Defining distance units for use in HR diagram code

Ciaran

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Throughtout this document I have referred to Carroll & Ostlie p.61 to base my definitions from.
One magnitude will indicate a difference of a factor e between fluxes,

$$\frac{F_2}{F_1} = e^{m_1 - m_2}. \tag{1}$$

Taking the logarithm (base e) we have

$$m_1 - m_2 = \log \frac{F_2}{F_1}. \tag{2}$$

We set our scale so that when $F = 1$ (units), we have $m = 0$. Hence,

$$m = -\log F. \tag{3}$$

Recall that we have

$$F = \frac{L}{4\pi r^2}. \tag{4}$$

Here I choose to define an absolute magnitude M to be the magnitude at 1pc (here we take a parsec to be an arbitrary but natural unit). Combining equations 2 and 4, we have

$$e^{m-M} = \frac{F_1}{F} = \left(\frac{d}{1\text{pc}}\right)^2. \tag{5}$$

This gives us an equation for the distance, where d is in parsecs,

$$d = e^{\frac{m-M}{2}}. \tag{6}$$

Finally we compute the distance modulus,

$$m - M = 2\log d, \tag{7}$$

which we can use to find that

$$M = m - 2\log d. \tag{8}$$