Prediction and Update steps

Optimal filtering is primarily about the prediction and update steps.

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})d\mathbf{x}_{k-1} \tag{1}$$

$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) \tag{2}$$

In the prediction step of filtering recursion, we know $p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})$ and have the motion model,

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{q}_{k-1} \tag{3}$$

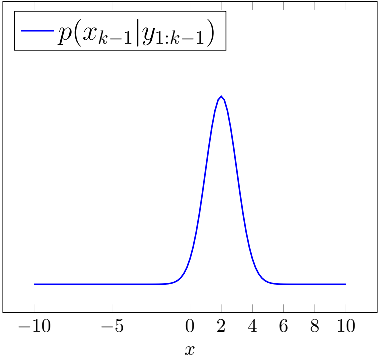
and we wish to compute $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$, using the Chapman-Kolomogorov equation

$$p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{y}_{1:k-1})d\mathbf{x}_{k-1} \tag{4}$$

Consider a 1-D random walk motion model

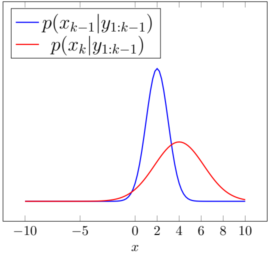
$$x_k = x_{k-1} + q_{k-1} \tag{5}$$

where $q_{k-1} \sim \mathcal{N}(0, 4)$. Then for the prior density, $p(x_{k-1}|y_{1:k-1}) = \mathcal{N}(x_{k-1}; 2, 1)$ depicted below, guess the predicted density:

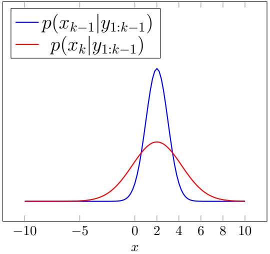


WHICH IS THE CORRECT PREDICTED DENSITY?

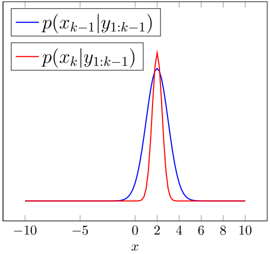
CHALMERS



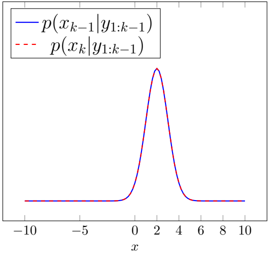
green



yellow



pink

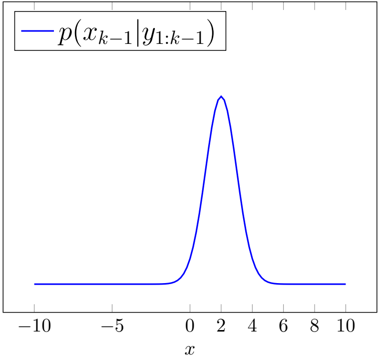


orange

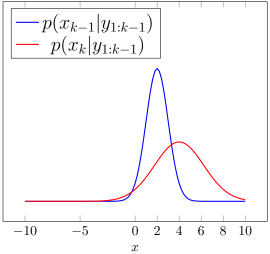
Consider a 1-D random walk motion model, now with a constant bias $b = 2$,

$$x_k = x_{k-1} + b + q_{k-1} \tag{6}$$

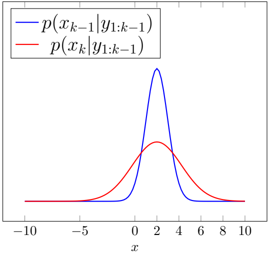
where $q_{k-1} \sim \mathcal{N}(0, 4)$. Then for the prior density, $p(x_{k-1}|y_{1:k-1}) = \mathcal{N}(x_{k-1}; 2, 1)$ depicted below, guess the predicted density:



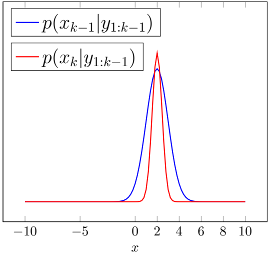
Which is the correct predicted density?



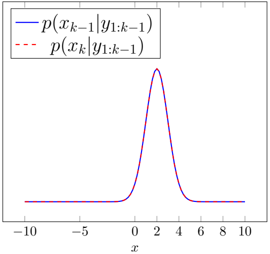
green



yellow



pink



orange

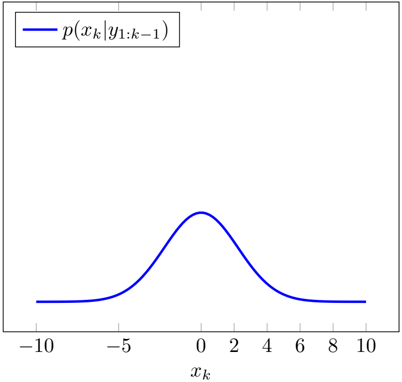
In the update step of filtering recursion, we know $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$ and have the measurement model,

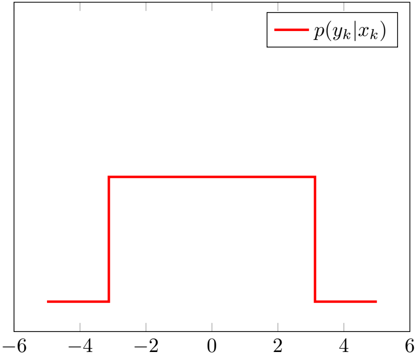
$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k \tag{7}$$

and we wish to compute $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$, using

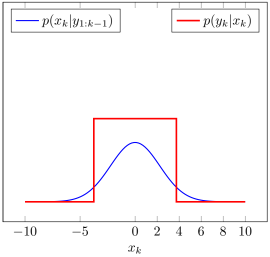
$$p(\mathbf{x}_k|\mathbf{y}_{1:k}) \propto p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{y}_{1:k-1}) \tag{8}$$

For the prior density and likelihood below, guess the filtered posterior density.

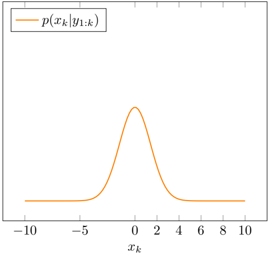




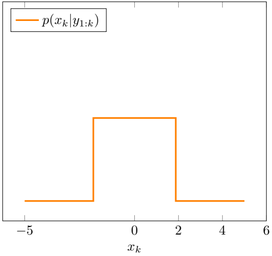
Guess the filtered posterior denisty



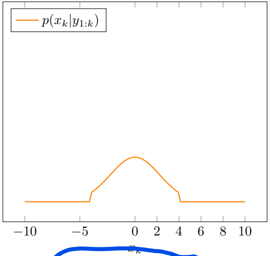
Predicted & likelihood



yellow



pink



orange

乘法