Electronic Supplementary Material for the paper: 'A Bayesian Quest for Finding a Unified Model for Predicting Volleyball Games'

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A. Sensitivity analysis

To account for robustness in our posterior estimates, we performed some sensitivity tests for the model's parameters (see Table 4 in the paper for a comprehensive summary of the final selected model). Sensitivity plots may be retrieved in the Supplementary Material folder available at `https://github.com/LeoEgidi/Bayesian-Volleyball-paper`. Specifically, we let vary:

- the standard deviations of the normal priors for the parameters $\mu, \theta, H^{point}, H^{set}, m$ (first five panels in Figure A.1);
- the scale parameter of the log-normal prior for λ (sixth panel in Figure A.1);
- the inverse-gamma parameters a_1, a_2, b_1, b_2 for the hyper-priors $\tau_\alpha^2 \sim \mathsf{InvGamma}(a_1, a_2)$, $\tau_\beta^2 \sim \mathsf{InvGamma}(b_1, b_2)$, assuming that the set and point abilities are assigned two normal priors, $\boldsymbol{\alpha} \sim \mathcal{N}(0, \tau_\alpha^2)$, $\boldsymbol{\beta} \sim \mathcal{N}(0, \tau_\beta^2)$, respectively. For simplicity, we assume $a_1 = a_2$, $b_1 = b_2$ (Figures A.2–A.3).

Red triangles on the x-axis in Figures A.1 denote the values used in the final model. As a general comment, the posterior estimates seem to be not much sensitive with respect to the hyperparameters choices. All selected values seem to lie on the "non-informative" region where the priors are stabilized to a specific posterior solely based on data information.

Regarding the point and set abilities in Figures A.2–A.3, we note that there are not sensitive changes of the posterior estimates in correspondence of distinct hyperparameters' values of the inverse-gamma distributions for $\tau_\alpha^2, \tau_\beta^2$.

Then, according to these checks, we feel our analysis are robust enough to justify the hyperparameters' choices adopted in the paper.

