Homework 1

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Problem1.1

1. Define $f_p(x;\mu)$

$$f_p(x;\mu)=\begin{cases}(x_1-1)^2+2(x_2-1)^2+\mu(1-x_1^2-x_2^2)^2, & 1-x_1^2-x_2^2<0\\ (x_1-1)^2+2(x_2-2)^2 & 1-x_1^2-x_2^2\geq 0\end{cases}$$

2.Compute the gradient

$$\frac{\partial f_p}{\partial x_1}=\begin{cases}2x_1-2-4\mu x_1+4\mu x_2^2x_1+4\mu x_1^3, & 1-x_1^2-x_2^2<0\\ 2x_1-2, & 1-x_1^2-x_2^2\geq 0\end{cases}$$

$$\frac{\partial f_p}{\partial x_2}=\begin{cases}4x_2-8-4\mu x_2+4\mu x_1^2x_2+4\mu x_1^3, & 1-x_1^2-x_2^2<0\\ 4x_2-8, & 1-x_1^2-x_2^2\geq 0\end{cases}$$

3.For the unconstrained situation

$$f(x_1,x_2)=(x_1-1)^2+2(x_2-1)^2$$

The unconstrained minimum occurs when $\begin{cases}2x_1-2=0\\ 4x_2-8=0\end{cases}$

Thus the unconstrained minimum $f_{min}(x_1,x_2)=f(1{,}2)=0$

The starting point is (1,2)

4.write the code

5. mu = 1 x(1) = 0.4337, x(2) = 1.2101

mu = 10 x(1) = 0.3313, x(2) = 0.9955

mu = 100 x(1) = 0.3137, x(2) = 0.9552

mu = 1000 x(1) = 0.3117, x(2) = 0.9507

Problem1.2

(a)

$$f(x_1,x_2)=4x_1^2-x_1x_2+4x_2^2-6x_2$$

$$\begin{cases}\dfrac{\partial f}{\partial x_1}=8x_1-x_2\\ \dfrac{\partial f}{\partial x_2}=-x_1+8x_2-6\end{cases}$$

$$8x_1=x_2$$

$$-x_1+8x_2-6=0$$

$$x_1 = \frac{2}{21}$$
$$x_2 = \frac{16}{21}$$
$$f\left(\frac{2}{21}, \frac{16}{21}\right) = -\frac{16}{7} = -2.2857$$

Three lines:

Line1: $x_1 = 0, 0 < x_2 < 1$
$$f = 4x_2^2 - 6x_2$$
when $x_2 = 0.75$ $f(0,0.75) = -2.25$

Line2: $0 < x_1 < 1, x_2 = 1$
$$f = 4x_1^2 - x_1 - 2$$
$$f' = 8x_1 - 1$$
when $x_1 = 0.125$ $f(0.125,1) = -2.0625$

Line3: $x_1 = x_2$
$$f = 7x_1^2 - 6x_1$$
when $x_1 = x_2 = \frac{3}{7}$ $f\left(\frac{3}{7}, \frac{3}{7}\right) = -1.2857$

Three points:
$$f(0,0) = 0$$
$$f(1,1) = 1$$
$$f(0,1) = -2$$

In conclusion, the minimum is
$$f\left(\frac{2}{21}, \frac{16}{21}\right) = -\frac{16}{7} = -2.2857$$

(b)
$$f(x_1, x_2) = 15 + 2x_1 + 3x_2$$
$$h(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 - 21 = 0$$
$$L(x_1, x_2, \lambda) = 15 + 2x_1 + 3x_2 + \lambda(x_1^2 + x_1x_2 + x_2^2 - 21)$$

$$\frac{\partial L}{\partial x_1} = 2 + 2\lambda x_1 + \lambda x_2 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2} = 3 + \lambda x_1 + 2\lambda x_2 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial \lambda} = x_1^2 + x_1x_2 + x_2^2 - 21 = 0 \qquad (3)$$

2(1)-(2)=0
$$1 + 3\lambda x_1 = 0$$

$$x_1 = -\frac{1}{3\lambda}$$

$$x_2 = -\frac{4}{3\lambda}$$

$$\lambda = \frac{1}{3} or -\frac{1}{3}$$

$$x_1 = -1 x_2 = -4$$

or

$$x_1 = 1 x_2 = 4$$

Thus the minimum is

$$f(-1,-4) = 1$$

Problem1.3

a)

populationSize	100
maximumVariableValue	5
numberOfGenes	50
numberOfVariables	2
tournamentSize	2
tournamentProbability	0.75
crossoverProbability	0.8
mutationProbability	0.02
numberOfGenerations	3000

X(1)	X(2)	g(x1, x2)
2.9999970794	0.4999993891	1.661435698996207e-12
2.9999923110	0.4999981970	9.710661292907745e-12
3.0000307560	0.5000077337	1.516671364243781e-10
2.9999717474	0.4999922365	1.410057771893535e-10
3.0000075102	0.5000020713	1.003546116753982e-11
2.9999970794	0.4999993891	1.661435698996207e-12
2.9999875426	0.4999970049	2.504323929284833e-11
2.9999967813	0.4999990910	1.941708556317401e-12
3.0000212193	0.5000053495	7.224513990809165e-11
2.9999976754	0.4999993891	8.923432350086872e-13

b)

