1º F (explicación) 1

②

Ñ falta

$$\lim_{n \to +\infty} \underbrace{\frac{n+2}{n+1}}_{1} \operatorname{sen}^2\left(\frac{n\pi}{4}\right)$$

$$\operatorname{sen}^2\left(\frac{n\pi}{4}\right)$$

- $n=1$ $\quad \operatorname{sen}^2\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$
- $n=2$ $\quad \operatorname{sen}^2\left(\frac{\pi}{2}\right) = 1$
- $n=3$ $\quad \operatorname{sen}^2\left(\frac{3\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$
- $n=4$ $\quad \operatorname{sen}^2(\pi) = 0$
- $n=5$ $\quad \operatorname{sen}^2\left(\frac{5\pi}{4}\right) = \left(\frac{-\sqrt{2}}{2}\right)^2 = \frac{1}{2}$
- $n=6$ $\quad \operatorname{sen}^2\left(\frac{6\pi}{4}\right) = (-1)^2 = 1$

Sublímites $\left\{0, \frac{1}{2}, 1\right\}$

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$\limsup x_n = \overline{\lim}\, x_n = 1$ o más de dos sol

①

$\liminf x_n = \underline{\lim}\, x_n = 0$

1) lim não é convergente pois

tem vários sub-limites

③ a) $\lim \dfrac{1}{5^n}\left(\dfrac{n+2}{n}\right)^{n+1}$

• $\lim \left(\dfrac{n+2}{n}\right)^{n+1} \Longrightarrow 1^{\infty}$

exponencial

$$\left(1+\frac{a}{n}\right)^n \to e^a$$

$$\left(\frac{n}{n}+\frac{2}{n}\right)^{n+1} = \Big[\underbrace{\left(1+\frac{2}{n}\right)^n}_{\downarrow}\Big]^{\frac{n+1}{n}}$$

$\lim \dfrac{n+1}{n} = 1$

$$(e^2)^1$$

$$\lim a^n \to \begin{cases} 0, & -1<a<1 \\ 1, & a=1 \\ +\infty, & a>1 \\ \nexists, & a\leq -1 \end{cases}$$

$$\left(\frac{n+2}{n}\right)^{n+1} \longrightarrow e^2$$

• $\lim \dfrac{1}{5^n} = \lim \left(\dfrac{1}{5}\right)^n = 0$

∴ o limite é $0 \times e^2 = 0$

b) $\lim \dfrac{\cancel{\sqrt[4]{n}} - \sqrt{3n}}{\cancel{\sqrt[4]{n}} + \sqrt{n}} = -\lim \sqrt{\dfrac{3n}{n}}$

$= -\sqrt{3}$

c) $\lim\limits_{n\to+\infty} 1 + \dfrac{1}{3^2} + \dots + \dfrac{1}{3^{2n}}$

$\lim\limits_{n\to+\infty} \sum\limits_{k=0}^{n} \dfrac{1}{3^{2k}}$

$= \sum\limits_{k=0}^{+\infty} \dfrac{1}{3^{2k}} = \sum\limits_{n=0}^{+\infty} \left(\dfrac{1}{3^2}\right)^n$

S. G. Razão $\dfrac{1}{9}$ Como $-1 < \frac{1}{9} < 1$ ✓
Converge

$S_n = \dfrac{1^{0}}{1-r} = \dfrac{1}{1-\frac{1}{9}} = \dfrac{1}{\frac{8}{9}} = \dfrac{9}{8}$

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②

(4)

$$\sum_{n=1}^{+\infty} \frac{1}{n(n+k)} = \frac{1}{k} \sum_{n=1}^{+\infty} \left(\underbrace{\frac{1}{n}}_{a_n} - \underbrace{\frac{1}{n+k}}_{a_{n+k}} \right)$$

S. Mengoli con $a_n = \frac{1}{n}$ e $k = k$

A soma é $S = a_1 + a_2 + \dots + a_k - k \lim a_n$

$$= 1 + \frac{1}{2} + \dots + \frac{1}{k} - k \underbrace{\lim \frac{1}{n}}_{0}$$

$$= 1 + \frac{1}{2} + \dots + \frac{1}{k}$$

= ~~[illegible]~~

A soma é $\frac{1}{k} \times \left(1 + \frac{1}{2} + \dots + \frac{1}{k} \right)$
original

