Primitivos

3ª f (eo operador)

$P_2 = 2x$

$\int 2\,dx = 2x$

$\int 7\,dt = 7t$

$$\boxed{\int x^a = \frac{x^{a+1}}{a+1}}$$

6.1

a) $$\int 3x+2\,dx$$

$$= 3\int x\,dx + \int 2\,dx$$

$$= 3\frac{x^2}{2} + 2x + C, \; C \in \mathbb{R}$$

d) $$\int \sqrt[5]{x^2}\,dx = \int x^{2/5}\,dx = \frac{x^{7/5}}{7/5}$$

$$= \frac{5}{7}\sqrt[5]{x^7} + C$$

$$\boxed{\int u' e^u = e^u}$$

$$\boxed{\int u' u^a = \frac{u^{a+1}}{a+1}}$$

e) $$\frac{1}{3}\int \underset{u'}{3x^2} e^{\overset{u}{x^3}}\,dx = \frac{1}{3}e^{x^3}$$

f) $$\int x\sqrt[3]{1+2x^2}\,dx = \frac{1}{4}\int \underset{u'}{4x}\,(\underset{u}{1+2x^2})^{1/3}\,dx$$

$$= \frac{1}{4}\,\frac{(1+2x^2)^{4/3}}{4/3} = \frac{3}{16}\sqrt[3]{(1+2x^2)^4} + C$$

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h) $\int \frac{1}{\sqrt[5]{2-3x}}\,dx = -\frac{1}{3}\int -3\underbrace{(2-3x)}_{u}^{-1/5 \nearrow a}\,dx$

$= -\frac{1}{3}\frac{(2-3x)^{4/5}}{4/5} = -\frac{5}{12}\sqrt[5]{(2-3x)^4} + C$

i) $\frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2|$

$$\boxed{\int \frac{u'}{u} = \ln|u|}$$

j) $\int \operatorname{tg} x\,dx = -\int \frac{-\operatorname{sen} x}{\cos x}\,dx = -\ln|\cos x| + C$

k) $\int \frac{x^2}{1+x^6}\,dx$

$$\boxed{\int \frac{u'}{1+u^2} = \operatorname{arctg} u}$$

$\frac{1}{3}\int \frac{\overset{u'}{3x^2}}{1+(\underbrace{x^3}_{u})^2} = \frac{1}{3}\operatorname{arctg}(x^3) + C$

l) $\frac{1}{2}\int 2e^{2x}\cos(\underbrace{e^{2x}}_{u})\,dx$

$= \frac{1}{2}\operatorname{sen}(e^{2x}) + C$

$$\boxed{\begin{array}{l}\int u'\cos u = \operatorname{sen} u \\ \hline \int u'\operatorname{sen} u = -\cos u\end{array}}$$

f) $\int \frac{1}{5+x^2} dx$

$\int \frac{1}{1+x^2} = \text{arctg}\, x$

$\int \frac{\mu'}{1+\mu^2} = \text{arctg}\, \mu$

$\int \frac{\mu'}{a^2+\mu^2} = \frac{1}{a} \text{arctg} \left(\frac{\mu}{a}\right)$

$\int \frac{1}{\underset{\downarrow a}{(\sqrt{5})^2} + \underset{\downarrow x}{x^2}} dx$

$\parallel$

$\frac{1}{\sqrt{5}} \text{arctg} \left(\frac{x}{\sqrt{5}}\right) + C$

Per Parti

$\int \mu' \times v = \mu v - \int \mu v'$

Criterio

μ'	v
e^x	$\log x$
$\cos x$	arc..
$\sin x$	polinomi

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d) $\int \underbrace{x^2}_{v} \underbrace{e^x}_{\mu'} dx$

$\mu' = e^x \xrightarrow{\text{Primitiva}} \mu = e^x$

$v = x^2 \longrightarrow v' = 2x$

$= e^x \times x^2 - \int \underbrace{e^x}_{\mu'} \times \underbrace{2x}_{v} dx$

$= e^x \times x^2 -$

$\mu' = e^x \longrightarrow \mu = e^x$

$v = 2x \longrightarrow v' = 2$

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②

$$= e^x x^2 - \left(e^x \times 2x - \int e^x \times 2 \, dx \right)$$

$$= e^x x^2 - e^x 2x + 2 \int e^x \, dx$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

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e) $\int \underbrace{1\times}_{u'} \underbrace{\ln(2x)}_{v} \, dx$

