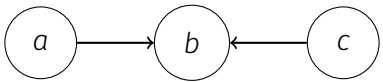
Bayesian networks vs factorisations

How can we factorise $p(a, b, c)$?



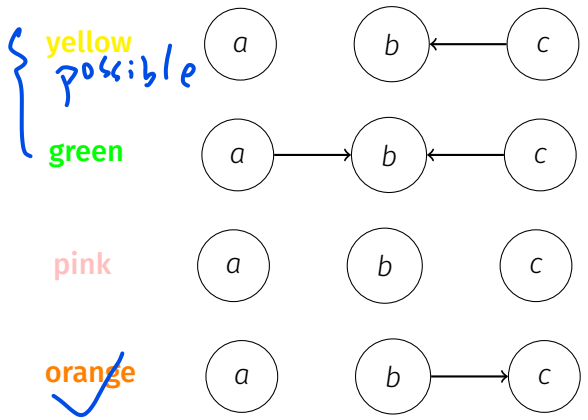
green: $p(a, b, c) = p(a)p(b)p(c)$

yellow: $p(a, b, c) = p(a)p(b|a)p(c|b)$

pink: $p(a, b, c) = p(b)p(a|b)p(c|b)$

orange: $p(a, b, c) = p(a)p(b|a, c)p(c)$

Which Bayesian network(s) corresponds to
$p(a, b, c) = p(a)p(c|b)p(b)$? $= p(a)\,p(b,c) = p(a)\,p(b|c)\,p(c)$



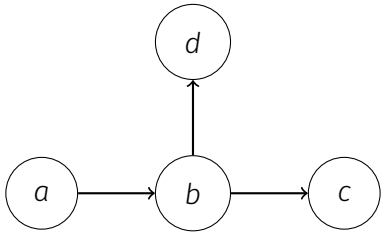
- A Bayesian network (BN) illustrates
  1. one possible factorisation
  2. certain conditional independencies (unless fully connected).

- **Example:** if $p(a, b)$ factorises according the BN below,

  it holds that
  1. $p(a, b) = p(a)p(b)$
  2. and, consequently, $p(b|a) = p(b)$ and $p(a|b) = p(a)$.

- A useful strategy when using BNs:
  - use the BN to factorize the joint distribution of all variables,
  - derive properties of interest from this factorisation.

How can we factorise $p(b, c, d)$?



green: $p(b, c, d) = p(b)p(b|c)p(c|d)$

yellow: $p(b, c, d) = p(d)p(c|d)p(b|c)$

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- We can follow the strategy mentioned earlier
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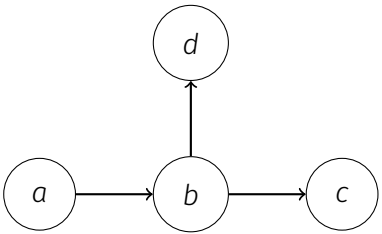

- According to the BN

$$p(a, b, c, d) = p(a)p(b|a)p(c|b)p(d|b)$$

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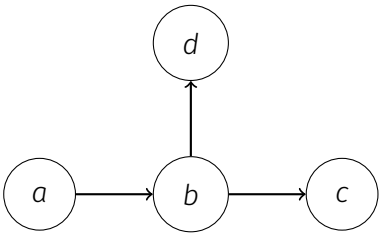

- According to the BN

$$p(a,b,c,d) = p(a)p(b|a)p(c|b)p(d|b)$$

$$\Rightarrow p(b,c,d) = \int p(a)p(b|a)\,\mathrm{d}a\; p(c|b)p(d|b)$$

$$= p(b)p(c|b)p(d|b)$$

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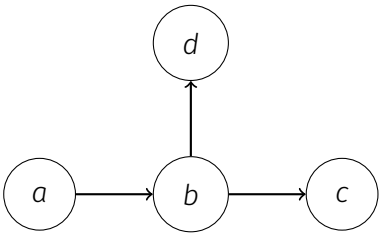

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Conclusion: the a variable and its node can be ignored here.

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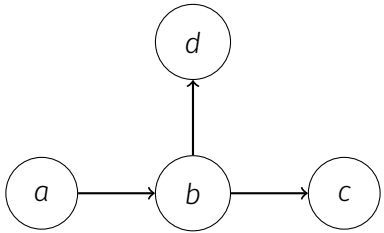

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