \frac{\Delta_t D_b}{\Delta_b H_{CT}^2} + \frac{\Delta_t \omega}{\Delta_b H_{CT}}\right)T_K^{new} - \frac{\Delta_t \omega}{\Delta_b H_{CT}}T_{w1}^{new} = T_K^{old} \tag{24}$$

Solution of the system, (22-24) also requires an equation for the bottom layer of the water column

$$H\Delta_w \partial_t T_{w1} = -\frac{2D_w}{\Delta_W H}\left(T_{w1} - T_{bw}\right) \tag{25}$$

which represents a fractional step accounting for bed-water column interaction subsequent to updating the water column temperature to account for vertical diffusion and thermal sources and sinks. Introducing the temperature flux, defined by (11) and (13) gives

$$r\partial_t T_{w1} = \frac{\omega}{H\Delta_w}\left(T_b - T_w\right) \tag{26}$$

or the discrete form

$$-\frac{\Delta_t \omega}{\Delta_w H} T_K^{new} + \left(r + \frac{\Delta_t \omega}{\Delta_w H} \right) T_{w1}^{new} = r T_{w1}^{old} \tag{27}$$

Equation (27) is made symmetric with respect to (24) by recasting it into the form

$$-\frac{\Delta_t \omega}{\Delta_b H_{CT}} T_K^{new} + \left(r \frac{\Delta_w H}{\Delta_b H_{CT}} + \frac{\Delta_t \omega}{\Delta_b H_{CT}} \right) T_{w1}^{new} = r \frac{\Delta_w H}{\Delta_b H_{CT}} T_{w1}^{old} \tag{28}$$

Equations (22-24) and (28) represent a system of K+1 tri-diagonal linear equations which are solved using a similar procedure employed for vertical diffusion in the water column.

Selection of the depth to constant temperature depth and the corresponding temperature are subject to initial approximation and subsequent calibration. The EFDC input parameter DABEDT defines the downward distance to the constant temperature surface relative to the deepest cell in the water body or minimum bottom elevation. Thus

$$H_{CT} = DABEDT + Z_b - Z_{bmin} \tag{29}$$

where Z_b corresponds to the bed elevation variable BELV in EFDC. Thus at the deepest point in the water body the constant temperature surface is *DABEDT* below the bottom. At shallower locations in the water body, H_{CT} will be larger. The bed thermal domain is flat bottomed but follows the bottom elevation profile of the water body. At present, horizontal heat transfer is neglected. Using a bed thermal diffusivity of 5E-7 m^2/s and a horizontal grid spacing of 100 meters, the time scale for horizontal heat transfer is 2E10 seconds or approximately 600 years. Thus it would be anticipated that any significant horizontal heat transfer under a water body would be associated with groundwater flow rather than pure conduction. Limited data in the literature suggest that constant temperatures are reached at 10 to 20 meters below ground surface. Given the implementation defined by (24), *DABEDT* could have an order of magnitude range from 1 to 10 meters. The formulation allows for T_{TC} to vary spatially or be set to a uniform value over the entire model domain. An approximate estimate of the constant temperature would be the mean of the minimum and maximum daily averaged ground surface temperature. If annual variations in the near bottom water temperature are available, a similar estimate based of minimum and maximum water temperature would be appropriate.

The EFDC bed thermal model is activated in the efdc.inp file when the temperature transport option in data set C6, ISTOPT(2) is set to 1. The number of layers in the bed plus one (K+1), is set by the variable KBH in data set C46, and should be less than or equal to the array dimension parameter KBHM in efdc.par. The initial bed temperature profile is very important in determining the accuracy of temperature simulations. For cold starts (ISRESTI = 0, data set C2), two options, set by the input variable ISBEDTEMI in Card image C46, are available to initialize the bed temperature profile. When ISBEDTEMI is step to 0, the initial temperature is defined by the input file tempb.inp. When ISBEDTEMI is set to 1, the initial temperature profile in the bed is internally set using the initial temperature in the bottom layer of the water column and the specified constant temperature at the defined depth of constant temperature. For hot starts, the initial bed temperature profile is defined in the restart file, temb.rst. Alternately, header lines can be added to tempb.rst to create a tempb.inp for subsequent cold start simulations. Additional input information for the bed thermal model are defined in the aser.inp and summarized in Table 1. It is noted that that a negative value of TCT in aser.inp indicated spatially varying constant in time temperature is read from file tempbbot.inp. Formats for tempb.inp and tempbbot.inp are shown in Table 2.

