Yearwork DMS 2025-2026

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Winter Session 2025-2026

1 Question 1

1) Define a finite element model valid in the frequency range $0 \div 30$ Hz (use 1.5 as the safety coefficient to define the maximum length allowed for the beam elements).

1.1 Answer

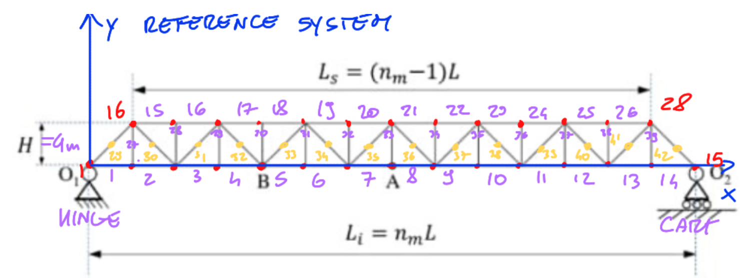

Figure 1: Finite element model of the bridge.

NODES: we are considering 28 nodes:

- 15 in the lower part (bottom) of the bridge;
- 13 above (upper part).

Remaining on the horizontal elements, we built them such that every 5 meters we have a node. Also at the start and the end of each vertical beam we have a node. Now switching to the diagonal elements of the bridge we need to consider something not very intuitive just seeing the image given by the delivery. We have to use 1.5 as a safety coefficient to define the maximum lentgh allowed for the beams element. In fact our doubt about diagonal's length is founded because:

$$diag_{Length} = \sqrt{4^2 + 5^2} = \sqrt{41} \cong 6.4 \tag{1}$$

While maximum allowed length for this type of element should be:

$$L_{diagmax} = \sqrt{\frac{\pi}{2\pi \cdot c \cdot f_{max}} \cdot \frac{\sqrt{E \cdot I}}{m}} \cong 4.78 \tag{2}$$

BEAMS: represented by each connection between 2 nodes; we consider as the picture shows a total of:

- 26 horizontal beams;
- 13 vertical;
- 28 diagonal.

The number 28 came from the computation made earlier in order to have a full diagonal connecting bottom with top that is divided into 2 pieces with a node at the center.
The frequency target (f) is computed using max frequency range value (30 Hz) and safety coefficient (1.5).

$$f_{target} = 30[Hz] \cdot 1.5 = 45[Hz] \tag{3}$$

So the computation of the maximum beam length type given f_{max} is

$$\overline{\Omega}_{1\text{min},j} = c \cdot 2\pi \cdot f_{max} = 1.5 \cdot 2\pi \cdot 30[Hz] = 282.74[s^{-1}] \tag{4}$$

Now we can compute the maximum length using the formula provided into the file for each beam type j:

$$L_{max,Top/Bottom} = \sqrt{\frac{\pi^2}{\overline{\Omega}_{1\text{min}}} \cdot \sqrt{\frac{9.929 \cdot 10^7}{90.09}}} = 6.0535 \tag{5}$$

Our horizontal beams are 5 meters, so $5 < 6.0535$ then they're okay.

$$L_{max,Vertcal} = \sqrt{\frac{\pi^2}{\overline{\Omega}_{1\text{min}}} \cdot \sqrt{\frac{1.599 \cdot 10^7}{59.9352}}} = 4.246 \tag{6}$$

Our vertical beam is $4 < 4.246$ so the model is valid. Finally

$$L_{max,Diag} = \sqrt{\frac{\pi^2}{\overline{\Omega}_{1\text{min}}} \cdot \sqrt{\frac{3.762 \cdot 10^7}{87.75}}} = 4.78 \tag{7}$$

But because we computed that each diagonal beam is 6.4 meters long we divide it into 2 pieces each of length 3.2 meters, so $3.2 < 4.78$ inequality is satisfied.

Let's dive into some property of the structural elements. In the .inp file used for the description of the bridge we have to insert Mass($\frac{kg}{m}$), EA(N) and EJ($N \cdot m^2$). We have ρ as volumetric density and the area (A) of the section for each beam. Then we can compute the linear density (μ) in this way:

$$\mu = \rho \cdot A \tag{8}$$

1.2 Horizontal beams, top and bottom

$\mu = 7.8 \cdot 10^3 \cdot 1.155 \cdot 10^{-2} = 90.09$
$EA = 2.06 \cdot 10^{11} \cdot 1.155 \cdot 10^{-2} = 2.379 \cdot 10^9$
$EJ = 2.06 \cdot 10^{11} \cdot 48200 \cdot 10^{-8} = 9.929 \cdot 10^7$
and doing the same for the other types we obtain the following values.

1.3 Vertical

$\mu = 59.93532$
$EA = 1.583 \cdot 10^9$
$EJ = 1.599 \cdot 10^7$

1.4 Diagonal

$\mu = 87.75$
$EA = 2.318 \cdot 10^9$
$EJ = 3.762 \cdot 10^7$

2 Question 2

Compute the natural frequencies and related modes of vibrations in the frequency range 0-30 Hz.
Up to now the information about our bridge are:

- Nodes = 42;
- Total D.O.F. = 123;
- Beam elements = 67;
- Total Mass = 22694.5685 [Kg].

