ldflow-3-4-test-site-data-analysis

August 28, 2024

[2]:
```
import os
import math
import datetime
import numpy as np
import pandas as pd
from math import pi
import scipy.integrate as spi
from scipy.integrate import quad
from matplotlib import pyplot as plt
from scipy.signal import savgol_filter
from scipy.ndimage import median_filter
from scipy.ndimage import gaussian_filter1d
from scipy.interpolate import UnivariateSpline
from sklearn.linear_model import LinearRegression
from matplotlib.ticker import (AutoMinorLocator, MultipleLocator)
```

0.1 1. Define Constants

Step 1: Define the constants

[3]:
```
# run tank dimensions
inner_diameter = 0.146304                                   # m
cyl_height = 0.912114                                       # m
V_total = 0.01533                                           # m^3
base_area = np.pi * ((inner_diameter/2)**2)                 # m^2
```

[4]:
```
# injector
hole_diameter = 2                                           # mm
num_holes = 16
```

[5]:
```
# input filename
input_data = 'COLDFLOW3.4.txt'
```

[6]:
```
# ambient temp
outdoor_temp = 17                                           # Celcius
```

0.2 2. Define Functions

[7]:
```
# get data in time range required
def get_data_range(data, time, start, end):
        return [d for t, d in zip(time, data) if start <= t <= end]
```

[8]:
```
# determine co2 vapour pressure, liquid density and gas density given␣
↪temperature
def carbon_dioxide_equations(temp_kelvin):
    # https://mychemengmusings.wordpress.com/2022/06/23/
↪five-short-equations-describing-the-saturated-density-and-enthalpy-of-carbon-dioxide-co2-va
↪
    ZsatT = 1 + (0.001613*(temp_kelvin*(304.128-temp_kelvin))**(0.6))-0.67508

    # Density of Saturated Liquid
    p_l =  ((-3.53267*(temp_kelvin-216.592))/(ZsatT**(0.646)))+1180.409

    # Density of Saturated Gas
    p_g = 467.6 * math.exp(-75.135 * (304.128-temp_kelvin)**0.68 /␣
↪temp_kelvin**1.15 / ZsatT**0.33  - 0.1855 )

    # Vapour Pressure
    p = ((p_g * ZsatT * temp_kelvin)/529.304)*100

    return p, p_l, p_g

# turn CO2 vapour pressure, liquid density and gas density into lists
def carbon_dioxide_equation_lists(temp_list):
    vap_list = []
    liquid_list = []
    gas_list = []
    for temp_kelvin in temp_list:
        p, p_l, p_g = carbon_dioxide_equations(temp_kelvin)
        vap_list.append(p)
        liquid_list.append(p_l)
        gas_list.append(p_g)

    return vap_list, liquid_list, gas_list
```

[9]:
```
# Determine liquid density from temperature. Finds key from closest value in␣
↪dictionary using list comprehension
def get_liquid_density_from_pressure(temp_liquid_density_dict,␣
↪pressure_temp_dict, pressure_key):
    pressure_KPa = pressure_key * 6.89476
    temp_K = pressure_temp_dict.get(pressure_KPa) or␣
↪pressure_temp_dict[min(pressure_temp_dict.keys(), key = lambda key:␣
↪abs(key-pressure_KPa))]
```

```
    liquid_density = temp_liquid_density_dict.get(temp_K) or␣
↪temp_liquid_density_dict[min(temp_liquid_density_dict.keys(), key = lambda␣
↪key: abs(key-temp_K))]

    return liquid_density
```

[10]:
```
# determine discharge coefficient given mass flow rate and combustion chamber␣
↪pressure
def calculate_discharge_coefficient(n_o, hole_diameter, num_holes, p_run_range,␣
↪run_temp_range, delta_p_run_cc, temp_liquid_density_dict):
    injection_area = (np.pi * ((hole_diameter/1000)/2)**2)
    discharge_coefficient = []
    for i in range(len(p_run_range)):
        liquid_density =␣
↪get_liquid_density_from_pressure(temp_liquid_density_dict,␣
↪pressure_temp_dict, p_run_range[i])
        discharge_coeff = n_o / (num_holes * injection_area * np.
↪sqrt(2*liquid_density*delta_p_run_cc[i]*6892.76))
        discharge_coefficient.append(discharge_coeff)

    return discharge_coefficient

def calculate_discharge_coefficient_uncertainty(discharge_coefficient,␣
↪hole_diameter, num_holes, p_run_range, delta_p_run_cc,␣
↪temp_liquid_density_dict, mass_flow_rate_std_error):
    injection_area = (np.pi * ((hole_diameter/1000)/2)**2)
    c_d_std_error = []
    for i in range(len(p_run_range)):
        liquid_density =␣
↪get_liquid_density_from_pressure(temp_liquid_density_dict,␣
↪pressure_temp_dict, p_run_range[i])
        discharge_coefficient_std_error = (1 / (num_holes * injection_area * np.
↪sqrt(2*liquid_density*delta_p_run_cc[i]*6892.76))) *␣
↪mass_flow_rate_std_error[i]
        c_d_std_error.append(discharge_coefficient_std_error)
    return c_d_std_error
```

[11]:
```
def get_carbon_dioxide_mass_unknown_ullage(pressure, m_total):

    temp_K = pressure_temp_dict.get(pressure) or␣
↪pressure_temp_dict[min(pressure_temp_dict.keys(), key = lambda key:␣
↪abs(key-pressure))]
    p, p_liq, p_gas = carbon_dioxide_equations(temp_K)
    x = (V_total-((m_total/p_gas)))/((m_total/p_liq)-(m_total/p_gas))       # x␣
↪= mass of liquid / total mass
```

```
    m_liq = x * m_total                                                          #␣
↪get mass of liquid in tank
    m_gas = (1-x) * m_total                                                      #␣
↪get mass of gas in tank
    V_liq = m_liq / p_liq                                                        #␣
↪get volume of liquid
    V_gas = m_gas / p_gas                                                        #␣
↪get volume of gas
    V_liq_height = V_liq / base_area                                             #␣
↪get height of liquid in tank
    dip_tube_length = cyl_height - V_liq_height                                  #␣
↪get diptube length
    ullage_percentage = (dip_tube_length / cyl_height) * 100

    return m_liq, m_gas, ullage_percentage

# def get_carbon_dioxide_mass_known_ullage(outside_temp, ullage_percentage):

#     p, p_liq, p_gas = carbon_dioxide_equations(outside_temp + 273.15)      #␣
↪get pressure and density values given known temperature
#     V_liq_height = cyl_height * (1-(ullage_percentage / 100))               #␣
↪get height of liquid given fixed 10% ullage
#     V_liq = V_liq_height * base_area                                        #␣
↪get volume of liquid in tank using liquid height and cylinder area of base
#     m_liq = V_liq * p_liq                                                   #␣
↪get mass of liquid given volume and density at given temperature
#     V_gas_height = cyl_height * (ullage_percentage / 100)                   #␣
↪get height of gas (ullage)
#     V_gas = V_gas_height * base_area                                        #␣
↪get volume of gas in tank using ullage and cylinder area of base
#     m_gas = V_gas * p_gas                                                   #␣
↪get mass of gas given volume and density at given temperature
#     m_total = m_liq + m_gas                                                 #␣
↪get total amount of carbon dioxide
#     ullage_percentage = (V_gas_height / cyl_height) * 100                   #␣
↪calculate ullage percentage

#     return m_liq, m_gas, ullage_percentage
```

