Here, I present three of the heuristic functions explored and the best performance is the compound function used in agent AB_Custom. Following are brief descriptions of the three heuristic functions:

Custom_2:
The function is defined as: (# of moves for `player`)-2x(# of moves for `opponent`). It enables `player` to take more aggressive moves against its opponent so that it will have more available moves.

Custom_3:
The function is defined as the Manhattan distance between `player` and `opponent`. This enables the `player` to take moves that re further away from opponent

Custom:
This is a compound heuristic function. It consists of three pasts depending on the # of empty spaces on the board.

When # of empty spaces on the board is more than three fifth of the total space, game is considered as beginning stage and the function is defined as : # of moves for `player` - # of moves for `opponent`
- This provides a rough approxiamation of the "goodness" of a board, but it is fast and enables a deeper search
- I chose search depth over heuristic quality for the beginning of the game because the board is very open

When # of empty spaces on the board is more than one fifth but less than three fifth of the total board space, game is considered as middle stage and the function is defined as weighted score of moves near wall or at corners. The general idea is that at the beginning of the middle stage, moves near wall or at corner is rewarded for the potential of occupying more spaces. But at later stages, such action is penalized for the potential of being cornered.
- This is a much better heuristic, and helps to find situations where opponents are completely walled off from each other
- This is slower than previous heuristics, but because it is only used in middle game, the board has fewer empty spaces, so it is not terrible

Lastly, at the final stage, the heuristics is chosen as: Length of longest path player can walk minus length of longest path opponent can walk
- This heuristic is perfectly accurate if the players are walled off from another, because whoever can make more moves will win
- This is an expensive heuristic that would fail for any reasonable depth in the beginning or middle game

Results of the three heuristics are shown in two tables below for 40 games. As depicted in the table, the compound heuristic performs the best and consistently outperform the other heuristic functions. This is expected since it takes into account more complicated scenarios and is therefore more effective in winning game

****
Playing Matches
****

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	17	3	19	1	19	1	18	2
2	MM_Open	13	7	14	6	16	4	16	4
3	MM_Center	18	2	19	1	19	1	15	5
4	MM_Improved	14	6	16	4	16	4	14	6
5	AB_Open	12	8	9	11	9	11	9	11
6	AB_Center	12	8	13	7	13	7	12	8
7	AB_Improved	8	12	12	8	14	6	9	11
	Win Rate:	67.1%		72.9%		75.7%		66.4%	

****
Playing Matches
****

Match #	Opponent	AB_Improved		AB_Custom		AB_Custom_2		AB_Custom_3	
		Won	Lost	Won	Lost	Won	Lost	Won	Lost
1	Random	18	2	19	1	19	1	18	2
2	MM_Open	16	4	17	3	16	4	15	5
3	MM_Center	18	2	17	3	17	3	17	3
4	MM_Improved	15	5	19	1	13	7	15	5
5	AB_Open	12	8	11	9	12	8	9	11
6	AB_Center	9	11	10	10	10	10	10	10
7	AB_Improved	7	13	9	11	9	11	6	14
	Win Rate:	67.9%		72.9%		68.6%		64.3%	