Natural frequencies are evaluated from the software given and resulting in:

- Mode n.1 - 2.3196 [Hz]
- Mode n.2 - 8.3107 [Hz]
- Mode n.3 - 12.7247 [Hz]
- Mode n.4 - 17.6562 [Hz]
- Mode n.5 - 26.4648 [Hz]

And relative images are:

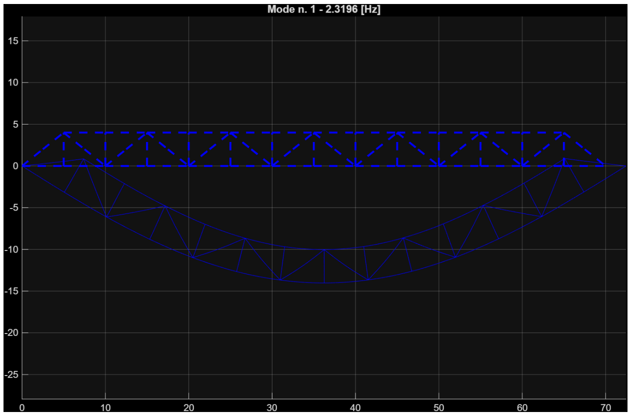

Figure 2: First mode

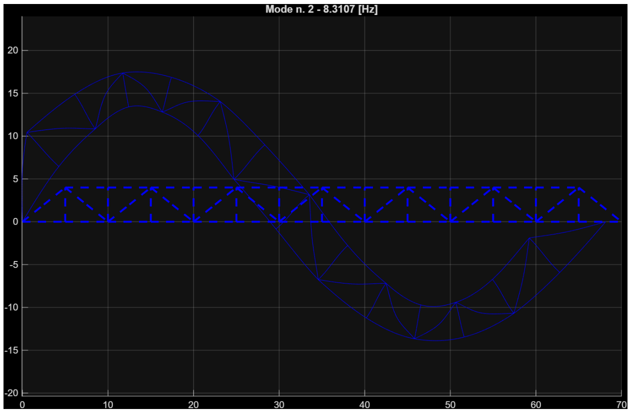

Figure 3: Second mode

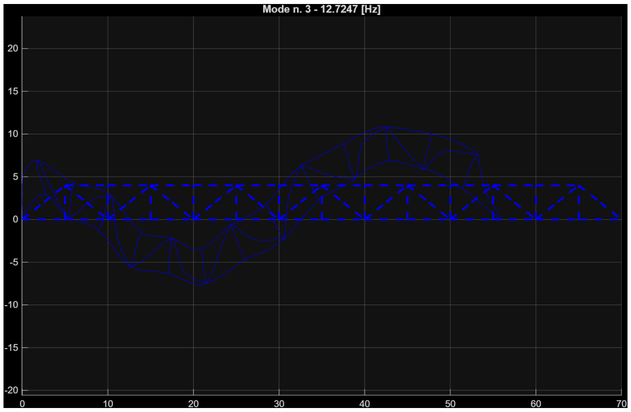

Figure 4: Third mode

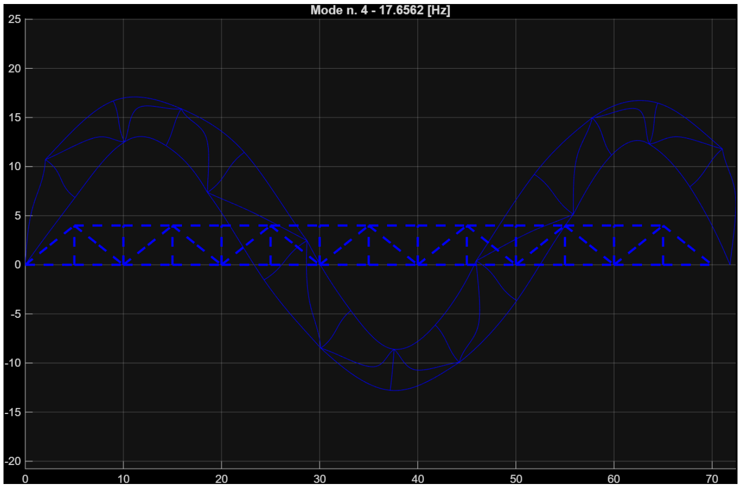

Figure 5: Fourth mode

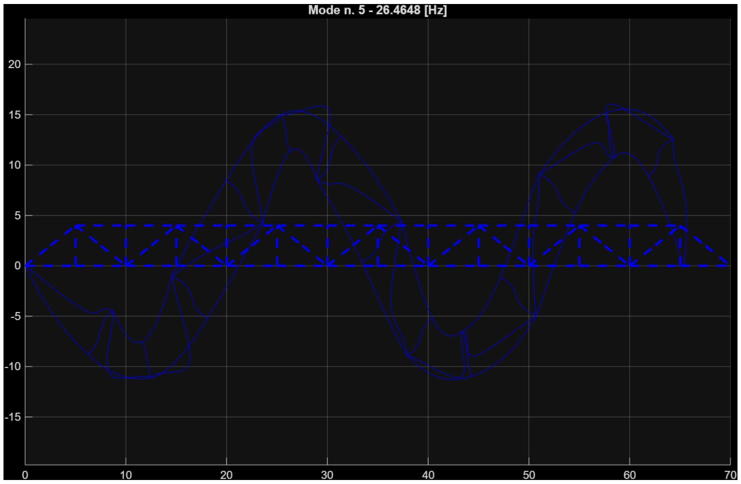

Figure 6: Fifth mode

3 Question 3

Compute the following frequency response function (FRF) in the frequency range $0 \div 30$ Hz with step 0.01 Hz:

- vertical displacement of point A produced by a vertical force on point A
- vertical displacement of point B produced by a vertical force on point A
- vertical acceleration of point A produced by a vertical force on point B
- vertical acceleration of point B produced by a vertical force on point B

Point A is at mid-span, while the distance between point B and O_1 is 2L.

