

Trabajo práctico obligatorio

Métodos de ordenamiento

Leo Joaquin Bruno - FAI 3268

Maria Elvira Monserrat Vidal - FAI 1829

Jeremias Ezequiel Herrera - FAI 3297

Enlace a GITHUB: <https://github.com/LeoBruno400/MetodosDeOrdenamientos>

1. Implemente el algoritmo de Búsqueda Secuencial.

(a) ¿A qué clase pertenece y por qué razón?

Pertenece a la clase de fuerza bruta ya que va recorriendo elemento por elemento hasta encontrar el valor deseado.

(b) Realice el algoritmo en pseudocódigo y java

Pseudocódigo

```
ALGORITMO ejercicio01 () RETORNA ∅  
    ENTERO [] arreglo ← CREAR ENTERO {4,5,10,-7,2,20,22,45,22,1,66,2,55,30}  
    ESCRIBIR(busquedaSecuencial(arreglo, -7))  
FIN ALGORITMO
```

```
MODULO busquedaSecuencial (ENTERO [] arreglo) RETORNA ENTERO  
    ENTERO posicion←-1  
    ENTERO i←0  
    REPETIR  
        i ← i + 1  
        MIENTRAS(i<longitud(arreglo) AND arreglo[i]<>buscado) HACER  
            SI(arreglo[i]=buscado)  
                posicion = i  
            FIN SI  
        FIN MIENTRAS  
    RETORNA posicion  
FIN MODULO
```

Java

```
public class busquedaSecuencial {  
    public static void main(String[] args) {  
        int [] arreglo = {4,5,10,-7,2,20,22,45,22,1,66,2,55,30};  
        System.out.println(busquedaSecuencial(arreglo, -7));  
    }  
  
    public static int busquedaSecuencial (int [] arreglo, int buscado){  
        int posicion=-1, i=0;  
        do{  
            i++;  
        }while(i<arreglo.length && arreglo[i]!=buscado);  
        if(arreglo[i]==buscado){  
            posicion = i;  
        }  
        return posicion;  
    }  
}
```

(c) Realice la traza para el arreglo:

arreglo:

| | | | | | | |
|----|----|----|----|----|----|----|
| 89 | 45 | 63 | 90 | 29 | 34 | 17 |
|----|----|----|----|----|----|----|

 MODULO busquedaSecuencial(arreglo, 63)

| arreglo | buscado | i | posicion | RETORNA |
|---------|---------|---|----------|---------|
| * | 63 | 0 | -1 | 2 |
| | | 1 | 2 | |
| | | 2 | | |

(d) Una variante del algoritmo sería aquel que realiza la búsqueda de atrás hacia adelante. Realice una implementación de esta variante.

```
public static int busquedaSecuencialReversa (int [] arreglo, int buscado){
    int posicion=-1, i=arreglo.length-1;
    do{
        i--;
    }while(i>=0 && arreglo[i]!=buscado);
    if(arreglo[i]==buscado){
        posicion = i;
    }
    return posicion;
}
```

(e) Calcule el Tiempo de Ejecución y Orden.

```
1 public static int busquedaSecuencialReversa (int [] arreglo, int buscado){
2     int posicion=-1, i=arreglo.length-1;
3     do{
4         i--;
5     }while(i>=0 && arreglo[i]!=buscado);
6     if(arreglo[i]==buscado){
7         posicion = i;
8     }
9     return posicion;
10 }
```

$$T_n = T_3 + T_4 = 1 + (5n + 10) = 5n + 11$$

$$T_3 = 1$$

$$T_4 = T_9 + T_{10} + T_{13} + T_{16} = 2 + (5n + 4) + 3 + 1 = 5n + 10$$

$$T_9 = 2$$

$$T_{10} = n * (4 + 1) + 4 = 5n + 4$$

$$T_{13} = 2 + T_{14} = 2 + 1 = 3$$

$$T_{14} = 1$$

$$T_{16} = 1$$

$$f(n) = 5n + 11$$

Complejidad lineal.

$$O(n)$$

2. Implemente el algoritmo de Búsqueda Binaria.

(a) ¿A qué clase pertenece y por qué razón?

Pertenece a la clase Divide y Vencerás porque un problema se divide en varios subproblemas más pequeños y fáciles de resolver, con temáticas similares . La solución se obtiene resolviendo cada uno de los subproblemas de manera independiente en forma recursiva o iterativa, y luego combinar sus soluciones para obtener la solución al problema original.

(b) Realice el algoritmo en pseudocódigo y java

Pseudocódigo

```
MODULO busquedaBinaria (ENTERO[] arreglo, ENTERO valorBuscado) RETORNA ENTERO
    LOGICO encontrado
    ENTERO inicio, fin, medio, posicion
    inicio ← 0
    fin ← LONGITUD(arreglo)
    encontrado ← falso
    posicion ← -1
    MIENTRAS inicio <= fin AND !encontrado HACER
        medio ← (inicio + fin)/2
        SI arreglo [medio] == valorBuscado ENTONCES
            posicion ← medio
            encontrado ← verdadero
        SINO
            SI arreglo[medio] > valorBuscado ENTONCES
                fin ← medio -1
            SINO
                inicio ← medio +1
            FIN SI
        FIN SI
    FIN MIENTRAS
    RETORNA posicion
FIN MODULO
```

