Cálculo diferencial: (teorema Lagrange → 2ª folha 3)

3ª F (resumos)

Regra cauchy: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ ou $\frac{\infty}{\infty} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

$0 \times \infty \begin{cases} \frac{0}{\frac{1}{\infty}} \to \frac{0}{0} \\ \frac{\infty}{\frac{1}{0}} \to \frac{\infty}{\infty} \end{cases}$

1^{∞}, 0^0 ou $\infty^0 \to e^{\ln(\infty^0)} = e^{0 \times \ln(\infty)} = e^{\frac{\ln(\infty)}{\frac{1}{0}}} = e^{\frac{\infty}{\infty}}$

Regra de L'Hôspital: $\frac{\infty}{\infty}$ ou $\frac{0}{0}$

$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \to g'(a) \neq 0$

$\lim_{x \to 0} \frac{\cos x - 1}{x^2} = \frac{-\operatorname{sen}(0)}{2x \to 2 \times 0}$ ✗

Regra Cauchy $\lim_{x \to 0} \frac{-\operatorname{sen} x}{2x} = \lim_{x \to 0} \frac{-\cos x}{2} = -\frac{1}{2}$

Fórmula de Taylor

(seja f uma função contínua e n vezes diferenciável num intervalo aberto I, a ∈ I. Tem-se que, para qualquer x ∈ I).

$f(x) = p_n(x) + R_n(x)$

$= f(a) + f'(a) \times (x-a) + \frac{f''(a)}{2!} \times (x-a)^2 + \dots + \frac{f^{(n-1)}(a)}{(n-1)!}(x-a)^{n-1} + \frac{f^{(n)}(a)}{n!} + R_n(x)$

Exemplo

$f(x) = \frac{1}{x}$ em $x = 2 = a$ $\quad f(2) = \frac{1}{2}$

$f'(x) = (x^{-1})' = -1x^{-2} \Rightarrow f'(2) = -1 \times 2^{-2} = -\frac{1}{4}$

$f''(x) = (-x^{-2})' = 2x^{-3} \Rightarrow f''(2) = 2 \times 2^{-3} = \frac{1}{4}$

$f'''(x) = (2x^{-3})' = -6x^{-4} \Rightarrow f'''(2) = -6 \times 2^{-4} = -\frac{3}{8}$

$f(x) = \frac{1}{2} + \left(-\frac{1}{4}\right)(x-2) + \frac{\frac{1}{4}}{2!}(x-2)^2 + \frac{-\frac{3}{8}}{3!}(x-2)^3 + \dots$

$\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3 + \dots$

$\sum_{n=0} \frac{(-1)^n}{2^{n+1}}(x-2)^n$

Assintotas:

A. Vertical $x=a$

$\lim_{x \to a^{\pm}} f(x) = \pm\infty$

A. Horizontal $y=b$

$\lim_{x \to \pm\infty} f(x) = b$

A. Obliqua $y=mx+b$

$m = \lim_{x \to \pm\infty} \frac{f(x)}{x}$ $\quad b = \lim_{x \to \pm\infty} (f(x) - mx)$

$\frac{1}{+\infty} = 0$ $\quad \ln\left(\frac{1}{+\infty}\right) = -\infty$ $\quad \ln(0^+) = -\infty$ $\quad \ln(e^-) = 0^+$

$\frac{+\infty}{0}$: $\frac{+\infty}{0^+} = +\infty$, $\frac{+\infty}{0^-} = -\infty$

$\lim_{x \to +\infty} \frac{a^x}{x^n} = +\infty \to \frac{x^n}{a^x} = 0$

$\lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$

Máximos e mínimos:

(fazer a derivada da função)

$f(x) = 3 - 2\cos x$

$f'(x) = (-2)' \times \cos x + (-2) \times (\cos x)'$

$= -2(-\operatorname{sen} x)$

$= 2\operatorname{sen} x$

$f'(0) = 2\operatorname{sen}(0)$

$= \operatorname{sen} x = 0$

$= x = 0 + k\pi, k \in \mathbb{Z}$

$= x = k\pi$

	0		π		2π
$f'(x)$	0	+	0	-	0
		↗	Máx.	↘	

$f(\pi) = 3 - 2\cos(\pi)$

$= 3 - 2(-1)$

$= 5$

Concavidade:

$f(x) = 3 - 2\cos x$

$f'(x) = 2\operatorname{sen} x$

$f''(x) = (2)' \times \operatorname{sen} x + 2 \times (\operatorname{sen} x)'$

$= 2\cos x$

$f''(0) = 2\cos(0)$

$= \cos x = 0$

$= x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

	$\frac{\pi}{2}$		$\frac{3\pi}{2}$		$\frac{5\pi}{2}$
$f''(x)$	0	-	0	+	0
$f(x)$		$\cap$	P.I.	$\cup$	

Tem $\cap$ $\left]\frac{\pi}{2}; \frac{3\pi}{2}\right[$

Tem $\cup$ $\left]\frac{3\pi}{2}; \frac{5\pi}{2}\right[$

P.I. $\left(\frac{3\pi}{2}; 3\right)$

$f\left(\frac{3\pi}{2}\right) = 3 - 2\cos\left(\frac{3\pi}{2}\right)$

$= 3$

Primitivação:

$P_2 = 2x$

$\int 2\,dx = 2x$

$\int 7\,dt = 7t$

$$\int 3x + 2\,dx = 3\int x\,dx + \int 2\,dx$$

$$= \frac{3x^2}{2} + 2x + c,\ c \in \mathbb{R}$$

$$\boxed{\int x^a = \frac{x^{a+1}}{a+1}}$$

$$\int \sqrt[5]{x^2}\,dx = \int x^{2/5}\,dx = \frac{x^{7/5}}{7/5}$$

$\frac{2}{5}+1=\frac{7}{5}$

$$= \frac{\sqrt[5]{x^7}}{\frac{7}{5}} = \frac{5}{7}\sqrt[5]{x^7} + c$$

—— // ——

$$\frac{1}{3}\int \underbrace{3x^2}_{u'} e^{\overbrace{x^3}^{u}}\,dx = \frac{1}{3}\ e^{x^3}$$

$$\boxed{\int u' e^u = e^u}$$

$$\int x\sqrt[3]{1+2x^2}\,dx = \frac{1}{4}\int \underbrace{4x}_{u'}\ \underbrace{(1+2x^2)}_{u}^{\overbrace{4/3}^{a}}\,dx$$

$$\boxed{\int u' u^a = \frac{u^{a+1}}{a+1}}$$

$$= \frac{1}{4}\,\frac{(1+2x^2)^{4/3}}{\frac{4}{3}} = \frac{1}{4}\ \frac{3}{4}\sqrt[3]{(1+2x^2)^4}$$

$$= \frac{3}{16}\sqrt[3]{(1+2x^2)^4} + c$$

$$\int \frac{1}{\sqrt[5]{2-3x}}\,dx = -\frac{1}{3}\int \underbrace{-3}_{u'}\underbrace{(2-3x)}_{u}^{\overbrace{-1/5}^{a}}\,dx$$

$-\frac{1}{5}+1 = \frac{-1+5}{5} = \frac{4}{5}$

$$= -\frac{1}{3}\ \frac{(2-3x)^{4/5}}{4/5} = -\frac{1}{3}\,\frac{5}{4}\sqrt[5]{(2-3x)^4}$$

$$= \frac{-5}{12}\left(\sqrt[5]{(2-3x)^4}\right) + c$$

$$\boxed{\int \frac{u'}{u} = \ln(u)}$$

—— // ——

$$\frac{1}{2}\int \frac{2x \to u'}{\underbrace{1+x^2}_{u}}\,dx = \frac{1}{2}\ln(1+x^2) + c$$

$$\int \operatorname{tg} x\,dx = -\int \frac{-\operatorname{sen} x \to u'}{\cos x \to u}\,dx = -\ln(\cos x) + c$$

$$\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2} = \frac{1}{3}\operatorname{arctg}(x^3)+c$$

$$\boxed{\int \frac{u'}{1+u^2} = \operatorname{arctg} u}$$

— // —

$$\frac{1}{2}\int \underbrace{2e^{2x}}_{u'} \cos(\underbrace{e^{2x}}_{u})\,dx = \frac{1}{2}\operatorname{sen}(e^{2x})+c$$

