Supporting Information for "The thermospheric auroral red line Angle of Linear Polarisation"

J. Lilensten,[1] Mathieu Barthélemy ,[1] G. Besson,[2] Hervé Lamy,[3] Magnar G. Johnsen,[4], and Jøran Moen[5]

Contents of this file

1. Text S1 to S5

Corresponding author: J. Lilensten, UJF-Grenoble 1 / CNRS-INSU, Institut de Planétologie et d'Astrophysique de Grenoble (IPAG), UMR 5274, Grenoble, F-38041, France (jean.lilensten@univ-grenoble-alpes.fr)

[1]UJF-Grenoble 1 / CNRS-INSU, Institut de Planétologie et d'Astrophysique de Grenoble (IPAG), UMR 5274, Grenoble, F-38041, France

[2]Institut Fourier, Université de Grenoble, France

[3]Belgian Institute for Space Aeronomy, Ringlaan-3-Avenue Circulaire, B-1180 Brussels, Belgium

[4]Tromsø Geophysical Observatory University of Tromsø, Norway

2. Figures S1 to S10

1. Introduction

Computing the apparent angle of the magnetic field as seen by the polarimeter in any direction is a classical spherical geometry problem. However, it is difficult to find the development in geophysical articles or textbooks. Therefore, we chose to provide it here. It is not something new, reason to write it as an appendix.

2. Geographical coordinates

The geography is plotted in Figure S1. The SPP is located at a point A characterized by a latitude ϕ_A and a longitude Ψ_A. It looks at the polarisation with an elevation ϵ_A and an azimuth α_A. The elevation is the angle with the plan which is tangent to the sphere in A and the azimuth is the angle with the meridian in A. It is taken equal to zero pointing South, to 90° West, 180° north and 270° East.

In [*Lilensten et al.*, 2015], we have shown that the theoretical polarisation equals the observations at the altitude at which the red line emission maximizes. Let us call h this altitude. In the above comparison, following [*Lilensten et al.*, 2015], $h = 220\ km$. We call H the point where the line of sight is at an altitude h. If O is the center of the Earth, the axis $\bar{OH}$ intersects the surface of the Earth at a point P of latitude ϕ_P and longitude

[5]Department of Physics, University of

Oslo, P.O. Box 1048, N-0316 Blindern, Oslo,

Norway

Ψ_P. The line of sight $\overrightarrow{AH}$ has a norm $|AH|$ and is carried by the unit vector $\overrightarrow{u}$.

The frame of reference is as follows: Axis x is the altitude, positive upward. Axis y is horizontal and points positively toward the east. Axis z is horizontal and points positively toward the north. Let us compute ϕ_P and Ψ_P. In the following, R_T^A is the radius of the Earth in A.

3. First step: A at the equator

We now consider that A is located at the equator such as shown in Figure S2, along the zeroth meridian. The reference frame in A is the same as the reference frame in O, the center of the Earth, translated by a distance R_t (the radius of the Earth) along the axis x. We call {X,Y,Z} the absolute frame centered around O. A magnification of the geometry around a is plotted in figure S3.

$$\left\{ \begin{array}{lcl} \phi_A & = & 0 \\ \Psi_A & = & 0 \end{array} \right. \tag{1}$$

The projection of $\overrightarrow{u}$ in the horizontal plane is (Figure S2):

$$\left\{ \begin{array}{lcl} u_y & = & -\sin(\alpha_{AH}) \\ u_z & = & -\cos(\alpha_{AH}) \end{array} \right. \tag{2}$$

With the choice of the azimuth oriented positively clockwise. Taking the elevation into account, one gets:

$$\left\{ \begin{array}{lclc} u_x & = & \sin(\epsilon_{AH}) & \text{altitude} \\ u_y & = & \sin(\alpha_{AH})\cos(\epsilon_{AH}) & \text{estward} \\ u_z & = & -\cos(\alpha_{AH})\cos(\epsilon_{AH}) & \text{northward} \end{array} \right. \tag{3}$$

The vector that links the center of the Earth O to the observed point H is $\overrightarrow{OH} = \overrightarrow{OA} + |AH| \ \overrightarrow{u}$. We can develop on the three axis:

$$\overrightarrow{OH} = \begin{cases} R_T^A + |AH| \sin(\epsilon_{AH}) \\ -|AH| \sin(\alpha_{AH}) \cos(\epsilon_{AH}) \\ -|AH| \cos(\alpha_{AH}) \cos(\epsilon_{AH}) \end{cases} \tag{4}$$

