Prediction is not everything, but everything is prediction

Abstract

Prediction is an unavoidable task for data scientists, and over the last few decades, statistics and machine learning have become the most popular 'prediction weapons' in many fields. However, prediction should always be associated with a measure of uncertainty - because from it we can only reconstruct and falsify the model/algorithm decisions. Machine learning methods offer many point predictions, but they rarely yield a measure of uncertainty, whereas statistical models usually do a poor job of communicating predictive results. According to Popper?s falsification philosophy, natural and physical sciences can be falsified on the grounds of incorrect predictions: however this is not always true in the social sciences. We move then to a weak instrumentalist philosophy: Predictive accuracy is not always constitutive of scientific success, especially in the social sciences.

Keywords: Prediction; Popper?s falsification philosophy; Weak instrumentalism; Predictive accuracy; Machine learning

1 Introduction

The ultimate goal of science is doubtless to find the truth while testing theories, and this sparked a lot of debate about two distinct positions in the philosophy of science, namely realism and empiricism (Sober, 2002). According to the former, the main scientific goal is to test and check which theories are true, whereas the latter suggests to discover which theories could be empirically suited for the problem at hand. In both cases, truth is the primary focus. According to another philosophical position, instrumentalism, science should instead provide accurate predictions rather than assessing whether a theory is true or not: in other words, the search for true theories and that for accurate predictive theories coincide; on the other hand, many philosophers of science claim that prediction has a primary role in the progress of science and, accordingly, how predictive accuracy is one of the ultimate goals in scientific inference (Forster, 2002). The interesting interplay existing between truth and prediction accuracy invokes falsificationism (Popper, 1934) with related criticisms (Kuhn, 1962; Lakatos, 1976). Popper argues in fact that theories, to be scientific, must be falsifiable on the ground of their predictions: wrong predictions should perhaps push scientists to reject their theories or to re-formulate them, conversely exact predictions should corroborate a scientific theory. Using the above terminology, we could say that Popper's philosophy is instrumentalist in a strong sense (Hitchcock and Sober, 2004) when applied to physical and natural sciences: predictive accuracy is constitutive of scientific success, not only symptomatic of it, and prediction works as a confirmation theory tool for science.

With regard to statistics, we are continuously pushed and asked to build, develop and test scientific theories by using a bag of tools containing distinct items, such as hypothesis testing, likelihood theory, Bayesian methods, randomized trials, non-parametric methods, and so on. As statisticians, we daily construct elegant devices (models) in many scientific disciplines, such as economics, psychology, physics, biology, education, and environmental sciences, designed to: (i) formulate a theory (ii) test the theory from some evidence (data) (iii) generalize the theory by induction (iv) confirm the theory. By adopting the above terminology, as statisticians we feel to belong to the tradition of empiricism, in line with the well-known George E. P. Box's quote "All models are wrong, but some are useful": we could then never 'sell' our models as true and real theories, rather we could just assess their empirical adequacy. However, this assessment is nowadays the matter of a vivid debate within the statistical community about the distinction between explanatory and predictive modeling (Shmueli, 2010; Hitchcock and Sober, 2004), particularly when the two dimensions—explanation and predictions—result to be in conflict with each other. To clarify, we define here explanation or explanatory modeling the use of statistical models for testing causal hypotheses or explanations—e.g. between a set of covariates and a response variable; we define prediction or predictive modeling the action of using a model (or device, algorithm) to produce predictions for new or future observations. The milestone of this debate can be considered as one of the most influential papers ever, Breiman et al. (2001), where Leo Breiman shed light on two distinct cultures, the modeling and the algorithmic one, which usually raised the importance of causal explanation and prediction, respectively.

However, these two dimensions often appear conflated, and any attempt to state the supremacy of one of the two entities over the other is in most cases misleading. For such reason, we think the debate about the supposed prevalence of one of the two dimensions is local and application-dependent and, in general, wasteful. Rather, we could try to go beyond, by incorporating prediction in the explanation step. As statisticians, we spent most of our efforts and our best years for reporting statistical estimates and standard errors,

along with measures of statistical significance (Gelman, 2016d), for unobservable parameters that do not exist in real life; rather, in this paper we maintain with Billheimer (2019) that inferential statistics is inherently predictive and should be focused on probabilistic predictions of future observable events and quantities. In such a way, the predictive inference paradigm (Bjornstad, 1990; Geisser, 2017), somehow in line with the prequentialism theory of Dawid (1984), allows the comparison of competing scientific theories by way of predictive accuracy and avoids to emphasize the estimation of virtual and artificial parameters. Moreover, from a communication point of view observed values are easier to interpret and much more accessible to non-statisticians than artificial parameters.

However, taking prediction to assess and explain model capability does not solve all the problems surrounding the statistical discipline. Nowadays, statistical models are currently used with application to very different disciplines, ranging from the so-called 'hard sciences' such as physics, biology and engineering to some social sciences such as psychology, education and economics. In this sense, we do not want to be entirely instrumentalist: rather, in this paper we encourage the use of predictive inference and propose the weak instrumentalism (herafter, WI), under which predictive accuracy is constitutive of scientific success only when the underlying statistical methods are falsifiable and transparently designed to predict future and new events. In other words, there are many contexts, especially in the social sciences, where falsification through the prediction's fallacy should be replaced by a more consistent idea of falsification: we believe this position may be beneficial for the socalled hard sciences as well. On one hand, mathematical and quantitative laws formulated by Galilei and Newton were physical and deterministic with which particular future facts could have been predicted with absolute precision; on the other hand, probabilistic and statistical laws designed to describe human behavior and social facts are stochastic laws,

with which particular future events could be predicted with an intrinsic amount of uncertainty. Of course, as statisticians we want to do our best in predict future social events, but we cannot entirely evaluate a model's performance only on the ground of its predictive accuracy. Using Popper's terminology, incorrect social sciences predictions should not be the only tool to falsify a theory.

