

Tarefa Básica 8

Matriz Inversa

$$1. \begin{pmatrix} x & 1 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 \\ y & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 3x + y = 1 \\ 15 + 3y = 0 \end{cases} \quad \begin{cases} -x + 2 = 0 \rightarrow -x = -2 \cdot (-1) \\ -5 + 6 = 1 \quad x = \boxed{2} \end{cases}$$

$$\rightarrow 3y = -15$$

$$y = \frac{-15}{3} = \boxed{-5}$$

$$x + y \rightarrow 2 - 5 = \boxed{-3}$$

(C)

2.

$$\det A^{-1} = \frac{1}{\det A}$$

$$\det A \neq 0$$

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ k & 1 & 3 & k & 1 \\ 1 & k & 3 & 1 & k \end{vmatrix}$$

$$3 + 0 + k^2$$

$$\det A \rightarrow k^2 - 3k + 2$$

$$k^2 - 3k + 2 = 0$$

$$\frac{1}{1} + \frac{2}{2} = 3$$

$$1 \cdot 2 = 2$$

$$\boxed{1 \text{ e } 2}$$

(C)

3.

$$A = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} \quad 12 - 10 = 2 \neq 0$$

$$B = \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix} \div \det A$$

(C)

$$B = \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix}$$

4.

$$\begin{vmatrix} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{vmatrix}$$

$$x^2 - 5x + 6 \neq 0$$

$$x^2 - 5x + 6 \neq 0$$

$$\frac{2}{2} + \frac{3}{3} = 5$$

$$\frac{2}{2} \cdot \frac{3}{3} = 6$$

$$\begin{matrix} x' \neq 2 \\ x'' \neq 3 \end{matrix}$$

(A)

$$5. \begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix} \begin{matrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{matrix} \rightarrow \det A = 1$$

$$A = \begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix} \quad A^{-1} = \begin{pmatrix} 1_2 & 0_3 & 1_4 \\ -1_3 & -1_4 & 0_5 \\ 0_1 & -2_5 & 1_6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} \div \det A \rightarrow \bar{A}^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A + A^{-1} = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$$

(B)

$$6. (X \cdot A)^T = B$$

$$((X \cdot A)^T)^T = B^T$$

$$X \cdot A = B^T$$

$$X = B^T \cdot A^{-1}$$

(B)

7. $A \cdot B = C$

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 4x + 5y \\ 5x + 6y \end{pmatrix}$$

↓

$$A = \begin{pmatrix} 4 & 5 \\ 5 & 6 \end{pmatrix}$$

$$\det A = 24 - 25 = -1$$

$$A^{-1} = \begin{pmatrix} 6 & -5 \\ -5 & 4 \end{pmatrix} \div \det A$$

$$\textcircled{1}$$

$$A^{-1} = \begin{pmatrix} -6 & 5 \\ 5 & -4 \end{pmatrix}$$

8.

$$A = \begin{pmatrix} 2 & k \\ -2 & 1 \end{pmatrix}$$

$$\det A = 2 + 2k = \det A^{-1}$$

$$(2 + 2k) \cdot (2 + 2k) = 1 \neq 0$$

$$(2 + 2k) \cdot (2 + 2k) \neq 0$$

$$4 + 4k + 4k + 4k^2 \neq 0$$

$$4k^2 + 8k + 4 \neq 0$$

$$\underline{-1} + \underline{-1} = -2$$

$$\underline{-1} \cdot \underline{-1} = 1$$

$$K' = -1$$

$$K'' = -1$$

$$-1 + -1 = \underline{-2}$$

\textcircled{B}

9.

a) $(A+B) \cdot (A-B)$

$$\boxed{A^2 - AB + BA - B^2}$$

b) $(A+B)^2 = A^2 + 2 \cdot A \cdot B + B^2$

$$(A+B) \cdot (A+B) = A^2 + 2AB + B^2$$

$$\cancel{A^2} + AB + BA + \cancel{B^2} = \cancel{A^2} + 2AB + \cancel{B^2}$$

$$AB + BA = 2AB$$

$$AB + BA = AB + AB$$

$$AB - AB + BA = AB \rightarrow \boxed{AB = BA}$$

c) $A = \begin{pmatrix} x & y \\ a & b \end{pmatrix} \quad \det A = xb - ay$

$$\frac{\det A}{\det -A} = \frac{xb - ay}{xb - ay}$$

$$= \boxed{1}$$

$-A = \begin{pmatrix} -x & -y \\ -a & -b \end{pmatrix} \quad \det -A = xb - ay$

d) $\det A \cdot \det A^{-1} = 1$

$$\det A^{-1} = \frac{1}{\det A}$$

\rightarrow

$$\boxed{\det B = \frac{1}{\det A}}$$