$u' = 1 \longrightarrow u = x$

$v = \ln(2x) \longrightarrow v' = \frac{2}{2x} = \frac{1}{x}$

$$= x \ln(2x) - \int x \times \frac{1}{x} \, dx$$

$$= x \ln(2x) - \int 1 \, dx$$

$$= x \ln(2x) - x + C$$

Extra

$$\int_0^2 7x \, e^{8-3x^2} \, dx$$

CA

$$\int 7x \, e^{\overbrace{8-3x^2}^{u}} \, dx = \frac{7}{-6} \int -6x \, e^{8-3x^2} \, dx$$

$$= -\frac{7}{6} e^{8-3x^2} + C$$

$$\int_0^2 7x e^{8-3x^2}dx = \left[-\frac{7}{6}e^{8-3x^2}\right]_0^2$$

$$= -\frac{7}{6}e^{8-3\times 2^2} - \left(-\frac{7}{6}e^{8-3\times 0^2}\right)$$

$$= -\frac{7}{6}e^{-4} + \frac{7}{6}e^{8} \;//$$

6.4

<u>Substituição</u>

o que chateia = t

5) $\int x\sqrt{1+3x}\,dx$

- $\sqrt{1+3x} = t$
  $1+3x = t^2$
- $x = \frac{t^2}{3} - \frac{1}{3}$
- $dx = \frac{2t}{3}\,dt$

$$\int\left(\frac{t^2}{3}-\frac{1}{3}\right)\times t \times \frac{2}{3}t\,dt$$

$$= \frac{2}{3}\int\left(\frac{t^2-1}{3}\right)t^2\,dt = \frac{2}{9}\int t^4 - t^2\,dt$$

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$$= \frac{2}{9}\left(\frac{t^5}{5}-\frac{t^3}{3}\right) = \frac{2}{9}\left(\frac{\sqrt{1+3x}^5}{5} - \frac{\sqrt{1+3x}}{3}\right) + C$$

△ Atenção

1) $\int \frac{x}{\sqrt{(1+x^2)^3}}\,dx =$

$\frac{1}{2}\int 2x\,\underset{u}{(1+x^2)}^{\overset{\rightarrow n}{-3/2}}\,dx = \frac{1}{2}\,\frac{(1+x^2)^{-1/2}}{-1/2}$ $\boxed{\sqrt{x} = x^{1/2}}$

$= -\frac{1}{\sqrt{1+x^2}} + C$ $\sqrt[3]{x^2} = x^{2/3}$

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c) $f'' = \frac{1}{1+x^2}$ $f'(0) = 2$ e $f(0) = -1$

$f = ?$

$f' = \int f'' = \int \frac{1}{1+x^2} = \operatorname{arctg}(x) + C$

Pode $f'(0) = 2 \Leftrightarrow \operatorname{arctg}(0) + C = 2 \Leftrightarrow C = 2$

$\therefore f'(x) = \operatorname{arctg} x + 2$

$f = \int f' = \int \operatorname{arctg} x + 2\,dx = \int \operatorname{arctg} x\,dx + \int 2\,dx$

Cá $\int \underset{u'}{1} \times \underset{v}{\underline{\operatorname{arctg} x}}\, dx$ $\qquad u' = 1 \longrightarrow u = x$

$\qquad\qquad v = \operatorname{arctg} x \longrightarrow v' = \frac{1}{1+x^2}$

$= x \operatorname{arctg} x - \int x \times \frac{1}{1+x^2}\, dx$

$= x \operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx$

$= x \operatorname{arctg} x - \frac{1}{2} \ln|1+x^2|$

$f = x \operatorname{arctg} x - \frac{1}{2} \ln|1+x^2| + 2x + D$

Pista

$f(0) = -1 \Rightarrow \cancel{0 \times \operatorname{arctg}(0)} \cancel{- \frac{1}{2} \ln|1+0^2|} + 2 \times 0 + D = -1$

$D = -1$

$\therefore f(x) = x \operatorname{arctg} x - \frac{1}{2} \ln|1+x^2| + 2x - 1$ ✓

(6.8)

b) $\int \frac{x^3}{x+1}\, dx$

grau cima ≥ baixo

⇓

Divide

$\overset{N}{x^3 + 0x^2 + 0x + 0} \;\; \underline{| \overset{D}{x+1}}$

$-x^3 - x^2 \qquad\qquad x^2 - x + 1$

$\qquad\qquad\qquad\qquad Q$

$-x^2$

$+x^2 \quad + x$

x

$-x \quad -1$

$\frac{N}{D} = Q + \frac{R}{D}$

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(4)

$$\int \frac{x^3}{x+1} = \int x^2 - x + 1 + \frac{-1}{x+1}\, dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \int \frac{1}{x+1}\, dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1|$$

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d) $$\int \frac{3x+1}{x^3 - x}\, dx$$

$$\frac{0x^2 + 3x+1}{x(x^2-1)} = \frac{3x+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$\times (x-1)(x+1)$ x^2-1

$\times x(x+1)$ x^2+x

$\times x(x-1)$ x^2-x

$$= \frac{Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx}{}$$

$$\Rightarrow \begin{cases} A+B+C = 0 \\ B - C = 3 \\ -A = 1 \end{cases} \Rightarrow \begin{cases} -1+3+C+C=0 \\ B = 3+C \\ A = -1 \end{cases}$$

$$\Rightarrow \begin{cases} C = -1 \\ B = 2 \\ A = -1 \end{cases}$$

$$\int \frac{3x+1}{x^3-x}\,dx = \int \frac{-1}{x} + \frac{2}{x-1} + \frac{-1}{x+1}\,dx$$

$$= -\ln|x| + 2\ln|x-1) - \ln|x+1| + C$$

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⑤