$$\boxed{\int u'\cos u = \operatorname{sen} u}$$

$$\boxed{\int u'\operatorname{sen} u = -\cos u}$$

— // —

$$\int \frac{1}{5+x^2}\,dx = \int \frac{\overset{u'}{1}}{\underbrace{(\sqrt{5})^2}_{a^2}+\underbrace{x^2}_{u^2}} = \frac{1}{\sqrt{5}}\operatorname{arctg}\left(\frac{x}{\sqrt{5}}\right)+c$$

$$\boxed{\int \frac{1}{1+x^2} = \operatorname{arctg} x}$$

$$\boxed{\int \frac{u'}{1+u^2} = \operatorname{arctg} u}$$

$$\boxed{\frac{u'}{a^2+u^2} = \frac{1}{a}\operatorname{arctg}(}$$

Por partes:

$$\int u' \times v \,dx = uv - \int uv'\,dx$$

critério

u'	v
e^u	log u / $\ln$
$\cos x$	polinómi
$\operatorname{sen} x$	arc

⚠ $\int \ln(x)\,dx = \int 1 \times \ln(x)\,dx$

$u' = 1 \quad v = \ln(x)$

$$\int \underbrace{x^2}_{v}\underbrace{e^x}_{u'}\,dx$$

$u' = e^x \longrightarrow u = e^x$

$v = x^2 \longrightarrow v' = 2x$

$$= e^x x^2 - \int \underbrace{e^x}_{u'}\,\underbrace{2x}_{v}\,dx$$

$u' = e^x \longrightarrow u = e^x$

$v = 2x \longrightarrow v' = 2$

$$= e^x x^2 - \left(e^x 2x - \int e^x 2\,dx\right)$$

$$= e^x x^2 - e^x 2x + 2\int e^x\,dx$$

$$= x^2 e^x - 2x e^x + 2e^x + c$$

$$\int \underbrace{1\times}_{u'} \underbrace{\ln(2x)}_{v}\, dx \qquad u' = 1 \longrightarrow u = x$$

$$v = \ln(2x) \longrightarrow v' = \frac{2}{2x} = \frac{1}{x}$$

$$= \cancel{1}\, x\ln(2x) - \int x \frac{1}{x}\, dx$$

$$= x\ln(2x) - \int 1\, dx$$

$$= x\ln(2x) - x + c$$

Extra

$$\int_0^2 7x\, e^{8-3x^2}\, dx$$

C.A.

$$\int \underbrace{7x}_{\to u'} e^{\overbrace{8-3x^2}^{u}}\, dx$$

$$\frac{7}{-6}\int -6x\, e^{8-3x^2}\, dx$$

$$= -\frac{7}{6} e^{8-3x^2} + c$$

$$= \left[-\frac{7}{6} e^{8-3x^2}\right]_0^2$$

$$= -\frac{7}{6} e^{8-3\times 2^2} - \left(-\frac{7}{6} e^{8-3\times 0^2}\right)$$

$$= -\frac{7}{6} e^{-4} + \frac{7}{6} e^{8}$$

Substituição: (o que chateia $= t$)

$$\int x\sqrt{1+3x}\, dx$$

- $\sqrt{1+3x} = t$

$1+3x = t^2$

- $x = \frac{t^2}{3} - \frac{1}{3} = \frac{t^2-1}{3}$

- $dx = \frac{2t}{3}\, dt$

$$\int \left(\frac{t^2-1}{3}\right) \times t \times \frac{2t}{3}\, dt$$

$$= \frac{2}{3}\int \left(\frac{t^2-1}{3}\right) t^2\, dt = \frac{2}{9}\int t^4 - t^2\, dt$$

$$= \frac{2}{9}\left(\frac{t^5}{5} - \frac{t^3}{3}\right) = \frac{2}{9}\left(\frac{(\sqrt{1+3x})^5}{5} - \frac{(\sqrt{1+3x})^3}{3}\right) + c$$

$$\int \frac{x}{\sqrt{(1+x^2)^3}}\,dx = \frac{1}{2}\int 2x\,\underbrace{(1+x^2)}_{u}^{-3/2} \quad = \frac{1}{2}\frac{(1+x^2)^{-1/2}}{-\frac{1}{2}}$$

$$= -(1+x^2)^{-1/2}$$

$$= -\frac{1}{\sqrt{1+x^2}} + C$$

$$f'' = \frac{1}{1+x^2} \qquad f'(0) = 2 \text{ e } f(0) = -1$$

$f = ?$

$$\hookrightarrow f' = \int f'' = \int \frac{1}{1+x^2} = \operatorname{arctg}(x) + c$$

$$f'(0) = 2 \Rightarrow \operatorname{arctg}(0) + c = 2 \Rightarrow c = 2$$

$$\therefore f'(x) = \operatorname{arctg} x + 2$$

$$f = \int f' = \int \operatorname{arctg} x + 2 = \int \operatorname{arctg} x\,dx + \int 2\,dx$$