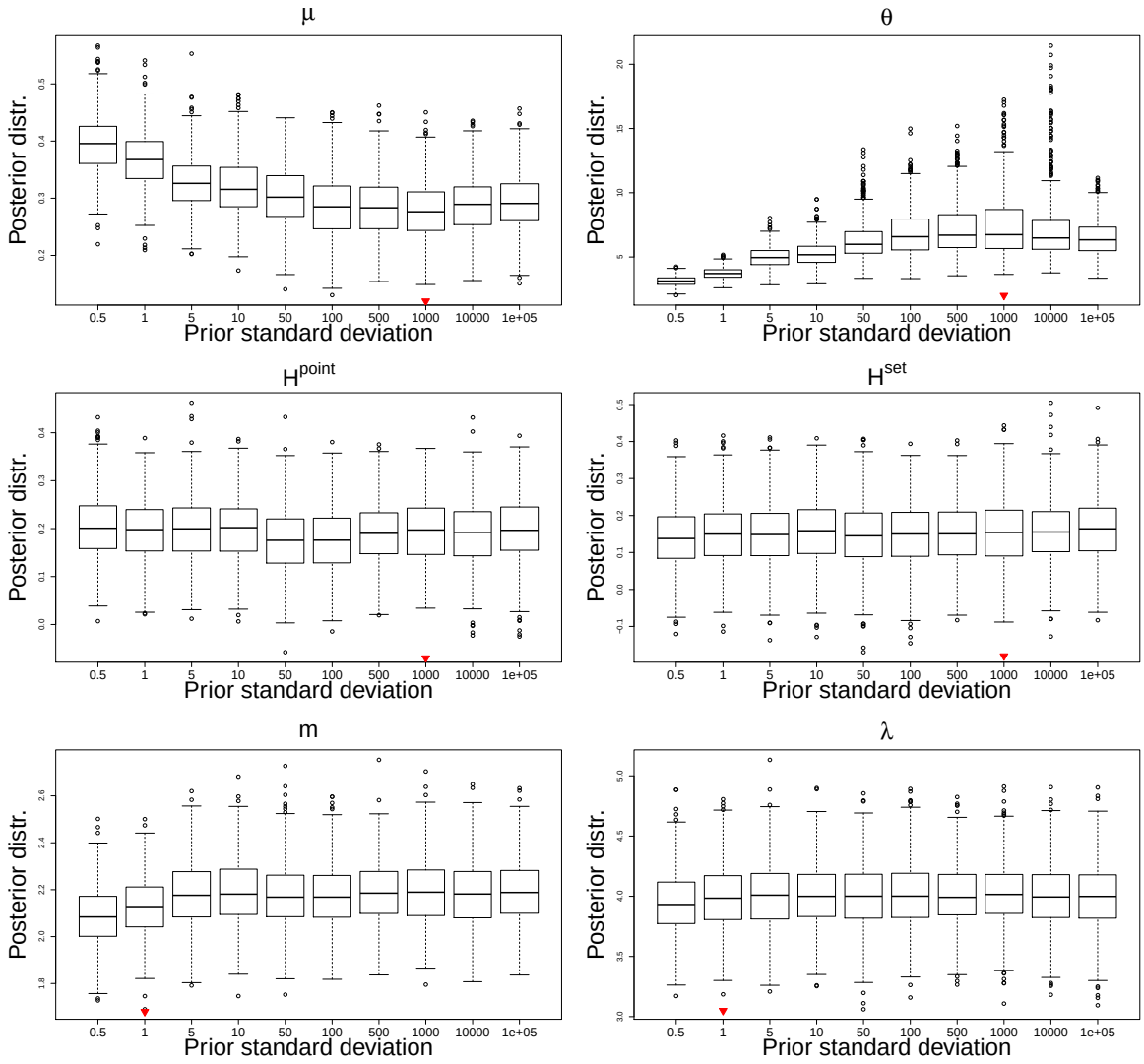

Figure A.1. Sensitivity tests for for the marginal posterior distributions of the parameters: $\mu, \theta, H^{point}, H^{set}, m, \lambda$ by varying the standard deviations of the normal priors and the scale parameter of the log-normal prior. `rjags`, 1000 MCMC iterations, burn-in period of 100 iterations.