From the arguments above it emerges clearly how prediction is relevant and instrumental for scientists working with data: however, it is not all we need, especially when framed in social science frameworks. Perhaps we need transparent assumptions, acknowledgment of variability in the distribution of the observables, considerations about the legitimation of the inductive reasoning. And it is clear how this legacy, in the modern big-data age with vast amounts of available data, should involve the whole data-science field, not only the theoretical statisticians or the *data-miners*: the WI philosophy should push model developers to criticize themselves and to eventually reformulate their algorithms in order to improve the scientific inference.

The remainder of this paper is organized as follows.

2 The role of prediction in hard and social sciences: a brief overview

In a very popular and well-known essay, Russell (1931) summarized the three main stages required to formulate a scientific law as follows: (1) observation of some relevant facts, (2) formulation of a hypothesis underlying and explaining the previously mentioned facts, and (3) deduction of some consequences from this hypothesis. As suggested by the same Russell, the modern scientific method was born with Galileo Galilei, father of the law

of falling bodies, and with Johannes Kepler, who discovered the three laws of planetary motion (from Russell (1931)):

Scientific method, as we understand it, comes into the world full-fledged with Galileo (1564-1642), and, to a somewhat lesser degree, in his contemporary, Kepler (1571-1630). [...] They proceeded from observation of particular facts to the establishment of exact quantitative laws, by means of which future particular facts could be predicted.

It is easy to see how the sequence of events that occurred over the next 250 years told us much about how scientific inference proceeds, with the law of universal gravitation of Isaac Newton embodying the two previous theories (18th century), and the theory of the general relativity of Albert Einstein (20th century) generalizing itself Newton's theory. It is then clear how in the last 500 years, physics—and, more generally, science—advanced by falsification and generalization of previous theories, by providing new and more exciting theories to predict new natural facts and highlighting the confirmation nature of prediction. Perhaps, as Hitchcock and Sober (2004) argue, mathematical descriptions of the invariant behaviour of a physical phenomenon are essentially predictive: further experiments and observations can validate these theories.

Aggiungere esempio su chimica e Mendeleev?

However, the role of prediction with regard to scientific progress is more ambiguous than what people are usually inclined to think. In general, is prediction a central step in science, or even, in the words of Forster (2002), is prediction the ultimate goal of science, according to the instrumentalist position (Sober, 2002; Hitchcock and Sober, 2004)? From an instrumental perspective, predictive success is not merely *symptomatic* of scientific success, is also *constitutive* of scientific success (Hitchcock and Sober, 2004). On the other

hand, for Popper (1934), a theory is scientific only if it is falsifiable, where the falsification of a theory is meant to be the possibility of comparing its predictions with the observed data. In his view, theories whose predictions conflict with any observed evidence must be rejected: prediction corroborates (or confirms) a theory when it survives an attempt at falsification; prediction delegitimizes a theory when it does not pass the falsification test.

To complicate the debate, over the last decades scientific predictions has become popular not only in the context of physics and natural science, but for the social sciences as well. As argued by Popper (1944, 1945) and Sarewitz and Pielke Jr (1999), the social sciences have long tried to emulate physical sciences in developing invariant mathematical laws of human behavior and interaction to predict economics quantities, elections, policies, etc., being the modern data-scientists increasingly asked to build 'weapons of mass prediction' in varying social contexts. Even though many scholars agree that a social theory should be judged on its power to predict (Friedman, 1953), we feel however that an instrumentalist position about predictive accuracy is still ambiguous in the social sciences, especially when the final actual observed outcome is somehow far away from the model's predictions. To take some explanatory and well-known examples, consider some very low-probability events, all actually occurred during 2016, such as the Donald Trump's win against Hillary Clinton in the US presidential elections, UK Brexit, and the Leicester's Premier League's win in soccer. None of the best data-scientists and modelers would bet even a dollar on each of these events happening—consult Gelman (2016a,b,c) for some more considerations about these facts. Can all of these rare, though actually observed, events falsify the finest algorithms and models that did not correctly predict their occurrence? Does it make sense to criticize such social science models/algorithms by retrospective evaluations of probabilistic predictions? We believe that social science methods require more valid motivations

to validate or falsify underlying theories beyond that of providing bad predictions for rare events (Gelman et al., 1998), such as a deep inspection of quality of data, psychological attitudes, study design, inclusion of more/other covariates and eventual interactions, and so on. Scientific falsification detected by wrong predictions (Popper, 1934) is a powerful and exceptional tool, but in this paper we caution its abuse/misuse. We give more details on this weak instrumentalist position in the next sections.

3 The role of prediction in statistical learning

Questa e' da riformulare. Probabilmente 3.1 si tiene, mentre 3.2 e 3.3 vanno ripensate e cambiate un po' di contenuti. 3.4 come e' al momento si toglie, e qualcosa si ricomprende da altre parti.

3.1 From the observed to the observable

As statisticians, we often deal with a double task: first, creating a sound mathematical model to accommodate the data and retrieve useful inferential conclusions from parameters' estimates—in this section, we make no distinction between classical and Bayesian paradigm; second, using this model to make predictions. From a practical point of view, inference and prediction should act sequentially and appear as "two sides of the same coin", or two dimensions in the spirit of Shmueli (2010), while contributing to the statistical workflow by coherently accounting for intrinsic model uncertainty.

