SSY316 - Python 4

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1 Gibbs sampler

The first part of this assignment was to implement a Gibbs sampler that samples μ and τ conditionally of the data y and τ. We are also given that the priors for these two random variables are $p(\mu) = N(0, \omega)$ and $p(\tau) = Gamma(\alpha, \beta)$

1.1 Activity 1

In Activity we implemented this Gibbs sampling using the derived conditional distributions of the following:

$$\mu | y, \tau \sim \mathcal{N}\left(\frac{\tau}{n\tau + \omega}\sum_{i=1}^{n} y_i, \frac{1}{n\tau + \omega}\right)$$

and

$$\tau | y, \mu \sim \text{Gamma}\left(\alpha + \frac{n}{2}, \beta + \frac{1}{2}\sum_{i=1}^{n}(y_i - \mu)^2\right)$$

The resulting mean and variance we got were: $mean \approx 4.974$ and $variance \approx 3.834$.

1.2 Activity 2

For activity 2 we wanted instead the distributions to have different priors and also which samples μ and β The derived conditional distributions to use were the following:

$$\mu | y, \beta \sim \text{Gamma}\left(2 + \sum_{i=1}^{N} y_i, N + \beta\right)$$

and

$$\beta | y, \mu \sim \text{Gamma}(3, 1 + \mu)$$

Using Gibbs sampler again we got the following mean and variance: $mean \approx 98463.833$ and $variance \approx 9791089880.444$.

2 Mean field approximation

The next part of the assignment was to get introduced to mean field approximation and implement the algorithm on our own. The algorithm can be seen in the code.

2.1 Activity 3

The results we got from activity 3 was:

	muf	tauf	tau muf
VB	4.967	1.238	124.995
MLE	5.018	1.790	178.993

2.2 Activity 4

For activity 4 we were given the Variational Bayes algorithm and were supposed to implement it on our own. The purpose is to approximate $\pi(\theta, y)$ using mean field approximation. The lambda and beta estimates we got can be seen below.

lambda estimate: 5063.73720119344
beta estimate: 5050.369526063156

3 Image de-noising with Gibbs sampling

This part consisted of using what we have learned prior about Gibbs sampling to de-noise an image similar to the previous assignment.

3.1 Activity 5

The resulting de-noised images can be seen in figure 1 and the NMSE score for varying noise levels can be seen in figure 2. Lastly we wanted to check the best fit with varying β and η which can be seen in figure 3

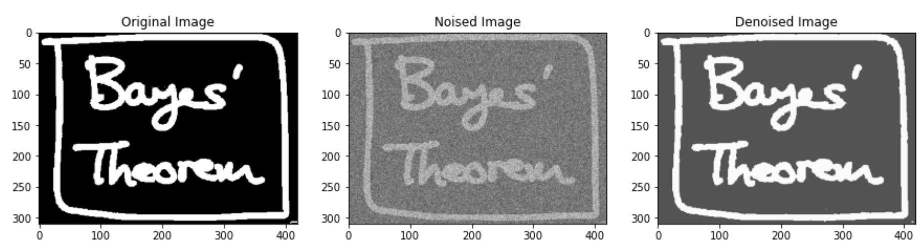

Figure 1: The denoised image using Gibbs sampling.

4 Image de-noising with mean field approximation

The last part of the assignment was to implement image de-noising using mean field approximation.

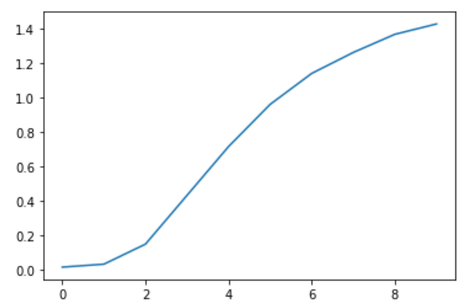

Figure 2: MNSE score with varying noise levels.

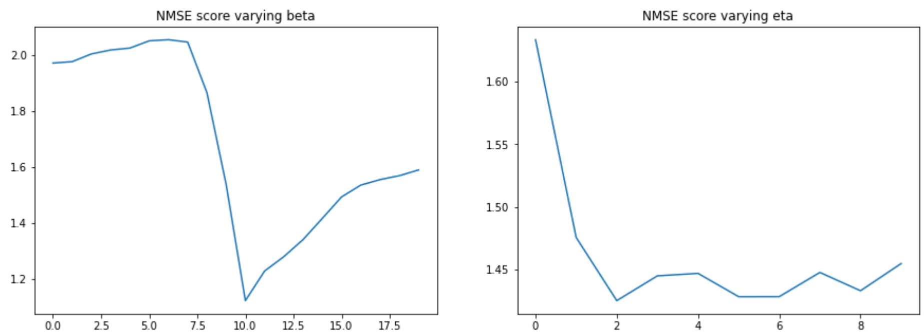

Figure 3: The MNSE score with varying betas (left) and etas (right).

4.1 Activity 6

The resulting denoised image using Mean-field approximation can be seen in figure 4.

4.2 Activity 7

The last activity was to compare the three de-noise algorithms we have implemented and in figure 5 one can see their MNSE scores with different noise levels. Judging from the graph we notice that Gibbs gave the best score overall.

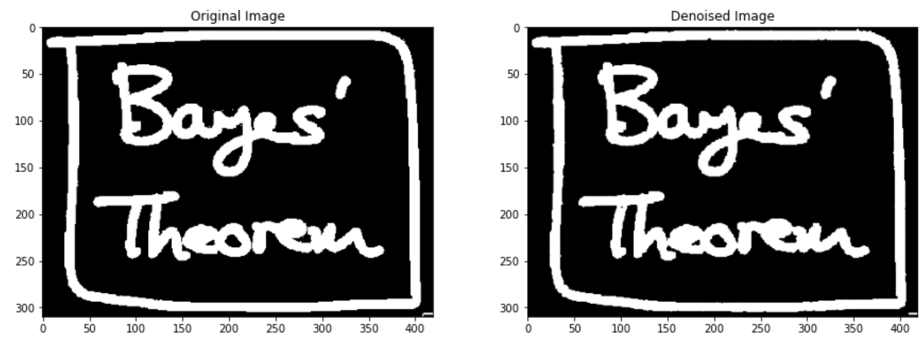

Figure 4: The denoised image using Mean-field approximation.

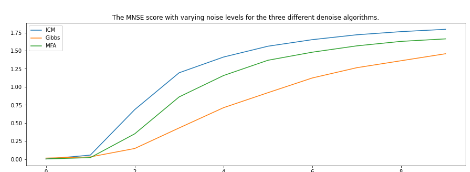

Figure 5: MNSE scores for three denoise algorithms with varying noise levels.