C.A. $\int \underset{u'}{1} \cdot \underset{v}{\operatorname{arctg} x}\,dx$ $\quad u' = 1 \to u = x$, $v = \operatorname{arctg} x \to v' = \frac{1}{1+x^2}$ $\quad = x\operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + 2x + D$

$$x\operatorname{arctg} x - \int x\frac{1}{1+x^2}\,dx$$

$$x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$$

$$x\operatorname{arctg} x - \frac{1}{2}\ln(1+x^2)$$

$f(0) = -1$

$$= 0\operatorname{arctg}(0) - \frac{1}{2}\ln(1+0) + 2(0) + D = -1$$

$$D = -1$$

$$f(x) = x\operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + 2x - 1$$

$$\int \frac{x^3}{x+1}\,dx$$

grau de cima ≥ baixo $\Downarrow$ divide

$$x^3 + 0x^2 + 0x + 0$$

$$\begin{array}{c|cccc|l}
 & 1 & 0 & 0 & 0 & N \\
D \quad -1 & & -1 & 1 & -1 & \\
\hline
 & 1 & -1 & 1 & \boxed{-1} & R
\end{array}$$

$x^2 - x + 1$

Q

$$\frac{N}{D} = Q + \frac{R}{D}$$

$$\int \frac{x^3}{x+1} = \int x^2 - x + 1 + \frac{-1}{x+1}\,dx$$

$$= \int x^2 - x + 1\,dx + \int \frac{-1}{x+1}\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \int \frac{1}{x+1}\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \ln(x+1)$$

$$\int \frac{0x^2\; 3x+1}{x^3-x}\,dx \quad 4$$

$$Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx$$

C.A. $\dfrac{3x+1}{x(x^2-1)} = \dfrac{3x+1}{x(x-1)(x+1)}$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$(x-1)(x+1)$ $\quad x(x+1)$ $\quad x(x-1)$

(x^2-1) $\quad (x^2+x)$ $\quad (x^2-x)$

$$\begin{cases} A + B + C = 0 \\ B - C = 3 \\ -A = 1 \end{cases} \qquad \begin{cases} -1 + 3 + C + C = 0 \\ B = 3 + C \\ A = -1 \end{cases} \qquad \begin{cases} C = -1 \\ B = 2 \\ A = -1 \end{cases}$$

$$\int \frac{3x+1}{x^3-x}\,dx = \int \frac{-1}{x} + \frac{2}{x-1} + \frac{-1}{x+1}\,dx$$

$$= \int -\frac{1}{x}\,dx + \int \frac{2}{x-1}\,dx + \int \frac{-1}{x+1}\,dx$$

$$= -\ln|x| + 2\ln(x-1) - \ln(x+1) + C$$

Cálculo integral:

Teorema fundamental do cálculo integral:

$$\left(\int_{a(x)}^{b(x)} f(t)\,dt\right)' = b' \times f(b) - a' \times f(a)$$

$$\lim_{x\to 0} \frac{\int_0^x \operatorname{sen} t^3\,dt}{x^4} = \frac{\int_0^0}{0} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\left(\int_0^x \operatorname{sen} t^3\,dt\right)'}{(x^4)'} \underset{\text{R. Cauchy}}{=} \lim_{x\to 0} \frac{1\times \operatorname{sen} x^3 - 0 \times \operatorname{sen} 0}{4x^3}$$

TFCI

$$= \frac{1}{4} \lim_{x\to 0} \frac{\operatorname{sen} x^3}{x^3} = \frac{1}{4} \times 1 = \frac{1}{4}$$