0.3 3. Determine Start and End Times

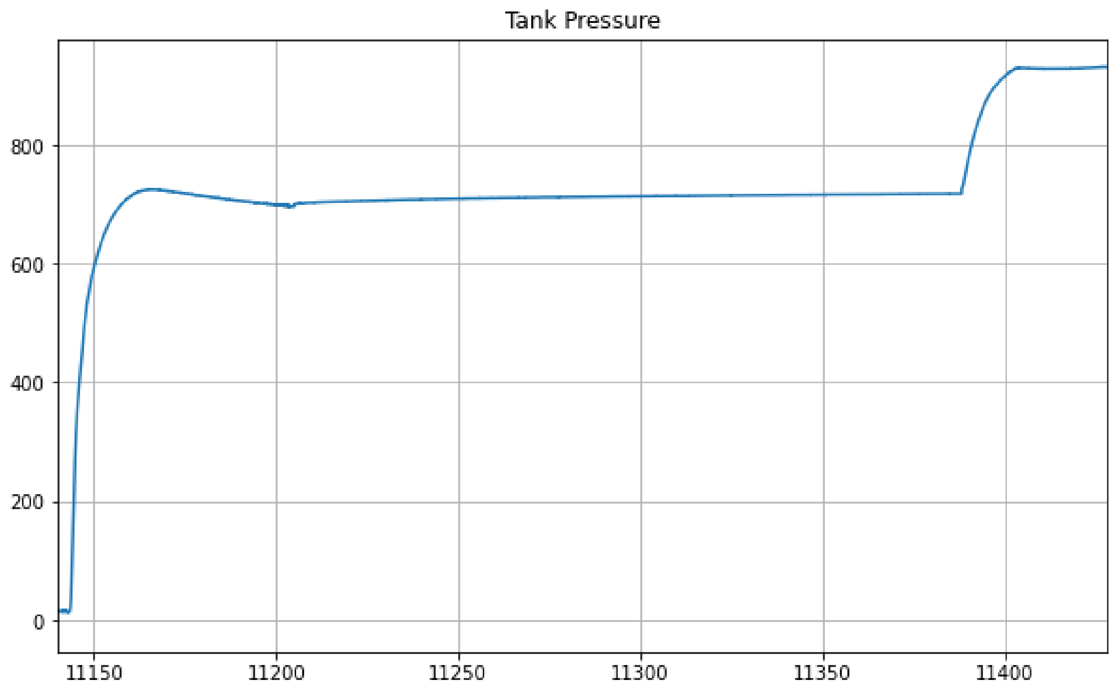
[12]:

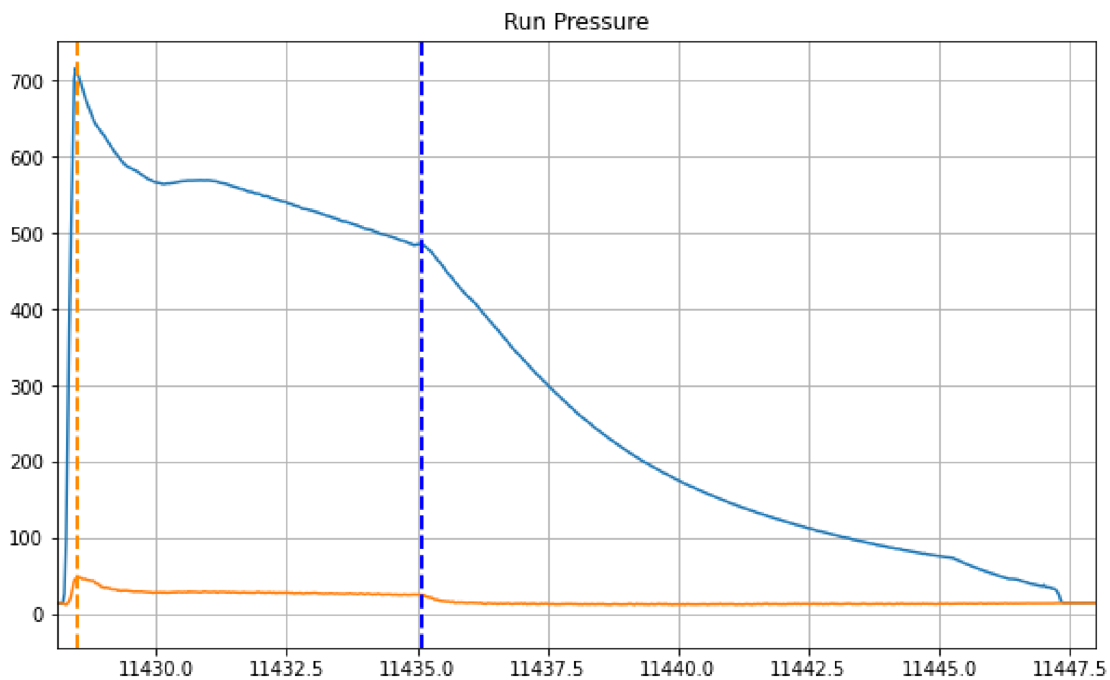
```
column_names = ['time_ms', 'run_pressure_V', 'fill_pressure_V',␣
↪'purge_pressure_V', 'tank_pressure_V', 'tank_mass_V', 'thrust_V',␣
↪'cc_pressure_V', 'tank_temp_V', 'run_temp_V', 'vent_temp_V', 'garbage',␣
↪'run_pressure_sw', 'fill_pressure_sw', 'purge_pressure_sw',␣
↪'tank_pressure_sw', 'tank_mass_sw', 'thrust_sw', 'cc_pressure_sw',␣
↪'run_temp_sw', 'tank_temp_sw', 'ven_temp_sw']
df = pd.read_csv(input_data, sep='\t', names=column_names, index_col=False,␣
↪dtype=np.float64, skip_blank_lines=True)
time, p_fill, p_tank, p_run, p_cc, tank_temp, run_temp, tank_mass, thrust =␣
↪(df['time_ms'] / 1000, df['fill_pressure_sw'], df['tank_pressure_sw'],␣
↪df['run_pressure_sw'], df['cc_pressure_sw'], df['tank_temp_sw'],␣
↪df['run_temp_sw'], df['tank_mass_sw'], df['thrust_sw'])
```

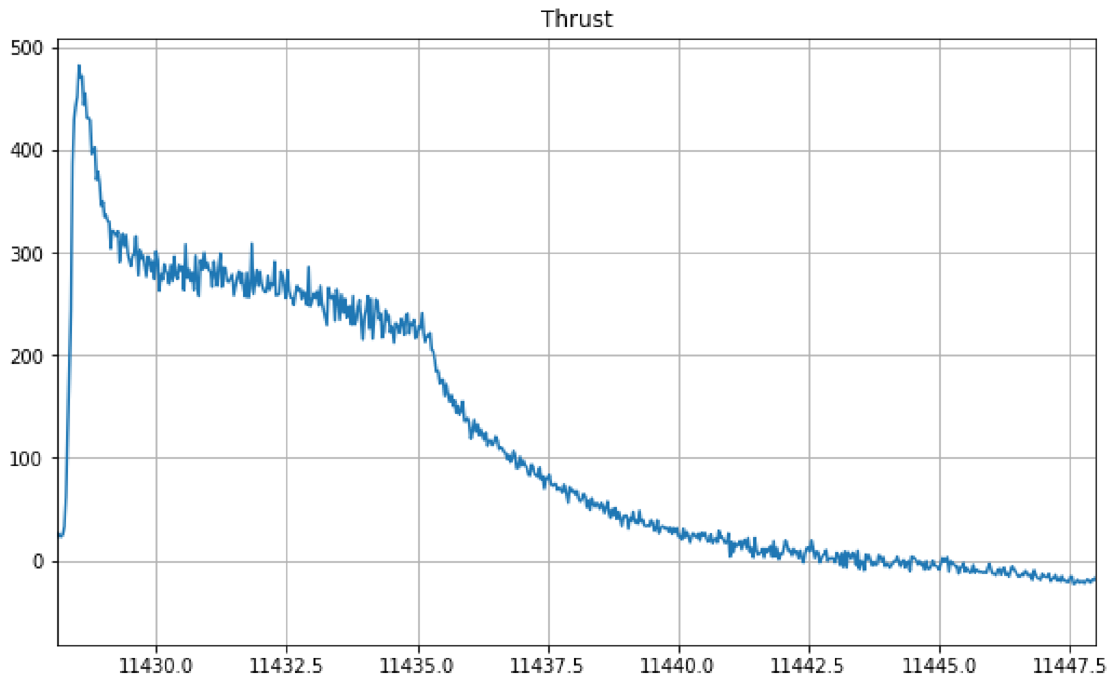
[13]:
```
# play around with these values until you find the correct times
start_time_fill = 11140
start_time = 11428.1
end_time = 11448
end_time_fill = start_time
start_time_liq_pre = 11428.5
end_time_liq_pre = 11435.1
start_time_liq = 0.4
end_time_liq = 7
```