The norm of $\overrightarrow{OH}$ is $|OH| = \sqrt{OH_x^2 + OH_y^2 + OH_z^2}$, which gives:

$$|OH| = \sqrt{(R_T^A)^2 + |AH|^2 + 2R_T^A |AH| \sin(\epsilon_{AH})} \tag{5}$$

We can now deduce the vector $\overrightarrow{OP}$ which is simply equal to $\overrightarrow{OH}\frac{R_T^A}{|OH|}$. To specify that this is at the equator, we call it $\overrightarrow{OP}(eq)$:

$$\overrightarrow{OP}(eq) = \frac{R_T^A}{\sqrt{(R_T^A)^2 + |AH|^2 + 2R_T^A |AH| \sin(\epsilon_{AH})}} \begin{cases} R_T^A + |AH| \sin(\epsilon_{AH}) \\ -|AH| \sin(\alpha_{AH}) \cos(\epsilon_{AH}) \\ -|AH| \cos(\alpha_{AH}) \cos(\epsilon_{AH}) \end{cases} \tag{6}$$

The height of the observation point H above the Earth is $|PH|$, which we take equal to 220 km. This value is the difference $|OH| - |OP(eq)|$. We suppose that the radius of the Earth is the same for $|OA|$ and $|OP(eq)|$ so that:

$$|PH| = \sqrt{(R_T^A)^2 + |AH|^2 + 2R_T^A |AH| \sin(\epsilon_{AH})} - R_T^A \tag{7}$$

from which we easily get the value of $|AH|$ (the distance between the observation point and the observed one) through:

$$|AH|^2 + 2|AH| \ R_T^A \sin(\epsilon_{AH}) - |PH|(|PH| + 2R_T^A) = 0 \tag{8}$$

The altitude component $OP_x(eq)$ is not equal to the radius of the Earth R_T^A since it is a projection on the altitude axis going through A. Only if $\epsilon_{AH} = \frac{\pi}{2}$, i.e. one looks to the zenith, one retrieves that $|OP(eq)| = R_T^A$ and $|PH| = |AH|$. This of course is checked in the code solving this set of equations.

From equations 3 and 8, one deduces the vector $\overrightarrow{AH}$:

$$\overrightarrow{AH} = |AH| \ \overrightarrow{u}. \tag{9}$$

4. Second step: At any location

The geometry of the problem is displayed in Figure S1. The SPP may be positioned anywhere on Earth.

From this vector $|OP(eq)|$, we can easily deduce the latitude $\phi_P(eq)$ and the longitude $\Psi_P(eq)$ of the point P where the red line is observed. Its elevation is still taken to 220 km (see Figure S3).

$$\begin{cases} \sin(\phi_P(eq)) & = \frac{OP_z(eq)}{|OP(eq)|} \\ \cos(\phi_P(eq)) & = \frac{\sqrt{OP_x^2(eq)+OP_y^2(eq)}}{|OP(eq)|} \end{cases} \tag{10}$$

$$\begin{cases} \sin(\Psi_P(eq)) & = \frac{OP_y(eq)}{\sqrt{OP_x^2(eq)+OP_y^2(eq)}} \\ \cos(\Psi_P(eq)) & = \frac{OP_x(eq)}{\sqrt{OP_x^2(eq)+OP_y^2(eq)}} \end{cases} \tag{11}$$

Retrieving the latitude and longitude of P at any location consists simply in adding the latitude and longitude of A:

$$\Psi_P = \Psi_P(eq) + \Psi_A \tag{12}$$

$$\Phi_P = \Phi_P(eq) + \Phi_A \tag{13}$$

The results above Ny-Ålesund are shown in Figure S4.

5. Apparent angle of observation

We use IGRF [*Olsen et al.*, 2000] to get the azimuth α_H and elevation ϵ_H at the observed point of coordinates ($latitude\ = \phi_P\ longitude\ = \Psi_P\ heigh = 220\ km$). The configuration is shown in Figure S5. The next question to solve is that of the apparent angle of the magnetic field in P seen from A.

IGRF provides the three coordinates of the magnetic field $\{B_x, B_y, B_y\}$ in the frame of P, characterized by three axis $\{x_P, y_P, z_P\}$. It is necessary to rotate these values in the reference frame of A, characterized by $\{x_A, y_A, z_A\}$. This is illustrated in Figure S6.