Java

```
public static int busquedaBinaria(int[] arreglo, int valorBuscado){
    //busca de manera binaria un valor numérico en un arreglo. Devuelve la posición de encontrarlo, -1 de no encontrarlo.
    boolean encontrado;
    int inicio, fin, medio, posicion;
    inicio = 0;
    fin = arreglo.length-1;
    encontrado = false;
    posicion = -1;
    while (inicio <= fin && !encontrado){
        medio= (inicio + fin)/2;
        if(arreglo[medio] == valorBuscado){
            posicion = medio;
            encontrado = true;
        }else{
            if(arreglo[medio]> valorBuscado){
                fin = medio -1;
            }else{
                inicio = medio +1;
            }
        }
    }
    return posicion;
}
```

(c) Realice la traza para el arreglo:

arreglo:

| | | | | | | | | | |
|---|---|---|---|----|----|----|----|----|----|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|----|----|----|----|----|----|

MODULO busquedaBinaria(arreglo, 12)

| encontrado | inicio | fin | medio | posicion | RETORNA |
|------------|--------|-----|-------|----------|---------|
| false | 0 | 9 | 4 | -1 | |
| | 5 | | | | |
| | | | 7 | | |
| | | 6 | | | |
| | | | 5 | | |
| | | | | 5 | |
| | | | | | 5 |

(d) Una variante del algoritmo sería aquel que realiza la búsqueda en un arreglo ordenado en forma decreciente. Realice una implementación de esta variante.

Pseudocódigo

```
MODULO busquedaBinariaInvertida (ENTERO[] arreglo, ENTERO valorBuscado) RETORNA ENTERO
    LOGICO encontrado
    ENTERO inicio, fin, medio, posicion
    inicio ← 0
    fin ← LONGITUD(arreglo)
    encontrado ← falso
    posicion ← -1
    MIENTRAS inicio <= fin AND !encontrado HACER
        medio ← (inicio + fin)/2
        SI arreglo [medio] == valorBuscado ENTONCES
            posicion ← medio
            encontrado ← verdadero
        SINO
            SI arreglo[medio] < valorBuscado ENTONCES
                fin ← medio -1
            SINO
                inicio ← medio +1
            FIN SI
        FIN SI
    FIN MIENTRAS
    RETORNA posicion
FIN MODULO
```

Java

```
public static int busquedaBinariaInvertida(int[] arreglo, int valorBuscado){
    //busca de manera binaria un valor numérico en un arreglo. Devuelve la posición de encontrarlo, -1 de no encontrarlo.
    boolean encontrado;
    int inicio, fin, medio, posicion;
    inicio = 0;
    fin = arreglo.length-1;
    encontrado = false;
    posicion = -1;
    while (inicio <= fin && !encontrado){
        medio= (inicio + fin)/2;
        if(arreglo[medio] == valorBuscado){
            posicion = medio;
            encontrado = true;
        }else{
            if(arreglo[medio] < valorBuscado){
                fin = medio -1;
            }else{
                inicio = medio +1;
            }
        }
    }
    return posicion;
}
```

(e) Calcule el Tiempo de Ejecución y Orden.

```
1 public static int busquedaBinaria(int[] arreglo, int valorBuscado){
2     //busca de manera binaria un valor numérico en un arreglo. De
3     boolean encontrado;
4     int inicio, fin, medio, posicion;
5     inicio = 0;
6     fin = arreglo.length-1;
7     encontrado = false;
8     posicion = -1;
9     while (inicio <= fin && !encontrado){
10         medio= (inicio + fin)/2;
11         if(arreglo[medio] == valorBuscado){
12             posicion = medio;
13             encontrado = true;
14         }else{
15             if(arreglo[medio] < valorBuscado){
16                 fin = medio -1;
17             }else{
18                 inicio = medio +1;
19             }
20         }
21     }
22     return posicion;
23 }
```

```
1 public static int busquedaBinaria(int[] arreglo, int valorBuscado){
2     //busca de manera binaria un valor numérico en un arreglo. De
3     boolean encontrado;
4     int inicio, fin, medio, posicion;
5     inicio = 0;
6     fin = arreglo.length-1;
7     encontrado = false;
8     posicion = -1;
9     while (inicio <= fin && !encontrado){
10         medio= (inicio + fin)/2;
11         if(arreglo[medio] == valorBuscado){
12             posicion = medio;
13             encontrado = true;
14         }else{
15             if(arreglo[medio] < valorBuscado){
16                 fin = medio -1;
17             }else{
18                 inicio = medio +1;
19             }
20         }
21     }
22     return posicion;
23 }
```

$$T_n = T_5 + T_6 + T_7 + T_8 + T_9 + T_{12}$$

$$T_5 = 1 \quad T_6 = 1 \quad T_7 = 1 \quad T_8 = 1 \quad T_{12} = 1$$

$$T_9 = \log n \cdot (3 + T_{10} + T_{11}) + 3 = \log n (3 + 3 + 6) + 3 = 12 \cdot \log n + 3$$

$$T_{10} = 3$$

$$T_{11} = 2 + \max(T_7, T_{15})$$

$$T_{12} = T_7 + T_{13} = 1 + 1 = 2$$

$$T_{11} = 2 + 4 = 6$$

$$T_{15} = 2 + \max(T_{16}, T_{18}) = 2 + 2 = 4$$

$$T_n = 1 + 1 + 1 + 1 + 1 + 12 \cdot \log n + 3 = 12 \cdot \log n + 8$$

$$T_{16} = 2 \quad T_{18} = 2$$

$$f(n) = 12 \cdot \log(n) + 8$$

Complejidad Logarítmica

$$O(\log n)$$

3. Implemente el algoritmo de Ordenamiento por Selección

(a) ¿A qué clase pertenece y por qué razón?

