Domínios

1ºF (explicação) 3

- $\frac{1}{u} \Longrightarrow u \neq 0$

- $\sqrt[\text{par}]{u} \Longrightarrow u \geq 0$

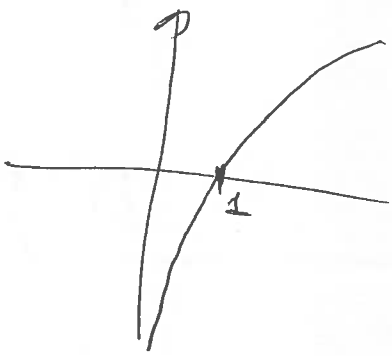
- $\log u \Longrightarrow u > 0$

- $\arcsin u$, $\arccos u \Longrightarrow -1 \leq u \leq 1$

$\cap$

4.1

d) $\ln(\ln x)$



$D = \{ x \in \mathbb{R} : \ln x > 0 \wedge x > 0 \}$

$x > e^0$

$x > 1$

$D = \left]1, +\infty\right[$

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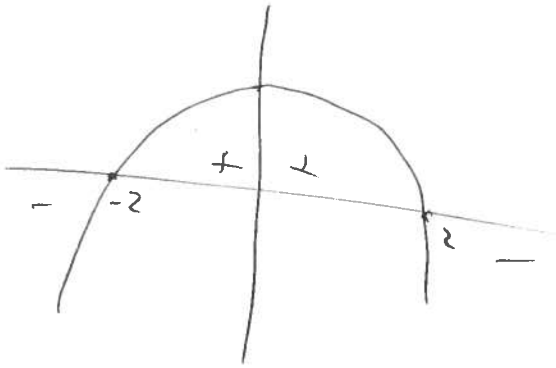
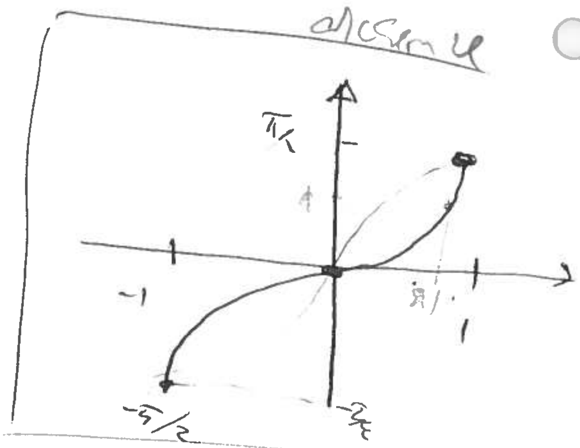
e) $f(x) = \dfrac{x}{\sqrt{4-x^2}}$

$D = \{ x \in \mathbb{R} : 4 - x^2 > 0 \} =]-2, 2[$

$4 - x^2 = 0 \Rightarrow x = \pm 2$



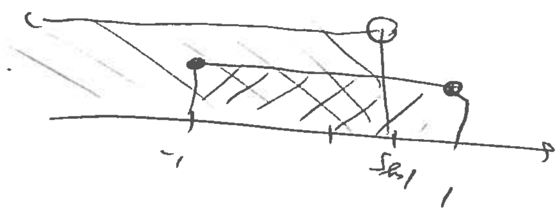


h) $f(x) = \ln(1 - \operatorname{arcsen}(x))$

$D = \{ x \in \mathbb{R} : 1 - \operatorname{arcsen} x > 0$ e $-1 \leq x \leq 1 \}$

$\operatorname{arcsen} x < 1$

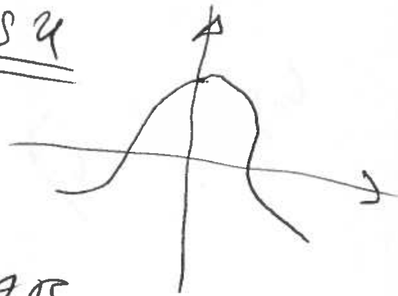
$\Rightarrow x < \operatorname{sen} 1$



$D = [-1, \operatorname{sen} 1[$

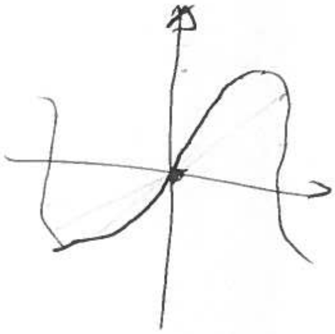
PARIDADE

$\cos x$



- $f(-x) = f(x)$ f é PAR

- $f(-x) = -f(x)$ f é ímpar



4.2

e) $f(x) = \ln\left(\dfrac{2+x}{2-x}\right)$

$\ln(a^b) = b\ln a$

$f(-x) = \ln\left(\dfrac{2-x}{2+x}\right)$

$= \ln\left(\left(\dfrac{2+x}{2-x}\right)^{-1}\right) = -\ln\left(\dfrac{2+x}{2-x}\right) = -f(x)$

$\therefore$ f é ímpar

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②

4.3

c) $f(x) = \operatorname{arctg}(3x)$

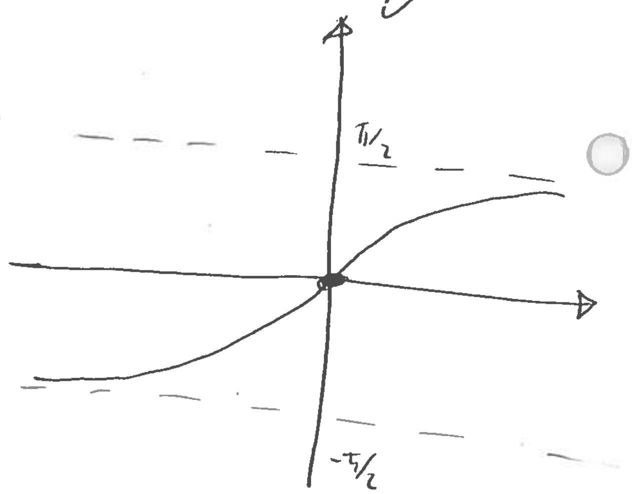
$\operatorname{arctg}(3x) = y \Leftrightarrow$