Table 1. Bed Thermal Model Input Parameters in Aser.inp

EFDC Input Parameter in ASER.INP	Variable	Range and Dimensions
DABEDT	See Equation (29)	1 to 10 meters
TBEDIT	T_{CT} (T_{CT}<0, read input file tempbbot.inp)	10 – 20^o C
HTBED1	$0.25\kappa\sqrt{c_b}$	0.001 to 0.005 (dimensionless)
HTBED2	D_b	1E-7 to 15E-7 m^2/s

Table 2. Format for Tempb.inp (k=1,KBT) and Tempbbot.inp

```
c file tempb.inp or tempbbot.inp (4 header lines)
c 1 st data line  istype  0 or 1
c next L=2,LA data lines, T(L,K=1,KBT) or L,I,J, T(L,K=1,KBT)
c
   0    !example for ISTYP=0
   T(2,K=1,KBM) or T(2)
                 :
   T(LA,K=1,KBM) or T(LA)

Or

   1    !example for ISTYP=1
   2   I   J     T(2,K=1,KBM) or T(2)
   :   :   :           :          :
   LA  I   J     T(LA,K=1,KBM) or T(LA)
```

Working Notes

Solution scheme as implemented in code

$$\left(\frac{1}{\Delta_t}+2\frac{D_b}{\Delta_b^2 H_{CT}^2}\right)T_1^{new}-\frac{D_b}{\Delta_b^2 H_{CT}^2}T_2^{new}=\frac{1}{\Delta_t}T_1^{old}+\frac{D_b}{\Delta_b^2 H_{CT}^2}T_{TC} \tag{18}$$

For an intermediate layer

$$-\frac{D_b}{\Delta_b^2 H_{CT}^2}T_{k-1}^{new}+\left(\frac{1}{\Delta_t}+2\frac{D_b}{\Delta_b^2 H_{CT}^2}\right)T_k^{new}-\frac{D_b}{\Delta_b^2 H_{CT}^2}T_{k+1}^{new}=\frac{1}{\Delta_t}T_k^{old} \tag{19}$$

For the top layer

$$-\frac{D_b}{\Delta_b H_{CT}^2}T_{K-1}^{new}+\left(\frac{1}{\Delta_t}+\frac{D_b}{\Delta_b H_{CT}^2}+\frac{\omega}{\Delta_b H_{CT}}\right)T_K^{new}-\frac{\omega}{\Delta_b H_{CT}}T_{w1}^{new}=\frac{1}{\Delta_t}T_K^{old} \tag{20}$$

which can be made symmetric with respect to (19) by recasting it into the form

$$-\frac{\omega}{\Delta_b H_{CT}}T_K^{new}+\left(\frac{r}{\Delta_t}\frac{\Delta_w H_w}{\Delta_b H_{CT}}+\frac{\omega}{\Delta_b H_{CT}}\right)T_w^{new}=\frac{r}{\Delta_t}\frac{\Delta_w H_w}{\Delta_b H_{CT}}T_w^{old} \tag{23}$$

$$\omega=2\frac{r\left(\frac{\kappa}{4}\sqrt{c_b}\,q+\frac{D_w}{\Delta_W H_W}\right)\frac{D_b}{\Delta_b H_{CT}}}{r\left(\frac{\kappa}{4}\sqrt{c_b}\,q+\frac{D_w}{\Delta_W H_W}\right)+\frac{D_b}{\Delta_b H_{CT}}} \tag{15}$$