However, the widespread feeling is that statistics has always been considered the *science of inference*, or *science of estimates*, and inference often considered the dominant side and seen as separate from prediction (Shmueli, 2010). Inference creates an underlying mathematical model of the data-generating process (Bzdok et al., 2018), its main task is

to formulate a theory that adequately captures an unknown mechanism connecting potentially influential predictors with a response variable, by explaining a theoretical causal relationship; the inferential laws should be as general as possible, ideally valid for the population of interest, and not symptomatic of the observed data (it is out of the scope of this paper to review the distinct inferential approaches). Prediction instead moves from the observed to the unobserved (though observable in the future), being the action designed to forecast future or new events without requiring a full understanding of the underlying data-generation process. Predictive actions can be daily shared and accepted from the human common sense: each person is in fact more or less confident with weather's predictions or with presidential election predictions, but rarely that person is aware of the underlying statistical model required to produce that forecast, unless he is a statistician/data scientist. In such a view, inference seems difficult and obscure, and prediction simple and transparent to the people.

This is a paradoxical argument, since inference is often associated to the action of explaining a given problem, and its results should be relevant and available to the majority of the population. However, statisticians operate like those magicians who resemble their decks of cards by fictitiously adding/removing extra cards that are not available to the audience. To continue with the metaphor, model parameters behave like these fake cards, which are incredibly relevant to build the trick (aka statistical theories), but do not exist in the real life; in brief, parameters are fictitious and technical devices used to explain and approximate complexity. Rather, only prediction links the observed with the observable and is accessible to the people: it is never a matter of parameters' interpretation (Billheimer, 2019), it only requires a check of the discrepancy between observed and future events, and can doubtless be done by anyone.

When framed in a predictive task, many technical questions arise. First, should we use all the data to build a reasonable/useful model, or take only a portion of the sample to accommodate the model (the training set), using the remaining values for validation and testing (the validation and test sets)? Is an overfitting model suited enough for predictive purposes in out-of-sample scenarios? Should we trust more a model that accurately accommodates the current data or a model/algorithm with an high predictive accuracy for future predictions? These and many other apparently naive questions pushed many scholars to debate about the supposed supremacy of prediction over accommodation (Maher, 1988; Hitchcock and Sober, 2004; Worrall, 2014). According to his own favoured epistemic point of view, the statistician should ask himself whether he wants models that are true—or approximately true—or predictively accurate.

There is not a clear domain of one approach over the other: inference and prediction are not enemies, but could be strong allies to reveal the truth. Moreover, even more importantly, we strongly believe that (almost) everything in statistics is predictive, or, at least, may be read from a predictive point of view. Even though many statisticians seek to mask their theories/models only by claiming the relevance of their estimation/inferential process, they are rarely aware of the predictive essence underlying their procedures. For illustrations purposes, consider a logistic/probit regression model for diagnosing diabetes testing positive probabilities using biochemical variables (such as glucose, insulin, mass, etc.) as predictors. A summary for these kinds of models is usually documented by the adoption of odds-ratios, confidence/predictive intervals for the parameters, p-values and other estimation-driven measures. However, the hidden task of such a model is intrinsically predictive: rather than hyperfocusing on the numerical impact of the classical/Bayesian estimates, the focus here should be on the general probability of being tested positive to

the diabetes, by considering existing or new values for glucose, insulin, etc.. Consider the usual randomized clinical trials set to assess some drugs' efficacy as another example: these studies are rarely conducted with the idea of predicting a useful and healthy behavior in the population (the hidden and final aim, according to us). Rather, they are built to find and display a statistically significant effect, according to the statistical significance of some parameters. This estimation obsession makes statistics obscure, whereas speaking in predictive terms, openly accessible to the population, would make statistics more transparent. This is the reason why we provocatively claim that prediction is not everything, but (almost) everything, in statistics, could be described in predictive terms.

3.2 Generalization performance of a statistical model

Assessing the generalization performance of a learning method related to the predictive performance on a test set is a central task in modern data-science. In this section we review some well-known approaches and highlight their main merits and weaknesses.

Suppose we use a set of observations $\tau = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ to fit (or train) the statistical model $Y = f(X) + \epsilon$, where the single x_i is the observed value of the predictor/covariate X_i , the single y_i is the observed value of the dependent/response variable Y_i , f is an unknown mathematical function of X, and ϵ is the random error. Fitting a model means producing an estimate \hat{f} for the unknown f: in the univariate linear regression case $f(X) = \beta_0 + \beta_1 X$, this is translated in estimating the parameters β_0, β_1 by finding $\hat{\beta}_0, \hat{\beta}_1$. Let (x_{n+1}, y_{n+1}) be a new observation not used to train the model, we then would like to take the expectation across all such new values and define the test mean square error (MSE), or generalization error:

