

Tarefa Básica 11

Teorema do Binômio

1. $(1 + 2x^2)^6$ coeficiente de x^8 ?

$$\binom{6}{k} \cdot 1^{6-k} \cdot (2x^2)^k \Rightarrow \binom{6}{k} 2^k \cdot x^{2k} \quad \begin{cases} 2k = 8 \\ k = 8/2 \\ k = 4 \end{cases} \rightarrow$$

$$\Rightarrow \binom{6}{4} 2^4 \cdot x^8 = \frac{6!}{4!(6-4)!} \cdot 16 \cdot x^8 = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2!} \cdot 16 \cdot x^8 \rightarrow$$

$$\Rightarrow \frac{30}{2} \cdot 16 \cdot x^8 = 15 \cdot 16 \cdot x^8 = \underline{\underline{240 x^8}} \quad \text{C}$$

2.

considerando x e y iguais a 1, podemos calcular a soma dos coeficientes:

$$(14x - 13y)^{237} = (14 \cdot 1 - 13 \cdot 1)^{237} = 1^{237} = \underline{\underline{1}} \quad \text{B}$$

3.

$$(x + 2)^{11}$$

$$\binom{11}{k} x^{11-k} \cdot 2^k = 1386 x^5$$

$$11 - k = 5$$

$$k = 6$$

$$\binom{11}{6} x^{11-6} \cdot 2^6 = 1386 x^5$$

$$\frac{11!}{6!(11-6)!} \cdot x^5 \cdot 2^6 = 1386 x^5 \rightarrow$$

\Rightarrow

$$\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 2^6 = 1386$$

$$\frac{55440}{120} \cdot 2^6 = 1386$$

$$462 \cdot 2^6 = 1386$$

$$2^6 = 1386$$

$$2^6 = 3$$

$$2 = \sqrt[6]{3} \quad \text{A}$$

$$4. \left(x + \frac{1}{x^2} \right)^9 \rightarrow (x + x^{-2})^9$$

$$\left. \begin{aligned} \binom{9}{k} \cdot x^{9-k} \cdot (x^{-2})^k \\ -3k = -9 \rightarrow k = 3 \end{aligned} \right\} \begin{aligned} 9-k-2k &= 0 \\ -3k &= -9 \end{aligned}$$

$$\boxed{\binom{9}{3}} \cdot x^{9-3} \cdot (x^{-2})^3$$

(D)

$$5. \left(x + \frac{1}{x^2} \right)^n \rightarrow (x + x^{-2})^n$$

$$\left. \begin{aligned} \binom{n}{k} \cdot x^{n-k} \cdot (x^{-2})^k \\ n-3k=0 \rightarrow 3k=n \rightarrow k = \frac{n}{3} \end{aligned} \right\} \begin{aligned} n-k-2k &= 0 \\ n-3k &= 0 \end{aligned}$$

(C)

se n é
divisível por 3

✓

$$6. \quad \left(3x^3 + \frac{2}{x^2} \right)^5 - \left(243x^{15} + 810x^{10} + 1080x^5 + \frac{240}{x^5} + \frac{32}{x^{10}} \right) \quad 1$$

$$1(3x^3)^5 + 5(3x^3)^4(2x^{-2}) + 10(3x^3)^3(2x^{-2})^2 + 10(3x^3)^2(2x^{-2})^3 + 5(3x^3)(2x^{-2})^4 + 1(2x^{-2})^5$$

$$1 \cdot 3^5 x^{15} + 5 \cdot 3^4 x^{12} \cdot 2x^{-2} + 10 \cdot 3^3 x^9 \cdot 2^2 x^{-4} + 10 \cdot 3^2 x^6 \cdot 2^3 x^{-6} + 5 \cdot 3x^3 \cdot 2^4 x^{-8} + 1 \cdot 2^5 x^{-10}$$

$$\{ 243x^{15} + 810x^{10} + 1080x^5 + 720x^0 + 240x^{-5} + 32x^{-10} \}$$

$$243x^{15} + 810x^{10} + 1080x^5 + 720x^0 + 240x^{-5} + 32x^{-10} - (243x^{15} + 810x^{10} + 1080x^5 + 240x^{-5} + 32x^{-10})$$

$$\cancel{243x^{15}} + \cancel{810x^{10}} + \cancel{1080x^5} + 720 \cdot 1 + \cancel{240x^{-5}} + \cancel{32x^{-10}} - \cancel{243x^{15}} - \cancel{810x^{10}} - \cancel{1080x^5} - \cancel{240x^{-5}} + \cancel{32x^{-10}}$$

$$720 \cdot 1 = \boxed{720}$$

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7. considerando x e y iguais a 1 ...

$$(2x + y)^5 = (2 \cdot 1 + 1)^5 = 3^5 = \boxed{243}$$

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