The first rotation is around the axis Z with an angle of $\Psi_A - \Psi_P$. It allows to bring the magnetic field coordinates at the longitude of A. The corresponding matrix is S_1:

$$S_1 = \begin{pmatrix} \cos(\Psi_A - \Psi_P) & \sin(\Psi_A - \Psi_P) & 0 \\ -\sin(\Psi_A - \Psi_P) & \cos(\Psi_A - \Psi_P) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{14}$$

The second rotation brings the field coordinate to the latitude of A. It consists in a rotation around the axis x_A of coordinates $\{\sin(\Psi_A), \cos(\Psi_A), 0\}$ with an angle $\Phi_A - \Phi_P$. The corresponding matrix is S_2:

$$S_2 = \begin{pmatrix} \sin^2(\Psi_A) + \cos^2(\Psi_A)\cos(\phi_A - \Phi_P) & -\sin(\Psi_A)\cos(\Psi_A)(1 - \cos(\phi_A - \Phi_P)) & \cos(\Psi_A)\sin(\phi_A - \Phi_P) \\ -\sin(\Psi_A)\cos(\Psi_A)(1 - \cos(\phi_A - \Phi_P)) & \cos^2(\Psi_A) + \sin^2(\Psi_A)\cos(\phi_A - \Phi_P) & \sin(\Psi_A)\sin(\phi_A - \Phi_P) \\ -\cos(\Psi_A)\sin(\phi_A - \Phi_P) & -\sin(\Psi_A)\sin(\phi_A - \Phi_P) & \cos(\phi_A - \Phi_P) \end{pmatrix} \tag{15}$$

So that the vector of the magnetic field in the reference frame of P $\overrightarrow{B_P(P)}$ can now be written in the reference frame of A:

$$\overrightarrow{B_A(P)} = S_2\ S_1\ \overrightarrow{B_P(P)} \tag{16}$$

Computing the magnetic angle seen by the SPP is performed through a last set of rotations that brings the face of the polarimeter in the same frame $\{x_A,y_A,z_A\}$ than $\overrightarrow{B_A(P)}$. The rotations are illustrated in Figure S7. It consists in rotating by an angle of $\pi - \alpha$ (where α is the azimuth) around the axis x_A followed by a rotation of an angle ϵ (elevation) around the axis y_{spp} of coordinates $\{0,\sin(\alpha),-\cos(\alpha)\}$.

The first rotation is S_3:

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\pi - \alpha) & \sin(\pi - \alpha) \\ 0 & -\sin(\pi - \alpha) & \cos(\pi - \alpha) \end{pmatrix} \tag{17}$$

The second rotation is S_4:

$$S_4 = \begin{pmatrix} \cos(\epsilon) & \cos(\alpha)\sin(\epsilon) & \sin(\alpha)\sin(\epsilon) \\ -\cos(\alpha)\sin(\epsilon) & \sin^2(\alpha) + \cos^2(\alpha)\cos(\epsilon) & -\sin(\alpha)\cos(\alpha)(1-\cos(\epsilon)) \\ -\sin(\alpha)\sin(\epsilon) & -\sin(\alpha)\cos(\alpha)(1-\cos(\epsilon)) & \cos^2(\alpha) + \sin^2(\alpha)\cos(\epsilon) \end{pmatrix} \tag{18}$$

So that we can now write the vector of the magnetic field $\overrightarrow{B(SPP)}$ as seen by the polarimeter:

$$\overrightarrow{B(SPP)} = S_4\ S_3\ \overrightarrow{B_A(P)} \tag{19}$$

The apparent angle of the magnetic field is η defined simply by:

$$\begin{cases} \sin(\eta) &= \frac{B_y(SPP)}{\sqrt{B_x^2(SPP)+B_y^2(SPP)}} \\ \cos(\eta) &= \frac{B_x(SPP)}{\sqrt{B_x^2(SPP)+B_y^2(SPP)}} \end{cases} \tag{20}$$

Figure S8 shows this angle as a function of the elevation in four geographic directions. Obviously, at a right angle elevation of 90°, all the values converge since this is a zenith pointing.

The wriggle at 85° in the south and north azimuth is due to the shape of the magnetic field, and more exactly to the eastward component of the magnetic field. As seen in Figure S9, this component turns from positive to negative for the northern azimuth above 86° while it turns from negative to positive for the southern azimuth.