$3x = \operatorname{tg} y$

$x = \dfrac{\operatorname{tg} y}{3}$

inversa

$f^{-1}(x) = \dfrac{\operatorname{tg} x}{3}$



$\operatorname{tg} x = \dfrac{\operatorname{sen} x}{\cos x}$

$D = \{x \in \mathbb{R} \ \cos x \neq 0\}$

$D = \{x \in \mathbb{R} : x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$

$D =]-\pi/2, \pi/2[$

4.4

$$f(x) = \frac{\pi}{4} - 3\,\mathrm{arcsen}(2x)$$

a) <u>Domínio</u>

$$-1 \leq 2x \leq 1$$

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$D = [-1/2, 1/2]$$

<u>Contradomínio</u>

$$-\pi/2 \leq \mathrm{arcsen}(x) \leq \pi/2$$

$$-\pi/2 \leq \mathrm{arcsen}(2x) \leq \pi/2$$

$$-\frac{3\pi}{2} \leq 3\,\mathrm{arcsen}(2x) \leq 3\frac{\pi}{2}$$

$$3\pi/2 \geq -3\,\mathrm{arcsen}(2x) \geq -3\pi/2$$

$$\frac{3\pi}{2} + \frac{\pi}{4} \geq \frac{\pi}{4} - 3\,\mathrm{arcsen}(2x) \geq -\frac{3\pi}{2} + \pi/4$$

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$$7\pi \ldots \qquad -5\pi \ldots$$

$CD_{f^{-1}} = D'_f = \left[-\frac{5\pi}{4}, \frac{7\pi}{4}\right]$

5) $f^{-1} = ?$

$$\frac{\pi}{4} - 3\arcsin(2x) = y$$

$$-3\arcsin(2x) = y - \frac{\pi}{4}$$

$$\arcsin(2x) = \frac{y - \pi/4}{-3}$$

$$2x = \operatorname{Sen}\left(\frac{y - \pi/4}{-3}\right)$$

$$x = \frac{\operatorname{Sen}\left(\frac{y - \pi/4}{-3}\right)}{2}$$

$$f^{-1}(x) = \frac{\operatorname{Sen}\left(\frac{x - \pi/4}{-3}\right)}{2}$$

i) $f(x) = \frac{\pi}{4} \Rightarrow$

$$\cancel{\frac{\pi}{4}} - 3\arcsin(2x) = \cancel{\frac{\pi}{4}}$$

$$-3\arcsin(2x) = 0$$

$$\arcsin(2x) = 0$$

$$2x = \sin 0$$

$$x = 0$$

4.8

b) c) l) $\Rightarrow$ é igual às sucessões

4.8

m) $$\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^3 - 3x^2 + 2} = \frac{0}{0}$$

Regra Cauchy ~~L'Hôpital~~

$$= \lim_{x \to 1} \frac{(x^2 - 3x + 2)'}{(x^3 - 3x^2 + 2)'}$$

$$= \lim_{x \to 1} \frac{2x - 3}{3x^2 - 6x} = \frac{-1}{-3} = \frac{1}{3}$$

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ou $x^2-3x+2=(x-1)(x-2)$

$x^3-3x^2+2=0$

$$\begin{array}{c|cccc} & 1 & -3 & 0 & 2 \\ 1 & & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & \boxed{0} \end{array}$$

$x^2-2x-2=0$

$$\boxed{\lim_{x\to 0}\frac{\operatorname{sen} x}{x}=1}$$

$(\operatorname{sen} x)'=\cos x$

$$\lim_{x\to 1}\frac{(x-1)(x-2)}{\cancel{(x-1)}(x^2-2x-2)}=\frac{-1}{-3}=\frac{1}{3}$$

h) $\lim_{x\to 0}\frac{\operatorname{sen}(\operatorname{sen} x)}{x}\overset{\frac{0}{0}}{=}\lim_{x\to 0}\frac{\operatorname{sen}(\operatorname{sen} x)}{\operatorname{sen} x}\times\frac{\operatorname{sen} x}{x}$

$\downarrow \quad \downarrow$

$1 \quad 1$

ou

Regra Cauchy $\lim_{x\to 0}\frac{\cos x\cos(\operatorname{sen} x)}{1}=\frac{1}{1}=1$

3.2 b) $$\sum \frac{\sqrt{n+2}-\sqrt{n}}{\sqrt{n(n+2)}} =$$

$$= \sum \frac{\cancel{\sqrt{n+2}}}{\sqrt{n}\,\cancel{\sqrt{n+2}}} - \frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\,\sqrt{n+2}}$$

$$= \sum_{n=1}^{+\infty} \Big(\underbrace{\frac{1}{\sqrt{n}}}_{a_n} - \underbrace{\frac{1}{\sqrt{n+2}}}_{a_{n+2}} \Big)$$ S. Mengoli com $k=2$

$$S = a_1 + a_2 - 2 \lim a_n$$

$$= \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - 2 \lim \frac{1}{\sqrt{n}}$$

$$= 1 + \frac{1}{\sqrt{2}}$$ Converge

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