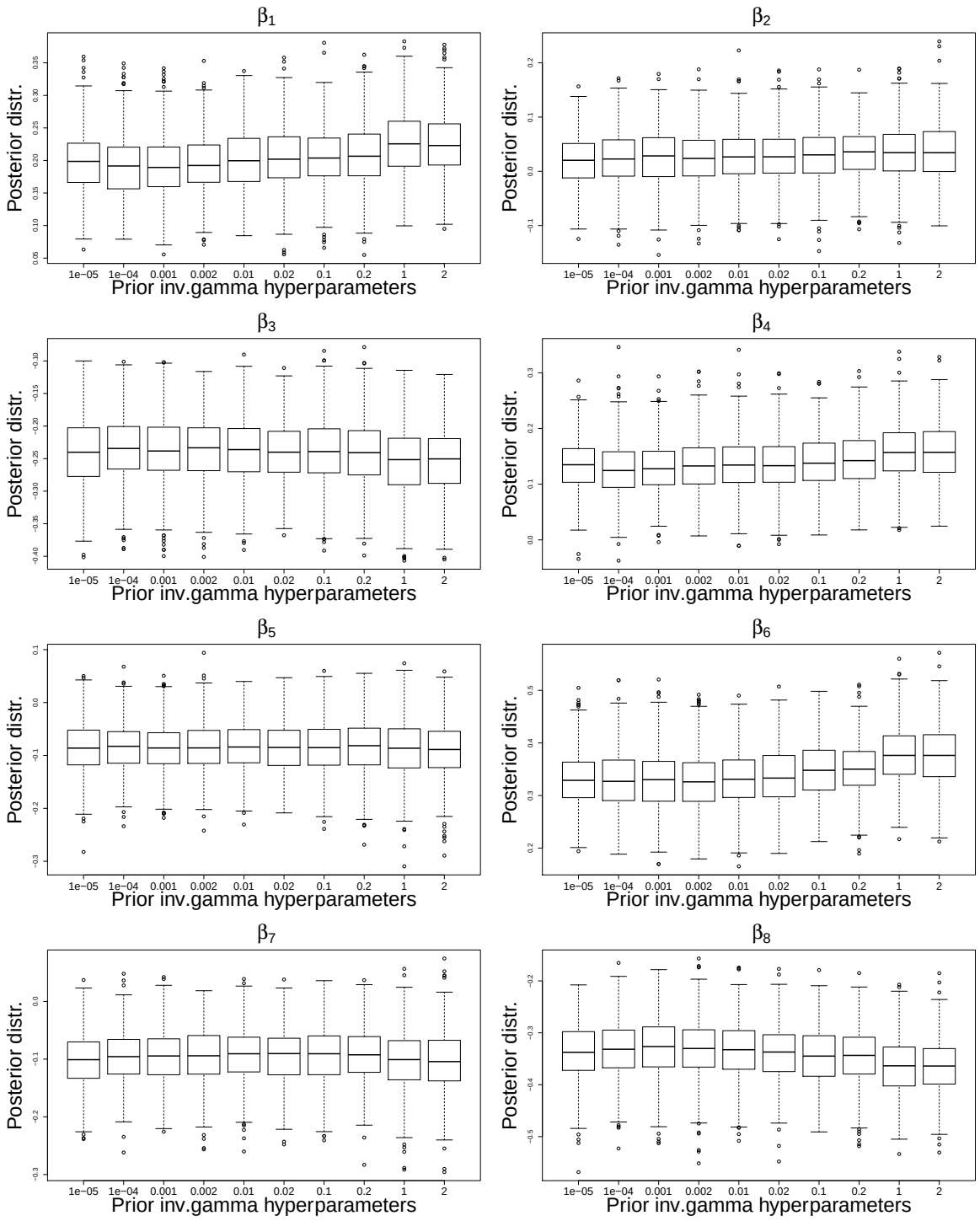

Figure A.2. Sensitivity tests for the marginal posterior distributions of the point abilities parameters $\beta_1, \ldots, \beta_8$ by varying the hyperparameters of the inverse-gamma prior assigned to τ_β^2. `rjags`, 1000 MCMC iterations, burn-in period of 100 iterations.