3.1 Answer

Analyzing points of interest we have A at x = 35m and B at x = 20m from the origin. Using the software "frequency domain analysis" we just put inside the box the info the algorithm needs obtaining the results we want. Steps are reported just for the first sub question then proceeding similarly for the others:

1. N. Forces: 1;
2. Forced Node: 8;
3. Force displacement component: 2 (y);
4. Force Amplitude: 1[N];
5. Initial frequency: 0 [Hz];
6. Final frequency: 30 [Hz];
7. Step: 0.01 [Hz];
8. Output node: 8;
9. Output displacement: 2 (y).

The result is this one: Without making explicit all the *iter* for the other 3 tasks I provide just the results.

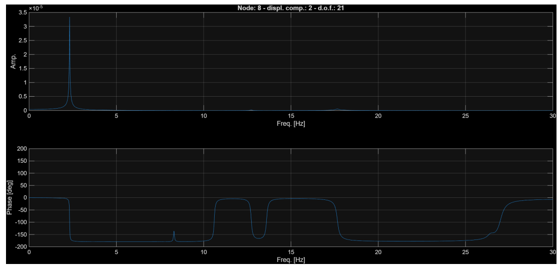

Figure 7: Vert disp. A force on A

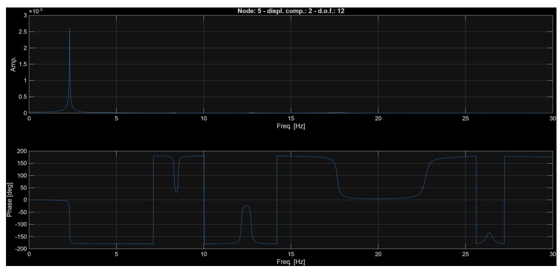

Figure 8: Vert disp. B force on A

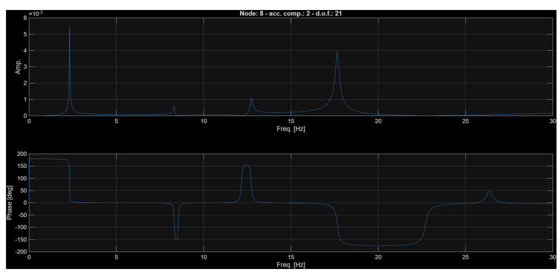

Figure 9: Vert acc. A force on B

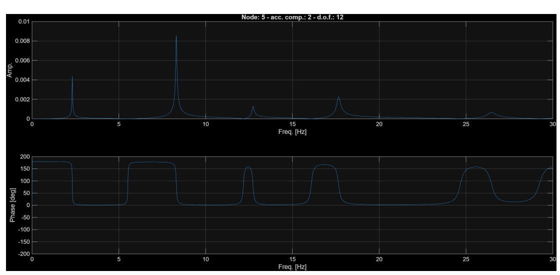

Figure 10: Vert acc. B force on B

4 Question 4

Compute the bridge response due to seismic motion of the ground, represented as vertical displacement of points O_1 and O_2 shown on the figure below:

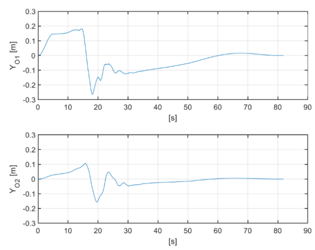

Figure 11: Seism

To solve the dynamic response of the bridge under non-uniform seismic excitation (support base displacement y_{O1} and y_{O2}), we adopted a Frequency Domain Analysis. This approach is computationally efficient and allows for the direct manipulation of the support excitation spectrum.

The general Equation of Motion (EoM) for the Multi-Degree-of-Freedom (MDOF) system is:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t) \tag{9}$$

Where $\mathbf{x}$ is the vector of displacements. Since the loading comes from prescribed support displacements rather than external forces on nodes, we partitioned the system matrices into *Free* (f) and *Constrained* (c) degrees of freedom:

$$\begin{bmatrix} \mathbf{M}_{ff} & \mathbf{M}_{fc} \\ \mathbf{M}_{cf} & \mathbf{M}_{cc} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_f \\ \ddot{\mathbf{x}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{ff} & \mathbf{C}_{fc} \\ \mathbf{C}_{cf} & \mathbf{C}_{cc} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}}_f \\ \dot{\mathbf{x}}_c \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ff} & \mathbf{K}_{fc} \\ \mathbf{K}_{cf} & \mathbf{K}_{cc} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_f \\ \mathbf{x}_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{R}_c \end{Bmatrix} \tag{10}$$

Where $\mathbf{x}_c$ represents the known seismic inputs (y_{O1}, y_{O2}) and $\mathbf{x}_f$ are the unknown responses of the bridge nodes. Considering the first row of the partitioned equation (dynamic equilibrium of free nodes), we obtain:

$$\mathbf{M}_{ff}\ddot{\mathbf{x}}_f + \mathbf{C}_{ff}\dot{\mathbf{x}}_f + \mathbf{K}_{ff}\mathbf{x}_f = -\left(\mathbf{M}_{fc}\ddot{\mathbf{x}}_c + \mathbf{C}_{fc}\dot{\mathbf{x}}_c + \mathbf{K}_{fc}\mathbf{x}_c\right) \tag{11}$$

To solve this, we applied the Fast Fourier Transform (FFT) to convert the time-domain signals into the frequency domain ($\mathbf{x}(t) \rightarrow \mathbf{X}(\omega)$). The differential equation transforms into an algebraic linear system for each frequency step ω_k:

$$\mathbf{A}_{sys}(\omega_k) \cdot \mathbf{X}_{free}(\omega_k) = \mathbf{Q}_{eff}(\omega_k) \tag{12}$$