(b) Realice el algoritmo en pseudocódigo y java

(c) Realice la traza para el arreglo:

| | | | | | | | | | |
|---|---|----|----|---|----|---|----|----|----|
| 7 | 6 | 11 | 17 | 3 | 15 | 5 | 19 | 30 | 14 |
|---|---|----|----|---|----|---|----|----|----|

(d) Una variante del algoritmo sería aquel que realiza el ordenamiento de mayor a menor. Realice una implementación de esta variante

a) Pertenece a la clase de fuerza bruta ya que debe recorrer todos los elementos si o si.

b)

```
public static void main(String[] args) {
    int [] arreglo = {4,5,10,-7,2,20,22,45,22,1,66,2,55,30};
    ordenarPorSeleccion(arreglo);
}

public static void ordenarPorSeleccion (int [] arreglo){
    int posMenor;
    for (int i = 0; i < arreglo.length; i++) {
        posMenor = buscarMenor(arreglo, i);
        if(arreglo[posMenor]<arreglo[i]){
            intercambiar(arreglo, posMenor, i);
        }
    }
}

public static void intercambiar(int [] arreglo,int posMenor,int i){
    int aux;
    aux = arreglo[i];
    arreglo[i] = arreglo[posMenor];
    arreglo[posMenor] = aux;
}

public static int buscarMenor (int[] arreglo, int i){
    int menorNum = 999999, posicion=i;
    for (int j = i; j < arreglo.length; j++) {
        if(arreglo[j]<menorNum){
            menorNum = arreglo[j];
            posicion = j;
        }
    }
    return posicion;
}
}
```

```
ALGORITMO ejercicio03 () RETORNA ∅
    ENTERO [] arreglo ← {4,5,10,-7,2,20,22,45,22,1,66,2,55,30}
    ordenarPorSeleccion(arreglo)
FIN ALGORITMO
```

```
MODULO ordenarPorSeleccion(ENTERO [] arreglo)
    ENTERO posMenor, i
    PARA i←0 HASTA longitud(arreglo)-1 PASO 1
        posMenor ←buscarMenor(arreglo, i)
        SI(arreglo[posMenor]<arreglo[i])
            intercambiar(arreglo, posMenor, i)
        FIN SI
    FIN PARA
FIN MODULO
```

```
MODULO intercambiar(ENTERO [] arreglo, ENTERO posMenor, ENTERO i)
    ENTERO aux
    aux ← arreglo[i]
    arreglo[i] ← arreglo[posMenor]
    arreglo[posMenor] ← aux
```

FIN MODULO

```
MODULO buscarMenor (ENTERO [] arreglo, ENTERO i)
    ENTERO menorNum ← 9999999
    ENTERO posicion ← i
    ENTERO j
    PARA j ← i HASTA longitud(arreglo)-1 PASO 1
        SI(arreglo[j] < menorNum)
            menorNum ← arreglo[j]
            posicion ← j
    FIN SI
FIN PARA
FIN MODULO
```

```
d)
public static void ordenarPorSeleccionVariante (int [] arreglo){
    int posMayor;
    for (int i = arreglo.length-1; i >= 0; i--) {
        posMayor = buscarMenorVariante(arreglo, i);
        if(arreglo[posMayor] < arreglo[i]){
            intercambiar(arreglo, posMayor, i);
        }
    }
}

public static int buscarMenorVariante (int[] arreglo, int i){
    int menorNum = 999999, posicion=i;
    for (int j = 0; j <= i; j++) {
        if(arreglo[j] < menorNum){
            menorNum = arreglo[j];
            posicion = j;
        }
    }
    return posicion;
}
```

| | | | | | | | | | |
|---|---|----|----|---|----|---|----|----|----|
| 7 | 6 | 11 | 17 | 3 | 15 | 5 | 19 | 30 | 14 |
|---|---|----|----|---|----|---|----|----|----|

| i | posMenor | [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | Cambio |
|----|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--------------------|
| 0 | 4 | 7 | 6 | 11 | 17 | 3 | 15 | 5 | 19 | 30 | 14 | Posicion 4 por 0 → |
| 1 | 6 | 3 | 6 | 11 | 17 | 7 | 15 | 5 | 19 | 30 | 14 | Posicion 6 por 1 → |
| 2 | 6 | 3 | 5 | 11 | 17 | 7 | 15 | 6 | 19 | 30 | 14 | Posicion 6 por 2 → |
| 3 | 4 | 3 | 5 | 6 | 17 | 7 | 15 | 11 | 19 | 30 | 14 | Posicion 4 por 3 → |
| 4 | 6 | 3 | 5 | 6 | 7 | 17 | 15 | 11 | 19 | 30 | 14 | Posicion 6 por 4 → |
| 5 | 9 | 3 | 5 | 6 | 7 | 11 | 15 | 17 | 19 | 30 | 14 | Posicion 9 por 5 → |
| 6 | 9 | 3 | 5 | 6 | 7 | 11 | 14 | 17 | 19 | 30 | 15 | Posicion 9 por 6 → |
| 7 | 9 | 3 | 5 | 6 | 7 | 11 | 14 | 15 | 19 | 30 | 17 | Posicion 9 por 7 → |
| 8 | 9 | 3 | 5 | 6 | 7 | 11 | 14 | 15 | 17 | 30 | 19 | Posicion 9 por 8 → |
| 9 | 9 | 3 | 5 | 6 | 7 | 11 | 14 | 15 | 17 | 19 | 30 | No hace cambio |
| 10 | | | | | | | | | | | | |

