

$$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^\top$$

$$\mathbf{A}^\top = \mathbf{V} \mathbf{\Sigma} \mathbf{U}^\top$$

$$\mathbf{A}^{-1} = \mathbf{V} \mathbf{\Sigma}^{-1} \mathbf{U}^\top$$

$$(\mathbf{A}^\top)^{-1} = (\mathbf{A}^{-1})^\top = \mathbf{U} \mathbf{\Sigma}^{-1} \mathbf{V}^\top$$

$$\mathbf{A}^\top \mathbf{A} = \mathbf{V} \mathbf{\Sigma}^2 \mathbf{V}^\top$$

$$\mathbf{A} \mathbf{A}^\top = \mathbf{U} \mathbf{\Sigma}^2 \mathbf{U}^\top$$

$$(\mathbf{A}^\top \mathbf{A})^{-1} = \mathbf{A}^{-1} (\mathbf{A}^{-1})^\top = \mathbf{V} \mathbf{\Sigma}^{-2} \mathbf{V}^\top$$

$$(\mathbf{A} \mathbf{A}^\top)^{-1} = (\mathbf{A}^{-1})^\top \mathbf{A}^{-1} = \mathbf{U} \mathbf{\Sigma}^{-2} \mathbf{U}^\top$$

$$\mathbf{A}^+ = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top = \mathbf{V} \mathbf{\Sigma}^{-1} \mathbf{U}^\top$$

$$= \mathbf{V} \mathbf{\Sigma}^+ \mathbf{U}^\top$$

(strictly speaking)

The last expression here is the Moore-Penrose pseudo-inverse.  $\mathbf{\Sigma}^{-1}$  is only defined if all the singular values,  $\sigma_i$ , are positive (when  $\mathbf{A}$  is invertible).  $\mathbf{\Sigma}^+$ , on the other hand, reciprocates only the positive singular values.