Here, $\mathbf{A}_{sys}$ is the Dynamic Stiffness Matrix of the structure, and $\mathbf{Q}_{eff}$ is the Effective Load Vector induced by the support motion:

$$\mathbf{A}_{sys} = -\omega^2\mathbf{M}_{ff} + i\omega\mathbf{C}_{ff} + \mathbf{K}_{ff} \tag{13}$$

$$\mathbf{Q}_{eff} = -\left(-\omega^2\mathbf{M}_{fc} + i\omega\mathbf{C}_{fc} + \mathbf{K}_{fc}\right) \cdot \mathbf{Z}_0(\omega) \tag{14}$$

Where $\mathbf{Z}_0$ contains the FFT of the prescribed displacements. The system was solved for every frequency component up to the Nyquist limit. Finally, the time-history response was reconstructed using the Inverse Fast Fourier Transform (IFFT). Acceleration spectra were derived directly in the frequency domain by multiplying the displacement spectra by $-\omega^2$, followed by a low-pass filter (cut-off at 30 Hz) to remove numerical noise before reconstruction.

Listing 1: MATLAB Script for Seismic Analysis

```
clear all;
close all;
clc;

%% 1. Load Data & Prepare
load('bridge_mkr.mat'); % Contains M, K, R, idb
data = load('seismic_displ.txt'); %our input

% Rename R to C (Damping Matrix)
C = R;

% Time Setup
t = data(:,1); %colonna tempi
y_O1 = data(:,2); %colonna disp y01
y_O2 = data(:,3); %colonna disp y02

dt = t(2) - t(1); %oppure dt = 0.01
Fs = 1/dt;          %sampling frequency 100 Hz
N = length(t);      %8192
df = Fs/N;          %frequency resolution
freq_vec = (0:N-1)*df;

%% --- INPUT VERIFICATION (Matches PDF Image) ---
figure(100)
subplot(2,1,1);
plot(t, y_O1, 'b'); grid on;
title('CHECK: Input Time History y_{O1}'); ylabel('Disp [m]');
subplot(2,1,2);
plot(t, y_O2, 'r'); grid on;
title('CHECK: Input Time History y_{O2}'); ylabel('Disp [m]'); xlabel('Time [s]');

%% 2. Matrix Partitioning (Using idb)
% Flatten idb to find indices
n_free = 123; % 126 total - 3 constraints
ind_f = 1:n_free; % Equation numbers 1 to 123 are free
```

```
% Identify the Prescribed Equation Numbers
% O1 Horizontal (Fixed=0) -> Node 1, DOF 1
eq_O1_x = idb(1, 1); %124-esimo
% O1 Vertical (Seismic y_O1) -> Node 1, DOF 2
eq_O1_y = idb(1, 2); %125-esimo
% O2 Vertical (Seismic y_O2) -> Node 15, DOF 2 (since 70m/5m = 14 steps + 1 = Node 15)
eq_O2_y = idb(15, 2); %126-esimo

% Prescribed Indices in global matrix (Must match order of z0 vector below)
ind_p = [eq_O1_x, eq_O1_y, eq_O2_y];

% Extract Matrices
MFF = M(ind_f, ind_f);
CFF = C(ind_f, ind_f);
KFF = K(ind_f, ind_f);

MFP = M(ind_f, ind_p);
CFP = C(ind_f, ind_p);
KFP = K(ind_f, ind_p);

%% 3. TASK A: Spectrum of Inputs

Y_O1_fft = fft(y_O1);
Y_O2_fft = fft(y_O2);

figure(1)
plot(freq_vec, 2*abs(Y_O1_fft)/N, 'b', 'LineWidth', 1.2); hold on;
plot(freq_vec, 2*abs(Y_O2_fft)/N, 'r', 'LineWidth', 1.2);
xlim([0 5]); grid on;
legend('Spectrum y_{O1}', 'Spectrum y_{O2}');
title('Spectrum of Input Displacements');
xlabel('Frequency [Hz]'); ylabel('|Y| [m]');

%% 4. Main Solver Loop (Frequency Domain)
X_free_fft = zeros(n_free, N); % Store response spectrum for all free DOFs
%matrice nostre risposte righe i gradi liberi 123, colonne N cioe N
%time-steps
i_c = sqrt(-1);

for k = 1:N/2+1 % da 1 alla frequenza di Nyquist oltre la quale e' speculare il valore
    f_hz = freq_vec(k);
    ome = 2*pi*f_hz;

```

```
    % Dynamic Stiffness Matrix
    A_sys = -ome^2 * MFF + i_c*ome * CFF + KFF;

    % Input Vector z0 corresponding to ind_p = [eq_O1_x, eq_O1_y, eq_O2_y]

    z0 = [0; Y_O1_fft(k); Y_O2_fft(k)]; %column vector? matrix 3x1

    % Effective Force
    Q0 = -(-ome^2 * MFP + i_c*ome * CFP + KFP) * z0;

    % Solve for Free DOFs
    x0 = A_sys \ Q0;

    X_free_fft(:, k) = x0; %righe 123 gradi liberi per k steps delle frequenze

    % Conjugate Symmetry (for correct IFFT)
    if k > 1 && k < N/2+1
        X_free_fft(:, N-k+2) = conj(x0);
    end
end

%% 5. Extract Results for Points A and B
% Node A (Mid-span, 35m) -> Node 8
% Node B (2L, 20m) -> Node 5
eq_A = idb(8, 2); % Vertical DOF of Node 8
eq_B = idb(5, 2); % Vertical DOF of Node 5

Spec_Disp_A = X_free_fft(eq_A, :);
Spec_Disp_B = X_free_fft(eq_B, :);

% Compute Acceleration Spectra in Frequency Domain (Acc = -w^2 * Disp)
ome_vec = 2*pi*freq_vec;
Spec_Acc_A = -(ome_vec.^2) .* Spec_Disp_A; %acc for point 4
Spec_Acc_B = -(ome_vec.^2) .* Spec_Disp_B;

% Find index for 30 Hz
idx_30Hz = find(freq_vec > 30, 1);

% Create a cleaned spectrum copy
Spec_Acc_A_Clean = Spec_Acc_A;
Spec_Acc_B_Clean = Spec_Acc_B;

% Zero out everything above 30 Hz
Spec_Acc_A_Clean(idx_30Hz:end) = 0;
Spec_Acc_B_Clean(idx_30Hz:end) = 0;