Test MSE
$$\equiv E\left[(y_{n+1} - \hat{f}(x_{n+1}))^2 | \tau\right],$$
 (1)

where $\hat{f}(x_{n+1})$ is the prediction for y_{n+1} produced by \hat{f} , the expectation is taken across all new unseen predictor-response pairs (x_{n+1}, y_{n+1}) and the training set τ is considered to be fixed. Unfortunately, it is commonly unfeasible to calculate the test MSE, because we are often in a situation in which we do not have any test data available. As suggested by Hastie et al. (2009), it does not seem possible to estimate the conditional error in (1) appropriately, given only the information in the same training set τ . For such reasons, we may introduce a related quantity, the expected test MSE, or expected generalization error, which is usually estimated by cross-validation (Hastie et al., 2009) and related statistical methods:

$$\exp \operatorname{test} \operatorname{MSE} \equiv \operatorname{E}[\operatorname{Test} \operatorname{MSE}] = \operatorname{Var}(\hat{f}(x_{n+1})) + \operatorname{Bias}^{2}(\hat{f}(x_{n+1})) + \operatorname{Var}(\epsilon), \tag{2}$$

where the expectation is taken across many training sets, $Var(\hat{f}(x_{n+1}))$ is the variance for $\hat{f}(x_{n+1})$, $Bias^2(\hat{f}(x_{n+1}))$ is the squared bias, $Var(\epsilon)$ is the error variance. This final term, known as the irreducible error, is the minimum lower bound for the test MSE. Since we only ever have access to the training data points (including the randomness associated with the ϵ values) we cannot ever hope to get a "more accurate" fit than what the variance of the residuals offer.

The predictive goal is to select the model where the expected test MSE is lowest: the statisticians should try to minimize exp test MSE by choosing the model that simultaneously has low variance and low bias. However, this is a relevant and difficult challenge, much debated within the statistical community: more complex models, with lower bias, tend to overfit the data, by yielding poor predictive results and then higher variance; conversely, too simple models tend to not fit the data adequately and have higher bias. Statistical

procedures often incur in the bias-variance trade-off (James et al., 2013), the challenge is to find a compromise by controlling for both bias and variance.

When building a model for real-life applications to extract information from the data, it is good practice to keep in mind this bias-variance trade-off. Nevertheless, it is often problematic to assess the performance of a statistical model by looking directly at the elements in Equation (2). For this reason many statistical methods such as cross-validation, bootstrap, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) have been proposed to provide reasonable estimates of the expected generalization error and help modelers choose among candidate models. However, not all of these methods are (always) effective in estimating the prediction error on out-of-sample datasets: AIC and Deviance Information Criterion (DIC) suffer from the conditioning on a point estimate, by estimating the performance of the plugin predictive density, as claimed by Gelman et al. (2014), whereas cross-validation is appealing but can be computationally expensive and also is not always well defined in dependent data settings.

3.3 Information criteria

In our practice, prediction should not be assimilated to "take a rabbit out of a hat", but look at its inherent uncertainty. Splitting the predictions' uncertainty in variance and squared bias has been proved to be useful from a theoretical point of view, however it can appear bogus and artificial when framed in practical data analysis: how long does it take control and lower the bias and the variance of a learning method? How much should the statistician stretch his model to avoid problematic bias-variance tradeoffs?

In the literature on predictive accuracy, as for the AIC (Akaike, 1973), there is no role played by model's uncertainty, since the measure of the model's accuracy is evaluated

conditionally on parameters' points estimates, the maximum likelihood point estimate. Even the DIC (Spiegelhalter et al., 2002), for many years a milestone for Bayesian model comparisons, is conditioned on a plugin estimate, the posterior mean, with the number of parameters of AIC replaced by a measure of effective number of parameters.

Rather, if we are framed in a Bayesian context we intend the unobserved and future values \tilde{y} to come from the posterior predictive distribution, denoted here by $p(\tilde{y}|y)$, which incorporates the intrinsic uncertainty propagating from the parameters—summarized by the posterior distribution—to the observable future values. Recent proposals such as the Watanabe-Akaike Information Criteria (WAIC) (Watanabe, 2010) and Leave-One-Out cross validation Information Criteria (LOOIC) (Vehtari et al., 2017) go in the direction of data granularity, by defining the expected log pointwise predictive density for a new dataset (ELPPD). These approaches require the computation of the log-pointwise predictive density $p(\tilde{y}_i|y)$ for each new observable value \tilde{y}_i and have the desirable property of averaging over the posterior distribution.

Although all the predictive information criteria may fail in some practical situations, LOOIC and WAIC offer the possibility of providing a measure of predictive accuracy based on single data points, in a computationally efficient way (both the methods are implemented in the loo R package (Vehtari et al., 2019)). Despite not conclusive for the predictive accuracy of a statistical model, these techniques allow in many situations to compare distinct models by acknowledging an intrinsic uncertainty propagating from the parameters to the observable future values: in such a viewpoint, observable values, and not parameters, are really relevant. A transparent predictive tool should encompass data, parameters and future data not focusing on parameters estimates/plugin predictive densities alone; in such a way, the falsification of a single piece makes the joint model falsifiable. In Section 4, we

make this point even more clear.

3.4 The two cultures

As brilliantly argued by Breiman et al. (2001), there are two cultures in the use of statistical modeling to reach conclusions from data: a stochastic data model consisting of predictors, parameters and random noise to explain the response variable y is adopted by the data modeling culture; a function of the predictors to predict the response variable y is assumed by the algorithmic modeling culture, also named machine learning (ML) culture. The two approaches strongly differ in their validation: goodness-of-fit tests vs. predictive accuracy on out-of-sample data. It is evident that the data modeling culture—linear regression, generalized linear models, Cox model, etc.—is aimed at extracting some information about how nature is associating the response variable to the dependent variable, whereas the algorithmic culture—decision and classification trees, neural nets—is more oriented to predicting future values of the response variable given the values of the predictors.