No hago traza de las invocaciones del resto de modulos porque sino se haria muy extenso

4. Implemente el algoritmo de Ordenamiento Burbuja.

(a) ¿A qué clase pertenece y por qué razón?

El ordenamiento burbuja pertenece a la clase de fuerza bruta porque enumera sistemáticamente todos los posibles candidatos para la solución de un problema con el fin de chequear si dicho candidato satisface la solución al mismo.

(b) Realice el algoritmo en pseudocódigo y java

Pseudocódigo

```
MODULO burbuja (ENTERO[] arreglo) RETORNA Ø
    ENTERO i, j, longitud, aux
    longitud = LONGITUD(arreglo)
    PARA i ← 0 HASTA longitud -1 PASO 1 HACER
        PARA j ← HASTA longitud - i - 2 PASO 1 HACER
            SI arreglo[j] > arreglo[j+1] ENTONCES
                aux ← arreglo[j]
                arreglo[j] ← arreglo[j+1]
                arreglo[j+1] ← aux
            FIN SI
        FIN PARA
    FIN PARA
FIN MODULO
```

Java

```
public static void burbuja (int[] arreglo){
    int i,j, longitud, aux;
    longitud = arreglo.length;
    for (i= 0; i < longitud; i++) {
        for(j=0; j<(longitud-i-1); j++){
            if(arreglo[j]>arreglo[j+1]){
                aux = arreglo[j];
                arreglo [j] = arreglo[j+1];
                arreglo [j+1] = aux;
            }
        }
    }
}
```

(c) Realice la traza para el arreglo:

arreglo:

| | | | | | | |
|----|----|----|----|----|----|----|
| 89 | 45 | 68 | 90 | 29 | 34 | 17 |
|----|----|----|----|----|----|----|

MODULO burbuja(arreglo)

| i | j | longitud | arreglo | | | | | | | | | | | | | | |
|----|----|----------|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| | | | <table><tr><td>89</td><td>45</td><td>68</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 89 | 45 | 68 | 90 | 29 | 34 | 17 | | | | | | | |
| 89 | 45 | 68 | 90 | 29 | 34 | 17 | | | | | | | | | | | |
| | | 7 | | | | | | | | | | | | | | | |
| 0 | | | | | | | | | | | | | | | | | |
| | 0 | | | | | | | | | | | | | | | | |
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| 45 | 45 | 68 | 90 | 29 | 34 | 17 | | | | | | | | | | | |
| 45 | 89 | 68 | 90 | 29 | 34 | 17 | | | | | | | | | | | |
| | 1 | | | | | | | | | | | | | | | | |
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| 45 | 68 | 68 | 90 | 29 | 34 | 17 | | | | | | | | | | | |
| 45 | 68 | 89 | 90 | 29 | 34 | 17 | | | | | | | | | | | |
| | 2 | | | | | | | | | | | | | | | | |
| | 3 | | | | | | | | | | | | | | | | |
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| 45 | 68 | 89 | 29 | 29 | 34 | 17 | | | | | | | | | | | |
| 45 | 68 | 89 | 29 | 90 | 34 | 17 | | | | | | | | | | | |
| | 4 | | | | | | | | | | | | | | | | |
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| 45 | 68 | 89 | 29 | 34 | 90 | 17 | | | | | | | | | | | |
| | 5 | | | | | | | | | | | | | | | | |
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|----|----|----|---|----|----|----|----|----|----|----|
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| 45 | 68 | 89 | 29 | 34 | 17 | 90 | | | | |
| | | | | | | | | | | |
| | 6 | | | | | | | | | |
| 1 | | | | | | | | | | |
| | 0 | | | | | | | | | |
| | 1 | | | | | | | | | |
| | 2 | | | | | | | | | |
| | | | <table><tr><td>45</td><td>68</td><td>29</td><td>29</td><td>34</td><td>17</td><td>90</td></tr></table> | 45 | 68 | 29 | 29 | 34 | 17 | 90 |
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| | 3 | | | | | | | | | |
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| | | | | | | | | | | |
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| 45 | 68 | 29 | 34 | 17 | 89 | 90 | | | | |
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| 45 | 29 | 34 | 68 | 17 | 89 | 90 | | | | |
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| 45 | 29 | 34 | 17 | 17 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 45 | 29 | 34 | 17 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | 45 | 34 | 17 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | 34 | 45 | 17 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 29 | 34 | 17 | 17 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | 34 | 17 | 45 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 29 | 17 | 17 | 45 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 29 | 17 | 34 | 45 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
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| 17 | 17 | 34 | 45 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 17 | 29 | 34 | 45 | 68 | 89 | 90 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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| 17 | 29 | 34 | 45 | 68 | 89 | 90 | | | | |