```

```
% Compute Time Histories (IFFT)
%Question For point 3 and half 4
time_disp_A = real(ifft(Spec_Disp_A));
time_disp_B = real(ifft(Spec_Disp_B));
time_acc_A  = real(ifft(Spec_Acc_A_Clean))*2;
time_acc_B  = real(ifft(Spec_Acc_B_Clean))*2;

%% 6. PLOTTING REMAINING TASKS

% TASK B: Spectrum of Vertical Displacements (A & B)
figure(2)
subplot(2,1,1);
plot(freq_vec, abs(Spec_Disp_A)/N, 'b'); xlim([0 15]); grid on;
title('Spectrum of Displacement - Point A'); ylabel('|X_A| [m]');
subplot(2,1,2);
plot(freq_vec, abs(Spec_Disp_B)/N, 'r'); xlim([0 15]); grid on;
title('Spectrum of Displacement - Point B'); ylabel('|X_B| [m]'); xlabel('Frequency [Hz]');

% TASK C: Time Histories of Vertical Displacements (A & B)
figure(3)
subplot(2,1,1);
plot(t, time_disp_A, 'b'); grid on;
title('Time History Displacement - Point A'); ylabel('Disp [m]');
subplot(2,1,2);
plot(t, time_disp_B, 'r'); grid on;
title('Time History Displacement - Point B'); ylabel('Disp [m]'); xlabel('Time [s]');

% TASK D: Spectrum & Time History of Vertical Accelerations
figure(4)
subplot(2,2,1);
plot(freq_vec, abs(Spec_Acc_A)/N, 'b'); xlim([0 30]); grid on;
title('Spectrum Acceleleration A'); ylabel('|Acc|');
subplot(2,2,2);
plot(t, time_acc_A, 'b'); grid on;
title('Time History Acceleleration A'); ylabel('m/s^2');

subplot(2,2,3);
plot(freq_vec, abs(Spec_Acc_B)/N, 'r'); xlim([0 30]); grid on;
title('Spectrum Acceleration B'); ylabel('|Acc|'); xlabel('Hz');
subplot(2,2,4);
plot(t, time_acc_B, 'r'); grid on;
title('Time History Accel B'); ylabel('m/s^2'); xlabel('s');
```

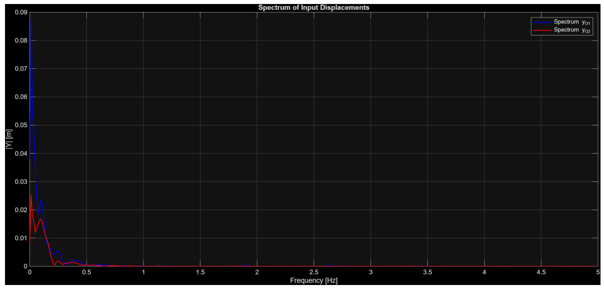

Figure 12: Spectrum of the input displacements YO1, YO2

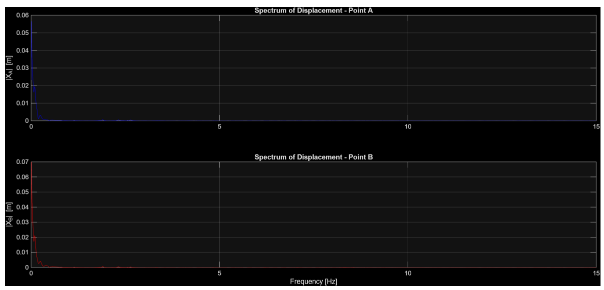

Figure 13: Spectrum of the vertical displacements of points A and B

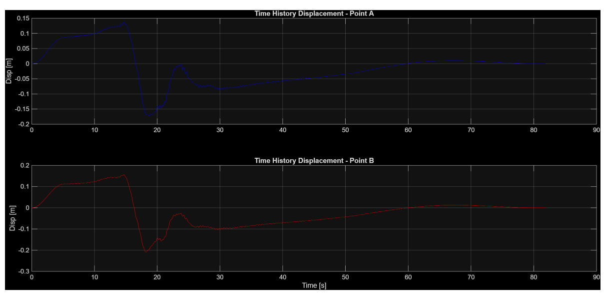

Figure 14: Time histories of the vertical displacements of points A and B

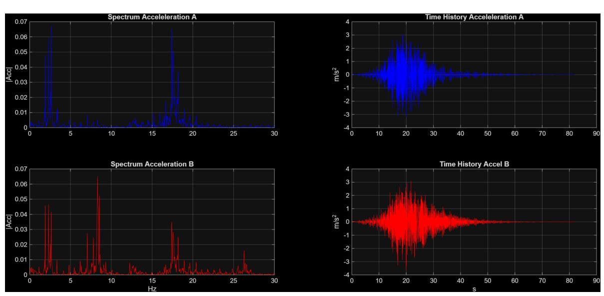

Figure 15: Spectrum and the time histories of the vertical accelerations of points A and B

5 Question 5

Considering the passage with constant speed V of a sequence of moving concentrated loads with distance of 26m one to the other, discuss the possibility of producing a resonance condition in the bridge for specified values of train speed V. To this end, consider an infinite sequence of moving load (approximation of a long train). Hint use the modal superimposition approach to answer this question.

