Diagonalizable

λ_1

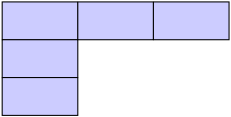
Minimal poly: $(x - \lambda_1)$

Char poly: $(x - \lambda_1)^3$

Geom mult: 3 (3 towers)

Non-Diagonalizable

λ_2



Minimal poly: $(x - \lambda_2)^3$

Char poly: $(x - \lambda_2)^5$

Geom mult: 3 (3 towers)

$$\Lambda_{\text{diag}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_1 \end{bmatrix} \qquad \Lambda_{\text{non-diag}} = \begin{bmatrix} \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 & 0 \\ 0 & 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & 0 & \lambda_2 \end{bmatrix}$$

Jordan blocks:

$$\begin{bmatrix}\lambda_1\end{bmatrix},\ \begin{bmatrix}\lambda_1\end{bmatrix},\ \begin{bmatrix}\lambda_1\end{bmatrix}$$

Jordan blocks:

$$\begin{bmatrix}\lambda_2\end{bmatrix},\ \begin{bmatrix}\lambda_2\end{bmatrix},\ \begin{bmatrix} \lambda_2 & 1 & 0 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

Generalized eigenspace for λ_j :

$$G_{\lambda_j} = \ker((T - \lambda_j I)^{k_j}), \quad V = \bigoplus_{j=1}^{s} G_{\lambda_j}$$

Theorem (Primary Decomposition). Let $T\colon V \to V$ be a linear operator on a finite-dimensional vector space V over $\mathbb{C}$, and let

$$m_T(x) = \prod_{j=1}^{s} (x - \lambda_j)^{k_j}$$

be the factorization of the minimal polynomial of T into powers of distinct linear factors. Then

$$V = \bigoplus_{j=1}^{s} V_j, \qquad \text{where } V_j = \ker\bigl((T - \lambda_j I)^{k_j}\bigr).$$

Each V_j is T-invariant, and the restriction $T|_{V_j}$ has minimal polynomial $(x - \lambda_j)^{k_j}$.