The historical appeal of the ML field dates back the mid-1980s, when neural nets and decision trees became incredibly popular (Breiman et al., 1984) in areas where parametric data models were not applicable, such as speech, image, and handwriting recognition, and prediction in financial markets. In analyzing real data from these fields, the only criterion to evaluate these algorithms was predictive accuracy: this is translated in finding an algorithm f(x) able to be a good predictor for y for future values of x, the so called test set. To alleviate the degree of overfitting and the lack of predictive robustness in decision trees, in the mid-1990s some data scientists argued that by aggregating many trees and perturbing the training set, using bagging (Breiman, 1996), boosting (Freund et al., 1996) or random forests (Ho, 1995), dramatically increased the predictive accuracy of the trees.

by decreasing the variance.

Data scientists are used to training their procedures on the training set, which is chosen at the beginning. A common strategy is to select the first half of a dataset to train the algorithm, and the second half to test it; another strategy consists of selecting only a percentage—say,75% of the dataset—and using the remaining 25% to test the algorithm. However, a small change in the dataset can cause a large change in the final predictions, and some adjustments are often required to increase the algorithm's robustness. This "training shaking" is popular despite controversy, and allows us to discuss the eventual supremacy of the prediction over model construction and interpretation.

In what follows, we relax these boundaries and merge the two cultures, by referring to this fusion as the field of statistical learning. The list of considerations contained in the next section is valid for algorithmic and stochastic modelers aimed at predictive purposes.

4 Predictive instrumentalism and how to make predictive models transparent and falsifiable

Diventa section 3. Da rivedere 4.1 e modificare abbastanza, la 4.2 probabilmente molto da accorciare/cambiare.

4.1 Weak instrumentalism philosophy

As statisticians and (data) scientists, demanded to build models for social and physical sciences, our efforts should be addressed to produce good, transparent and well posed algorithms/models, and make them falsifiable upon a strong check (Gelman and Shalizi, 2013).

However, lately the need for powerful prediction weapons emerged, especially in the machine learning field, and the goodness of a modeling procedure is often associated with its predictive ability on out-of-sample scenarios. As a consequence, only good predictive models are retained, whereas the others, even when sophisticated and well built, are discarded; predictive accuracy became the only discrimination's tool to decide between good and bad statistical models/algorithms. We refer to this philosophical position as strong instrumentalism, for which the predictive accuracy carried out by the algorithms is constitutive—and not only symptomatic—of broader scientific success. People strictly adhering to this philosophical perspective are usually inclined towards some "munging" procedures, such as "shaking the training set", or "over-tuning" some tuning parameters to ensure lower variance and higher accuracy; for the most of the time, these data scientists seem apparently ready to do "whatever it takes" to improve over the previous methods.

It is worth stressing that evaluating a model/algorithm in light of its ability to predict future data is not shameful at all; conversely, it turned out to be beneficial in many areas, for instance where a parametric stochastic model failed to be really generative and useful. However, even if predictions of future data were good tools to falsify a posed theory, some strong instrumentalist techniques lack a general and valid theoretical framework. As an illustrative example, the number of predictors at each split of a random forest is a tuning parameter fixed at \sqrt{p} in most cases, but in practice the best values for these parameters will depend on the problem; if the method (the theory) is tuned and selected on the ground of its predictive accuracy, the underlying theory to be falsified is bogus, and not posed in a transparent way.

As mentioned above, our skepticism considers the recently dominant role of prediction in falsifying our models, for such a reason we adhere to the *weak instrumentalism* position: in

brief, predictions and predictive accuracy are a central task of science, but only sometimes do they constitute scientific success.

4.2 The falsificationist Bayesianism framework: going beyond inference and prediction

Gelman and Shalizi (2013) argue that a key part of Bayesian data analysis regards the model checking through posterior predictive checks. In such a view, the prior is seen as a testable part of the Bayesian model and is open to falsification: from such intuition, Gelman and Hennig (2017) name this framework falsificationist Bayesianism.

As stated by Gelman et al. (2013), the process of Bayesian data analysis can be idealized by dividing it into the following three steps:

- 1. Setting up a full probability model?—a joint probability distribution—for all observable and unobservable quantities in a problem. The model should be consistent with knowledge about the underlying scientific problem and the data collection process.
- 2. Conditioning on observed data: calculating and interpreting the appropriate posterior distribution i.e. the conditional probability distribution of the unobserved quantities of ultimate interest, given the observed data.
- 3. Evaluating the fit of the model and the implications of the resulting posterior distribution: How well does the model fit the data, are the substantive conclusions reasonable, and how sensitive are the results to the modeling assumptions in step (a)? In response, one can alter or expand the model and repeat the three steps.

In the above paradigm, predictions are never mentioned. However this does not mean that predictions are not relevant in the Bayesian paradigm. Denoted by \tilde{y} the unobserved vector

Table 1: Weak instrumentalism summary

General science

- p1 Predictive accuracy is not always constitutive of scientific success
- p2 Scientific falsification on the ground of wrong predictions is sometimes misleading, especially in social sciences (Trump's election, Leicester win, Brexit)
- p3 Supposedly valid scientific theories should exist before the future data have been revealed
- p4 Prediction is not explicitly part of the formulation of a scientific hypothesis at the time the law is posed, but it becomes relevant and relevant as science advances

Statistics

- p5 Take care of variability in the statistical predictions
- p6 If necessary, go beyond the distinction between inference and prediction, and consider a joint model for data, parameters and future data (falsificationist Bayes)
- p7 Rather than reasoning in terms of variance and bias, reason more in terms of predictive information criteria and posterior predictive distribution