5.1 Answer

The primary dynamic concern for high-speed railway bridges is Resonance, a phenomenon where the excitation frequency of the moving load coincides with one of the bridge's natural frequencies ($f_{exc} \approx f_n$). This leads to a theoretical unbounded increase in amplification, limited only by the system's damping. The train does not act as a single moving force but as a series of repeating loads (axles/bogies). This periodicity creates a forcing function that operates at specific frequencies related to the train speed (V) and the characteristic length of the carriages (d).

Mathematical Formulation of Excitation to identify the critical speeds, we employed the analytical approach derived by Wu & Zhang (2022)[1]. The excitation frequencies f_{exc} induced by a train with characteristic repeating length d are given by:

$$f_{exc,k} = k \cdot \frac{V}{d} \tag{15}$$

Where:

- V is the train speed [m/s].
- d is the characteristic repeating length ($d = 26$m for standard carriages).
- k is the integer harmonic order ($k = 1, 2, 3 \dots$).

This linear relationship indicates that as the train accelerates, the excitation frequency increases. When this diagonal "excitation ray" intersects with the horizontal line of a structural natural frequency (f_n), resonance occurs. The critical speed (V_{crit}) for a specific mode n and harmonic k is therefore:

$$V_{crit} = \frac{f_n \cdot d}{k} \cdot 3.6 \quad \text{[km/h]} \tag{16}$$

Methodology: Modal Superposition to simulate the bridge response, we utilize the Modal Superposition Method rather than direct time integration. This approach assumes the total response $\nu(x, t)$ is a weighted sum of the mode shapes $\phi_i(x)$:

$$\nu(x, t) \approx \sum_{k=1}^{N} \phi_k(x) q_k(t) \tag{17}$$

This method allows us to:

1. Filter High-Frequency Noise: We restrict the analysis to the range 0-30 Hz, ignoring higher modes that contribute negligible displacement.

2. Capture Global Continuity: Although the load is applied locally to specific nodes, the use of global mode shapes (ϕ_k) ensures that the coupling between elements (Node k influencing Node $k+1$) is mathematically preserved via the stiffness matrix $\mathbf{K}$.

```matlab
%% Question 5: Final Campbell Diagram (Interference Plot)
clc; close all;

% 1. Input Data from Yearwork and previous points [cite: 68, 18]
d = 26; % Axle spacing [m]
V_kmh = 0:1:400; % Speed range [km/h]
V_ms = V_kmh / 3.6; % Convert to m/s

% Natural Frequencies [Hz] provided from your Question 2 analysis
fn = [2.3196, 8.3107, 12.7247, 16.5432, 26.4648];
num_harmonics = 8;

% 2. Figure Setup
figure('Color', [0.15 0.15 0.15], 'Name', 'Campbell Diagram - Question 5');
ax = axes('Color', [0.1 0.1 0.1], 'XColor', 'w', 'YColor', 'w');
hold on; grid on;
ax.GridColor = [0.3 0.3 0.3];

% 3. Plot Natural Frequencies (Horizontal White Lines) [cite: 715]
for i = 1:length(fn)
    yline(fn(i), 'w--', 'LineWidth', 1.2, 'Alpha', 0.5);
end

% 4. Force Natural Frequency Values onto the Y-Axis
standard_ticks = 0:5:35;
all_ticks = unique(sort([standard_ticks, fn]));
set(ax, 'YTick', all_ticks);
set(ax, 'XTick', 0:50:400);

% 5. Plot Loading Harmonics and Intersections [cite: 868]
colors = hsv(num_harmonics);
h_harmonics = gobjects(num_harmonics, 1);
h_res_point = []; % Dummy for legend

for k = 1:num_harmonics
    f_exc = k * (V_ms / d); % f_k = k * V / d
```

```matlab
    h_harmonics(k) = plot(V_kmh, f_exc, 'Color', colors(k,:), 'LineWidth', 2, ...
        'DisplayName', sprintf('Harmonic k = %d', k));

    % Mark Resonance Points
    for n = 1:length(fn)
        v_res_kmh = (fn(n) * d / k) * 3.6;
        if v_res_kmh <= 400
            h_res_point = plot(v_res_kmh, fn(n), 'wo', 'MarkerSize', 5, ...
                'MarkerFaceColor', 'w', 'HandleVisibility', 'off');
        end
    end
end

% 6. Add Resonance Point to Legend
% Creating a dummy plot for the legend entry
h_legend_point = plot(nan, nan, 'wo', 'MarkerFaceColor', 'w', 'DisplayName', 'Resonance Points');

% 7. Labels and Aesthetics
xlabel('Train Speed V [km/h]', 'FontSize', 11, 'FontWeight', 'bold');
ylabel('Frequency [Hz]', 'FontSize', 11, 'FontWeight', 'bold');
title('Campbell Diagram: Bridge resonance vs Train harmonics', 'Color', 'w', 'FontSize', 13);

% Combine harmonic handles and the resonance point handle for the legend
legend([h_harmonics; h_legend_point], 'TextColor', 'w', 'Color', 'none', ...
    'Location', 'northwest', 'NumColumns', 2);

xlim([0 400]); ylim([0 35]);

% Validity Limit
yline(30, 'r--', 'FE Validity Limit (30 Hz)', 'Color', 'r', 'LabelVerticalAlignment', 'bottom');
```

