Explanation of the use of the function randn() and poisson() of the subpacket numpy.random

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Use of the function poisson()

The function poisson of the sub-packet numpy.random receives two arguments: the lambda and the number of values you want to simulate, which is an optional parameter. This function can be used as a numpy ufunc, allowing you to use a matrix as input and evaluate the function at each position, returning an array with the same shape and the output of each evaluation in the corresponding position.

This is exactly what we need to simulate the detector noise for the image, to generate a random value that follows the poission distribution using the intensity of each position as lambda.

Use of the function randn()

The function randn() of the sub-packet generate values following the Standard Normal Distribution, which is the normal distribution with mean 0 and 1 as standard deviation. This function receives as argument the number of values you want generate, and also allow you to give him the shape of the output you like.

For this simulation we want the variables to distribute with mean equals to the intensity of each point and standard deviation equals to the root of this intensity. Therefore, in order to use randn() to simulate the noise we need to change the parameters of the distribution, and for this aim we will use the following property:
Let $X \sim N(\lambda_0, \sigma_0^2)$, let $Y = aX + b$, then $Y \sim N(\lambda_1, \sigma_1^2)$ where :

$$\lambda_1 = a\lambda_0 + b \tag{1}$$
$$\sigma_1^2 = a^2\sigma_0^2 \tag{2}$$

Then if we work in the previous expression to get the values that we need we get the following:

$$a = \frac{\sigma_1}{\sigma_0} \iff \sigma_1 = a\sigma_0 \Rightarrow \sigma_1^2 = a^2\sigma_0^2 \tag{3}$$
$$b = \lambda_1 - a\lambda_0 \tag{4}$$

Substituting the values of the mean and the variance of the Standard Normal Distribution we obtain:

$$a = \sigma_1 \quad b = \lambda_1 \tag{5}$$

Substituting the mean and the standard deviation we want:

$$a = \sqrt{N} \quad b = N \tag{6}$$

Where N is the intensity.
Hence a way to do what is to generate random variable following the Standard Normal Distribution using as shape of the output the same of the image, then we can use the sqrt and multiplication ufunc to multiply each generated value by the square root of the intensity in each position, and finally use the addition ufunc to sum the intensity