Journal of VLSI Signal Processing 25, 101–105, 2000

The Birth of CORDIC

JACK E. VOLDER

Abstract. The very earliest history of the CORDIC computing technique—a highly efficient method to compute elementary functions—is presented. The CORDIC technique was born out of necessity, the incentive being the replacement of the analog navigation computer of the B-58 aircraft by a high accuracy, high-performance digital computer. The revolutionary development of the CORDIC technique is presented, along with details of the very first implementations: the CORDIC I prototype and the CORDIC II airborne digital navigation computer.

In early 1956 the aeroelectronics department of Convair, Fort Worth, was given the task of determining the feasibility of replacing the analog computer-driven navigation system of the B-58 bomber (see Fig. 1) with a digital computer. This replacement effort was deemed necessary because of the limited accuracy of analog computing elements.

At that time, digitalization of the B-58 navigation system was considered a formidable task without any assurance of worthwhile results. Transistors were new and limited to a 250 KHz logic rate. The main challenge was the real-time calculation of the complicated navigation equations required for determining present position on a spherical earth.

By then, digital differential analyzers had been developed that were capable of efficiently solving continuous navigation problems, but they could not produce solutions in real-time during flights near the North Pole. Also, they were too slow in providing solutions for the discontinuous problems of fix-taking from either star-tracking or radar ground sightings. Therefore, an entire-word transfer type of computation was definitely necessary. However, the existing trigonometric algorithms necessary for navigation were too time-consuming for the real-time requirements of the B-58.

Most navigation system specialists agreed that the existing B-58 navigation computer was an ingenuous device utilizing analog resolvers to compute, in real time, the complex trigonometric relationships necessary for navigation over a spherical earth. Each resolver was capable of performing either a rotation of input coordinates or inversely determining the magnitude and angle of the vector defined by the input coordinates—also called vectoring—as shown in the diagram of Fig. 2.

Trying to solve navigation problems without resolver capabilities is an extremely complicated problem. Resolver capability allows solution flow diagrams to be drawn with interconnected resolvers as was shown in the original CORDIC paper [1]. For example, in great circle navigation, the solution of course angle and distance to destination requires a network of only 5 interconnected resolvers.

1. Digitalization of the Analog Resolver

At that time there were no known digital operators equivalent to the analog resolver. Digital computation of navigation problems required either series expansions, approximations or table look-up of individual trigonometric functions.

At Convair, the first brain-storming effort toward digitalization was directed toward either encoding the sine and cosine functions or some intermediate function on optical encoders. This effort was soon abandoned. Afterwards, I began looking for some clue to solving this problem in the trigonometric equations tabulated in my 1946 edition of the Handbook of Chemistry and Physics. This led to the massaging of the basic angle addition equations to obtain the following interesting equations. If $\tan(\phi) = 2^{-n}$, then:

$$K_n R \sin(\theta \pm \phi) = R \sin(\theta) \pm 2^{-n} R \cos(\theta) \quad (1)$$

$$K_n R \cos(\theta \pm \phi) = R \cos(\theta) \mp 2^{-n} R \sin(\theta)\,, \quad (2)$$

where $K_n = \sqrt{1 + 2^{-2n}}$.

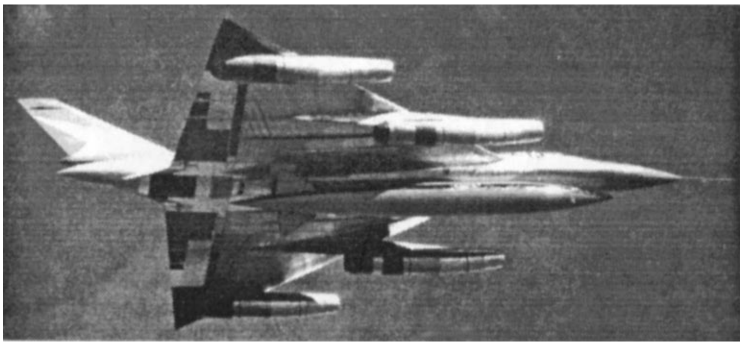


Figure 1. The B-58 supersonic bomber in flight. (Reprinted with kind permission of General Dynamics.)