What are we expecting? We can check easily and manually some results of the first mode intercepts: 1^{st} on 2.3196 Hz k=1:

$$V_1 = \frac{2.3196 \cdot 26}{1} = 60.3096 \text{m/s} \cong 217.115 \text{km/h} \tag{18}$$

and for k=2 and k=3:

$$V_1 = \frac{2.3196 \cdot 26}{2} = 30.1548 \text{m/s} \cong 108.557 \text{km/h} \tag{19}$$

$$V_1 = \frac{2.3196 \cdot 26}{3} = 20.1032\text{m/s} \cong 72.3715\text{km/h} \tag{20}$$

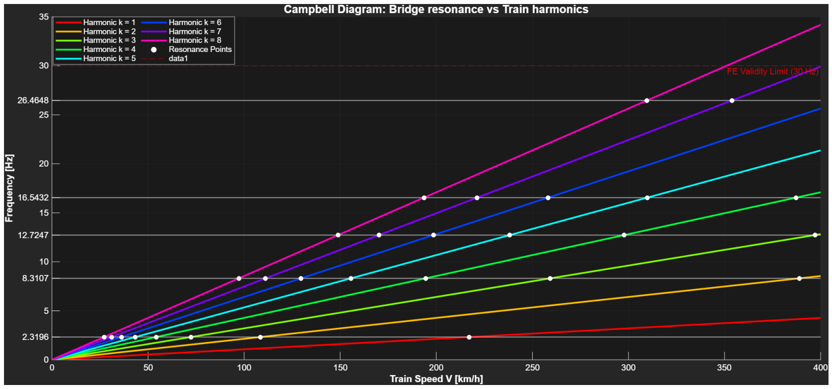

Figure 16: Speeds at which the train generates resonance

Interpretation of Results (The Interference Diagram) The resulting Interference Diagram plots the bridge's natural frequencies (horizontal lines) against the train's excitation harmonics (diagonal lines).

- Primary Harmonics: We placed particular emphasis on the first eight harmonics ($k = 1$ to 8), as these carry the highest energy content and pose the greatest risk to structural safety.
- Critical Intersections: The "dots" on the diagram represent the exact speeds where resonance is predicted. These mathematical predictions serve as the inputs for the Time History Analysis in the subsequent step.

Validation: The observed linear increase of excitation frequency with speed confirms the "Loading Rate Resonance" mechanism described by [1], distinct from the "Speed Parameter Resonance" associated with the crossing time of the span.

6 Question 6

Define a structural change of the bridge that allows for a 15% reduction of the maximum amplitude of the vertical acceleration of point B produced by a vertical force on point B. The maximum allowed increase of the total mass of the bridge is +5%. It is not allowed to introduce additional constraints acting on the structure.

6.1 Answer

Let's reasonate; the total mass increase allowed is 5% so being the total mass approximately 22690.7 Kg (22694.5 on the software) we can add up to 1134.535

Kg. Translating the weight in number of beams for each type we obtain:

$$N_{Horizontal} = \frac{1134.535}{90.09 \cdot 5} = 2.52 \sim 2 \tag{21}$$

$$N_{Vertical} = \frac{1134.535}{59.9352 \cdot 4} = 4.73 \sim 4 \tag{22}$$

$$N_{Diagonal} = \frac{1134.535}{87.75 \cdot 3.2} = 4.04 \sim 4 \tag{23}$$

The local solution we found based on the data we have is the following one shown in the figure:

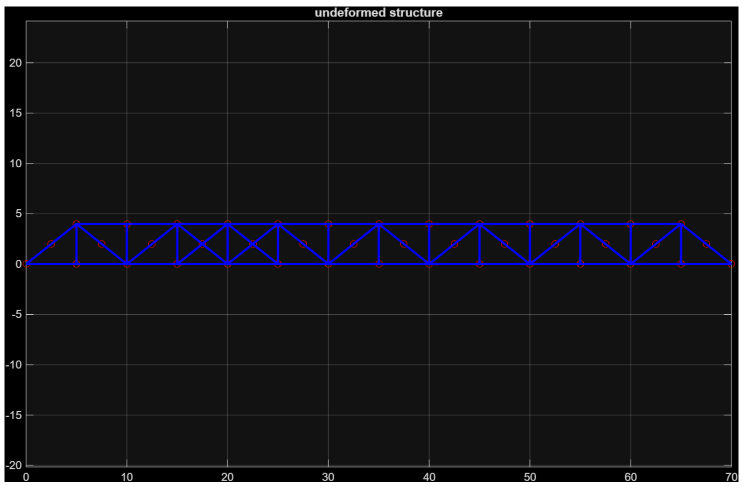

Figure 17: Local Solution

Having as results of the vertical acceleration of point B (node 5) produced by a vertical force on point B this:

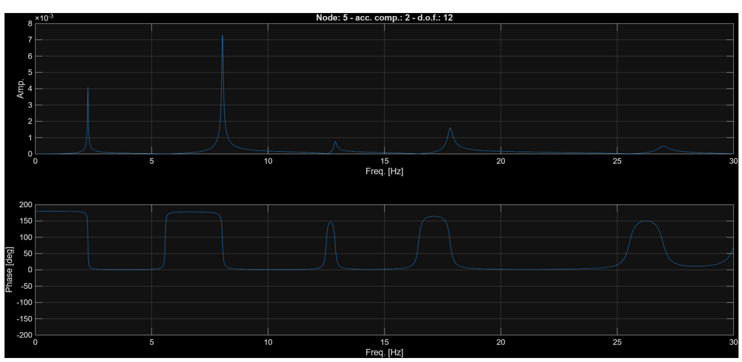

Figure 18: Vertical acc. node B

We got as maximum value on y-axis 0.00729 $\frac{m}{s^2}$; so we lowered the peak of $0.0086 - 0.00729 \cong 0.0013$ so at the end enough to stay in the specific.

$$Reduction_{\%} \cong \frac{1.3 \cdot 10^{-5}}{8.6 \cdot 10^{-3}} = 15.11\% \tag{24}$$

6.2 Symmetrical (Global) solution

We are not satisfied with the previous solution because it's not symmetrical and elegant, but it represents a valid local solution. Thinking about something more general we have to start from this image in order to do a reasoning.

