Interpretation of densities involved in the Kalman filter recursion

For the linear and Gaussian motion and measurement models below,

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{q}_{k-1} \tag{1}$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k \tag{2}$$

where $\mathbf{q}_{k-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q_{k-1}})$ and $\mathbf{r}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R_k})$.

The Kalman filter yields recursive, closed-form expressions to perform prediction and update steps.

Prediction:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \tag{3}$$

$$\mathbf{P}_{k|k-1} = \mathbf{A}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{A}_{k-1}^{T} + \mathbf{Q}_{k-1} \tag{4}$$

Update:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{v}_k \tag{5}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{S}_K\mathbf{K}_k^{T} \tag{6}$$

where the Kalman gain, $\mathbf{K}_k$, the innovation, $\mathbf{v}_k$, and the innovation covariance, $\mathbf{S}_k$, are:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^{T}\mathbf{S}_k^{-1} \tag{7}$$

$$\mathbf{v}_k = \mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \tag{8}$$

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{T} + \mathbf{R}_k \tag{9}$$

Which of the following correspond/s to the filtering density:

green: $p(\mathbf{x}_k|\mathbf{y}_{1:K}), \quad K > k$ → smoothing

yellow: $p(\mathbf{x}_k|\mathbf{y}_{1:K}), \quad K = k$

pink: $p(\mathbf{x}_k|\mathbf{y}_{1:K}), \quad K < k$ → prediction

orange: $p(\mathbf{x}_k|\mathbf{y}_K), \quad K = k$

Which of the following densities are computed in the Kalman filter recursion:

green: $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$

yellow: $p(\mathbf{v}_k|\mathbf{y}_{1:k-1})$

pink: $p(\mathbf{x}_k|\mathbf{x}_{k-1})$

orange: $p(\mathbf{y}_k|\mathbf{y}_{1:k-1})$

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The innovation $\mathbf{v}_k$ is

$$\begin{aligned}\mathbf{v}_k &= \mathbf{y}_k - \mathbb{E}\{\mathbf{y}_k|\mathbf{y}_{1:k-1}\} \\ &= \mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1},\end{aligned}$$

then which of the following is true?

green: $\mathbb{E}\{\mathbf{v}_k|\mathbf{y}_{1:k-1}\} = \hat{\mathbf{y}}_k$

yellow: $\mathbb{E}\{\mathbf{v}_k|\mathbf{y}_{1:k-1}\} = 0$

pink: $\mathbb{E}\{\mathbf{v}_k|\mathbf{y}_{1:k-1}\} = \hat{\mathbf{x}}_{k|k-1}$

orange: $\mathbb{E}\{\mathbf{v}_k|\mathbf{y}_{1:k-1}\} = \mathbb{E}\{\mathbf{x}_k|\mathbf{y}_{1:k-1}\}$