Machine Learning

- p8 'Shaking the training set' to improve predictive accuracy is an obscure step
- p9 Avoid to tune the algorithm with the only task to improve predictive accuracy
- p10 To be falsifiable, ML techniques need 19 be transparently posed

of future values, we may derive the posterior predictive distribution as

$$p(\tilde{y}|y) = \int p(\tilde{y}|\theta)p(\theta|y)d\theta, \tag{3}$$

where $p(\theta|y)$ is the posterior distribution for θ , whereas $p(\tilde{y}|\theta)$ is the likelihood function for future observable values. Equation (3) can be resembled in the following way:

$$p(\tilde{y}|y) = \frac{p(\tilde{y},y)}{p(y)} = \frac{1}{p(y)} \int p(\tilde{y},y,\theta) d\theta.$$
 (4)

From Equation (4) we immediately notice that whenever we are interested in predictions, we need to consider a joint model $p(\tilde{y}, y, \theta)$ for both the observed data y and the unobserved quantities \tilde{y}, θ . This joint model incorporates both the likelihood and the prior, being $p(\tilde{y}, y, \theta) = p(\tilde{y}|\theta)p(y|\theta)p(\theta)$. Thus, the joint model for the predictions, the data and the parameters is transparently posed, and open to falsification when the observable \tilde{y} becomes known.

To summarize the above discussion, we collect in Table 1 the main points that follow from the weak instrumentalist philosophy. We divided them into three categories: the first one collects general considerations about the role of prediction in modern science and data science, whereas the second and the third one propose some tips for statisticians and machine learners, respectively.

5 Applied example: Russia World Cup 2018

Diventa section 4: non mi piace esempio, da cambiare, probabilmente il voto presidenziale 2016 in america.

In this section we review and motivate a simple example of football prediction in light of the weak instrumentalist philosophy proposed in the previous section and summarized in Section ??. In particular, we put in evidence the influence of the training set for future predictions by revealing some paradoxical considerations in ML results from a small-sample case. We consider here the dataset containing the results of all 64 tournament's matches (48 of the group stages, and 16 of the knockout stage) for the FIFA World Cup 2018 hosted in Russia and won by France.

Let (y_n^H, y_n^A) denote the observed number of goals scored by the home and away team in the *n*-th game, respectively. A general bivariate Poisson model allowing for goals' correlation (Karlis and Ntzoufras, 2003) is the following:

$$Y_n^H, Y_n^A | \lambda_{1n}, \lambda_{2n}, \lambda_{3n} \sim \mathsf{BivPoisson}(\lambda_{1n}, \lambda_{2n}, \lambda_{3n})$$

$$\log(\lambda_{1n}) = \theta + \mathsf{att}_{h_n} + \mathsf{def}_{a_n} + \frac{\gamma}{2} w_n$$

$$\log(\lambda_{2n}) = \theta + \mathsf{att}_{a_n} + \mathsf{def}_{h_n} - \frac{\gamma}{2} w_n$$

$$\log(\lambda_{3n}) = \beta_0,$$
(5)

where the case $\lambda_{3n}=0$ is reduced to the double Poisson model (Baio and Blangiardo, 2010). $\lambda_{1n}, \lambda_{2n}$ represent the scoring rates for the home and the away team, respectively, where: θ is the common baseline parameter; the parameters att_T and def_T represent the attack and the defense abilities, respectively, for each team $T, T=1,\ldots,N_T$; the nested indexes $h_n, a_n=1,\ldots,N_T$ denote the home and the away team playing in the n-th game, respectively; the only predictor is $w_n=(\operatorname{rank}_{h_n}-\operatorname{rank}_{a_n})$, the difference of the FIFA World Rankings (https://www.fifa.com/fifa-world-ranking/)—expressed in FIFA ranking points divided by 10^3 —between the home and the away team in the n-th game, multiplied by a parameter $\gamma/2$. This last term tries to correct for the well-known phenomenon of draw inflation (Karlis and Ntzoufras, 2003), favoring draw occurrence when teams are close in terms of their FIFA rankings. The value of the FIFA ranking difference w included in

the models was considered on June 7th, only a bunch of days before the tournament took place. In a Bayesian framework, attack and defence parameters are usually assigned some noninformative prior distributions (Baio and Blangiardo, 2010) and imposed a sum-to-zero constraint to achieve identifiability.

We decided to train our statistical models/ML techniques on distinct portions of matches from the group stage, where teams are more heterogeneous in terms of their FIFA rankings and actual strengths. To assess predictive performance between statistical models and ML algorithms in predicting football outcomes, we compare the double Poisson and the bivariate Poisson model, fitted by rstan package (Stan Development Team, 2018), with five ML procedures: Random Forest, Classification and Regression Trees (CART), Bagged CART, Multivariate Adaptive Regression Splines (MARS) and Neural Network, according to their standard use as provided by the caret package (Kuhn, 2019). The three different prediction scenarios are as:

A Train 75% of randomly selected group stage matches Test Remaining 25% group stage matches

B Train Group stage matches

Test Knockout stage

C Train Group stage matches for which both teams have a Fifa ranking greater than 1

Test Knockout stage.