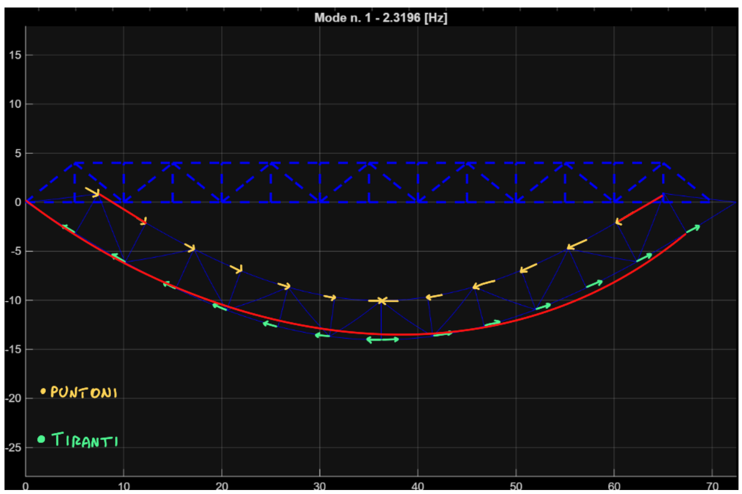

Figure 19: First mode bridge's deformation

The idea is to change bottom horizontal beams in order to erode mass that we can put for diagonal beams near points B and B', and if we have enough mass then adding diagonal toward the extremity. It should be enough just change the horizontal bottom beams. Why horizontal and not vertical, because as it can be seen from the image the lower beams are tie beams so they have to resist traction more than compression, therefore can be more tiny; instead the upper beams are struts and cannot be changed cause it's more dangerous.
We arrived switching the IPE500 with IPE360 having the following characteristics:

- Area (A) = 72.73 cm^2
- Bending Stiffness (J) = 16300 cm^4
- Linear mass = 57.1 $\frac{kg}{m}$

So computing EA and EJ we got respectively $1.498 \cdot 10^9$ and $3.3578 \cdot 10^7$; then we insert those parameter in the new .inp file.

Now we can compute the mass gained:

$$(90.09 - 57.1) \cdot 5 \cdot 14 = 2309.3[Kg] \tag{25}$$

$$1134.535 + 2309.3 = 3443.835[Kg] \tag{26}$$

$$\frac{3443.835}{280.8} = 12.26 \sim 12 \tag{27}$$

So we can add up to 12 diagonal HEA300 beams. We do it obtaining the following bridge:

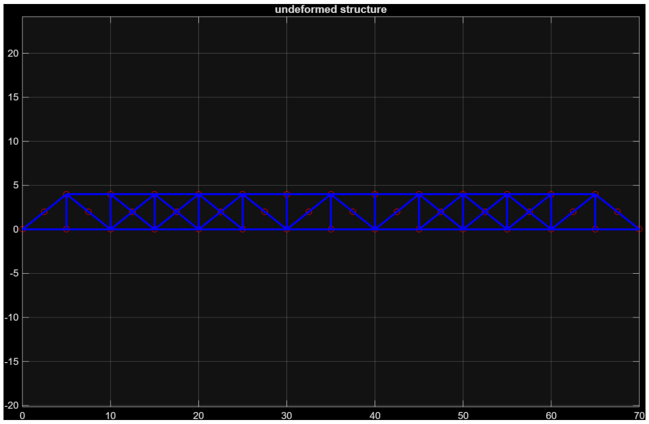

Figure 20: General solution bridge

The total weight is $23756[kg] < 23829[kg]$ and most important the response in acceleration of point B is shown in the picture:

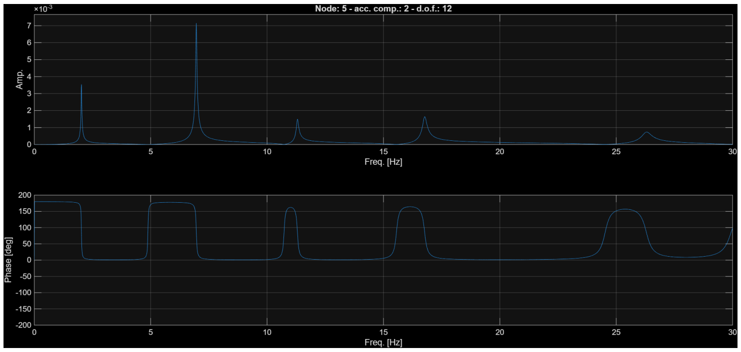

Figure 21: Caption

The top value on y-axis reached by point B is $0.00713983 \sim 0.00714$ so the difference with basic bridge is $0.00857 - 0.00174 = 0.00143$ that in percentage

(%) it's equivalent to:

$$\frac{0.00143 \cdot 10^2}{0.00857} = 16.68 \sim 16.7\% > 15\% \tag{28}$$

We have found a solution that is feasible, simple and realistic. Not too expensive and without changing specific constraints of the bridge neither other fixed parameter as length of the bridge and of the beams.

References

[1] Jinbao Yao Zhaozhi Wu, Nan Zhang and Vladimir Poliakov. Wavelet time-frequency analysis on bridge resonance in train-track-bridge interactive system. Applied Sciences, Beijing, China, 2022.