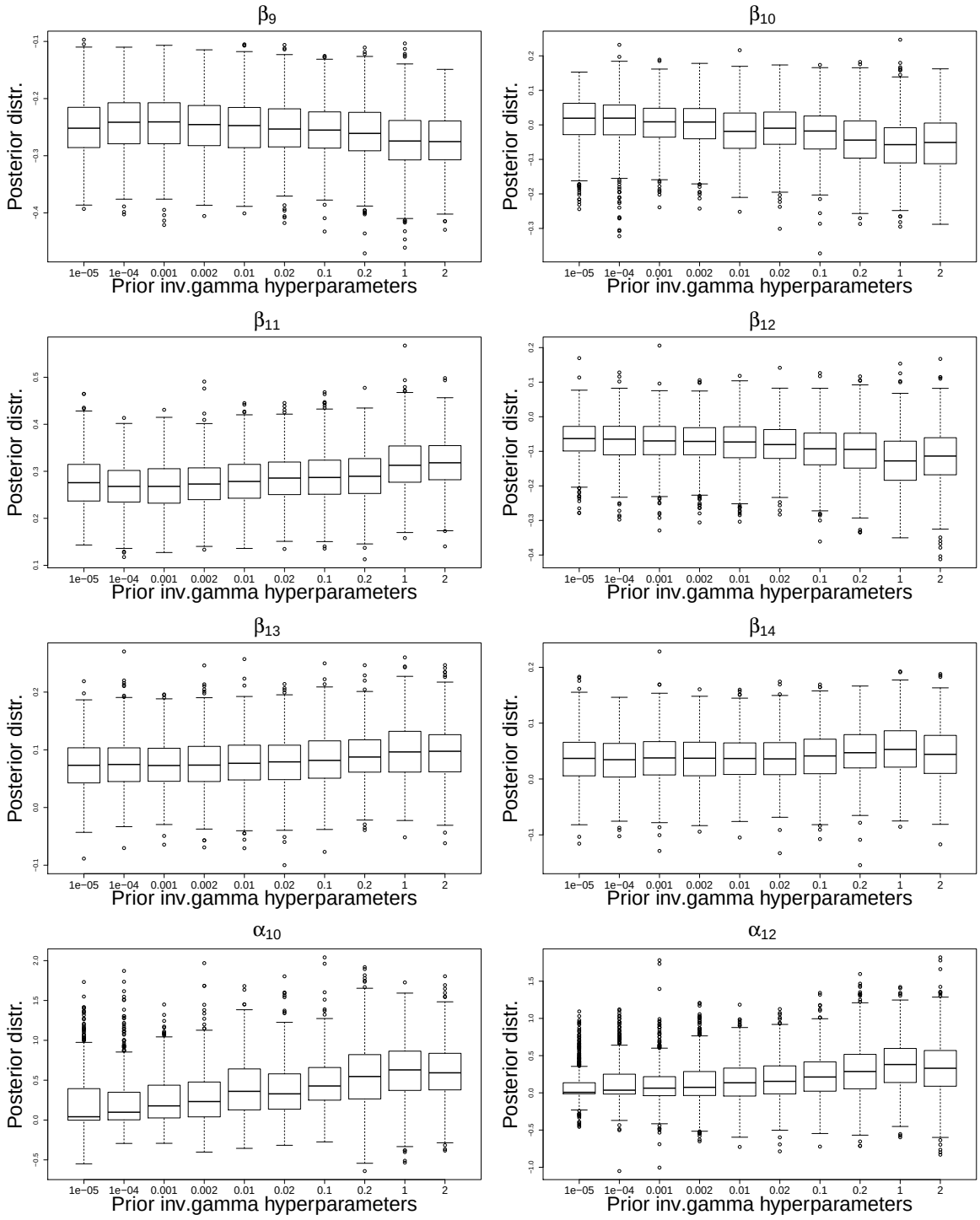

Figure A.3. Sensitivity tests for the marginal posterior distributions of of the point abilities parameters $\beta_9, \ldots, \beta_{14}$ and the extra set abilities α_{10}, α_{12} by varying the hyperparameters of the inverse-gamma priors assigned to the parameters: $\tau^2_\alpha, \tau^2_\beta$. `rjags`, 1000 MCMC iterations, burn-in period of 100 iterations.

B. Set dynamic abilities

For completeness, we depict in Figure B.1 the predictive intervals for the dynamic set abilities (model 11 of Table 2 in the paper), with $\sigma^2_\alpha \sim \mathsf{InvGamma}(2,2)$.