(d) Una variante del algoritmo sería aquel que realiza el ordenamiento de mayor a menor. Realice una implementación de esta variante

Pseudocódigo
MODULO burbujalInvertida (ENTERO[] arreglo) RETORNA Ø
 ENTERO i, j, longitud, aux
 longitud = LONGITUD(arreglo)
 PARA i ← 0 HASTA longitud -1 PASO 1 HACER
 PARA j ← HASTA longitud - i - 2 PASO 1 HACER
 SI arreglo[j] < arreglo[j+1] ENTONCES
 aux ← arreglo[j]
 arreglo[j] ← arreglo[j+1]
 arreglo[j+1] ← aux
 FIN SI
 FIN PARA
FIN PARA
FIN MODULO

Java
public static void burbujalInvertida (int[] arreglo){
 int i,j, longitud, aux;
 longitud = arreglo.length;
 for (i= 0; i < longitud; i++) {
 for(j=0; j<(longitud-i-1); j++){
 if(arreglo[j] < arreglo[j+1]){
 aux = arreglo[j];
 arreglo [j] = arreglo[j+1];
 arreglo [j+1] = aux;
 }
 }
 }
}

(e) Calcule el Tiempo de Ejecución y Orden.

```

1  public static void burbuja (int[] arreglo){
2      int i,j, longitud, aux;
3      longitud = arreglo.length;
4      for (i= 0; i < longitud; i++) {
5          for(j=0; j<(longitud-i-1); j++){
6              if(arreglo[j]>arreglo[j+1]){
7                  aux = arreglo[j];
8                  arreglo [j] = arreglo[j+1];
9                  arreglo [j+1] = aux;
10             }
11         }
12     }
13 }

```

```

1  public static void burbuja (int[] arreglo){
2      int i,j, longitud, aux;
3      longitud = arreglo.length;
4      for (i= 0; i < longitud; i++) {
5          for(j=0; j<(longitud-i-1); j++){
6              if(arreglo[j]>arreglo[j+1]){
7                  aux = arreglo[j];
8                  arreglo [j] = arreglo[j+1];
9                  arreglo [j+1] = aux;
10             }
11         }
12     }
13 }

```

$$T_n: T_3 + T_4 \quad T_3 = 2$$

$$T_4 = 1 + \sum_{i=0}^n (1 + T_5 + 2) + 1$$

$$T_5 = 1 + \sum_{j=0}^{n-i-1} (3 + T_6 + 2) + 1$$

$$T_6 = 4 + T_7 + T_8 + T_9 = 4 + 2 + 4 + 3 = 13$$

$$T_7 = 2 \quad T_8 = 4 \quad T_9 = 3$$

$$T_5 = 2 + \sum_{j=0}^{n-i-1} (3 + 13 + 2) = 2 + \sum_{j=0}^{n-i-1} 18 = 2 + 18(n-i-1) = 18n - 18i - 16$$

$$T_4 = 2 + \sum_{i=0}^n (3 + 18n - 18i - 16) = 2 - 13n + 18n^2 - 18 \sum_{i=0}^n i =$$

$$= 18n^2 - 13n + 2 - 18 \left(\frac{n^2 + n}{2} \right) = 18n^2 - 13n + 2 - 9n^2 - 9n = 9n^2 - 22n + 2$$

$$T_n = 2 + 9n^2 - 22n + 2 = 9n^2 - 22n + 4$$

$$f(n) = 9n^2 - 22n + 4$$

Complejidad cuadrática

$$O(n^2)$$

5. Implemente el algoritmo de Ordenamiento por Inserción.

(a) ¿A qué clase pertenece y por qué razón?

Pertenece a la clase Fuerza Bruta ya que recorre todo ordenando los elementos tantas veces como sea necesario para que quede ordenado

(b) Realice el algoritmo en pseudocódigo y java

Pseudocódigo

```
MODULO insercion(ENTERO[] arr) RETORNA Ø
    ENTERO i, j, aux
    ENTERO n ← LONGITUD(arr) - 1
    PARA i ← 1 HASTA n PASO 1 HACER
        aux ← arr[j]
        j ← i
        MIENTRAS (j > 0 AND arr[j-1] > aux ) HACER
            arr[j] ← arr[j-1]
            j ← j-1
        FIN MIENTRAS
        arr[j] ← aux
    FIN PARA
FIN MODULO
```

Java

```
public static void insercion(int[] arr) {
    int i, j, aux;
    int n = arr.length - 1;

    for (i = 1; i <= n; i++) {
        aux = arr[i];
        j = i;
        while (j > 0 && arr[j - 1] > aux) {
            arr[j] = arr[j - 1];
            j = j - 1;
        }
        arr[j] = aux;
    }
}
```

(c) Realice la traza para el arreglo:

arr:

| | | | | | | |
|----|----|----|----|----|----|----|
| 89 | 45 | 68 | 90 | 29 | 34 | 17 |
|----|----|----|----|----|----|----|