6. Angle between the line of sight and the magnetic field in P versus the SPP elevation.

In *Lilensten et al.* [2015], we computed the theoretical Degree of Linear Polarisation. This value depends on the the angle Θ between the line of sight $|AH|$ (equation 9) and the local magnetic field line in P $\overrightarrow{B_P(P)}$ (equation 16). It is simply computed through:

$$\cos(\Theta) = \frac{\overrightarrow{AH}.\overrightarrow{B_A(P)}}{|AH||B_A(P)|} \tag{21}$$

In Figure S10, we show this angle. Because we are still relatively low at 220 km, the magnetic field lines are almost vertical straight lines. At an elevation of 30° as in the present study, this angle is 67.2° in the west azimuth and 52° in the Eastward direction. North and south directions give roughly the same angle of 60°. This contradicts our first articles [*Lilensten et al.*, 2008] where we mentioned that our north pointing was close to a right angle. The error was due to the fact that we had used a simplified geometry. This has a effect on the published values of the DoLP (out of the scope of this article). The measured $DoLP_{obs}$ is a projection of the absolute value $DoLP_{abs}$ on the line of sight. We have therefore:

$$DoLP_{abs} = DoLP_{obs} \sin \Theta \tag{22}$$

In our case, $\sin \Theta = 0.92$, introducing an error of 8% when a right angle is considered.

References

Lilensten, J., J. Moen, M. Barthélemy, R. Thissen, C. Simon, D. A. Lorentzen, O. Dutuit, P. O. Amblard, and F. Sigernes (2008), Polarisation in aurorae: A new dimension for space environments studies, *Geophysical Research Letters*, *35*, 8804, doi: 10.1029/2007GL033006.

Lilensten, J., V. Bommier, M. Barthélemy, D. Bernard, H. Lamy, J. Moen, M. Johnsen, U. Løvhaug, and F. Pitout (2015), The auroral red line polarisation: modelling and measurements, *Journal of Space Weather and Space Climate*, *5*, A26, doi: 10.1051/swsc/2015027.

Olsen, N., T. J. Sabaka, and L. Tøffner-Clausen (2000), Determination of the IGRF 2000 model, *Earth, Planets, and Space*, *52*, 1175–1182, doi:10.1186/BF03352349.