Figure 1 displays for each scenario the values for the FIFA rankings for the training set matches (blue points) and the test set matches (orange points), along with the line Rank 1 = Rank 2, implying that the ranking difference is w = 0. In Scenario A, the test

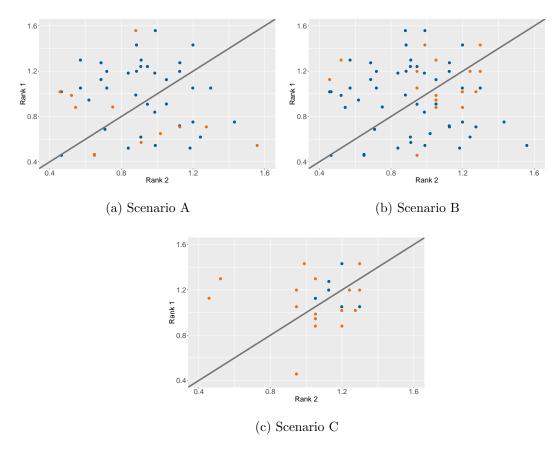


Figure 1: For each prediction scenarios, the values of the FIFA rankings for each match are shown in blue for the training set and in orange for the test set.

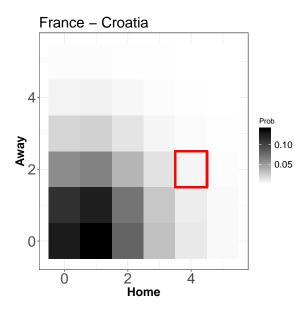


Figure 2: Posterior prediction distribution of the goals for the final France-Croatia

set matches are randomly selected from the group stage, and they do not show any particular pattern around line w=0. In Scenarios B and C, test set matches belong to the knockout stage, where the teams are expected to be stronger and closer to each other in terms of their rankings. In fact, the majority of the orange points (13 out of 16) is displayed towards the bottom right corner—higher rankings—and closer to the line w=0—closer strengths. Scenario B uses more and more data to predict test set results—all 48 group stage matches—whereas Scenario C only six matches.

Figure 2 depicts the posterior predictive distribution (p5 and p7) of the number of goals scored by France and Croatia during the final from the bivariate Poisson model. Darker regions are associated with higher probabilities, whereas the red square corresponds with the observed result, 4-2. From this plot, one could be tempted to conclude that the bivariate Poisson model completely failed to predict the match; however, the global probability of France winning within the 90 minutes—obtained summing the single probabilities over the lower triangle of the plot—is about 42%, against the 29% chance of winning for Croatia (p1 and p2). Fro this plot only we can acknowledge the intrinsic variability in our model predictions (p5).

To have a glimpse into statistical and ML procedures' predictive performance, Table 2 shows the accuracy in the predictions for the seven methods and the three scenarios. Assuming that higher predictive accuracies should not entirely suggest the best scientific methods (p1), we analyse the performance of the methods by focusing on pro and cons. As suggested by Figure 1a, Scenario A is the noisiest in terms of rankings' differences, with the test set constituted by matches randomly chosen from the group stage, without any kind of pattern. As is intuitive, ML techniques (Random Forest and Neural Nets), perform better, since they "shake" the training set (p8) in such a way as to retrieve the

Table 2: Prediction accuracy for the selected methods, according to three prediction sce-

narios.

Train	75% group	100% group	rank > 1
Test	25% group	knockout	knockout
Random forest	0.67	0.25	0.44
Bagged CART	0.67	0.31	0.37
CART	0.58	0.31	0.19
MARS	0.58	0.38	0.49
NN NN	0.67	0.25	0.44
Double Pois.	0.58	0.50	0.56
Biv. Pois.	0.58	0.56	0.56

highest predictive accuracy. The ML performances dramatically decrease in Scenario B and C, where learning from the training set should be focused on predicting the knockout stage. ML algorithms learn less and in a very random way, but it is not clear why (p10). As already argued, the choice of the training and the test set can dramatically change the predictive performance of the ML algorithms, which over-perform statistical models only when considering a portion of the group stage to predict the remaining group stage matches. Should we perhaps conclude that statistical models are better scientific tools to predict the World Cup? Not at all (p1), but we can learn from this example to improve over the next World Cups (p4).

By concluding, from this simple case study we cannot openly falsify our statistical/ML techniques on the ground of future predictions. However, Poisson models seem to be less sensitive to the training set structure, and then falsifiable in a broader sense.

6 Discussion

Diventa section 5. Da scrivere ex-novo.

Prediction is central in the progress of science and has become even more relevant in statistics and data science, as the availability of new computational tools has become common to accommodate data and predict new events. The entire field of science changed drastically over the last decades, new disciplines entered the scientific environment, and social sciences became a new frontier where predictive accuracy was required.

Natural and physical sciences progressed with Popper's falsificationism a main consequence of which is the strong predictivism: scientific theories should be falsified in light of wrong predictions. However, social sciences are not falsifiable in the same way: some social events—presidential elections, football results, and policy effects—are not perfectly predictable for many reasons, such as data origins and unpredictable human behaviors. In this paper, we relax the assumptions behind strong instrumentalism and we provide a several points (see Table 1) to frame statistical and ML techniques within a weak instrumentalist philosophy, in which the main proposals regard algorithm transparency and variability in predictions.

As statisticians required to build good models to accommodate complex data, we must warn statisticians and data science users about the role of prediction. Predictive accuracy is not always constitutive of scientific success: Prediction is not everything, but is vital, and it is our responsibility to choose the gun or the bazooka.