Converge

(5) b) $\sum \frac{1}{n^3+2}$

$b_{dx} = 3$
$a_n = 0$

$3-0=3$

$\lim \frac{\frac{1}{n^3+1}}{\frac{1}{n^3}} = \lim \frac{n^3}{n^3+1} = 1 > 0 \in \mathbb{R}^+$

$\therefore$ Pelo comparação as séries $\sum \frac{1}{n^3+2}$ e $\sum \frac{1}{n^3}$ têm a mesma natureza. Como $\sum \frac{1}{n^3}$ converge (S. Dirichlet com $\alpha > 1$) a nossa tb converge

c) $\sum \left(\frac{2n-3}{2n}\right)^{n^2}$

$$\left(\frac{2n-3}{2n}\right)^{n^2} = \left[\left(1+\frac{-3}{2n}\right)^{2n}\right]^{\frac{n^2}{2n}}$$

$$\left(e^{-3}\right)^{+\infty} = e^{-\infty} \Rightarrow 0$$

Nada a concluir

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(3)

Critério da Raiz

$$\lim \sqrt[n]{\left(\frac{2n-3}{2n}\right)^{n^2}} = \lim \left(\frac{2n-3}{2n}\right)^{n} =$$

$$= \dots \lim \left[\left(1 + \frac{-3}{2n}\right)^{2n}\right]^{\frac{n}{2n}}$$

$$\downarrow$$

$$\left(e^{-3}\right)^{1/2} = e^{-3/2}$$

$$= \frac{1}{\sqrt{e^3}} < 1 \text{, pelo c. da Raiz}$$

a série converge

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d) $\sum (-1)^n \operatorname{tg}\left(\frac{1}{n}\right)$

→ Conv. absoluta $\sum \left|(-1)^n \operatorname{tg}\left(\frac{1}{n}\right)\right| = \sum \operatorname{tg}\left(\frac{1}{n}\right)$

$$\operatorname{tg}\left(\frac{1}{n}\right) = \frac{\operatorname{sen}\left(\frac{1}{n}\right)}{\cos\left(\frac{1}{n}\right)} = \operatorname{sen}(1/n) \times \frac{1}{\cos(1/n)}$$

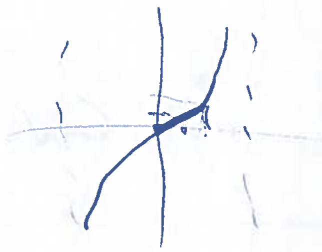
$$\lim_{n \to +\infty} \frac{\sin(1/n) \times \frac{1}{\cos(1/n)}}{\frac{1}{n}} = 1 \times 1 = 1 > 0 \text{ e } f$$

$\therefore$ Pn crit $\sum tg(1/n)$ e $\sum \frac{1}{n}$

têm a mesma natureza. Como $\sum \frac{1}{n}$ diverge

(S. Dirichlet com $\alpha \leq 1$) a nossa tb diverge

$\therefore$ $\overline{N}$ conv. absolutamente



→ Conv. simples $\quad \sum (-1)^n \underbrace{tg(1/n)}_{a_n}$

· $a_n \to 0$ ✓

· a_n decrescente $\qquad a_n \overset{?}{\geq} a_{n+1}$

$$tg\left(\tfrac{1}{n}\right) \overset{?}{\geq} tg\left(\tfrac{1}{n+1}\right) ✓$$

Pelo c. Leibniz conv. simplesmente

19,

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II

① b) $$\sum (-1)^n \frac{1}{n^3+n}$$

Conv. absolut: $$\sum \left| \ldots \right| = \sum \frac{1}{n^3+n}$$

+PC. VA converge

$\therefore$ Conv. absolut mult

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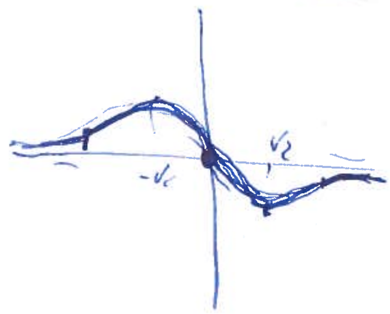
③



$$f' = \frac{-2(x^2+1)+2x\cdot 2x}{(x^2+1)^2} = \frac{2x^2-4}{\ldots}$$

0^o 1^o 2^o

③

$$\lim_{x\to 0} \frac{\operatorname{sen}(4x)}{\operatorname{arctg}(3x)} = \frac{0}{0} \to R.C.$$

$$\lim_{x\to 0} \frac{4\cos(4x)}{\frac{3}{1+(3x)^2}} = \frac{4}{3}$$

$(\operatorname{arctg} x)' = \frac{x'}{1+x^2}$

4) $$\lim_{x\to+\infty} \frac{1}{\sqrt{x^2+1}} \times \underbrace{\cos\left(\frac{\pi}{2}-x\right)}_{\text{limit.}} = 0$$