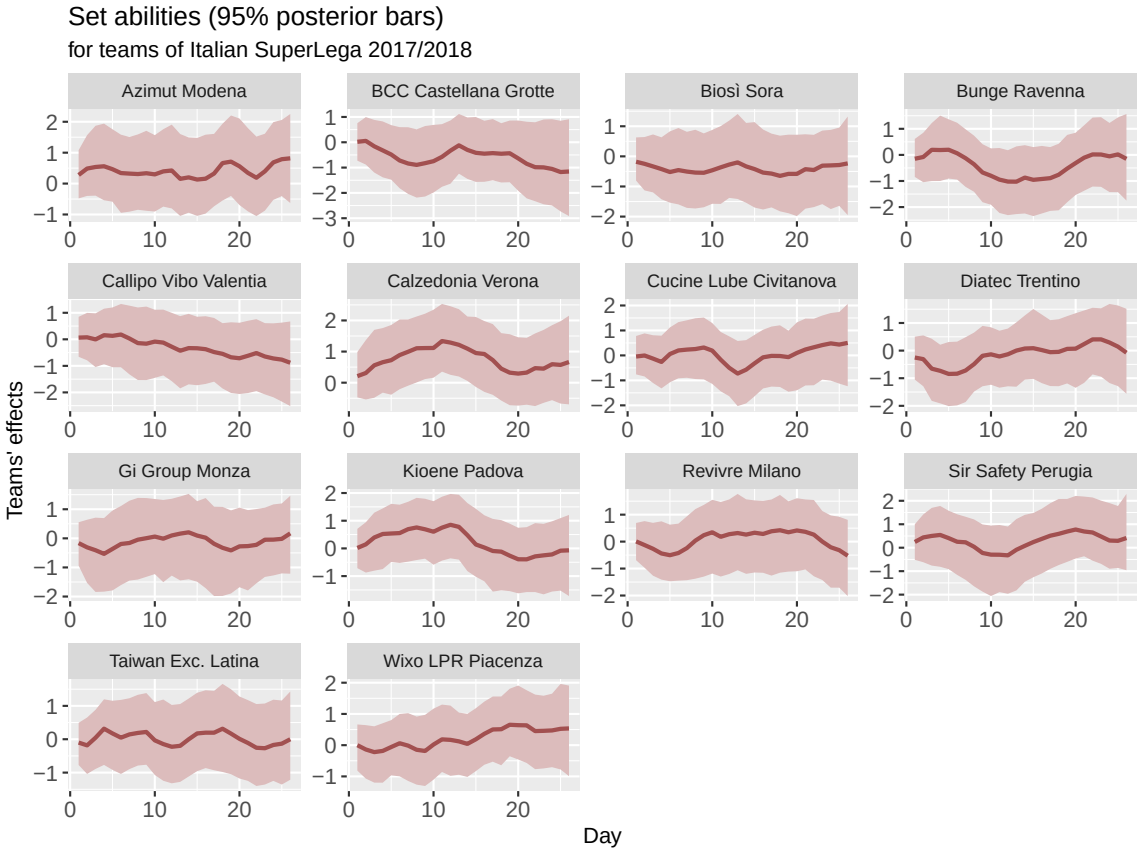

Figure B.1. Posterior mean and 95% density interval for the dynamic set abilities parameters α, Italian SuperLega 2017/2018 (model 11 in Table 2 of the paper).

C. Graphical convergence diagnostics

We depict in Figure C.1–C.2 the trace and the density plots from the MCMC sampling for the parameters: $\mu, \theta, H^{point}, H^{set}, \lambda, m$ (`rjags`, 1000 iterations, burn-in of 100, 3 chains).