SUPPLEMENTARY MATERIAL

References

- Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In In: Petrov, B.N., Csaki, F. (eds.) Proceedings of the Second International Symposium on Information Theory, pp. 267281. Akademiai Kiado, Budapest (1973). Reprinted in: Breakthroughs in Statistics, pp. 610–624. Springer, New York (1992).
- Baio, G. and M. Blangiardo (2010). Bayesian hierarchical model for the prediction of football results. *Journal of Applied Statistics* 37(2), 253–264.
- Billheimer, D. (2019). Predictive inference and scientific reproducibility. *The American Statistician* 73(sup1), 291–295.
- Bjornstad, J. F. (1990). Predictive likelihood: A review. Statistical Science, 242–254.
- Breiman, L. (1996). Bagging predictors. Machine learning 24(2), 123–140.
- Breiman, L. et al. (2001). Statistical modeling: The two cultures (with comments and a rejoinder by the author). Statistical science 16(3), 199–231.
- Breiman, L., J. Friedman, R. Olshen, and C. Stone (1984). Classification and regression trees. Wadsworth.
- Bzdok, D., N. Altman, and M. Krzywinski (2018). Points of significance: statistics versus machine learning.
- Dawid, A. P. (1984). Present position and potential developments: Some personal views statistical theory the prequential approach. *Journal of the Royal Statistical Society:*Series A (General) 147(2), 278–290.

Forster, M. R. (2002). Predictive accuracy as an achievable goal of science. *Philosophy of science* 69(S3), S124–S134.

Freund, Y., R. E. Schapire, et al. (1996). Experiments with a new boosting algorithm. In *icml*, Volume 96, pp. 148–156. Citeseer.

Friedman, M. (1953). Essays in positive economics. University of Chicago Press.

Geisser, S. (2017). Predictive inference: an introduction. Chapman and Hall/CRC.

Gelman, A. (2016a).

Gelman, A. (2016b). Election surprise, and three ways of thinking about probability.

Gelman, A. (2016c). Explanations for that shocking 2% shift.

Gelman, A. (2016d). The problems with p-values are not just with p-values. The American Statistician 70(10).

Gelman, A., J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin (2013).

Bayesian data analysis. Chapman and Hall/CRC.

Gelman, A. and C. Hennig (2017). Beyond subjective and objective in statistics. *Journal* of the Royal Statistical Society: Series A (Statistics in Society) 180(4), 967–1033.

Gelman, A., J. Hwang, and A. Vehtari (2014). Understanding predictive information criteria for bayesian models. *Statistics and computing* 24(6), 997–1016.

Gelman, A., G. King, and W. J. Boscardin (1998). Estimating the probability of events that have never occurred: when is your vote decisive? *Journal of the American Statistical Association* 93(441), 1–9.

- Gelman, A. and C. R. Shalizi (2013). Philosophy and the practice of bayesian statistics.

 British Journal of Mathematical and Statistical Psychology 66(1), 8–38.
- Hastie, T., R. Tibshirani, and J. Friedman (2009). The elements of statistical learning:

 data mining, inference, and prediction. Springer Science & Business Media.
- Hitchcock, C. and E. Sober (2004). Prediction versus accommodation and the risk of overfitting. The British journal for the philosophy of science 55(1), 1–34.
- Ho, T. K. (1995). Random decision forests. In *Proceedings of 3rd international conference* on document analysis and recognition, Volume 1, pp. 278–282. IEEE.
- James, G., D. Witten, T. Hastie, and R. Tibshirani (2013). An introduction to statistical learning, Volume 112. Springer.
- Karlis, D. and I. Ntzoufras (2003). Analysis of sports data by using bivariate poisson models. Journal of the Royal Statistical Society: Series D (The Statistician) 52(3), 381–393.
- Kuhn, M. (2019). caret: Classification and Regression Training. R package version 6.0-84.
- Kuhn, T. S. (1962). The structure of scientific revolutions. Chicago and London.
- Lakatos, I. (1976). Falsification and the methodology of scientific research programmes. In Can theories be refuted?, pp. 205–259. Springer.
- Maher, P. (1988). Prediction, accommodation, and the logic of discovery. In *PSA: Proceedings of the Biennial meeting of the philosophy of science association*, Volume 1988, pp. 273–285. Philosophy of Science Association.
- Popper, K. (1934). The logic of scientific discovery. Routledge.

Popper, K. (1944). The poverty of historicism, ii. a criticism of historicist methods. *Economica* 11(43), 119–137.

Popper, K. (1945). The poverty of historicism, iii. Economica 12(46), 69–89.

Russell, B. (1931). The scientific outlook. Routledge.

Sarewitz, D. and R. Pielke Jr (1999). Prediction in science and policy. *Technology in Society* 21(2), 121–133.

Shmueli, G. (2010). To explain or to predict? Statistical science 25(3), 289–310.

Sober, E. (2002). Instrumentalism, parsimony, and the akaike framework. *Philosophy of Science* 69(S3), S112–S123.

Spiegelhalter, D. J., N. G. Best, B. P. Carlin, and A. Van Der Linde (2002). Bayesian measures of model complexity and fit. *Journal of the royal statistical society: Series b* (statistical methodology) 64(4), 583–639.

Stan Development Team (2018). RStan: the R interface to Stan. R package version 2.18.2.

Vehtari, A., J. Gabry, Y. Yao, and A. Gelman (2019). loo: Efficient leave-one-out cross-validation and waic for bayesian models. R package version 2.1.0.

Vehtari, A., A. Gelman, and J. Gabry (2017). Practical bayesian model evaluation using leave-one-out cross-validation and waic. *Statistics and computing* 27(5), 1413–1432.

Watanabe, S. (2010). Asymptotic equivalence of bayes cross validation and widely applicable information criterion in singular learning theory. *Journal of Machine Learning Research* 11(Dec), 3571–3594.

Worrall, J. (2014). Prediction and accommodation revisited. Studies in History and Philosophy of Science Part A 45, 54–61.