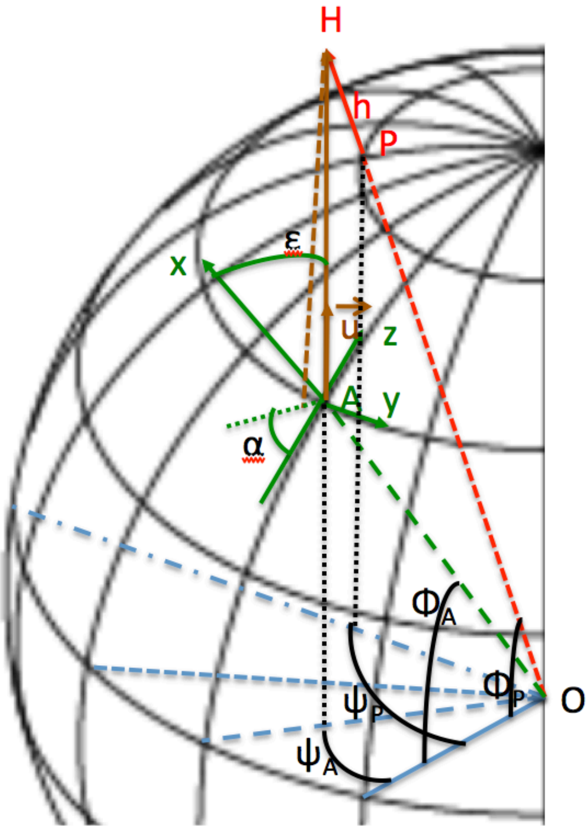

Figure S1. Geometry of the problem

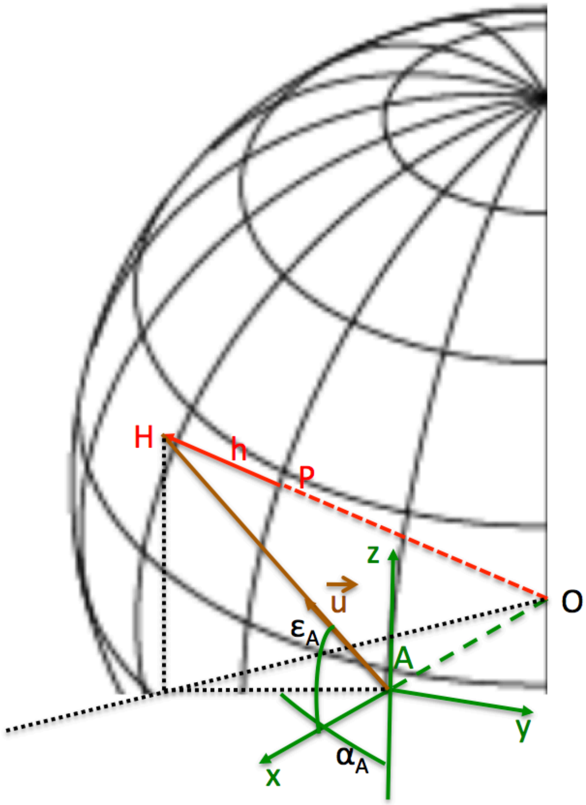

Figure S2. Geometry of the problem when the instrument is at the equator.

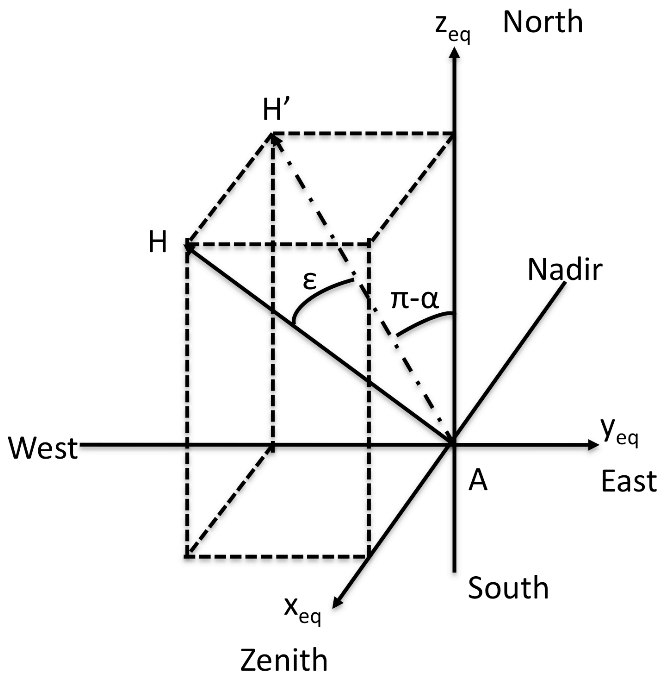

Figure S3. Magnification around A of the geometry of the problem when the instrument is at the equator.

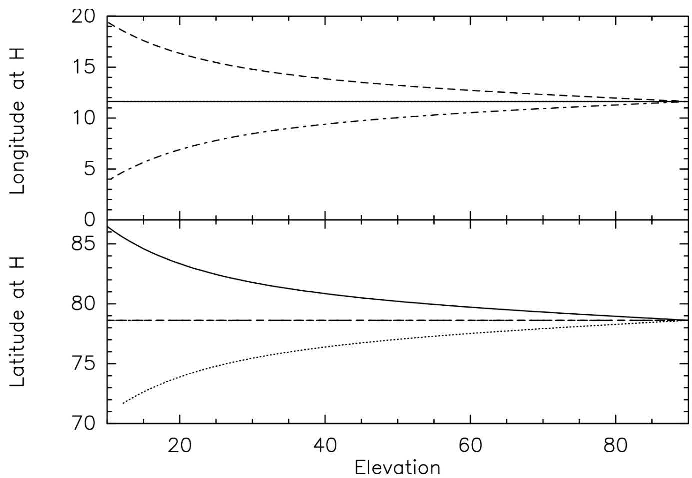

Figure S4. Latitude (lower panel) and longitude (upper panel) of the point H at 220 km above the ground in the line of sight of the SPP versus its elevation at Svalbard (latitude 78.62 °, longitude 11.63 °).. The azimuth are north (continuous line), East (dashed line), South (dotted line) and West (dash-dotted line)