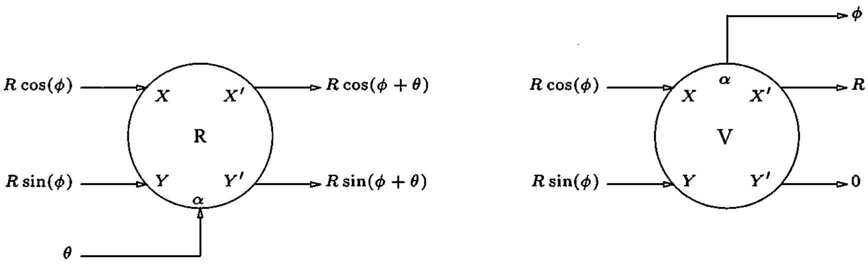


Figure 2. Two analog resolver modes: rotation and vectoring.

The above equations can be used to implement the rotation of the vector R (as defined by its orthogonal coordinates) through either a positive or negative angle equal in magnitude to $\tan(2^{-n})$, therefore why couldn't a sequence of rotations with increasing powers of n be used to rotate R through any desired angle?

EUREKA!... Based on these two equations a new algorithm that was the digital equivalent of an analog resolver was developed. The resulting special purpose computer design for efficiently computing this special algorithm was "officially" designated as the COordinate Rotation DIgital Computer and referred to by its acronym: CORDIC. Without this special algorithm, the task of digitizing the B-58 navigation system would probably have been abandoned. In summary, it can be said that the discovery of the CORDIC computing technique obeyed the popular axiom;

Necessity is the mother of invention.

The CORDIC algorithm and a computer design for implementing this algorithm were presented to Convair management in the form of an internal technical report [6] in June 1956. In the process of preparing the report, I realized that this same computer design could be easily modified to compute hyperbolic coordinate rotation, logarithms and exponential functions. These capabilities were also described in this technical report. By this time it was obvious that the same design could easily perform multiplication and division. Dan Daggett, a co-worker, also developed algorithms on the same CORDIC basis to convert between (internal) binary and (external) binary-coded decimal (BCD) number representation.

Unfortunately, the initial acceptance of the CORDIC algorithm was slow, partly because of doubting peers and partly because of a new interest by the Air Force toward development of atomic radiation resistant computer elements.

2. Building the CORDIC I Prototype

Finally, in late 1958, permission was obtained from both Convair and the Air Force to present a technical description of CORDIC at the March 1959 Western Joint Computer Conference [2]. Also, permission was obtained to build a demonstrational limited capability navigation system using the new CORDIC computing concept. This system, identified as CORDIC I, was completed in early 1960 and is shown in Fig. 3.

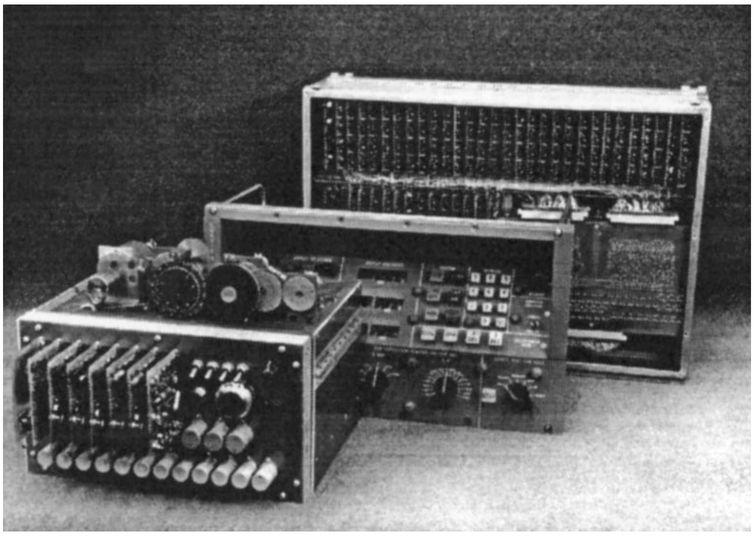


Figure 3. The CORDIC I Digital Astro-Computer System. (Reprinted with kind permission of General Dynamics.)