MODULO insercion(arr)

| i | j | aux | longitud | arr | | | | | | | |
|----|----|-----|----------|---|----|----|----|----|----|----|----|
| | | | | <table><tr><td>89</td><td>45</td><td>68</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 89 | 45 | 68 | 90 | 29 | 34 | 17 |
| 89 | 45 | 68 | 90 | 29 | 34 | 17 | | | | | |
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| | | 45 | | | | | | | | | |
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| 89 | 89 | 68 | 90 | 29 | 34 | 17 | | | | | |
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| 2 | | | | | | | | | | | |
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| 45 | 68 | 89 | 90 | 29 | 34 | 17 | | | | | |
| 4 | | | | | | | | | | | |
| | | 29 | | | | | | | | | |
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| 45 | 68 | 89 | 90 | 90 | 34 | 17 | | | | | |
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| 45 | 68 | 89 | 89 | 90 | 34 | 17 | | | | | |
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| 45 | 68 | 68 | 89 | 90 | 34 | 17 | | | | | |
| | 1 | | | <table><tr><td>45</td><td>45</td><td>68</td><td>89</td><td>90</td><td>34</td><td>17</td></tr></table> | 45 | 45 | 68 | 89 | 90 | 34 | 17 |
| 45 | 45 | 68 | 89 | 90 | 34 | 17 | | | | | |
| | 0 | | | <table><tr><td>29</td><td>45</td><td>68</td><td>89</td><td>90</td><td>34</td><td>17</td></tr></table> | 29 | 45 | 68 | 89 | 90 | 34 | 17 |
| 29 | 45 | 68 | 89 | 90 | 34 | 17 | | | | | |
| 5 | | | | | | | | | | | |
| | | 34 | | | | | | | | | |
| | 5 | | | <table><tr><td>29</td><td>45</td><td>68</td><td>89</td><td>90</td><td>90</td><td>17</td></tr></table> | 29 | 45 | 68 | 89 | 90 | 90 | 17 |
| 29 | 45 | 68 | 89 | 90 | 90 | 17 | | | | | |
| | 4 | | | | | | | | | | |

| | | | | | | | | | | |
|---|---|----|--|----|----|----|----|----|----|----|
| | | | | 29 | 45 | 68 | 89 | 89 | 90 | 17 |
| | 3 | | | 29 | 45 | 68 | 68 | 89 | 90 | 17 |
| | 2 | | | 29 | 45 | 45 | 68 | 89 | 90 | 17 |
| | 1 | | | 29 | 34 | 45 | 68 | 89 | 90 | 17 |
| 6 | | | | | | | | | | |
| | | 17 | | | | | | | | |
| | 6 | | | 29 | 34 | 45 | 68 | 89 | 90 | 90 |
| | 5 | | | 29 | 34 | 45 | 68 | 89 | 89 | 90 |
| | 4 | | | 29 | 34 | 45 | 68 | 68 | 89 | 90 |
| | 3 | | | 29 | 34 | 45 | 45 | 68 | 89 | 90 |
| | 2 | | | 29 | 34 | 34 | 45 | 68 | 89 | 90 |
| | 1 | | | 29 | 29 | 34 | 45 | 68 | 89 | 90 |
| | 0 | | | 17 | 29 | 34 | 45 | 68 | 89 | 90 |

(d) Una variante del algoritmo sería aquel que realiza el ordenamiento de mayor a menor. Realice una implementación de esta variante.

Pseudocódigo

```

MODULO insercion(ENTERO[] arr) RETORNA Ø
    ENTERO i, j, aux
    ENTERO n ← LONGITUD(arr) - 1
    PARA i ← 1 HASTA n PASO 1 HACER
        aux ← arr[i]
        j ← i
        MIENTRAS (j > 0 AND arr[j-1] < aux ) HACER
            arr[j] ← arr[j-1]
            j ← j-1
        FIN MIENTRAS
        arr[j] ← aux
    FIN PARA
FIN MODULO

```

Java

```
public static void insercion(int[] arr) {
    int i, j, aux;
    int n = arr.length - 1;

    for (i = 1; i <= n; i++) {
        aux = arr[i];
        j = i;
        while (j > 0 && arr[j - 1] < aux) {
            arr[j] = arr[j - 1];
            j = j - 1;
        }
        arr[j] = aux;
    }
}
```

(e) Calcule el Tiempo de Ejecución y Orden.

```
1 public static void insercion(int[] arr) {
2     int i, j, aux;
3     int n = arr.length - 1;
4
5     for (i = 1; i <= n; i++) {
6         aux = arr[i];
7         j = i;
8         while (j > 0 && arr[j - 1] > aux) {
9             arr[j] = arr[j - 1];
10            j = j - 1;
11        }
12        arr[j] = aux;
13    }
14 }
```

$$T_N = T_3 + T_5$$

$$T_3 = 3$$

$$T_5 = 1 + \sum_{i=1}^N (T_6 + T_7 + T_8 + T_{12} + 2 + 2) + 2$$

$$T_6 = 2 \quad T_7 = 1$$

$$T_8 = N \cdot (5 + (T_9 + T_{10})) + 5$$

$$T_9 = 4 \quad T_{10} = 2 \quad T_{12} = 2$$

$$T_8 = N \cdot (5 + (4 + 2)) + 5 = N \cdot (5 + 6) + 5 = 16N$$

$$T_5 = 3 + \sum_{i=1}^N (2 + 1 + 16N + 2 + 4) = 3 + \sum_{i=1}^N (16N + 9) = 3 + N \cdot (16N + 9) = 16N^2 + 9N + 3$$

$$T_N = 16N^2 + 9N + 3 + 3 = 16N^2 + 9N + 6$$

$$f(n) = 16n^2 + 9n + 3$$

Complejidad cuadrática.

$$O(n^2)$$

6. Implemente el algoritmo de Ordenamiento Burbuja Mejorado.

(a) ¿A qué clase pertenece y por qué razón?