Monotonia e extremos de $I(x) = \int_2^{e^x} \frac{1}{\ln t}\,dt$

$$I' = 0 \Leftrightarrow \left(\int_2^{e^x} \frac{1}{\ln t}\,dt\right)' = 0 \underset{\text{TFCI}}{\Leftrightarrow} e^x \times \frac{1}{\ln(e^x)} - 0 = 0 \Leftrightarrow \frac{e^x}{x} = 0$$

I'

$e^x = 0$

Não tem extremo)

	$-\infty$	0	$+\infty$
I'	$-$	S.S.	$+$
I	$\searrow$		$\nearrow$

Mínimo

Critérios de convergência dos integrais impróprios

$$\int_1^{+\infty} \frac{\sqrt{1+2x+x^2}}{1+x^3}\,dx \qquad x^{2 \cdot 1/2} = x$$

g. cima = 1 ; g. baixo = 3

baixo − cima = 2

↳ converge

$$\boxed{\int_1^{+\infty} \frac{1}{x^\alpha} \quad \alpha > 1 \text{ conv.} \quad \alpha \leq 1 \text{ div.}}$$

$$\lim_{x\to +\infty} \frac{\frac{\sqrt{1+2x+x^2}}{1+x^3}}{1/x^2} = \frac{\sqrt{x^2} \times x^2}{x^3} = 1 > 0 \text{ e finito}$$

∴ Por comparação, os integrais $\int_1^{+\infty} \frac{\sqrt{1+2x+x^2}}{1+x^3}$ e $\int_1^{+\infty} \frac{1}{x^2}$ têm a mesma natureza.

∴ Como $\int_1^{+\infty} 1/x^2$ converge, então a nossa também.

Integrais impróprios:

1ª espécie (um ou dois extremos com ∞)

Se for finito converge

infinito diverge

$$\boxed{\int_0^{+\infty} \frac{1}{x}\,dx} \to \lim_{b\to+\infty} \int_1^b \frac{1}{x}\,dx =$$

$$= \lim_{b\to+\infty} \left[\ln x\right]_1^b = \lim_{b\to+\infty} \ln(b)-\ln(1) = \ln(+\infty)$$

$$= +\infty$$

diverge

2ª espécie (extremos limitados) ($D = \mathbb{R}\setminus\{0\}$)

$$\int_0^3 \frac{7{,}6}{x}\,dx = \lim_{c\to0^+} \int_c^3 \frac{7{,}6}{x}\,dx = 7{,}6 \lim_{c\to+0} \int_c^3 \frac{1}{x}\,dx =$$

$$7{,}6\,\left[\ln(x)\right]_{0^+}^3 = 7{,}6 \times \left(\ln(3) - \ln(0^+)\right) = 7{,}6\times(\ln(3) - (-\infty))$$

$$= 7{,}6 \times +\infty = +\infty \text{ Diverge}$$

*

Regra de Barrow:

$$\int_a^b f(x)\,dx = \left[f(x)\right]_a^b = f(b) - f(a)$$

Valor médio:

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Quando os valores são infinitos a média é dada por:

$$\bar{f} = \frac{\int_a^b f(x)\,dx}{b-a} \qquad [a,b]$$

$$f(c) = \frac{\int_a^b f(x)\,dx}{b-a} \quad \text{ou} \quad f(c)\times(b-a) = \int_a^b f(x)\,dx$$

Ex: Valor médio da função e^x em $[0,1]$

$$\int_0^1 e^x\,dx = \left[e^x\right]_0^1 = e - 1$$

$$f(c)\times(1-0) = \int_0^1 e^x\,dx \Leftrightarrow f(c) = e-1$$

* Mistes: (1.ª e 2.ª)

$$\int_{-\infty}^{1} (x-1)^{-2}\,dx =$$

$$= \int_{-\infty}^{0} (x-1)^{-2}\,dx + \int_{0}^{1} (x-1)^{-2}\,dx = \lim_{a\to-\infty} \int_{a}^{0} (x-1)^{-2}\,dx + \lim_{c\to 1^-} \int_{0}^{1} (x-1)^{-2}\,dx$$

$$= \lim_{a\to-\infty} \left[-\frac{1}{(x-1)}\right]_a^0 + \lim_{c\to 1^-} \left[-\frac{1}{(x-1)}\right.$$