CORDIC I only solved the problem of fix-taking from inputs of a simulated star-tracker. Although only a laboratory prototype operating at a clock rate of 96.5773 KHz, it created significant interest with Air Force, and management persons concerned with navigation.

3. The CORDIC II Airborne Navigation Computer

As a result of the CORDIC I demonstration, Convair was given authorization to develop a flyable demonstration model that could solve the radar fix-taking problem. This new model, identified as CORDIC II, was highly advanced for its time. Its performance was estimated at a factor of seven times that of contemporary general purpose computers, mainly thanks to the revolutionary development of the CORDIC algorithm.

Operating at a clock frequency of 198.4 KHz, The CORDIC II was fully transistorized, using diode-transistor logic (DTL) circuits. It had a high-density assembly of miniature printed circuit boards, each containing in the order of 10 gates (multi-input ANDs, ORs, invertors). For storage, it employed a magnetic drum with multiple read/write heads, and rotating at 6000 rpm. The main frame of the CORDIC II is shown in Fig. 4, partially showing the drum memory on the top.

The CORDIC II was highly versatile, and could be applied to a wide variety of navigation and control applications. It had an instruction set of 31 operations including rotation, vectoring, binary to BCD conversion, multiplication, division, addition and subtraction. It was highly reliable, capable of maintaining fault-free operation under the severe conditions (vibration, pressure, temperature) of the environment aboard the B-58 aircraft. It was even capable of withstanding the shock of explosive decompression and crash landings. It successfully passed the airborne radar fix-taking test in early 1962 [3].

The CORDIC II came in two models, A and B. Model A was the most versatile and was used in ground setups. Model B was more compact, and suitable for airborne operation. The characteristics of the CORDIC II, model B are summarized in Table 1.

Dan Daggett and Harry Schuss, two of the engineers responsible for the construction of both CORDICs, are still employed at the same facility where it all took place, and which is now managed by Lockheed Martin.

4. Beyond CORDIC I and II

Before completion of CORDIC I, the editors of the IRE Transaction on Electronic Computers requested permission to reprint my WJCC paper in their September 1959 issue [1]. It was published back-to-back with Dan Daggett's publication on using CORDIC for the conversion between binary to BCD representation [4]. Industry acceptance of these technical papers was immediate, especially by those involved in military navigation computers. Versions of CORDIC were soon

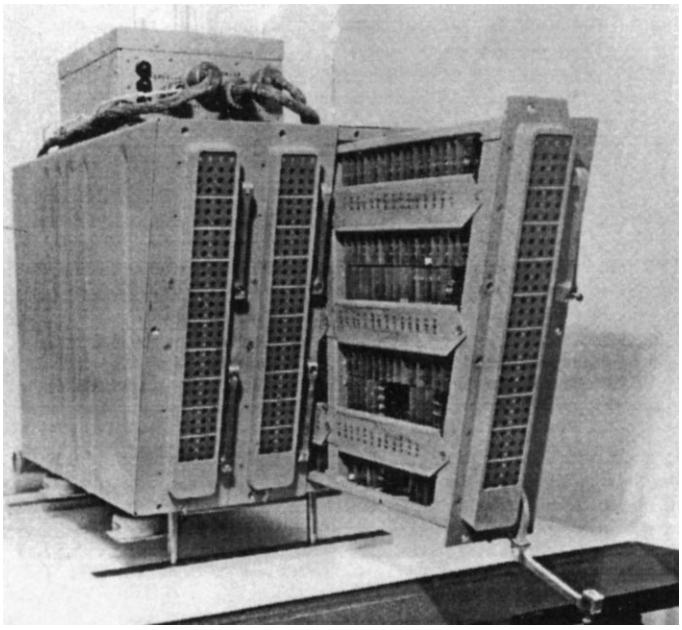


Figure 4. The CORDIC II Digital Computer System. (Reprinted with kind permission of General Dynamics.)

Table 1. Characteristics of the CORDIC II, model B computer.