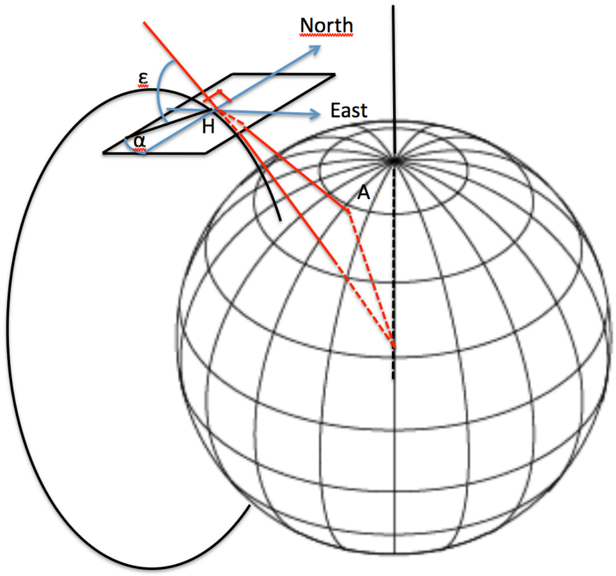

Figure S5. Geometry of the problem with the magnetic field in H

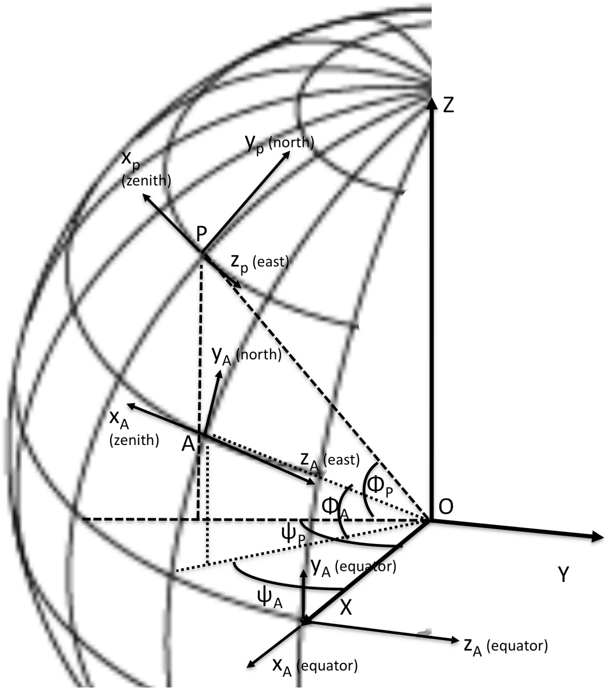

Figure S6. The reference frames for the rotations.

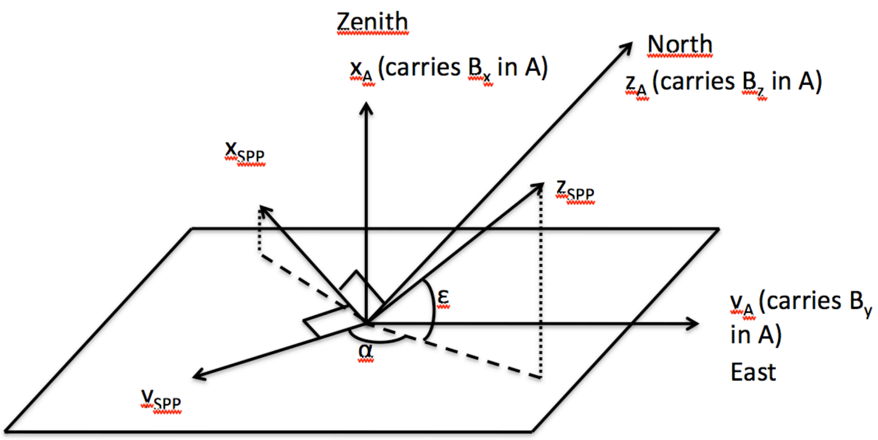

Figure S7. The frame of the polarimeter (labelled spp) must be rotated in the frame of A.