$\downarrow$
0

infinitésimo × limitada = infinitésimo

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④ Teo. Rolle
- f cont. em $[a,b]$
- f dif. em $]a,b[$
- $f(a)=f(b)$

$\Longrightarrow \exists\ a<c<b : f'(c)=0$

$[-2,-1]$
- h cont. em $[-2,-1]$ ✓
- h dif. em $]-2,-1[$ ✓
- $h(-2)=0=h(-1)$

$\Rightarrow \exists\ -2<c_1<-1 : h'(c_1)=0$

$[-1,0]$
- ✓
- ✓

$h(-1)=0=h(0)$

$\Rightarrow \exists\ -1<c_2<0 : h'(c_2)=0$

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⑤

$[0,1]$ ✓ ✓

$f(0) = 0 = f(1)$

$\exists\ 0 < c_3 < 1 : f'(c_3) = 0$

$[1,2]$ ✓ ✓

$f(1) = 0 = f(2)$

$\exists\ c_4\ \ 1 < c_4 < 2 : f'(c_4) = 0$

f' tem 4 raízes distintas c_1, c_2, c_3, c_4

III

① $\int \cos(\sqrt{x})\, dx$

Subs $\sqrt{x} = t$
$x = t^2$
$dx = 2t\, dt$

$\int \underbrace{\cos t}_{u'} \times \underbrace{2t}_{v}\, dt$

$u' = \cos t \Rightarrow u = \sin t$
$v = 2t \Rightarrow v' = 2$

$= \sin t \times 2t - \int \sin t \times 2\, dt =$

$= 2t \sin t + 2\cos t$

Voltar a x

$= 2\sqrt{x}\,\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + C$

Pist. $g\left(\frac{\pi^2}{4}\right) = 2 \Rightarrow 2 \times \frac{\pi}{2} \sin\frac{\pi}{2} + 2\cancel{\cos\frac{\pi}{2}} + C = 2 \Rightarrow C = 2 - \pi$

$\therefore g(x) = 2\sqrt{x}\,\sin(\sqrt{x}) + 2\cos(\sqrt{x}) + 2 - \pi$

② TAYLOR em $x = a$ Resto Lagrange

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(\xi)}{3!}(x-a)^3$$

$$F(x) = \int_2^x \sqrt{t^4+9}\, dt \quad \text{em} \quad x = 2$$

- $F(2) = \int_2^2 \ldots = 0$

- $F'(x) = 1 \times \sqrt{x^4+9} - \cancel{0 \times \sqrt{2^4+9}}$

TFCi

$$F'(2) = \sqrt{2^4+9} = \sqrt{25} = 5$$

- $F''(x) = \left((x^4+9)^{1/2}\right)' = \frac{1}{2}(x^4+9)^{-1/2} \times 4x^3$

$$F''(2) = \frac{1}{2}(2^4+9)^{-1/2} \times 4 \times 2^3 = \frac{1}{2}\frac{1}{\sqrt{25}} \times 32$$

$$= \frac{16}{5}$$

$$\therefore F(x) = 0 + 5(x-2) + \frac{16/5}{2}(x-2)^2 + \frac{F'''(\xi)}{3!}(x-2)^3$$

16/16 ⑥

③ $\lim_{x\to 0} \frac{\int_{x/2}^{x} e^{t^2}\,dt}{2x} = \frac{0}{0}$

$\underline{\underline{R.C.}} \; \underline{\underline{TFCI}} = \lim_{x\to 0} \frac{1 \times e^{x^2} - \frac{1}{2} e^{(x/2)^2}}{2} = \frac{1 - 1/2}{2} = \frac{1}{4}$

④ a) $\int_0^1 x \operatorname{arctg}(x)\,dx$

$\int \underbrace{x}_{u'} \underbrace{\operatorname{arctg} x}_{v}\,dx$ $\quad u' = x \longrightarrow u = \frac{x^2}{2}$