[14]:
```
plt.figure(figsize=(10, 6))
plt.plot(time, p_tank)
plt.title('Tank Pressure')
plt.grid(True)
plt.xlim(start_time_fill, start_time)
plt.show()

plt.figure(figsize=(10, 6))
plt.plot(time, p_run)
plt.plot(time, p_cc)
plt.axvline(x= start_time_liq_pre, color='darkorange', linestyle='--',␣
↪linewidth=2)
plt.axvline(x= end_time_liq_pre, color='blue', linestyle='--', linewidth=2)
plt.title('Run Pressure')
plt.grid(True)
plt.xlim(start_time, end_time)
plt.show()

plt.figure(figsize=(10, 6))
plt.plot(time, thrust)
#plt.axvline(x= liq_start, color='darkorange', linestyle='--', linewidth=2)
#plt.axvline(x= liq_end, color='blue', linestyle='--', linewidth=2)
plt.title('Thrust')
plt.grid(True)
```

```
plt.xlim(start_time, end_time)
plt.show()
```







0.4 4. Extract Data

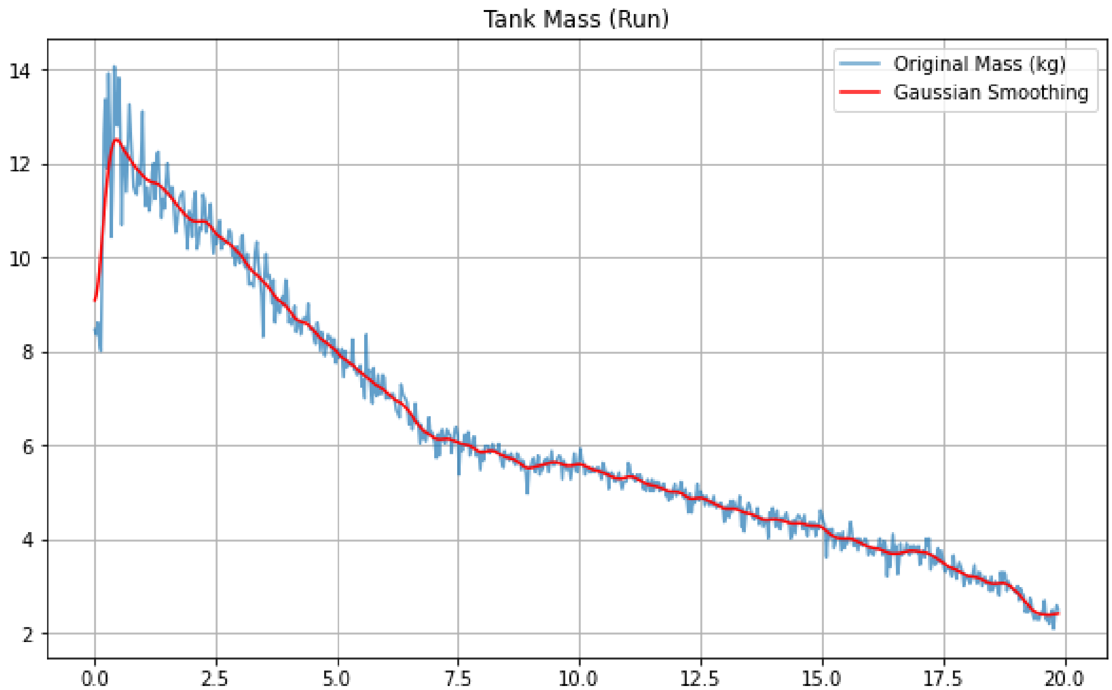
[15]:
```
# create dictionary of carbon_dioxide liquid density and temperature values (as␣
↪solving for liquid density from temp is too mathematically intensive)
temp_list = np.linspace(183, 309, num = 10000)
vap_list, liquid_list, gas_list = carbon_dioxide_equation_lists(temp_list)
temp_liquid_density_dict = dict(zip(temp_list, liquid_list))
pressure_temp_dict = dict(zip(vap_list, temp_list))

# get time ranges
time_range = [t - start_time for t in time[(time >= start_time) & (time <=␣
↪end_time)]]
time_range_fill = [t - start_time_fill for t in time if start_time_fill <= t <=␣
↪end_time_fill]
time_range_full = [t - start_time_fill for t in time[(time >= start_time_fill)␣
↪& (time <= end_time)]]

# get data ranges in time frame
p_fill_range = get_data_range(p_fill, time, start_time_fill, end_time_fill)
p_tank_fill_range = get_data_range(p_tank, time, start_time_fill, end_time_fill)
p_tank_range = get_data_range(p_tank, time, start_time, end_time)
p_run_range = get_data_range(p_run, time, start_time, end_time)
p_cc_range = get_data_range(p_cc, time, start_time, end_time)
tank_temp_range = get_data_range(tank_temp, time, start_time, end_time)
```

```
run_temp_range = get_data_range(run_temp, time, start_time, end_time)
thrust_range = get_data_range(thrust, time, start_time, end_time)
mass_range = get_data_range(tank_mass, time, start_time_fill, start_time)
mass_run_range = get_data_range(tank_mass, time, start_time, end_time)

# calculate pressure deltas
delta_p_tank_fill = [pt - pf for pt, pf in zip(p_tank_fill_range, p_fill_range)]
delta_p_tank_run = [pt - pr for pt, pr in zip(p_tank_range, p_run_range)]
delta_p_run_cc = [pr - pc for pr, pc in zip(p_run_range, p_cc_range)]
```

```
/var/folders/yn/w934gfts1gv92p3xhy03fn880000gn/T/ipykernel_87504/3557967768.py:4
: RuntimeWarning: invalid value encountered in double_scalars
  ZsatT = 1 + (0.001613*(temp_kelvin*(304.128-temp_kelvin))**(0.6))-0.67508
/var/folders/yn/w934gfts1gv92p3xhy03fn880000gn/T/ipykernel_87504/3557967768.py:1
0: RuntimeWarning: invalid value encountered in double_scalars
  p_g = 467.6 * math.exp(-75.135 * (304.128-temp_kelvin)**0.68 /
temp_kelvin**1.15 / ZsatT**0.33  - 0.1855 )
```

0.5 5. Mass Data Fitting and Mass Estimates

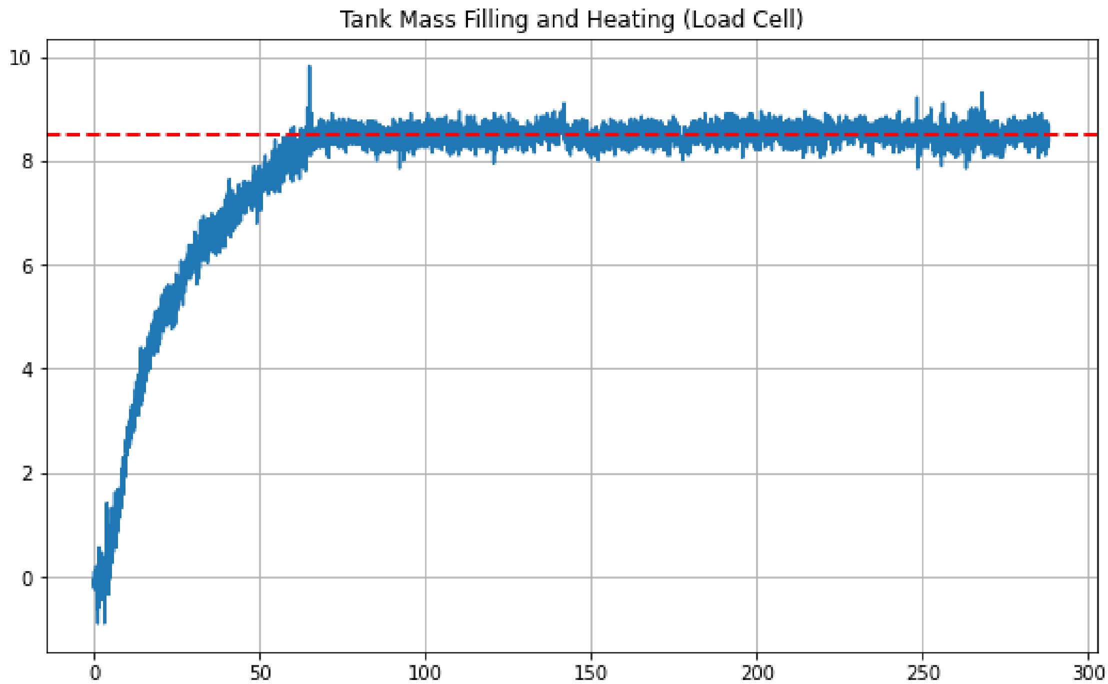
[16]:
```
# Low Pass Filter and Gaussian Smoothing (ideal)
smoothed_mass_gaussian = gaussian_filter1d(mass_run_range, sigma=4) # Gaussian␣
 ↪Smoothing

plt.figure(figsize=(10, 6))
plt.plot(time_range, mass_run_range, label='Original Mass (kg)', alpha=0.7)
plt.plot(time_range, smoothed_mass_gaussian, label='Gaussian Smoothing',␣
 ↪color='red')
plt.title('Tank Mass (Run)')
plt.grid(True)
plt.legend()
plt.show()
```