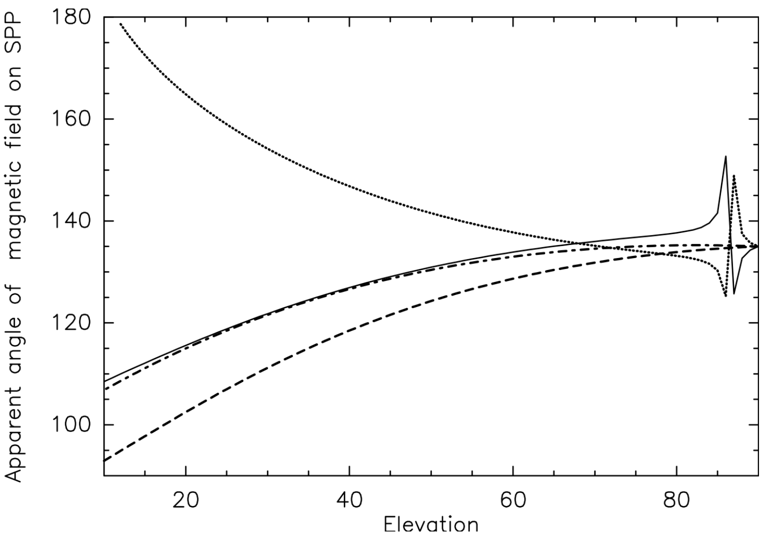

Figure S8. Apparent angle of the magnetic field (η) projected on the SPP lens (in degrees) as a function of the elevation. The azimuth are north (continuous line), East (dashed line), South (dotted line) and West (dash-dotted line).

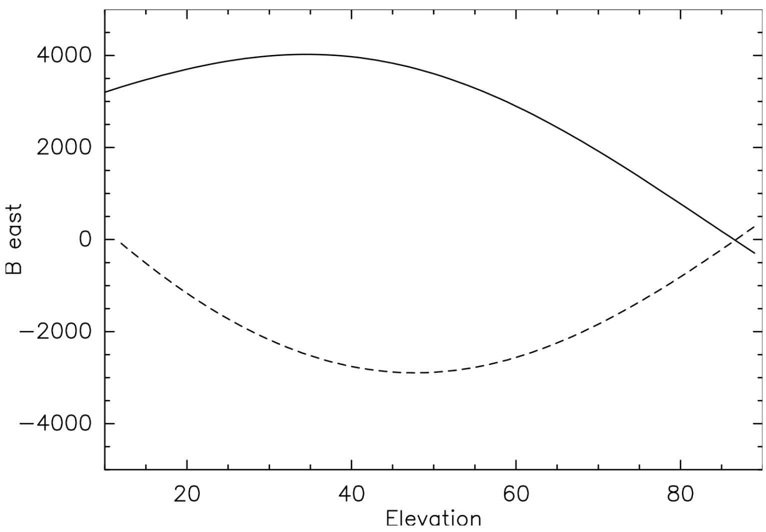

Figure S9. Eastward component of the magnetic field projected on the SPP for the north and south azimuths versus the SPP elevation. Continuous line: the SPP pointing North; dashed line: the SPP pointing south.

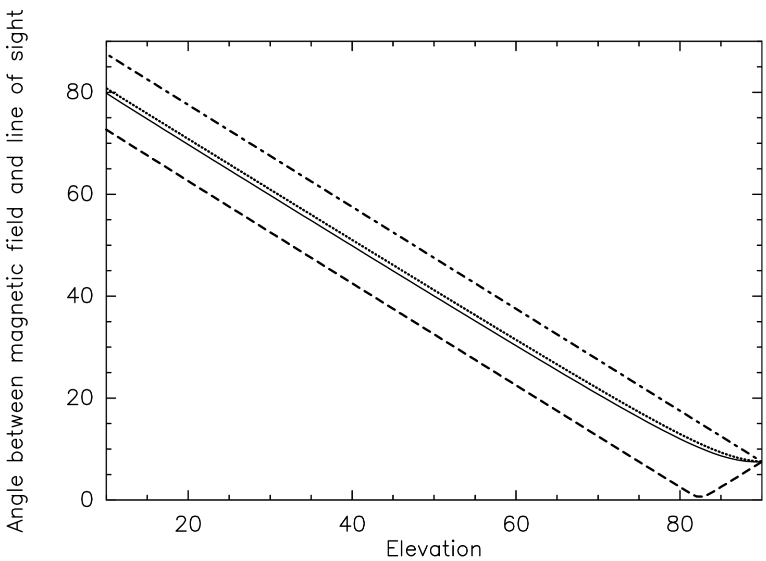

Figure S10. Angle between the magnetic field at the emission point H and the SPP line of sight. Continuous line: the SPP pointing North; dashed line: East; dotted line : South; dash-dotted line: West