Integração por partes.

$$\int_1^4 \underbrace{x}_{u'} \underbrace{\ln x}_{v}\,dx \qquad u' = x \to u = \frac{x^2}{2} \qquad v = \ln x \to v' = \frac{1}{x}$$

$$\int_1^4 x\ln x\,dx = \left[\frac{x^2}{2}\ln x\right]_1^4 - \int_1^4 \frac{x^{\cancel{2}}}{2} \times \frac{1}{\cancel{x}}\,dx =$$

$$= \frac{4^2}{2}\ln 4 - \frac{1^2}{2}\ln(1) - \int_1^4 \frac{x}{2}\,dx = 8\ln 4 - \left[\frac{x^2}{4}\right]_1^4$$

$$= 8\ln 4 - \left(\frac{4^2}{4} - \frac{1^2}{4}\right) = 8\ln 4 - \frac{15}{4}$$

Integração por substituição:

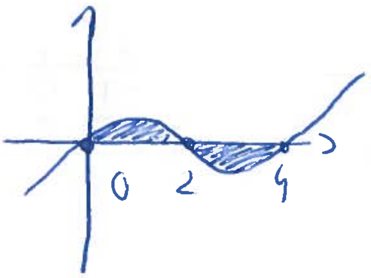
$$\int_0^1 \frac{e^{2x}}{2e^x + e^{2x}} \qquad \cdot\ t = e^x \quad \cdot\ x = \ln(t) \quad \cdot\ dx = \frac{1}{t}\,dt \qquad x = 1 \to t = e \quad x = 0 \to t = 1$$

$$\int_0^1 \frac{e^{2x}}{2e^x + e^{2x}}\,dx = \int_1^{e} \frac{t^2}{2t + t^2} \times \frac{1}{t}\,dt = \int_1^{e} \frac{1}{2+t}\,dt = \left[\ln(2+t)\right]_1^{e}$$

$$= \ln(2+e) - \ln 3$$

Área

$f(x) = x^3 - 6x^2 + 8x$ e eixo xx



cima baixo

Área $\int_0^a x^3 - 6x^2 + 8x - 0 \, dx + \int_a^b 0 - (x^3 - 6x^2 + 8x) \, dx$

C.A.

$\boxed{a \text{ e } b}$

$x^3 - 6x^2 + 8x = 0 \Rightarrow x(x^2 - 6x + 8)$

$x = 0 \vee x = \underset{a}{2} \vee x = \underset{b}{4}$

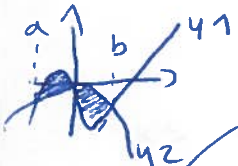
$$\int_0^2 x^3 - 6x^2 + 8x - 0 \, dx + \int_2^4 0 - (x^3 - 6x^2 + 8x) \, dx$$

$$= \left[\frac{x^4}{4} - \frac{\cancel{6}x^3}{\cancel{3}} + \frac{\cancel{8}x^2}{\cancel{2}}\right]_0^2 + \left[-\frac{x^4}{4} + \frac{6x^3}{3} - \frac{8x^2}{2}\right]_2^4$$

$$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2 + \left[-\frac{x^4}{4} + 2x^3 - 4x^2\right]_2^4$$

$$= \left[\left(\frac{2^4}{4} - 2\times(2)^3 + 4(2)^2\right) - (0)\right] + \left[\left(-\frac{4^4}{4} + 2(4)^3 - 4(4)^2\right) - \left(-\frac{2^4}{4} + 2\times 2^3 - 4(2)^2\right)\right]$$

$$= 4 - \cancel{16} + \cancel{16} + (-64 + 128 - 64) - (-4 + \cancel{16} - \cancel{16}) = 8$$



$y_1 = x^3 - 2x^2 - 2x$ e $y_2 = -x^2$

cima baixo cima baixo

Área $\int_a^0 x^3 - 2x^2 - 2x - (-x^2) \, dx + \int_0^b -x^2 - (x^3 - 2x^2 - 2x) \, dx$