[17]:
```
# Mass Loss Estimate
#Note: coldflow ran for 15 seconds. This is hardcoded for now.
print(time_range[15])
print(time_range[-156])

mass_loss_estimate = smoothed_mass_gaussian[15] - smoothed_mass_gaussian[-156]
```

```
0.48099999999976717
15.003999999998996
```

[18]:
```
# Total Mass (end of fill)
m_total_start = 8.5

plt.figure(figsize=(10, 6))
plt.plot(time_range_fill, mass_range, label='Mass (kg)')
plt.axhline(y= m_total_start, color='red', linestyle='--', linewidth=2)
plt.title('Tank Mass Filling and Heating (Load Cell)')
#plt.xlim(1750,2000)
#plt.ylim(8, 9)
plt.grid(True)
plt.show()
```



[19]:
```
# Mass from diptube length (ullage) estimate
# m_liq, m_gas, ullage_percentage =␣
 ↪get_carbon_dioxide_mass_known_ullage(outdoor_temp, 25)
# diptube_mass = m_liq + m_gas
```

[20]:
```
# mass estimates
print('Load Cell Mass (end of fill) =', m_total_start, 'kg')
print('Mass Loss Estimate =', mass_loss_estimate, 'kg')
# print('Filled to diptube =', diptube_mass, 'kg')
```

```
Load Cell Mass (end of fill) = 8.5 kg
Mass Loss Estimate = 8.259596709509006 kg
```

0.6 6. Calculate Important Parameters

[21]:
```
# fill time
fill_time = start_time - start_time_fill
```

[22]:
```
# peak tank pressure
peak_tank_pressure = max(p_tank_range)

# peak_tank_temperature
peak_tank_temp = max(tank_temp_range)
```

```
# peak run pressure
peak_run_pressure = max(p_run_range)

# peak combustion chamber pressure
peak_cc_pressure = max(p_cc_range)
```

[23]:
```
# ullage factor, liquid and gaseous carbon_dioxide mass
m_liq, m_gas, ullage_percentage =␣
 ↪get_carbon_dioxide_mass_unknown_ullage(peak_tank_pressure, m_total_start)
#m_liq, m_gas, ullage_percentage =␣
 ↪get_carbon_dioxide_mass_known_ullage(outdoor_temp)
```

[24]:
```
# mass flow rate
time_derivative = np.gradient(smoothed_mass_gaussian, time_range)
peak_mass_flow_rate = min(time_derivative)
average_mass_flow_rate = np.mean([value for t, value in zip(time_range,␣
 ↪time_derivative) if start_time_liq <= t <= end_time_liq])



# discharge coefficient
average_c_d = calculate_discharge_coefficient(-average_mass_flow_rate,␣
 ↪hole_diameter, num_holes, p_run_range, run_temp_range, delta_p_run_cc,␣
 ↪temp_liquid_density_dict)
peak_c_d = calculate_discharge_coefficient(-peak_mass_flow_rate, hole_diameter,␣
 ↪num_holes, p_run_range, run_temp_range, delta_p_run_cc,␣
 ↪temp_liquid_density_dict)
```

```
/var/folders/yn/w934gfts1gv92p3xhy03fn880000gn/T/ipykernel_87504/1248393918.py:7
: RuntimeWarning: invalid value encountered in sqrt
  discharge_coeff = n_o / (num_holes * injection_area *
np.sqrt(2*liquid_density*delta_p_run_cc[i]*6892.76))
```

[25]:
```
# print values
print('Ullage Factor =', ullage_percentage, '%')
print('Fill Time =', fill_time, 's')
print('Peak Tank Pressure =', peak_tank_pressure, 'psi')
print('Peak Tank Temp =', peak_tank_temp, 'C')
print('Peak Run Pressure =', peak_run_pressure, 'psi')
print('Peak CC Pressure =', peak_cc_pressure, 'psi')
print('Peak Mass Flow Rate =', - peak_mass_flow_rate, 'kg/s')
print('Average Mass Flow Rate =', - average_mass_flow_rate, 'kg/s')
print('Mass of Liquid =', m_liq, 'kg')
print('Mass of Gas =', m_gas, 'kg')
```

```
Ullage Factor = 51.83327877606979 %
Fill Time = 288.10000000000036 s
Peak Tank Pressure = 930.932787 psi
```

```
Peak Tank Temp = 28.361517 C
Peak Run Pressure = 716.462599 psi
Peak CC Pressure = 48.442868 psi
Peak Mass Flow Rate = 1.9014299870717082 kg/s
Average Mass Flow Rate = 0.9559481153823655 kg/s
Mass of Liquid = 8.307423275653472 kg
Mass of Gas = 0.19257672434652912 kg
```

0.7 7. Mass Error Propogation

[164]:
```
# Mass Error (standard deviation and standard error)
#mass_data = np.array([m_total_start, abs(integrated_mass), diptube_mass])
mass_data = np.array([m_total_start, mass_loss_estimate])
mean_mass = np.mean(mass_data)
std_dev = np.std(mass_data, ddof=1)
std_error = std_dev / np.sqrt(len(mass_data))

print("Mean Mass:", mean_mass, "kg")
print("Standard Deviation:", std_dev, "kg")
print("Standard Error:", std_error, "kg")
```

```
Mean Mass: 8.379798354754502 kg
Standard Deviation: 0.16999079692574104 kg
Standard Error: 0.1202016452454968 kg
```

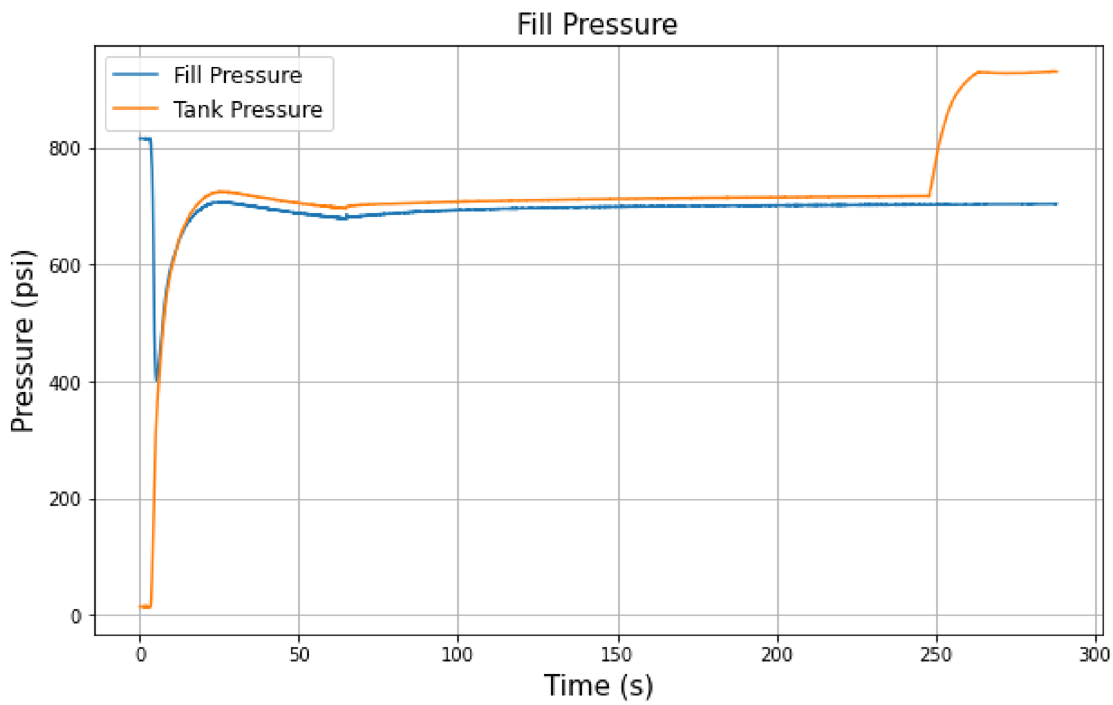
[156]:
```
# Mass Flow Rate Error
mass_flow_rate_std_error = np.abs(time_derivative) * std_error
```

