HAMT

Heat and Mass Transfer

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1 Introduction

Text [1].

2 Boundary Conditions

2.1 Heat Flux

Given a boundary segment we can prescribe the a heat flux normal to that boundary as

$$\lambda \left(\vec{n} \cdot \nabla T\right) = \dot{q}_{\perp} \tag{2.1}$$

where λ is the thermal conductivity, $\vec{n}$ the normal vector of the boundary, ∇T the temperature gradient at a given point in the cell and $\dot{q}_{\perp}$ the differential heat flux perpendicular to the boundary.

We use linear element as our ansatz function for T and triangles as cells which means that $\vec{n}$ and ∇T are constants. If we now assume a per cell constant thermal conductivity λ this means that the differential heat flux $\dot{q}_{\perp}$ must be constant over a given cell side.

2.2 Radiation

The net radiation heat flux from surface 1 to surface 2 using grey body radiation can be calculated as

$$\dot{Q}_{1\to 2} = A_1 F_{1\to 2} E_1 - A_2 F_{2\to 1} E_2. \tag{2.2}$$

using the formula for emission of grey bodies

$$E_i = \epsilon_i \sigma T_i^4 \tag{2.3}$$

and the reciprocity rule for configuration factors $A_1 F_{1\to 2} = A_2 F_{2\to 1}$ we can write

$$\dot{Q}_{1\to 2} = \sigma A_1 F_{1\to 2} \left(\epsilon_1 T1^4 - \epsilon_2 T_2^4\right) \tag{2.4}$$

and $\dot{Q}_{1\to 2} = -\dot{Q}_{2\to 1}$. Given two line segments as seen in Fig. 2.1

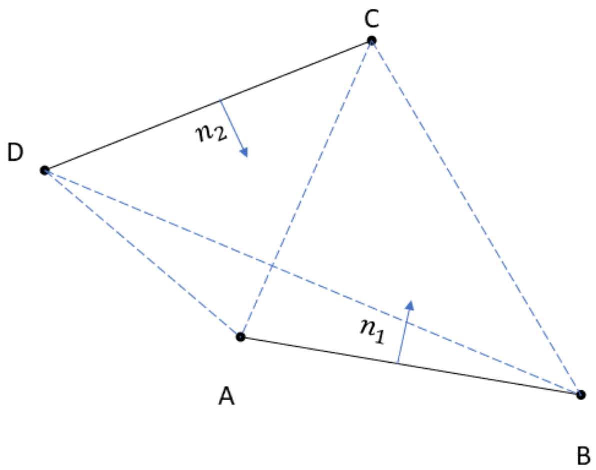

Figure 2.1: Radiation.

the configuration factor from surface $\overline{\mathrm{AB}}$ to surface $\overline{\mathrm{CD}}$ can be calculated as

$$F_{\overline{\mathrm{AB}}\to\overline{\mathrm{CD}}} = \frac{\overline{\mathrm{AC}} + \overline{\mathrm{BD}} - \overline{\mathrm{AD}} - \overline{\mathrm{BC}}}{2\overline{\mathrm{AB}}} \tag{2.5}$$

where $\overline{\mathrm{XY}}$ is the distance from X to Y. The total heat flux from or to a single surface is the sum of the heat fluxes to other surfaces plus the heat flux to the background

$$\dot{Q}_{tot} = \sigma A_1 \left\{ \epsilon_1 T_1^4 \sum_i F_{1\to i} - \sum_i \epsilon_i F_{1\to i} T_i^4 \right\} + \dot{Q}_{backgr} \tag{2.6}$$

$$\dot{Q}_{tot} = \sigma A_1 \left\{ \epsilon_1 T_1^4 \sum_i F_{1\to i} - \sum_i \epsilon_i F_{1\to i} T_i^4 + F_{1\to bg}(\epsilon_i T_1^4 - \epsilon_{bg} T_{bg}^4) \right\} \tag{2.7}$$

Since $F_{1\to bg} = 1 - \sum_i F_{1\to i}$ we get

$$\dot{Q}_{tot} = \sigma A_1 \left\{ \epsilon_1 T_1^4 - \sum_i \epsilon_i F_{1\to i} T_i^4 - F_{1\to bg})\epsilon_{bg} T_{bg}^4 \right\} \tag{2.8}$$

In terms of boundary conditions, using $\dot{q}_{1\to 2} = \dot{Q}_{1\to 2}/A_1$,we ca write

$$\lambda\,(\vec{n}\cdot\nabla T) = \dot{q}_{1\to 2} \tag{2.9}$$

Bibliography

[1] Hans Dieter Baehr and Karl Stephan. *Wärme- und Stoffübertragung.* Berlin, Heidelberg: Springer Berlin Heidelberg, 2019. ISBN: 978-3-662-58440-8 978-3-662-58441-5. DOI: 10.1007/978-3-662-58441-5. URL: http://link.springer.com/10.1007/978-3-662-58441-5 (visited on 10/20/2024).