$v = \operatorname{arctg} x \longrightarrow v' = \frac{1}{1+x^2}$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2}{1+x^2}\,dx =$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int 1 - \frac{1}{1+x^2}\,dx$

$= \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg} x$

$= \left[\qquad \right]_0^1 = \ldots$

3) $\displaystyle\int_{-1}^{0} \frac{2e^x}{e^{2u}-2e^u+1}\,du = 2\int_{-1}^{0} \frac{e^u}{(e^u-1)^2}\,du$

$$\downarrow$$

$$(e^u-1)^2$$

$$= 2\int_{-1}^{0} e^u\,(e^u-1)^{-2}\,du$$

$$= 2\left[\frac{(e^u-1)^{-1}}{-1}\right]_{-1}^{0} = 2\left(0 - \frac{(e^{-1}-1)^{-1}}{-1}\right)$$

$$= \frac{2}{1/e - 1}$$

ou Subs

$e^u = t$

$e^{2u} = t^2$

$u = \ln t$

$du = \frac{1}{t}\,dt$

1) $\displaystyle\int_{0}^{+\infty} \frac{1}{\sqrt{u}}\,du \equiv \sum_{n=0}^{+\infty} \frac{1}{\sqrt{n}}$ diverg

Midt

$$\int_{0}^{10} \frac{1}{\sqrt{u}}\,du + \int_{10}^{+\infty} \frac{1}{\sqrt{u}}\,du$$

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⑥

$$A = \int_0^{1/c} x^2 - cx^3 \, dx$$

$$\left[\frac{x^3}{3} - \frac{cx^4}{4} \right]_0^{1/c} =$$

$$= \frac{\left(\frac{1}{c}\right)^3}{3} - \frac{c\left(\frac{1}{c}\right)^4}{4} - (0-0)$$

$$= \underset{\times 4}{\frac{1}{3c^3}} - \underset{\times 3}{\frac{1}{4c^3}} = \frac{4}{12c^3} - \frac{3}{12c^3} = \frac{1}{12c^3}$$

$$\frac{1}{12c^3} = \frac{2}{3} \Rightarrow 12c^3 = \frac{3}{2}$$

$$c^3 = \frac{3}{2 \times 12} \Rightarrow c^3 = \frac{1}{8} \Rightarrow c = \sqrt[3]{\frac{1}{8}}$$

$$c = \frac{1}{2}$$

$D \Rightarrow \quad 3e^{x}-1>0 \;\Rightarrow\; e^{x}>\frac{1}{3}$

$x > \ln(1/3)$

$D = \left]\ln(1/3), +\infty\right[$

- Pode haver A. Vertical em $x = \ln(1/3)$

$$\lim_{x\to \ln(1/3)^+} \ln(3e^{x}-1) = \ln\left(3\times\frac{1}{3}-1\right) = \ln(0) = -\infty$$

Há A. Vertical $x = \ln(1/3)$

- A. Horizontal   Se $\exists \lim\limits_{x\to\pm\infty}$

$$\lim_{x\to+\infty} \ln(3e^{x}-1) = \ln(+\infty) = +\infty$$

$\bar{N}$ há A. Horizontal

- A. oblíqua $m = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x}$

$$m = \lim_{x\to+\infty} \frac{\ln(3e^{x}-1)}{x} \overset{\frac{\infty}{\infty}}{\underset{\text{R.C.}}{=}} \lim_{x\to+\infty} \frac{\frac{3e^{x}}{3e^{x}-1}}{1} = 1$$

$10/1$

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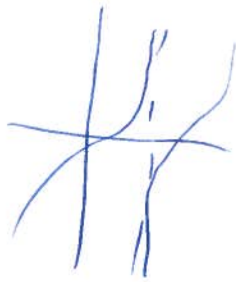
$$b = \lim_{x\to\pm\infty} f(x) - mx$$

$$\lim_{x\to+\infty} \ln(3e^x - 1) - 1x = +\infty - \infty$$

$$= \ln 3$$

$$\div x \quad \cancel{\lim_{x\to+\infty} \frac{\ln(3e^x-1)}{x}}$$



$$\ln(3e^x - 1) = \ln(e^{x+\ln 3} - 1)$$

A. obl. $y = 1x + \ln 3$

$$\lim_{x\to+\infty} \frac{\ln(x+1)}{x}$$

$$\lim_{x\to+\infty} \frac{\ln\left((3e^x-2)+1\right)}{3e^x-2} \times (3e^x - 2)$$