[180]:
```
# Discharge Coefficient Error
average_c_d_std_error =␣
↪calculate_discharge_coefficient_uncertainty(average_c_d, hole_diameter,␣
↪num_holes, p_run_range, delta_p_run_cc, temp_liquid_density_dict,␣
↪mass_flow_rate_std_error)
peak_c_d_std_error = calculate_discharge_coefficient_uncertainty(peak_c_d,␣
↪hole_diameter, num_holes, p_run_range, delta_p_run_cc,␣
↪temp_liquid_density_dict, mass_flow_rate_std_error)
```

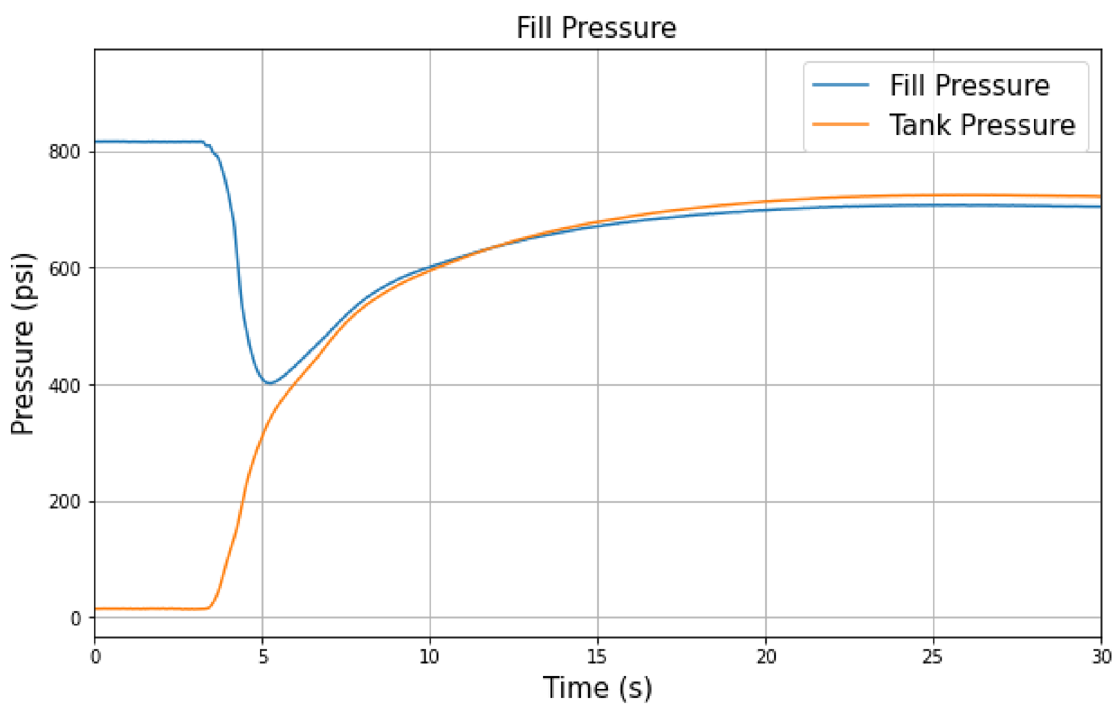
```
/var/folders/yn/w934gfts1gv92p3xhy03fn880000gn/T/ipykernel_82637/1248393918.py:1
7: RuntimeWarning: invalid value encountered in sqrt
  discharge_coefficient_std_error = (1 / (num_holes * injection_area *
np.sqrt(2*liquid_density*delta_p_run_cc[i]*6892.76))) *
mass_flow_rate_std_error[i]
```

0.8 8. Plots

[114]:
```
# Fill Pressure
plt.figure(figsize=(10, 6))
plt.plot(time_range_fill, p_fill_range, label='Fill Pressure')
plt.plot(time_range_fill, p_tank_fill_range, label='Tank Pressure')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Pressure (psi)', fontsize = 15)
plt.title('Fill Pressure', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 12)
plt.gca().set_facecolor('white')
plt.savefig('fill_pressure.png', facecolor='white')
plt.show()

# Fill Pressure Zoomed
plt.figure(figsize=(10, 6))
plt.plot(time_range_fill, p_fill_range, label='Fill Pressure')
plt.plot(time_range_fill, p_tank_fill_range, label='Tank Pressure')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Pressure (psi)', fontsize = 15)
plt.title('Fill Pressure', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 15)
plt.xlim(0,30)
#plt.ylim(0,500)
plt.gca().set_facecolor('white')
plt.savefig('fill_pressure_zoomed.png', facecolor='white')
plt.show()
```

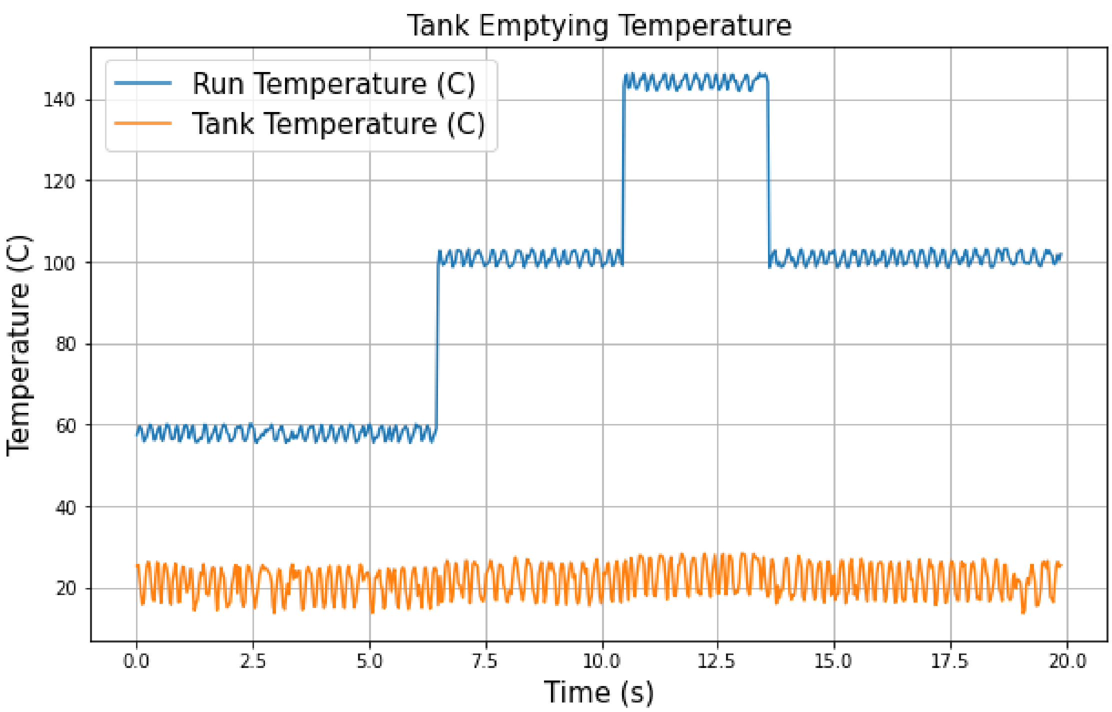


Fill Pressure
Fill Pressure
Tank Pressure
Pressure (psi)
Time (s)
0
200
400
600
800
0
50
100
150
200
250
300