Pertenece a la clase fuerza bruta, ya que recorre el arreglo las veces que sea necesario para dejar los elementos más grandes al final

(b) Realice el algoritmo en pseudocódigo y java

Pseudocódigo

```
MODULO burbujaMejorado(ENTERO[] arr) RETORNA Ø
    LOGICO ordenado ← falso
    ENTERO i ← 0, j, aux
    ENTERO n ← LONGITUD(arr)

    MIENTRAS i < n-1 AND no ordenado HACER
        ordenado ← verdadero //considero que esta ordenado
        PARA j ← 0 HASTA n-i-2 PASO 1 HACER
            SI a[j] > a[j+1] ENTONCES
                ordenado ← falso
                aux = arr[j];
                arr [j] = arr[j+1];
                arr [j+1] = aux;
            FIN SI
        FIN PARA
        i ← i + 1
    FIN MIENTRAS
FIN MODULO
```

Java

```
public static void burbujaMejorada(int[] arr) {
    boolean ordenado = false;
    int i = 0, j, aux;
    int k = arr.length;
    int n = k-1-i;

    while (i < k - 1 && !ordenado) {
        ordenado = true;
        for (j = 0; j < n; j++) {
            if (arr[j] > arr[j + 1]) {
                ordenado = false;
                aux = arr[j];
                arr[j] = arr[j + 1];
                arr[j + 1] = aux;
            }
        }
        i++;
    }
}
```

(c) Realice la traza para el arreglo:

arr:

| | | | | | | |
|----|----|----|----|----|----|----|
| 89 | 45 | 68 | 90 | 29 | 34 | 17 |
|----|----|----|----|----|----|----|

| ordenado | i | j | aux | n | arreglo | | | | | | | |
|----------|----|----|-----|----|---|----|----|----|----|----|----|----|
| false | 0 | | | 8 | | | | | | | | |
| | | | | 7 | | | | | | | | |
| true | | | | | | | | | | | | |
| | | 0 | | | | | | | | | | |
| false | | | | | | | | | | | | |
| | | | 89 | | | | | | | | | |
| | | | | | <table><tr><td>45</td><td>45</td><td>68</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 45 | 45 | 68 | 90 | 29 | 34 | 17 |
| 45 | 45 | 68 | 90 | 29 | 34 | 17 | | | | | | |
| | | | | | <table><tr><td>45</td><td>89</td><td>68</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 45 | 89 | 68 | 90 | 29 | 34 | 17 |
| 45 | 89 | 68 | 90 | 29 | 34 | 17 | | | | | | |

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|-------|----|----|----|----|---|----|----|----|----|----|----|----|
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| | | | | | <table><tr><td>45</td><td>68</td><td>68</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 45 | 68 | 68 | 90 | 29 | 34 | 17 |
| 45 | 68 | 68 | 90 | 29 | 34 | 17 | | | | | | |
| | | | | | <table><tr><td>45</td><td>68</td><td>89</td><td>90</td><td>29</td><td>34</td><td>17</td></tr></table> | 45 | 68 | 89 | 90 | 29 | 34 | 17 |
| 45 | 68 | 89 | 90 | 29 | 34 | 17 | | | | | | |
| | | 2 | | | | | | | | | | |
| | | 3 | | | | | | | | | | |
| | | | 90 | | | | | | | | | |
| | | | | | <table><tr><td>45</td><td>68</td><td>89</td><td>29</td><td>29</td><td>34</td><td>17</td></tr></table> | 45 | 68 | 89 | 29 | 29 | 34 | 17 |
| 45 | 68 | 89 | 29 | 29 | 34 | 17 | | | | | | |
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| 45 | 68 | 89 | 29 | 90 | 34 | 17 | | | | | | |
| | | 4 | | | | | | | | | | |
| | | | 90 | | | | | | | | | |
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| 45 | 68 | 89 | 29 | 34 | 34 | 17 | | | | | | |
| | | | | | <table><tr><td>45</td><td>68</td><td>89</td><td>29</td><td>34</td><td>90</td><td>17</td></tr></table> | 45 | 68 | 89 | 29 | 34 | 90 | 17 |
| 45 | 68 | 89 | 29 | 34 | 90 | 17 | | | | | | |
| | | 5 | | | | | | | | | | |
| | | | 90 | | | | | | | | | |
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| 45 | 68 | 89 | 29 | 34 | 17 | 90 | | | | | | |
| | | 6 | | | | | | | | | | |
| | 1 | | | | | | | | | | | |
| true | | | | | | | | | | | | |
| | | 0 | | | | | | | | | | |
| | | 1 | | | | | | | | | | |
| | | 2 | | | | | | | | | | |
| false | | | | | | | | | | | | |
| | | | 89 | | | | | | | | | |
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| 45 | 68 | 29 | 29 | 34 | 17 | 90 | | | | | | |

