Topics for the practice session on L4

Basic information

One purpose with the flipping the classroom model used in this course is to enable us to dedicate our time in class to active learning. We call these sessions our practice sessions. During these sessions we intend to focus on problem solving, peer-instruction, multi choice questions and discussions on the fundamental concepts that we study. **These classes are for you** – take the opportunity to ask questions and discuss whatever elementary or advanced aspect of this course that you are interested in at the moment.

Today we focus on the material in **lecture** 4. To boost our discussions I have prepared a set of problems that we can look at. Though we do not have to cover all of them, I will try to limit the time that we spend on each of them to roughly 15 minutes.

Problem 1 – Kalman gain

In this problem we will try to get a feeling for how to interpret the Kalman gain with respect the different sensors / sensor models.

Problem setting: Sara is a particle physicist who is trying to measure the position, p, of a very short lived particle after a particle collision. To her disposal she has a-priori knowledge of what p ought to be (i.e. a prior) and 6 sensors to choose from, all with slightly different characteristics but all having linear measurement models. As the particle is so short lived Sara only has time to make one observation.

Having some knowledge about Bayesian estimation, she naturally wants to use the update step in the Kalman filter to estimate the position of the particle. However, the sensors are very expensive so she wants to make sure that she buys the best. She figures that she can get a feeling for how informative the sensors are compared to her prior by calculating and comparing the Kalman gain for each of the sensors. Is this a good strategy?

Model assumption: Let us assume that Sara, without making any observations, knows that a good probabilistic model of the position of the particle is

$$p \sim \mathcal{N}(\bar{p}, 1) \tag{1}$$

Additionally, from each sensor supplier she is given the following sensor models

s_1: $y = p + r, \quad r \sim \mathcal{N}(0, 100)$ 4 $\qquad$ s_2: $y = 0.1p$ 1 $K_2 = 10$

s_3: $y = p + r, \quad r \sim \mathcal{N}(0, 10)$ 3 $\qquad$ s_4: $y = p + r, \quad r \sim \mathcal{N}(0, 1)$ 2

s_5: $y = p$ 1 $K_5 = 1$ $\qquad$ s_6: $y = 100(p + r), \quad r \sim \mathcal{N}(0, 1)$. 2

Your assignment:

Kalman gain K_n

a) Order the sensors starting with the most accurate sensor.

b) Without making any calculations, discuss what the Kalman gain should be for each sensor.

c) Confirm your intuition by calculating the Kalman gain for each sensor. Was your intuition correct?

   Hint: $K = P_{k|k-1} H_k S_k^{-1} = P_{xy} P_{yy}^{-1}$.

d) Use your result to simplify the kalman update equation when performing an measurement update with a observation from s_2 and s_4. Does the result seem reasonable?

   Hint: The Kalman update equation is $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k \hat{x}_{k|k-1})$.

e) What conclusion can you draw regarding what the Kalman gain does?