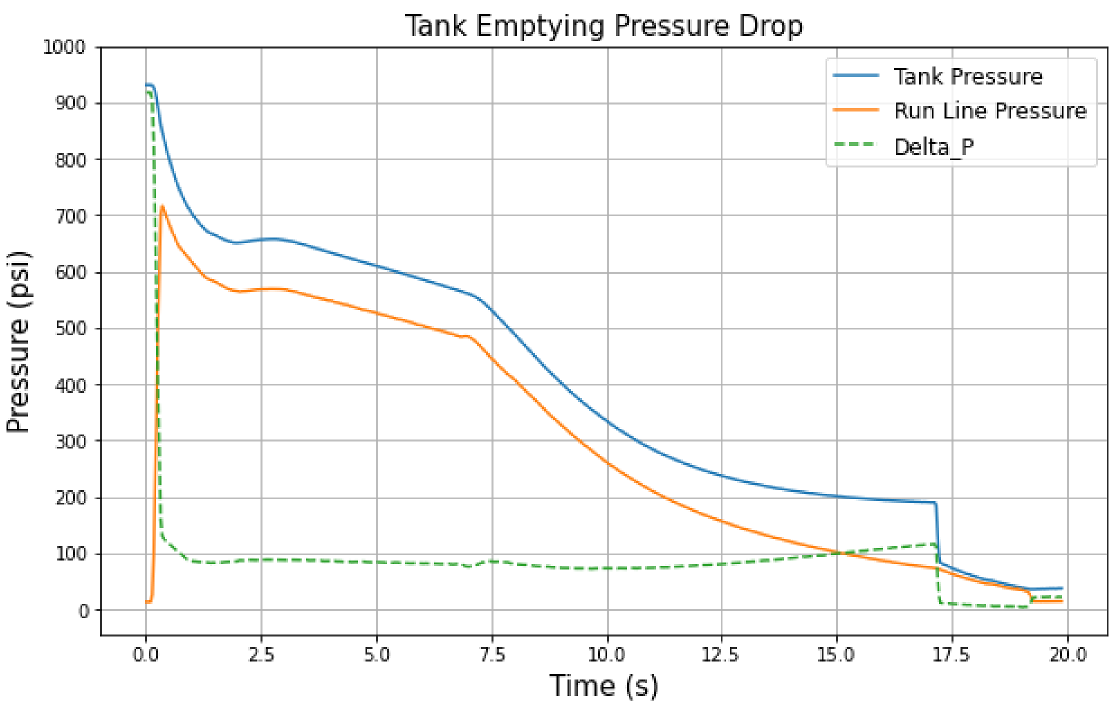
Fill Pressure
Fill Pressure
Tank Pressure
Pressure (psi)
Time (s)
0
200
400
600
800
0
5
10
15
20
25
30

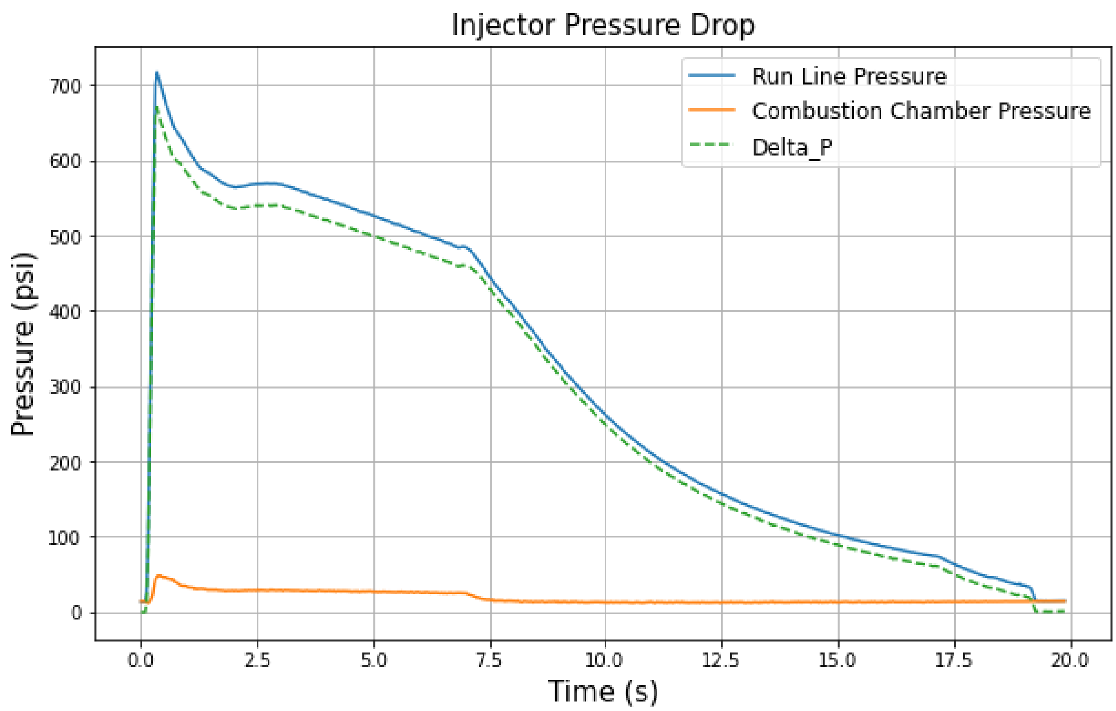
[115]:
```python
# Temperature
plt.figure(figsize=(10, 6))
plt.plot(time_range, run_temp_range, label='Run Temperature (C)')
plt.plot(time_range, tank_temp_range, label='Tank Temperature (C)')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Temperature (C)', fontsize = 15)
plt.title('Tank Emptying Temperature', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 15)
plt.gca().set_facecolor('white')
plt.savefig('tank_emptying_temp.png', facecolor='white')
plt.show()
```



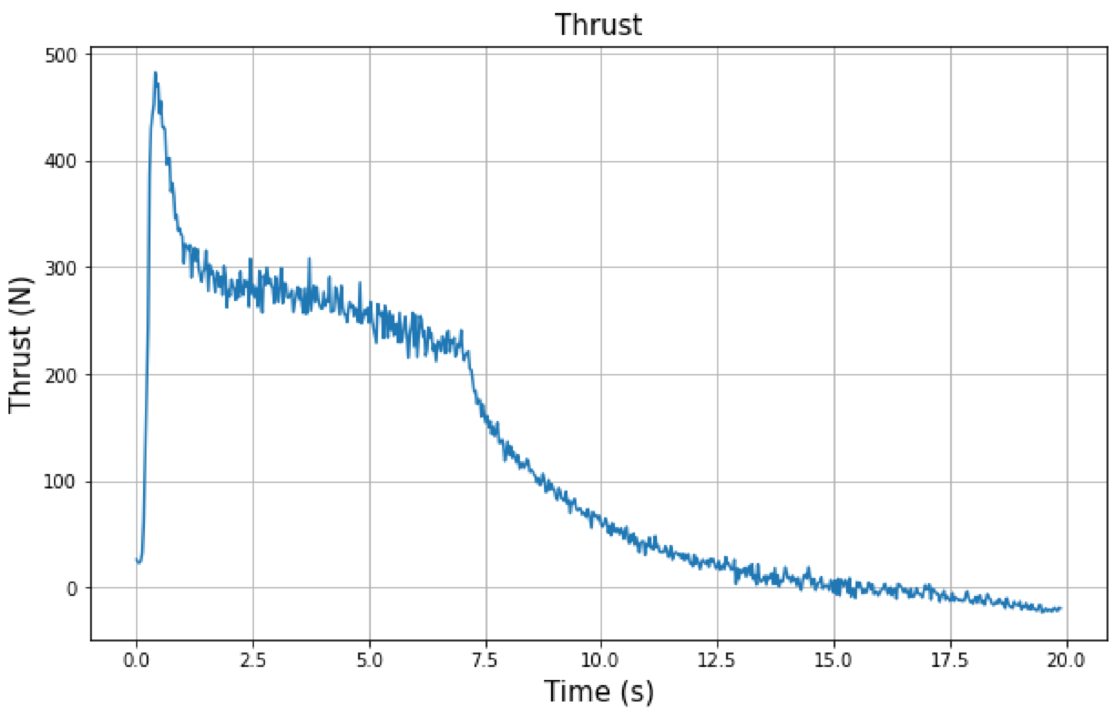
[116]:
```python
# Tank/Run Pressure
plt.figure(figsize=(10, 6))
plt.plot(time_range, p_tank_range, label='Tank Pressure')
plt.plot(time_range, p_run_range, label='Run Line Pressure')
plt.plot(time_range, delta_p_tank_run, linestyle='--', label='Delta_P')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Pressure (psi)', fontsize = 15)
plt.title('Tank Emptying Pressure Drop', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 12)
plt.gca().set_facecolor('white')
```

```python
plt.yticks(np.arange(0, max(p_tank_range) + 100, 100))
plt.savefig('tank_emptying_pressure.png', facecolor='white')
plt.show()

# Injector Pressure Drop
plt.figure(figsize=(10, 6))
plt.plot(time_range, p_run_range, label='Run Line Pressure')
plt.plot(time_range, p_cc_range, label='Combustion Chamber Pressure')
plt.plot(time_range, delta_p_run_cc, linestyle='--', label='Delta_P')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Pressure (psi)', fontsize = 15)
plt.title('Injector Pressure Drop', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 12)
plt.gca().set_facecolor('white')
plt.savefig('injector_pressure_drop.png', facecolor='white')
plt.show()
```