| | | | | | | | | | | | | |
|-------|----|----|----|----|---|----|----|----|----|----|----|----|
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| 45 | 68 | 29 | 89 | 34 | 17 | 90 | | | | | | |
| | | 3 | | | | | | | | | | |
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| 45 | 68 | 29 | 34 | 89 | 17 | 90 | | | | | | |
| | | 4 | | | | | | | | | | |
| | | | 89 | | | | | | | | | |
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| 45 | 68 | 29 | 34 | 17 | 17 | 90 | | | | | | |
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| 45 | 68 | 29 | 34 | 17 | 89 | 90 | | | | | | |
| | | 5 | | | | | | | | | | |
| | 2 | | | | | | | | | | | |
| true | | | | | | | | | | | | |
| | | 0 | | | | | | | | | | |
| | | 1 | | | | | | | | | | |
| false | | | | | | | | | | | | |
| | | | 68 | | | | | | | | | |
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| 45 | 29 | 34 | 68 | 17 | 89 | 90 | | | | | | |
| | | 3 | | | | | | | | | | |
| | | | 68 | | | | | | | | | |
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|-------|---|---|----|--|----------------|
| | | | | | 45293417178990 |
| | | | | | 45293417688990 |
| | | 4 | | | |
| | 3 | | | | |
| true | | | | | |
| | | 0 | | | |
| false | | | | | |
| | | | 45 | | |
| | | | | | 29293417688990 |
| | | | | | 29453417688990 |
| | | 1 | | | |
| | | | 45 | | |
| | | | | | 29343417688990 |
| | | | | | 29344517688990 |
| | | 2 | | | |
| | | | 45 | | |
| | | | | | 29341717688990 |
| | | | | | 29341745688990 |
| | | 3 | | | |
| | 4 | | | | |
| true | | | | | |
| | | 0 | | | |
| | | 1 | | | |
| false | | | | | |
| | | | 34 | | |
| | | | | | 29171745688990 |
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| | | | | | | | | | | | | |
|-------|----|----|----|----|---|----|----|----|----|----|----|----|
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| 29 | 17 | 34 | 45 | 68 | 89 | 90 | | | | | | |
| | | 2 | | | | | | | | | | |
| | 5 | | | | | | | | | | | |
| true | | | | | | | | | | | | |
| | | 0 | | | | | | | | | | |
| false | | | | | | | | | | | | |
| | | | 29 | | | | | | | | | |
| | | | | | <table><tr><td>17</td><td>17</td><td>34</td><td>45</td><td>68</td><td>89</td><td>90</td></tr></table> | 17 | 17 | 34 | 45 | 68 | 89 | 90 |
| 17 | 17 | 34 | 45 | 68 | 89 | 90 | | | | | | |
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| 17 | 29 | 34 | 45 | 68 | 89 | 90 | | | | | | |
| | | 1 | | | | | | | | | | |
| | 6 | | | | | | | | | | | |

(d) Una variante del algoritmo sería aquel que realiza el ordenamiento de mayor a menor (con la mejora). Realice una implementación de esta variante.

```
Java
public static void burbujaMejoradaInvertida(int[] arr) {
    boolean ordenado = false;
    int i = 0, j, aux;
    int k = arr.length;
    int n = k-1-i;

    while (i < k - 1 && !ordenado) {
        ordenado = true;
        for (j = 0; j < n; j++) {
            if (arr[j] < arr[j + 1]) {
                ordenado = false;
                aux = arr[j];
                arr[j] = arr[j + 1];
                arr[j + 1] = aux;
            }
        }
        i++;
    }
}
```

(e) Calcule el Tiempo de Ejecución y Orden.

```
1 public static void burbujaMejorada(int[] arr) {
2     boolean ordenado = false;
3     int i = 0, j, aux;
4     int k = arr.length;
5     int n = k-1-i;
6
7     while (i < k - 1 && !ordenado) {
8         ordenado = true;
9         for (j = 0; j < n; j++) {
10             if (arr[j] > arr[j + 1]) {
11                 ordenado = false;
12                 aux = arr[j];
13                 arr[j] = arr[j + 1];
14                 arr[j + 1] = aux;
15             }
16         }
17         i++;
18     }
19 }
```

$$T_N = T_3 + T_4 + T_5 + T_7$$

$$T_3 = 1 \quad T_4 = 2 \quad T_5 = 3$$

$$T_7 = N \cdot (4 + (T_8 + T_9 + T_{17})) + 4$$

$$T_8 = 1 \quad T_{17} = 2$$

$$T_9 = 1 + \sum_{j=1}^{N-1} (T_{10} + 1 + 2) + 1$$

$$T_{10} = 4 + T_{11} + T_{12} + T_{13} + T_{14}$$

$$T_{10} = 4 + 1 + 2 + 4 + 3 = 14$$

$$T_{11} = 1 \quad T_{12} = 2 \quad T_{13} = 4 \quad T_{14} = 3$$

$$T_9 = 2 + \sum_{j=1}^{N-1} (14 + 1 + 2) = 2 + \sum_{j=1}^{N-1} 17 = 2 + 17 \cdot (N-1) = 2 + 17N - 17 = 17N - 15$$

$$T_7 = N(4 + (1 + 17N - 15 + 2)) + 4 = 17N^2 - 8N + 4$$

$$T_N = 1 + 2 + 3 + 17N^2 - 8N + 4 = 17N^2 - 8N + 10$$

$$f(n) = 17n^2 - 8n + 10$$

Complejidad cuadrática.

$$O(n^2)$$