PMut = 0.00: Median: 0.9948040890

PMut = 0.01: Median: 0.9998381649

PMut = 0.02: Median: 0.9999999932

PMut = 0.05: Median: 0.9999970378

PMut = 0.10: Median: 0.9999498222

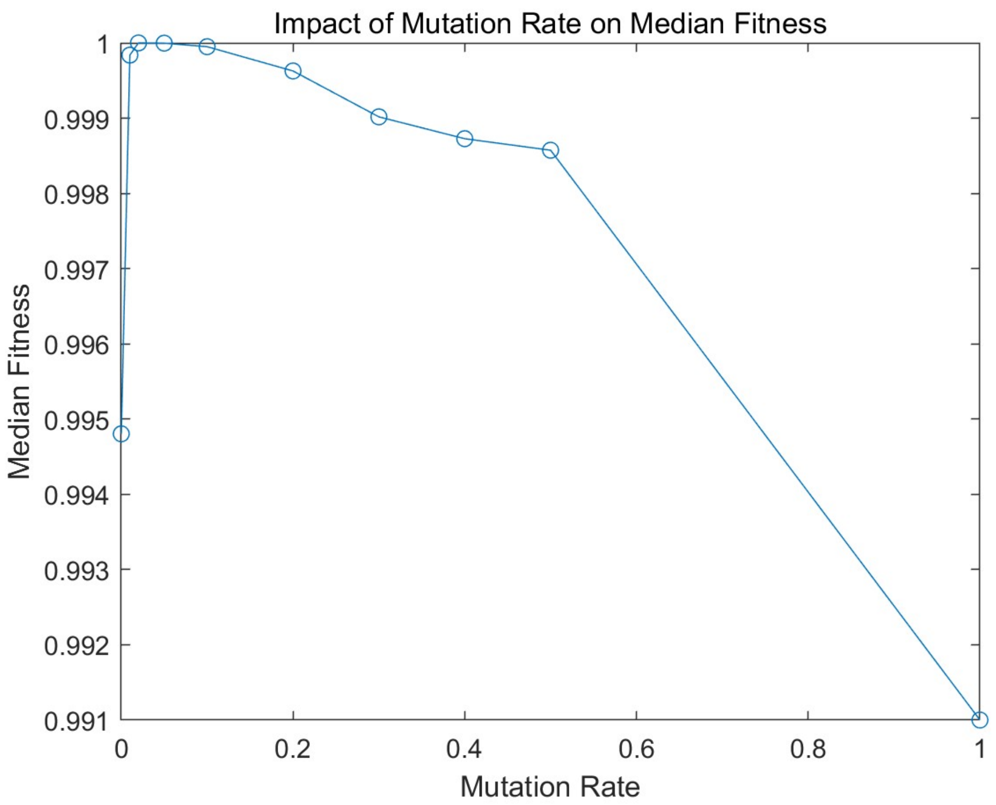
PMut = 0.20: Median: 0.9996273420

PMut = 0.30: Median: 0.9990174824

PMut = 0.40: Median: 0.9987268949

PMut = 0.50: Median: 0.9985738552

PMut = 1.00: Median: 0.9910002302



Summary

- Low mutation rates (around 0.01 to 0.02) seem to yield the best performance for this specific problem.
- Too much mutation (e.g., 0.4, 0.5, or 1) can have a detrimental effect on the performance.
- No mutation (0) also doesn't yield the best result.

c)

To find out whether the point $(x_1^*, x_2^*) = (3,0.5)$ is a stationary point for the function

$$g(x_1, x_2) = (1.5 - x_1 + x_1x_2)^2 + (2.25 - x_1 + x_1x_2^2)^2$$
$$+ (2.625 - x_1 + x_1x_2^3)^2$$

The first-order partial derivatives are:

$$\frac{\partial_g}{\partial_{x_1}} = 2(1.5 - x_1 + x_1x_2)(-1 + x_2) + 2(2.25 - x_1 + x_1x_2^2)(-1 + x_2^2)$$
$$+ 2(2.625 - x_1 + x_1x_2^3)(-1 + x_2^3)$$

$$\frac{\partial_g}{\partial_{x_2}} = 2(1.5 - x_1 + x_1x_2)(x_1) + 2(2.25 - x_1 + x_1x_2^2)(2x_1x_2) + 2(2.625$$
$$- x_1 + x_1x_2^3)(3x_1x_2^2)$$

$\frac{\partial_g}{\partial_{x_1}}$=2(1.5−3+3×0.5)(−1+0.5)+2(2.25−3+3×0.25)(−1+0.25)+2(2.625−3+3×0.125)(−1+0.125)

=2(−1+1.5)(−0.5)+2(−0.75+0.75)(−0.75)+2(−0.375+0.375)(−0.875)

=0

$\frac{\partial_g}{\partial_{x_2}}$=2(1.5−3+3×0.5)(3)+2(2.25−3+3×0.25)(2×3×0.5)+2(2.625−3+3×0.125)(3×0.25)

=2(0)(3)+2(0)(3)+2(0)(0.75)

=0

Since both $\frac{\partial_g}{\partial_{x_1}}$ and $\frac{\partial_g}{\partial_{x_2}}$ are zero at this point, we can confirm that (3,0.5) is a stationary point of g(x1,x2).