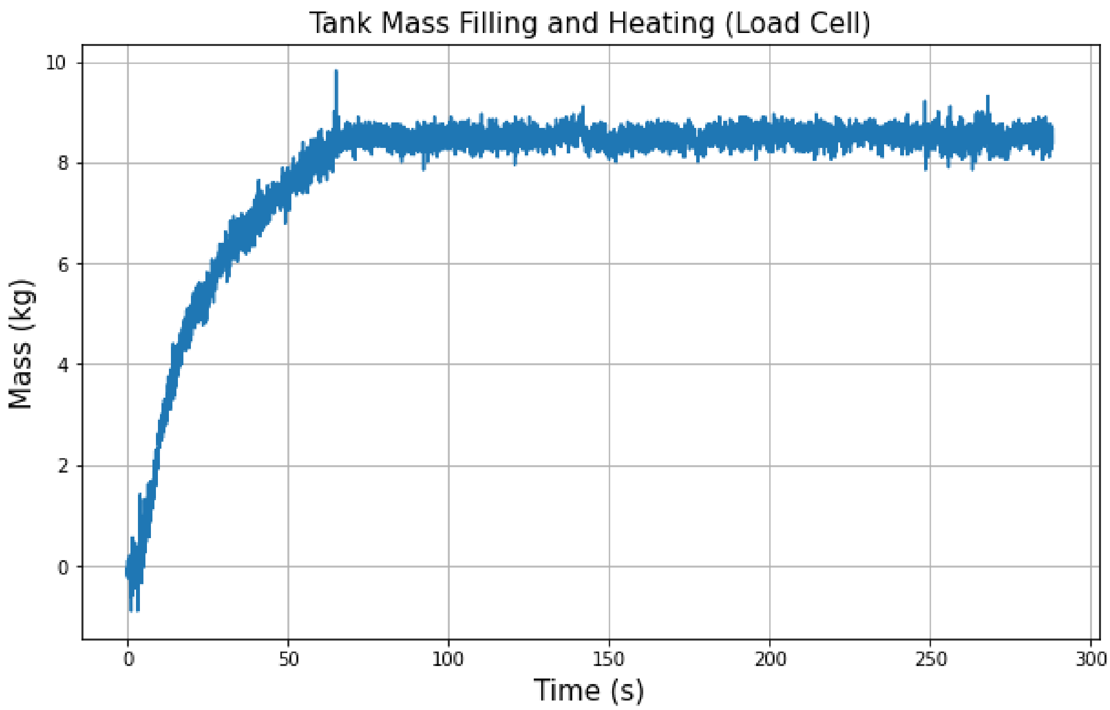
[117]:
```
# Thrust
plt.figure(figsize=(10, 6))
plt.plot(time_range, thrust_range, label='Thrust (N)')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Thrust (N)', fontsize = 15)
plt.title('Thrust', fontsize = 15)
plt.grid(True)
plt.gca().set_facecolor('white')
plt.savefig('thrust.png', facecolor='white')
plt.show()
```

