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MSE 120: *Electronic Properties of Materials* Professor Ya-Hong Xie
Winter Quarter, 2020

Suggested Problem Set No. 2

1. Evaluate the Fermi function for an energy k_BT above the Fermi energy. Find the temperature at which there is a 1% probability that a state, with an energy 0.5 eV above the Fermi energy, will be occupied by an electron.

2. Ultraviolet light of 0.2 μm wavelength is incident upon a metal. Which of the metals listed in Table below will emit electrons in response to the input light?

Table 6.2 *Work functions of metals*

Metal	Work function (eV)
Li	2.48
Na	2.3
K	2.2
Cs	1.9
Cu	4.45
Ag	4.46
Au	4.9
Mg	3.6
Ca	3.2
Ba	2.5
Al	4.2
Cr	4.6
Mo	4.2
Ta	4.2
W	4.5
Co	4.4
Ni	4.9
Pt	5.3

3. Determine the density of occupied states at an energy k_BT above the Fermi level. Fine the energy below the Fermi level which will yield the same density of un-occupied states.

4. Show that the average kinetic energy of free electron, following Fermi-Dirac statistics, is $(3/5)E_F$ at T = 0 K.

5. X-ray measurements show that electrons in the conduction band of lithium have energies up to 4.2 eV. Take this as the highest filled energy level in the band. If you further illustrate this energy level with the Fermi level, what average effective mass will give you the same result from free-electron theory? Assume one free electron per atom. The atomic weight of lithium is 6.94, and its density is 530 kg/m^3.

6. Suppose that all the vibrational branches of a one-dimensional crystal consisting of two different masses with equal spacing, a=3.0 can be represented by a single optical mode branch and a single acoustical mode branch. If the frequency of the long-wavelength optical waves is $2x10^{13}$ rad/s and the maximum frequency of acoustical waves is 10^{13} rad/s, what is the group velocity of long wavelength acoustical waves?

7. Write out the equation for thermal conductivity. Sketch three separate graphs that show the log of the heat capacity, the log of the phonon mean free path and the log of the thermal conductivity vs. log of temperature. Be sure you understand the interpretation of each part of each plot.

8. Consider a longitudinal wave

$$u_s = ucos(\omega t - sKa)$$

Which propagates in a monatomic linear lattice of atoms of mass M, spacing a, and nearest-neighbor interaction C.

a) Show that the total energy of the wave is

$$E = \frac{1}{2}M\sum_s (du_s/dt)^2 + \frac{1}{2}C\sum_s (u_s - u_{s+1})^2$$

Where s runs over all atoms.

b) By substitution of u_s in the expression, show that the time-average total energy per atom is

$$\frac{1}{4}M\omega^2u^2 + \frac{1}{2}C(1 - cosKa)u^2 = \frac{1}{2}M\omega^2u^2$$

Where in the last step we have used the dispersion relation for the problem. Dispersion relation:

$$\omega^2 = \frac{4C}{M}sin^2\frac{1}{2}Ka$$

9. Show that for long wavelengths the equation of motion (2)

$$M\frac{d^2u_s}{dt^2} = C(u_{s+1} + u_{s-1} - 2u_s)$$

reduces to the continuum elastic wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

Where v is the velocity of sound.