

Tarefa Básica 6

- Regra de Cramer -

1. a)

$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$$

$$x = \frac{3}{5} \quad y = \frac{-4}{5}$$

$$V = \left\{ \left(\frac{3}{5}, \frac{-4}{5} \right) \right\}$$

b)

$$\begin{cases} 3x - y + z = 1 \\ 2x + 0 + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = 3(0 - 3) - 1(-2 - 12) + 1(2 - 4) = -9 + 10 - 2 = -1$$

$$D_x = \begin{vmatrix} -1 & 1 & 1 \\ 0 & 3 & -1 \\ 1 & -2 & 1 \end{vmatrix} = -1(3 - 1) - 1(3 - 1) + 1(-3 - 1) = -2 - 2 - 4 = -8$$

$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} = 3(-2 - 21) - 1(-4 - 12) + 1(14 - 8) = -69 + 16 + 6 = -47$$

$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 1 \end{vmatrix} = 3(0 - 1) - 1(2 - 4) + 1(2 - 4) = -3 + 2 - 2 = -3$$

$$x = \frac{-8}{-1} = 8 \quad y = \frac{-47}{-1} = 47 \quad z = \frac{-3}{-1} = 3$$

$$V = \{ (8, 47, 3) \}$$

2.

$$\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \quad D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 4 & -1 & 3 & 4 \\ 4 & 5 & 2 & 4 & 5 \\ 1 & -2 & 3 & 1 & -2 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 3 & -1 & 3 \\ 4 & 2 & 4 \\ 1 & 3 & 8 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 3 & 3 & -1 \\ 4 & 2 & 4 & 4 & 2 \\ 1 & 3 & 8 & 1 & 3 \end{vmatrix} \quad y = \frac{30}{30} = 1$$

$$78 - 48 = 30$$

A

3.

$$\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & 1 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 2 & 1 & -11 \\ 1 & 3 & -1 \\ -1 & -22 & -6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & -11 & 2 & 1 \\ 1 & 3 & -1 & 1 & 3 \\ -1 & -22 & -6 & -1 & -22 \end{vmatrix} \quad x = \frac{-1}{1} = -1$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} \quad y = \frac{1}{1} = 1$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} \quad z = \frac{0}{1} = 0$$

C

$$a + b + c = 1 + 1 + 0 = 0$$

4.

$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{vmatrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{vmatrix} \quad -130 + 146 = 16 \quad x = \frac{16}{16} = 1$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{vmatrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{vmatrix} \quad 26 + 54 = 80 \quad y = \frac{80}{16} = 5$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix} \quad 3 - 99 = -96 \quad z = \frac{-96}{16} = -6$$

(A)

$$5. \begin{cases} 2x + y + z = 5 \\ 0 + 2y + 2z = 3 \\ 3x + 2y + z = 7 \end{cases} \quad D = \begin{vmatrix} 2 & 1 & 1 \\ 0 & 2 & 2 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 5 & 1 & 1 \\ 3 & 2 & 2 \\ 7 & 2 & 1 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{vmatrix} \quad 17 - 13 = 4 \quad D_y = \begin{vmatrix} 2 & 5 & 1 \\ 0 & 3 & 2 \\ 3 & 7 & 1 \end{vmatrix} \begin{vmatrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix} \quad 37 - 42 = -5 \quad x = \frac{4}{3} \quad y = \frac{7}{3} \quad z = \frac{-5}{3}$$

$$\left[\frac{4}{3}; \frac{7}{3}; -\frac{5}{3} \right]$$

(D)

$$\begin{cases} x & = 3 \\ 2x + y & = 7 \\ -x + 2y + 2z & = -1 \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix}$$

$$Dy = \begin{array}{ccc|ccc} & 1 & 3 & 0 & 1 & 3 & 12 \\ 2 & 2 & 0 & 2 & 7 & 14 & -32 = \boxed{2} \\ -1 & -1 & 2 & -1 & -1 & & \end{array}$$

$$D_R = \begin{array}{ccc|cc} & & & -3 & 14 & 0 \\ 1 & 0 & 3 & 1 & 0 & \\ \hline 2 & 1 & 7 & 2 & 1 & 11-11=0 \\ \hline 1 & 2 & -1 & -1 & 2 & \\ & & & -1 & 0 & 12 \end{array}$$

$$x = \frac{6}{2} = \boxed{3} \quad y = \frac{2}{2} = \boxed{1} \quad z = \frac{0}{2} = \boxed{0}$$

E

Tarefa Básica 6 - Escalonamento -

1.

$$S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{array}{l} -2 \left[\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right] \begin{array}{l} 3 \\ 0 \\ -7 \end{array} \left[\begin{array}{ccc|c} 0 & -7 & -1 & -27 \\ 0 & -7 & -1 & -27 \\ 0 & -3 & -4 & -8 \end{array} \right] \sim \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 25 & -25 \end{array} \right) \quad \begin{array}{l} 25z = -25 \\ \boxed{z = -1} \end{array} \quad \begin{array}{l} -3y + 4 = -8 \\ -3y = -12 \\ \boxed{y = 4} \end{array}$$

$$2x - y + 3 = -5$$

$$2x = -4$$

$$\boxed{x = -2}$$

$$\boxed{x = -2; y = 4; z = -1}$$

2.