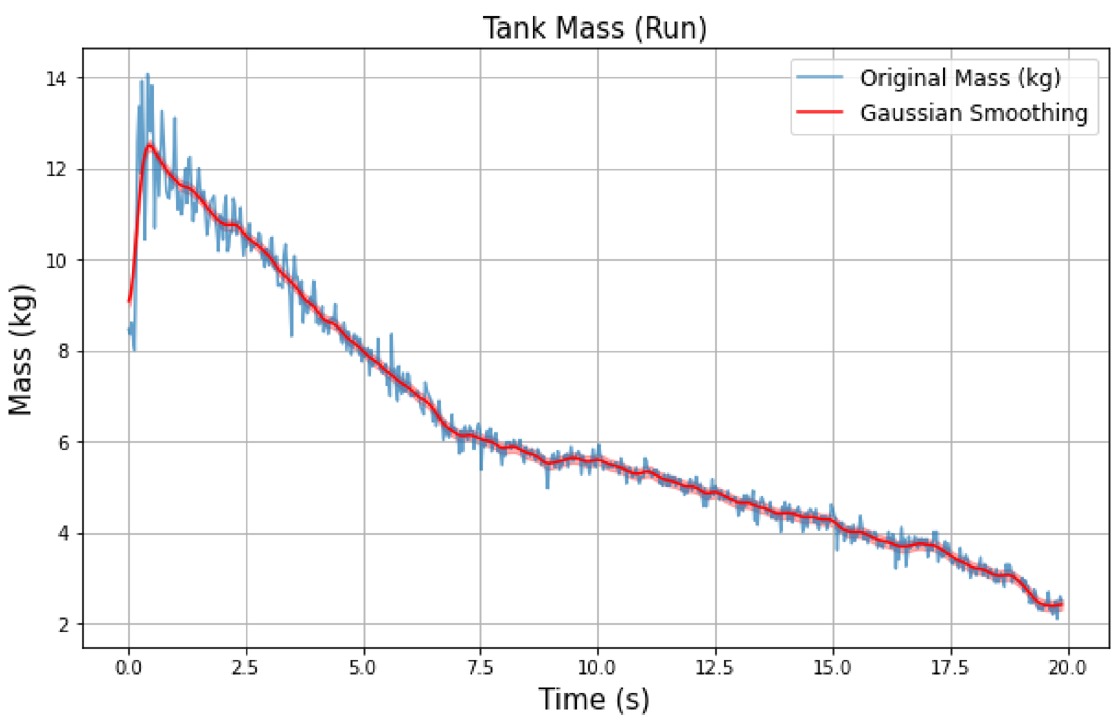


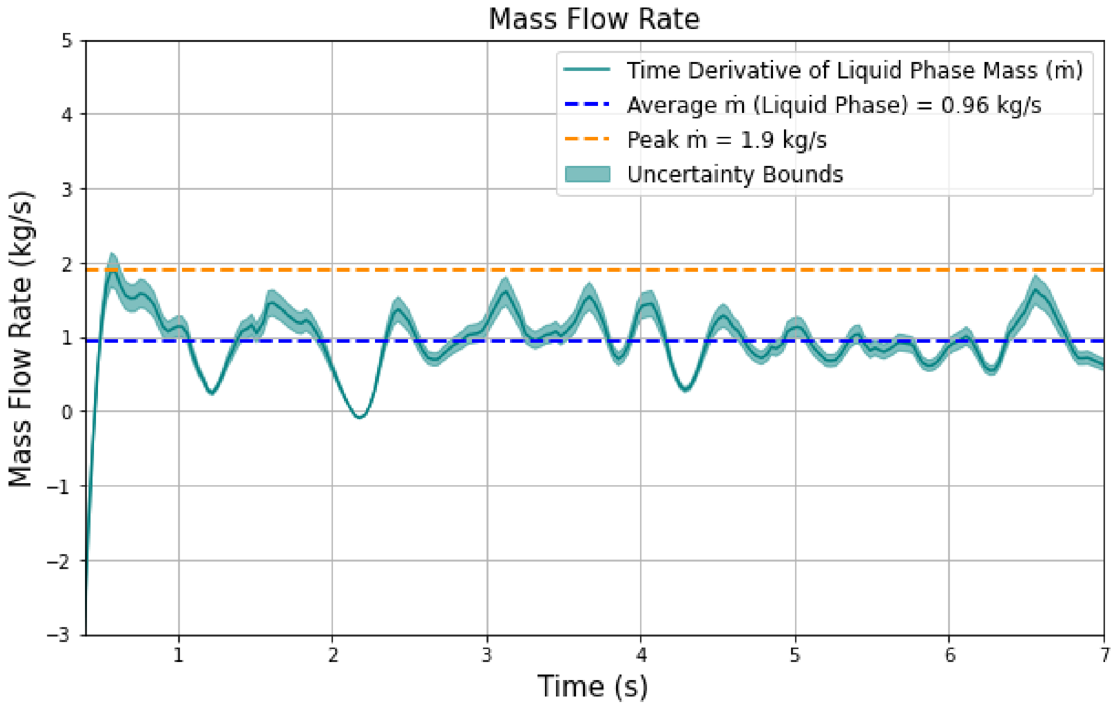
[194]:
```
# Mass
plt.figure(figsize=(10, 6))
plt.plot(time_range_fill, mass_range, label='Mass (kg)')
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Mass (kg)', fontsize = 15)
plt.title('Tank Mass Filling and Heating (Load Cell)', fontsize = 15)
plt.grid(True)
plt.gca().set_facecolor('white')
plt.savefig('mass_fill.png', facecolor='white')
plt.show()


# 2.9 Mass (Run)
plt.figure(figsize=(10, 6))
plt.plot(time_range, mass_run_range, label='Original Mass (kg)', alpha=0.7)
plt.plot(time_range, smoothed_mass_gaussian, label='Gaussian Smoothing',␣
 ↪color='red')
upper_bound = smoothed_mass_gaussian + std_error
lower_bound = smoothed_mass_gaussian - std_error
plt.fill_between(time_range, lower_bound, upper_bound, color='red', alpha=0.3)
plt.title('Tank Mass (Run)', fontsize = 15)
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Mass (kg)', fontsize = 15)
plt.grid(True)
```

```python
plt.gca().set_facecolor('white')
plt.legend(fontsize = 12)
plt.savefig('mass_tank_emptying.png', facecolor='white')
plt.show()

# Mass Flow Rate
plt.figure(figsize=(10, 6))
plt.plot(time_range, - time_derivative, label='Time Derivative of Liquid Phase␣
↪Mass (ṁ)', color='teal')
plt.axhline(y= - average_mass_flow_rate, color='blue', linestyle='--',␣
↪label="Average ṁ (Liquid Phase) = 0.96 kg/s", linewidth=2)
plt.axhline(y= - peak_mass_flow_rate, color='darkorange', linestyle='--',␣
↪label="Peak ṁ = 1.9 kg/s", linewidth=2)
upper_bound = time_derivative + mass_flow_rate_std_error
lower_bound = time_derivative - mass_flow_rate_std_error
plt.fill_between(time_range, -upper_bound, -lower_bound, color='teal', alpha=0.
↪5, label='Uncertainty Bounds')
plt.title('Mass Flow Rate', fontsize = 15)
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Mass Flow Rate (kg/s)', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 12)
plt.xlim(start_time_liq, end_time_liq)
plt.ylim(-3,5)
plt.gca().set_facecolor('white')
plt.savefig('injector_mass_flow_rate.png', facecolor='white')
plt.show()

# Discharge Coefficient Estimation
plt.figure(figsize=(10, 6))
plt.plot(time_range, average_c_d, label='Average ṁ = 0.96 kg/s (CO2)', color =␣
↪'blue')
plt.plot(time_range, peak_c_d, label='Peak ṁ = 1.9 kg/s (CO2)', color =␣
↪'darkorange')
#upper_bound_avg = average_c_d + average_c_d_std_error
#lower_bound_avg = average_c_d - average_c_d_std_error
#plt.fill_between(time_range, -upper_bound_avg, -lower_bound_avg, color='teal',␣
↪alpha=0.5, label='Uncertainty Bounds')
plt.title('Discharge Coefficient Estimation', fontsize = 15)
plt.xlabel('Time (s)', fontsize = 15)
plt.ylabel('Discharge Coefficient', fontsize = 15)
plt.grid(True)
plt.legend(fontsize = 12)
plt.xlim(start_time_liq, end_time_liq)
plt.ylim(0,0.8)
plt.gca().set_facecolor('white')
```

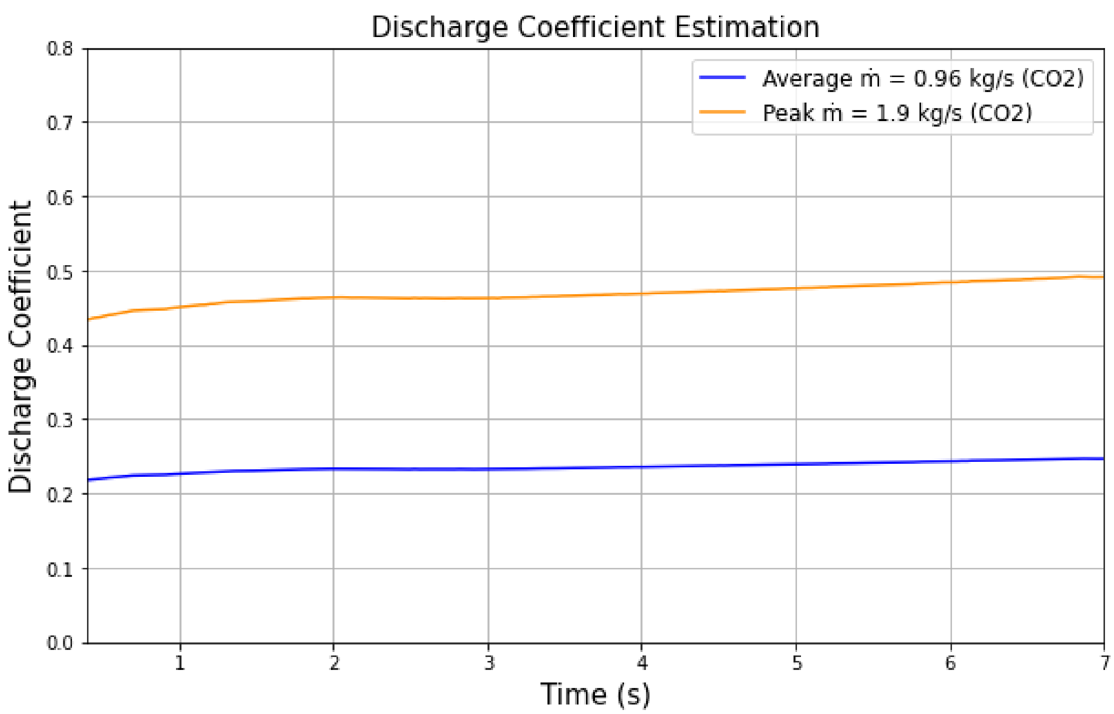
```
plt.savefig('discharge_coefficient.png', facecolor='white')
plt.show()
```







Mass Flow Rate
Time Derivative of Liquid Phase Mass (ṁ)
Average ṁ (Liquid Phase) = 0.96 kg/s
Peak ṁ = 1.9 kg/s
Uncertainty Bounds
Mass Flow Rate (kg/s)
Time (s)



Discharge Coefficient Estimation
Average ṁ = 0.96 kg/s (CO2)
Peak ṁ = 1.9 kg/s (CO2)
Discharge Coefficient
Time (s)