Clock frequency	198.4 KHz
Number representation	Binary, two's complement
Precision	30 bits, including sign
Computation	Bit-serial
Instruction set	31 operations: 5 CORDIC operations and 26 auxiliary operations
Time for addition	156.25 μs
Time for multiplication or division	5 ms
Time for CORDIC operation	5 ms
Performance	6400 additions per second, 200 CORDIC operations, multiplications, or divisions per second.
Storage type	Magnetic drum, 6000 rpm, 73 heads
Storage capacity	1346 words of 30 bits, of which 1024 reserved for program storage.
Logic family	Diode-transistor logic (DTL)
Number of semiconductors	2498 transistors, 4265 diodes
Size	3.17 cubic feet
Weight	134 pounds
Power consumption	510 Watts (includes the rest of the fix-taking equipment), forced air cooling
Reliability	MTBF $>$ 100 hours

incorporated into the navigation computers built by Martin-Orlando, Computer Control, Litton, Kearfott, Lear-Siegler, Sperry, Raytheon, Collins Radio, and later by Univac and Hughes. In 1966 Hewlett Packard developed a decimal version that computed trigonometric and hyperbolic solutions in their scientific calculators [5].

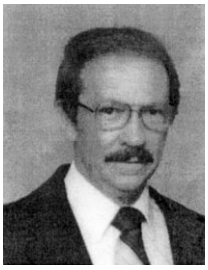
However, in spite of the success of the CORDIC solution for digitizing the B-58 navigation system, the Air Force terminated construction of B-58 bombers in favor of support for the development of ballistic missile systems. Out of the 117 B-58's that were ever built, only 5 survived to present day at various aircraft musea, the remainder being scrapped in the 1970's.

The CORDIC approach continued to find use in other navigation systems, Loran C systems and numerical co-processors for general purpose computers.

References

1. J.E. Volder, "The CORDIC Trigonometric Computing Technique," *IRE Transactions on Electronic Computers*, vol. EC-8, no. 3, 1959, pp. 330–334.
2. J.E. Volder, "The CORDIC Computing Technique," *Proceedings of the Western Joint Computer Conference*, 1959, pp. 257–261.

3. Advanced Systems Group, “Technical Description of Fix-taking Tie-in Equipment,” General Dynamics Aerosystems Report FZE-052, 6 Aug. 1962.
4. D.H. Daggett, “Decimal-Binary Conversion In CORDIC,” *IRE Transactions on Electronic Computers*, vol. EC-8, no. 3, 1959, pp. 335–339.
5. D.S. Cochran, “About utilizing CORDIC for computing transcendental functions in BCD,” Private communication to Jack Volder, 14 March 1966.
6. J.E. Volder, “Binary Computation Algorithms for Coordinate Rotation and Function Generation,” Convair internal report IAR-1.148, Aeroelectronics group, 15 June 1956.



Jack Volder Jack volder served an a B-24 flight engineer during World War II. After his discharge, he enrolled in Texas Technological College where he received his Bachelor of Science degree in Electrical Engineering in 1949. His first professional employment was with the Allis Chalmers manufacturing company in Milwaukee, Wisconsin. In 1951 he joined Convair, Fort Worth as a B-36 field service representative. In 1956, as a senior engineer in the aeroelectronics department, he discovered algorithms for computing transcendental functions and then developed the basic CORDIC computer design suitable for the digital solution of real-time navigation problems. His original internal Convair paper, IAR 1.148 dated 15 June 1956, also included algorithms for hyperbolic coordinate rotation and exponential and logarithmic function generation. He left Convair before the CORDIC I computer was completed and, except for occasional consulting assignments, never actively participated in any more CORDIC projects. Some time later, he privately developed a CORDIC type algorithm for use in a 4 axis coordinate system. After joining Collins Radio, he was active in computer design and programming for several computer controlled data transmission systems. In 1971 he joined Litton Data Systems where he was responsible for the AN/UYK-7 computer program for the control and monitoring of communications equipment on Navy LHA ships. After joining Hughes Aircraft in 1975, he first tackled the problem of computer security in the communication systems of the U.S. Strategic Air Command. Later, he assisted in the problems of converting map coordinates for the U.S. Army Position Location Reporting System (PLRS). He is presently enjoying retirement in Yorba Linda, California with his wife of 51 years and his children and grandchildren. His present technical interests are mainly directed towards the computer simulation of natural intelligence.