$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \quad \begin{matrix} 2y = 3z \\ z = \frac{2y}{3} \end{matrix} \quad \begin{matrix} x + y + z = 11 \\ 2y + y + \frac{2y}{3} = 11 \\ \frac{3y + 3y + 2y}{3} = 11 \end{matrix}$$

$$\begin{matrix} x = 2y & 2y = 3z & 6y + 3y + 2y = 33 \\ \underline{x = 6} & 3z = 6 & \cancel{3} \quad \quad \quad \cancel{3} \\ & \underline{z = 2} & 11y = 33 \\ & & \underline{y = 3} \end{matrix}$$

$$x + 2y + 3z$$

$$6 + 6 + 6 = \underline{18}$$

B

3.

$$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 0 + 6y + 3z = -12 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ -3 & 0 & 6 & -12 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \\ -3 & -3 & 3 & -12 \end{array} \right] \sim$$

$$\left[\begin{array}{ccc|c} & & & \\ & & & \\ -15 & 0 & 0 & -15 \end{array} \right] \quad \begin{matrix} -15x = -15 \\ \underline{x = 1} \end{matrix} \quad \begin{matrix} -3 + 3y = -12 \\ 3y = -9 \\ \underline{y = -3} \end{matrix} \quad \begin{matrix} 1 - 3 + z = 0 \\ \underline{z = 2} \end{matrix}$$

D

4.

$$\begin{cases} A + B + C = 68 \\ 0,2C + B = A \\ 0,2A + C = 3B \end{cases}$$

$$0,2A + C = 3B$$

$$\textcircled{I} \rightarrow 0,2A + C = 3(34 - 0,6C)$$

$$0,2A + C = 102 - 1,8C$$

$$2,8C + 0,2A = 102$$

$$(0,2C + B) + B + C = 68$$

$$1,2C + 2B = 68$$

$$2B = 68 - 1,2C$$

$$\textcircled{I} B = 34 - 0,6C$$

$$2,8C + 0,2(0,2C + B) = 102$$

$$2,8C + 0,04C + 0,2B = 102$$

$$2,84C + 0,2B = 102$$

$$B = 34 - 0,6 \cdot 35$$

$$\textcircled{II} 2,84C + 0,2(34 - 0,6C) = 102$$

$$B = 34 - 21$$

$$2,84C - 0,12C + 6,8 = 102$$

$$\underline{B = 13}$$

$$2,72C = 95,2$$

$$\underline{C = 35}$$

$$A + B + C = 68$$

$$A + 13 + 35 = 68$$

$$A = 68 - 48$$

$$\underline{A = 20}$$

$$Al: = R\$ 20,00$$

$$Bia = R\$ 13,00$$

$$Caco = R\$ 35,00$$

Al: tem 7 reais a mais
que Bia e 15 reais a
menos que Caco.

A

5.

$$\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

D =

$$\begin{array}{ccc|ccc} 0 & 3 & 4 & 0 & 3 & \\ 1 & 0 & 5 & 1 & 0 & 134 \\ 2 & 1 & 0 & 2 & 1 & 48 \end{array}$$

$$D_x = \begin{array}{ccc|ccc} 134 & 3 & 4 & 134 & 3 & \\ 115 & 0 & 5 & 115 & 0 & \\ 48 & 1 & 0 & 48 & 1 & \end{array}$$

$$1180 - 670 = 510$$

$$D_y = \begin{array}{ccc|ccc} 0 & 134 & 4 & 0 & 134 & \\ 1 & 115 & 5 & 1 & 115 & \\ 2 & 48 & 0 & 2 & 48 & \end{array}$$

$$1532 - 920 = 612$$

$$D_z = \begin{array}{ccc|ccc} 0 & 3 & 134 & 0 & 3 & \\ 1 & 0 & 115 & 1 & 0 & \\ 2 & 1 & 48 & 2 & 1 & \end{array}$$

$$824 - 144 = 680$$

$$x = \frac{510}{34} = 15$$

$$y = \frac{612}{34} = 18$$

$$z = \frac{680}{34} = 20$$

$$x + y + z$$

$$15 + 18 + 20 = R\$ 53,00$$

A