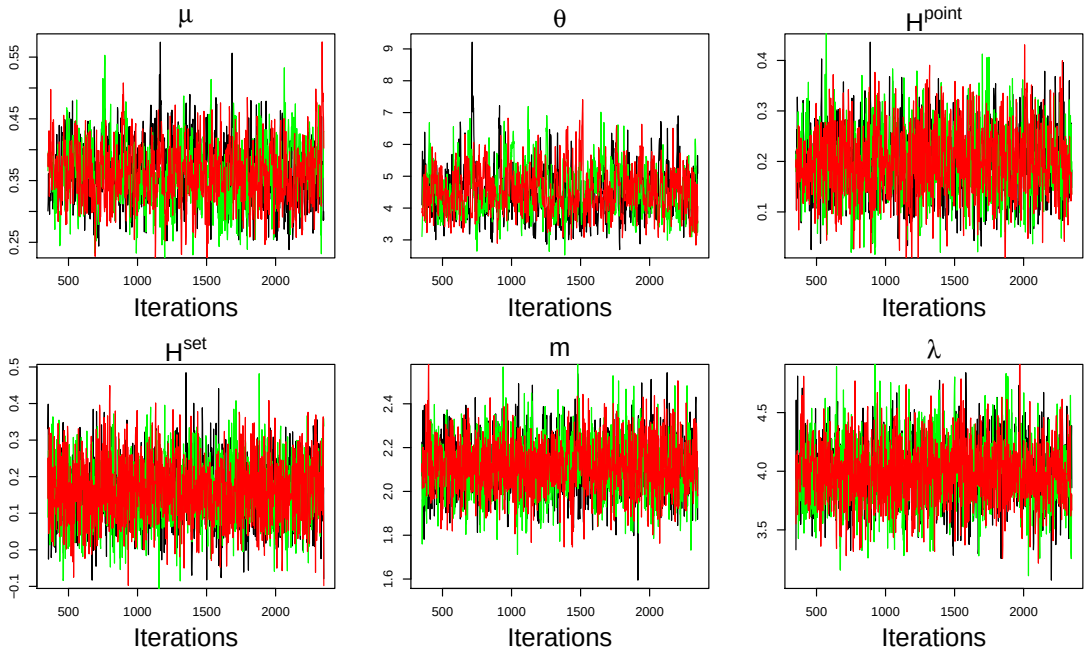

Figure C.1. Trace plots for the parameters: $\mu, \theta, H^{point}, H^{set}, \lambda, m$ (model 9 in the paper).

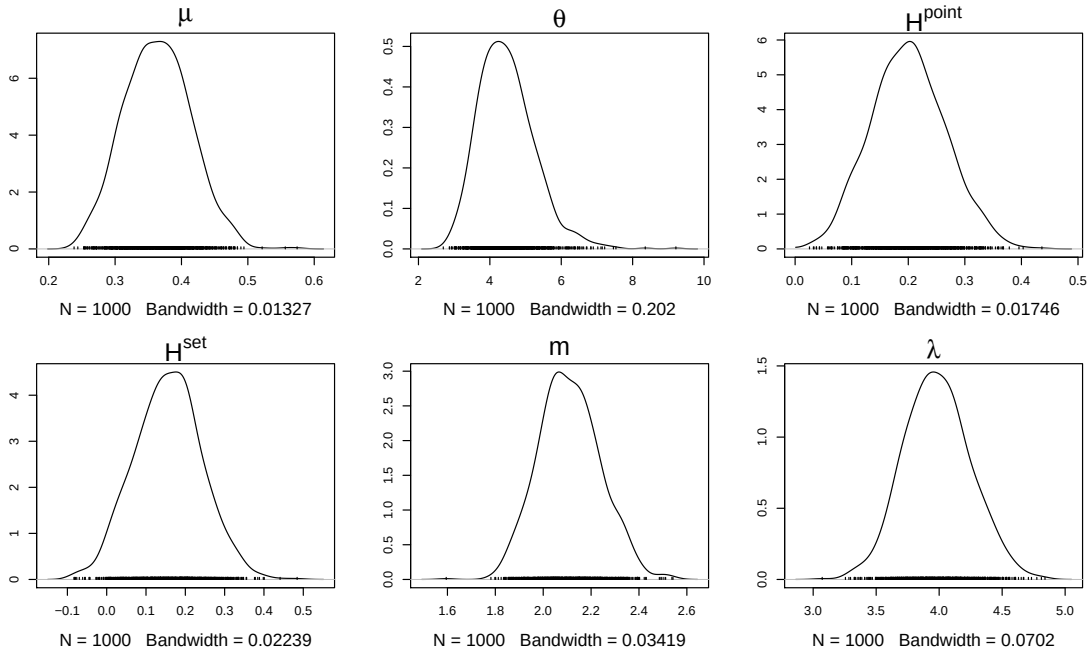

Figure C.2. Density plots for the parameters: $\mu, \theta, H^{point}, H^{set}, m, \lambda$ (model 9 in the paper).