$\boxed{a \text{ e } b}$

$-x^2 = x^3 - 2x^2 - 2x$

$x^3 - x^2 - 2x = 0 \Leftrightarrow x(x^2 - x - 2) = 0$

$x = 0 \vee x = \underset{b}{2} \vee x = \underset{a}{-1}$

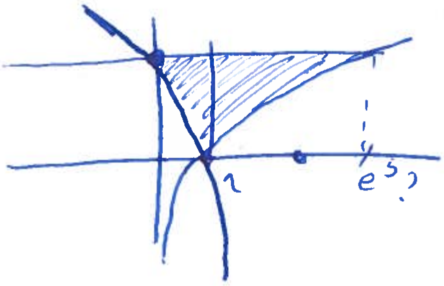
$$\int_{-1}^0 x^3 - x^2 - 2x \, dx + \int_0^2 -(x^3 - x^2 - 2x) \, dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}}\right]_{-1}^0 + \left[-\frac{x^4}{4} + \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}}\right]_0^2$$

$$= [(0) - (1/4 - 1/3 - 1)] + \left[-\frac{2^4}{4} + \frac{2^3}{3} + 2^2 - (0)\right]$$

$$= \frac{5}{12} + \frac{8}{3} = \frac{5 + 32}{12} = \frac{37}{12}$$

$y \leq 5$ $\qquad$ $y \geq -5x+5$ $\qquad$ $y \geq \ln x$ $\qquad$ $\ln x$



x	y
0	5
1	0

x	y
1	0
e^5	5

$$\text{Área} = \int_0^1 5-(-5x+5)\,dx + \int_1^{e^5} 5-\ln x\,dx$$

$$\int x \cdot \ln|x|\,dx$$

$$= \left[5x + \frac{5x^2}{2} - 5x\right]_0^1 + \left[5x + x\ln|x| + x\right]_1^{e^5}$$

$$x\ln(x) - x$$

$$= \left[\left(5+\frac{25}{2}-5\right)-(0)\right] + \left[\left(5e^5 + e^5 5 + e^5\right) - (5+1)\right]$$

$$= \cancel{5} + \frac{25}{2} - \cancel{5} + 11e^5 - 6 = 11e^5 + \frac{25}{2} - 6$$

Comprimento de arco:

$$L = \int_a^b \sqrt{1+(f')^2}\,dx$$

$$f(x) = \ln(\cos x)$$

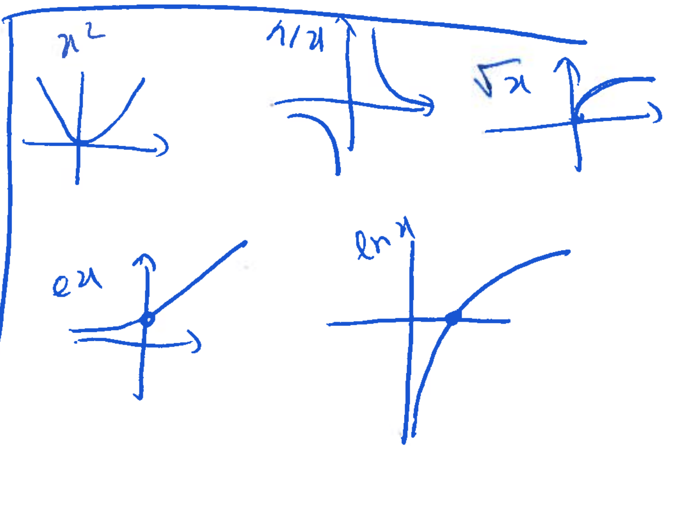
$$x = 0 \;\; e \;\; x = \frac{\pi}{4}$$

$$f'(x) = \frac{-\operatorname{sen} x}{\cos x}$$

$$L = \int_0^{\pi/4} \sqrt{1+\left(\frac{-\operatorname{sen} x}{\cos x}\right)^2}\,dx = \int_0^{\pi/4} \sqrt{\frac{1}{\cos^2 x}}\,dx = \int_0^{\pi/4} \frac{1}{\cos x}\,dx \ldots$$

Volumes:

$$V = \pi \int_a^b f^2 - g^2\,dx$$



$$V = \pi \int_0^{\pi/2} (\operatorname{sen} x \cos x)^2 - 0^2\,dx$$

$$= \pi \int_0^{\pi/2} \operatorname{sen}^2 x \cos^2 x\,dx = \ldots$$

$$\cancel{\operatorname{sen}(2x)} = \frac{1}{2}$$